4

The Scots College

2001
TRIAL HSC EXAMINATION

Mathematics
Extension 1

General Instructions

- Reading time - 5 minutes
- Working time - 2 hours
- Write using a blue or black pen
- Board-approved calculators may be used
- A table of standard integrals is provided on page 8
- All necessary working should be shown in every question
- Start each question in a new booklet.

Total Marks: (84)
Weighting: 35% HSC

- Attempt Questions 1 - 7
- All questions are of equal value

Total marks (84)
Attempt Questions 1 – 7
All questions are of equal value

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Question 1 (12 marks) Use a SEPARATE writing booklet.

a. Evaluate $\int_0^{2\sqrt{3}} \frac{dx}{4+x^2}$ **2**

b. Differentiate $\cos^3 x$ **2**

c. Find the point which divides the line joining (4, 6) and (13, 5) externally in the ratio 4:1 **2**

d. Write down the equation of the vertical asymptote of $y=\frac{2x}{3x-1}$ **1**

e. Solve for x: $\frac{3}{x+5} \le 1$ **2**

f. Evaluate $\int_0^{\frac{1}{\sqrt{2}}} \frac{2x^3}{\sqrt{1-x^4}}dx$ using the substitution $u=x^4$ **3**

End of Question 1

Question 2 (12 marks) Use a SEPARATE writing booklet. **Marks**

a. Using all the letters, how many different arrangements can be made from the word MATHEMATICS ? **2**

b. Find all values of θ in the range $0 \le \theta \le 2\pi$ for which $\sin\theta + \sqrt{3}\cos\theta = 1$ **4**

c. i. Show that the function $f(x) = 2x^2 + x - 2$ cuts the x axis between $x = 0$ and $x = 1$ **1**

ii. Use the method of halving the interval twice to find an approximation to the root of this equation. **3**

iii. Starting with a value of $x = 0.7$ use Newton's method once to find an approximation to this root correct to 3 decimal places. **2**

End of Question 2

Question 3 (12 marks) Use a SEPARATE writing booklet.

Marks

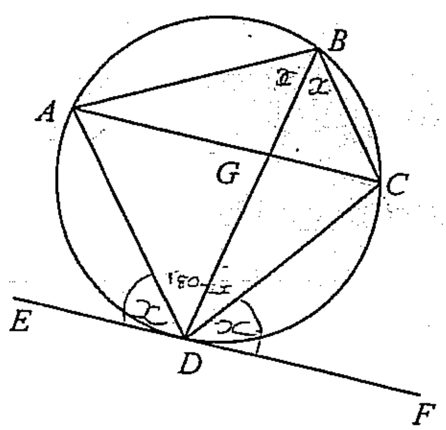
a. The region R is bounded by the curve $y = \cos x$, $x = 0, x = \frac{\pi}{2}$ and the $x-$axis.

i. Sketch R **1**

ii. Find the exact volume of the solid generated when the region R is rotated about the $x-$axis. **2**

b. If $\alpha, \beta, \gamma,$ are the roots of the cubic polynomial equation $x^3 + 4x^2 - 6x - 8 = 0$
Find the value of $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$ **2**

c.



$ABCD$ is a cyclic quadrilateral. EF is a tangent at D. If BD bisects $\angle ABC$, prove that AC is parallel to EF **2**

d. i. By equating coefficients, find the values of A and B in the identity

$$A(2\sin x + \cos x) + B(2\cos x - \sin x) \equiv 7\sin x + 11\cos x$$ **2**

ii. Hence show that $\int_0^{\frac{\pi}{2}} \frac{7\sin x + 11\cos x}{2\sin x + \cos x}\,dx = \frac{5\pi}{2} + \ln 8$ **3**

End of Question 3

Question 4 (12 marks) Use a SEPARATE writing booklet. **Marks**

a. P is a variable point on the parabola $x^2 = 8y$ with parameter p. The normal at P cuts the y axis at A and R is the midpoint of AP.

i. Show that the normal at P has equation $x + py = 4p + 2p^3$ **2**

ii. Show that R has coordinates $(2p, 2p^2 + 2)$ **2**

iii. Show that the locus of R is a parabola and show that the vertex of this parabola is the focus of the parabola $x^2 = 8y$. **3**

b. i. Evaluate $\int_1^3 \frac{dx}{x}$ **1**

ii. Use Simpson's rule with 3 function values to approximate $\int_1^3 \frac{dx}{x}$ **2**

iii. Use your results to parts i and ii to obtain an approximation for e. Give your answer correct to 3 decimal places. **2**

End of Question 4

Question 5 (12 marks) Use a SEPARATE writing booklet. **Marks**

a. Prove by induction that, for all integers $n \geq 1$,

$$\frac{1}{1\times 2}+\frac{1}{2\times 3}+\frac{1}{3\times 4}+\ldots+\frac{1}{n(n+1)}=\frac{n}{n+1}$$ 3

b. i. Find the domain over which the function $y = x^2 + 6x$ is monotonic increasing. 2

ii. Find the inverse function over this restricted domain, and sketch a graph of this inverse function clearly showing its domain and range. 3

iii Evaluate $\cos\left[\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)\right]$ 1

iv. Sketch the graph of $y = 3\sin^{-1}\left(\frac{x}{2}-1\right)$ 3

End of Question 5

Question 6 (12 marks) Use a SEPARATE writing booklet.

Marks

a. When the temperature T of a certain body is 65°C it is cooling at the rate of 1°C per minute.

Assuming Newton's law of cooling: $\frac{dT}{dt} = -k(T - S)$ where

T is the temperature of the body at time t minutes
S is the temperature of the surrounding medium
k is a constant

i. Verify that $T = S + Ae^{-kt}$ is a solution of the given differential equation, where A is a constant. **2**

ii. Determine the value of k given that S, which is constant, is 15°C. **2**

iii. Find T when $t = 20$ minutes, giving your answer to the nearest degree **2**

iv. How long will it take for the temperature of the body to fall to 35°C? **2**

b. The acceleration of a particle P, moving along a straight line has an acceleration given by

$$\frac{d^2x}{dt^2} = -4\left(x + \frac{16}{x^3}\right)$$

i. Given that P is initially at rest at the point $x = 2$ m, show that the velocity v m/s at any time is given by **3**

$$v^2 = 4\left(\frac{16 - x^4}{x^2}\right)$$

ii. Hence, or otherwise, show that when P is halfway to the origin, the speed is given by $2\sqrt{15}$ m/s **1**

End of Question 6

Question 7 (12 marks) Use a SEPARATE writing booklet. **Marks**

a. An arrow is fired horizontally at 60ms^{-1} from the top of a 20m high wall. Taking $g = 10\text{ ms}^{-2}$

i. Show, using calculus, that the horizontal and vertical components of the arrows motion are given by

$$x = 60t$$
$$y = -5t^2 + 20$$
3

ii. Find the time taken for the arrow to hit the ground. **2**

iii. Find the distance that the point of impact will be from the base of the wall. **1**

iv. Find the angle with which the arrow will strike the ground. **2**

b. A squad of 8 is chosen at random from 3 baseball teams A, B and C with 10 players in each team.

i. If 5 of the squad are chosen from the A team, 2 from the B team and 1 is chosen from the C team, in how many ways can the squad be formed? **2**

ii. Find the probability that Joe from the B team and Fred from the A team will be chosen. **2**

End of paper

The Scots College Mathematics Department 2001
Written by: D Hamaty
Assessed by: D Scardino

Standard Integrals

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + C, \quad n \neq -1; x \neq 0, \text{if } n < 0$$

$$\int \frac{1}{x} dx = \ln x + C, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax} + C, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a}\sin ax + C, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a}\cos ax + C, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a}\tan ax + C, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a}\sec ax + C, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a}\tan^{-1}\frac{x}{a} + C, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\frac{x}{a} + C, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln\left(x + \sqrt{x^2 - a^2}\right) + C, \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln\left(x + \sqrt{x^2 + a^2}\right) + C$$

NOTE : $\ln x \equiv \log_e x, \quad x > 0$

Mathematics Extension 1 Trial HSC 2001

Question 1

a. $\int_0^{2\sqrt{3}} \frac{dx}{4+x^2} = \left[\frac{1}{2} \tan^{-1}\left(\frac{x}{2}\right) \right]_0^{2\sqrt{3}}$

$= \frac{1}{2}\tan^{-1}\sqrt{3} - 0$

$= \frac{\pi}{6}$ //

[2]

b. $\frac{d}{dx}\left(\cos^3 x\right) = 3\sin x \cdot \cos^2 x$

[2]

c. $x_1 = 4 \quad x_2 = 13$

$y_1 = 6 \quad y_2 = 5$

$m = 4 \quad n = -1$

$x = \frac{(-1)(4) + (4)(13)}{4-1}$

$= 16$

$y = \frac{(-1)(6) + (4)(5)}{4-1}$

$= \frac{14}{3}$

$\therefore$ the point required is $(16, \frac{14}{3})$

[2]

d. $y = \frac{2x}{3x-1}$

$x = \frac{1}{3}$ is the equation of the vertical asymptote.

[1]

e

$x \geq -2$ or $x \leq -5$

f. $\int_0^{1/\sqrt{2}} \frac{2x^3}{\sqrt{1-x^4}} \cdot dx$ $\quad u = x^4$, $\frac{du}{dx} = 4x^3$

$= \frac{1}{2}\int_0^{1/4} \frac{1}{\sqrt{1-u}} \cdot du$ $\quad u_1 = \left(\frac{1}{\sqrt{2}}\right)^4 = \frac{1}{4}$

$= \frac{1}{2}\int_0^{1/4} (1-u)^{-1/2} \cdot du$ $\quad u_2 = 0^4 = 0$

$= \frac{1}{2}\left[-\frac{2}{3}(1-u)^{3/2} \right]_0^{1/4}$

$= \frac{1}{2}\left(-\frac{\sqrt{3}}{4}\right) - \frac{1}{2}\left(-\frac{2}{3}\right)$

$= -\frac{\sqrt{3}}{8} + \frac{1}{3}$ //

[3]

Question 2

a. # arrangements $= \dfrac{9!}{2!2!2!}$

$= 45360$ //

[2]

b. $\sin\theta + \sqrt{3}\cos\theta = 1 \quad 0 \leq \theta \leq 2\pi$

let $t = \tan\dfrac{\theta}{2}$

$\Rightarrow \dfrac{2t}{1+t^2} + \sqrt{3}\left(\dfrac{1-t^2}{1+t^2}\right) = 1$

$2t + \sqrt{3} - \sqrt{3}.t^2 = 1 + t^2$

$(-\sqrt{3}-1)t^2 + 2t + (\sqrt{3}-1) = 0$

$t = \dfrac{-2 \pm \sqrt{4 + (\sqrt{3}+1)(\sqrt{3}-1)}}{-2(\sqrt{3}+1)}$

$= \dfrac{-2 \pm \sqrt{12}}{-2(\sqrt{3}+1)}$

$= 1$ or $\dfrac{\sqrt{3}-1}{-\sqrt{3}-1}$

$\underline{\underline{t=1}}$: $\tan\dfrac{\theta}{2} = 1 \quad 0 \leq \dfrac{\theta}{2} \leq \pi$

$\dfrac{\theta}{2} = \dfrac{\pi}{4}$

$\theta = \dfrac{\pi}{2}$ //

$t = \dfrac{\sqrt{3}-1}{-\sqrt{3}-1}$:

$=$

$\tan\dfrac{\theta}{2} = \dfrac{\sqrt{3}-1}{-\sqrt{3}-1} \quad 0 \leq \dfrac{\theta}{2} \leq \pi$

$\dfrac{\theta}{2} = \dfrac{11\pi}{12}$

$\theta = \dfrac{11\pi}{6}$

$\therefore \theta = \dfrac{\pi}{2}, \dfrac{11\pi}{6}$ // [4]

c.

i $f(x) = 2x^2 + x - 2$

$f(0) = 2(0)^2 + 0 - 2$

$= -2$

$f(1) = 2(1)^2 + 1 - 2$

$= 1$

so $f(0) < 0$ and $f(1) > 0$

$\therefore f(x)$ must cut the x-axis between $x=0$ and $x=1$.

[1]

ii $f\left(\frac{0+1}{2}\right) = f(0.5)$
$= 2(0.5)^2 + 0.5 - 2$
$= -1$

$\therefore$ root lies between $x = 0.5$ and $x = 1$

$\therefore$ choose $x = \frac{0.5+1}{2} = 0.75$

$f(0.75) = 2(0.75)^2 + 0.75 - 2$
$= -0.125$

$\therefore$ root lies between $x = 0.75$ and $x = 1$
$x = 0.75$ is our approximation.
[3]

iii $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$f(x) = 2x^2 + x - 2$
$f'(x) = 4x + 1$

choosing $x_1 = 0.7$ ...

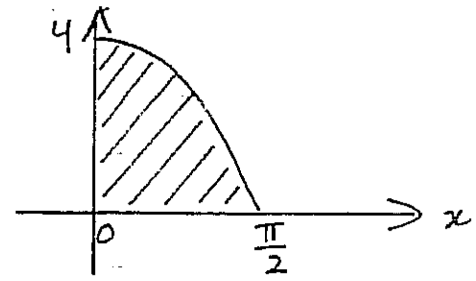
$x_2 = 0.7 - \frac{f(0.7)}{f'(0.7)}$

$= 0.7 - \left(\frac{-0.32}{3.8}\right)$

$= 0.784$ // (3 d.p's)
[2]

Question 3:

a. i



ii $V = \pi \int_0^{\pi/2} \cos^2 x \cdot dx$

$= \frac{\pi}{2} \int_0^{\pi/2} \cos 2x + 1 \cdot dx$

$= \frac{\pi}{2} \left[\frac{1}{2} \sin 2x + x \right]_0^{\pi/2}$

$= \frac{\pi}{2}\left(\frac{\pi}{2}\right) - \frac{\pi}{2}(0)$

$= \frac{\pi}{4}$ ~~sq~~ cu. units. //

b. $x^3 + 4x^2 - 6x - 8 = 0$
let the roots be α, β, γ

$\alpha + \beta + \gamma = -4$
$\alpha\beta + \alpha\gamma + \beta\gamma = -6$
$\alpha\beta\gamma = 8$

$$\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}$$

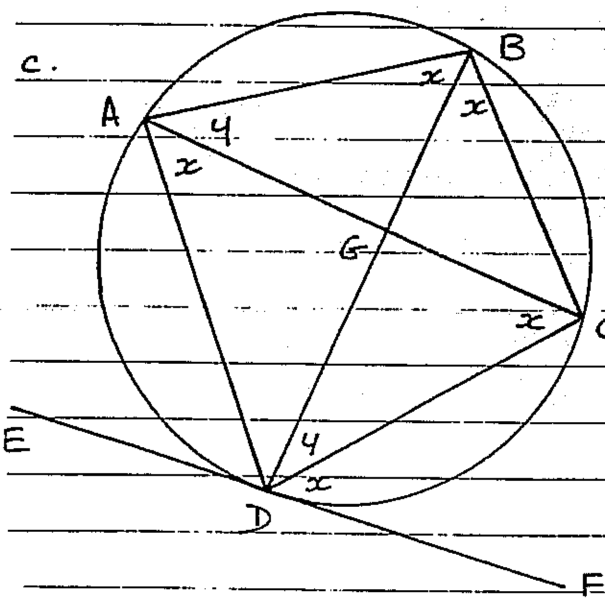
$$= \frac{\beta\gamma+\alpha\gamma+\alpha\beta}{\alpha\beta\gamma}$$

$$= \frac{-6}{8}$$

$$= -\frac{3}{4} \quad //$$

[2]

c.



let $\angle DBC = x$

$\angle DAC = x$ ($\angle$'s on same arc)

$\angle ABD = x$ (BD bisects $\angle ABC$)

$\angle ACD = x$ ($\angle$'s on same arc)

let $\angle BAC = y$

$\angle BDC = y$ ($\angle$'s on same arc)

and $\angle BDF = x+y$ ($\angle$ in alt. seg.)

$= \angle BAD$

$\therefore \angle CDF = \angle BDF - \angle BDC$

$= (x+y) - y$

$= x$

$= \angle ACD$

$\therefore$ alternate $\angle$'s $\angle CDF$ and $\angle ACD$ are equal

$\therefore AC \parallel EF$ //

[2]

d.

i

$A(2\sin x+\cos x)+B(2\cos x-\sin x) \equiv 7\sin x + 11\cos x$

$2A\sin x + A\cos x + 2B\cos x - B\sin x \equiv 7\sin x + 11\cos x$

$(2A-B)\sin x + (A+2B)\cos x \equiv 7\sin x + 11\cos x$

$\Rightarrow$ $2A - B = 7$ — (1)

$A + 2B = 11$ — (2)

from (1)

$B = 2A - 7$

subb'ing into (2) ...

$A + 2(2A-7) = 11$

$A + 4A - 14 = 11$

$5A = 25$

$A = 5$

$2(5) - B = 7$

$B = 3$

$\therefore A = 5, B = 3$ //

[2]

ii

$$\int_0^{\pi/2} \frac{7\sin x + 11\cos x}{2\sin x + \cos x} \cdot dx$$

$$= \int_0^{\pi/2} \frac{5(2\sin x + \cos x) + 3(2\cos x - \sin x)}{2\sin x + \cos x} \cdot dx$$

$$= \int_0^{\pi/2} 5 + 3\left(\frac{2\cos x - \sin x}{2\sin x + \cos x}\right) \cdot dx$$

$$= \Big[5x + 3\ln(2\sin x + \cos x) \Big]_0^{\pi/2}$$

$$= \left(\frac{5\pi}{2} + 3\ln 2\right) - \left(0 + 3\ln 1\right)$$

$$\frac{5\pi}{2} + 3\ln 2$$

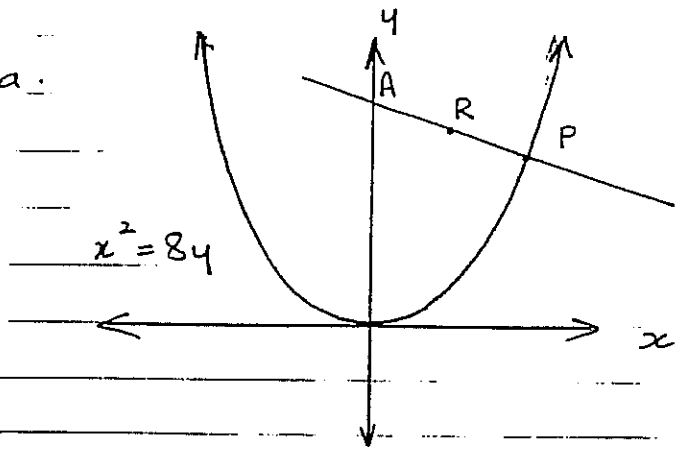
$$\frac{5\pi}{2} + \ln 2^3$$

$$\frac{5\pi}{2} + \ln 8 \;//$$

[3]

Question 4:

a.



i coords of P : $(2ap, ap^2)$
where $a = 2$
$\therefore$ P is $(4p, 2p^2)$.

gradient of normal $= -\frac{1}{p}$

$y - 2p^2 = -\frac{1}{p}(x - 4p)$

$py - 2p^3 = -x + 4p$

$\therefore x + py = 4p + 2p^3$ is the equation of the normal at P.

[2]

ii when $x = 0$, $y = 4 + 2p^2$
$\therefore$ A is $(0, 4 + 2p^2)$

coords of R : $\left(\frac{4p}{2}, \frac{4 + 2p^2}{2}\right)$

$= (2p, 2 + 2p^2)$

[2]

iii

Coords of R:

$x = 2p$ — ①
$y = 2 + 2p^2$ — ②

from ① $p = \frac{x}{2}$

subbing into ② gives

$y = 2 + 2\left(\frac{x}{2}\right)^2$

$= 2 + \frac{2x^2}{4}$

$= 2 + \frac{x^2}{2}$

$x^2 = 2(y - 2)$ is the locus of R.
This is a parabola with vertex $(0, 2)$.

now focus of $x^2 = 8y$ is $(0, a)$ where $a = 2$ ie $(0, 2)$ which is the same as the vertex of $x^2 = 2(y - 2)$ //.

b. i $\displaystyle\int_1^3 \frac{dx}{x}$

$= \Big[\ln x \Big]_1^3$

$= \ln 3 - \ln 1$

$= \ln 3$ [1]

ii

$\displaystyle\int_1^3 \frac{dx}{x} \doteqdot \frac{1}{3}\left\{f(1) + 4f(2) + f(3)\right\}$

$f(x) = \frac{1}{x}$

$f(1) = 1 \;;\; f(2) = \frac{1}{2} \;;\; f(3) = \frac{1}{3}$

$\therefore \displaystyle\int_1^3 \frac{dx}{x} \doteqdot \frac{1}{3}\left\{1 + 2 + \frac{1}{3}\right\}$

$\doteqdot \frac{10}{9}$ [2]

iii from i and ii

$\ln 3 \doteqdot \frac{10}{9}$

$\therefore\; 3 \doteqdot e^{\frac{10}{9}}$

$\Rightarrow e \doteqdot 3^{\frac{9}{10}}$

$\doteqdot 2.688$ (3 d.p's) [2]

Question 5:

a. Prove...

$\frac{1}{1\times 2} + \frac{1}{2\times 3} + \frac{1}{3\times 4} + \cdots + \frac{1}{n(n+1)}$

$= \frac{n}{n+1}$ for $n \geq 1$

<u>let $n=1$</u>:

$LHS = \frac{1}{1\times 2} = \frac{1}{2}$

$RHS = \frac{1}{1+1} = \frac{1}{2} = LHS$

$\therefore$ true for $n=1$

<u>Assume true for $n=k$</u>

ie $\frac{1}{1\times 2} + \frac{1}{2\times 3} + \cdots + \frac{1}{k(k+1)} = \frac{k}{k+1}$

Prove true for $n = k+1$

$$HS = \frac{1}{1\times 2} + \frac{1}{2\times 3} + \cdots + \frac{1}{k(k+1)} + \frac{1}{(k+1)(k+2)}$$

$$= \frac{k}{k+1} + \frac{1}{(k+1)(k+2)}$$

$$= \frac{k(k+2)+1}{(k+1)(k+2)}$$

$$= \frac{k^2+2k+1}{(k+1)(k+2)}$$

$$= \frac{(k+1)^2}{(k+1)(k+2)}$$

$$= \frac{k+1}{k+2}$$

$$RHS = \frac{k+1}{(k+1)+1}$$

$$= \frac{k+1}{k+2}$$

$$= LHS$$

$\therefore$ true for $n = k+1$

$\therefore$ since true for $n = 1$

then true for $n = 2$, $n = 3$, ...

$\therefore$ true for all $n \geq 1$ // [3]

b.

i $y = x^2 + 6x$

$$\frac{dy}{dx} = 2x + 6$$

for monotonic increasing ... $\frac{dy}{dx} > 0$

$2x + 6 > 0$

$2x > -6$

$x > -3$

$\therefore$ the function is monotonic increasing when $x > -3$

[2]

ii let $x = y^2 + 6y$

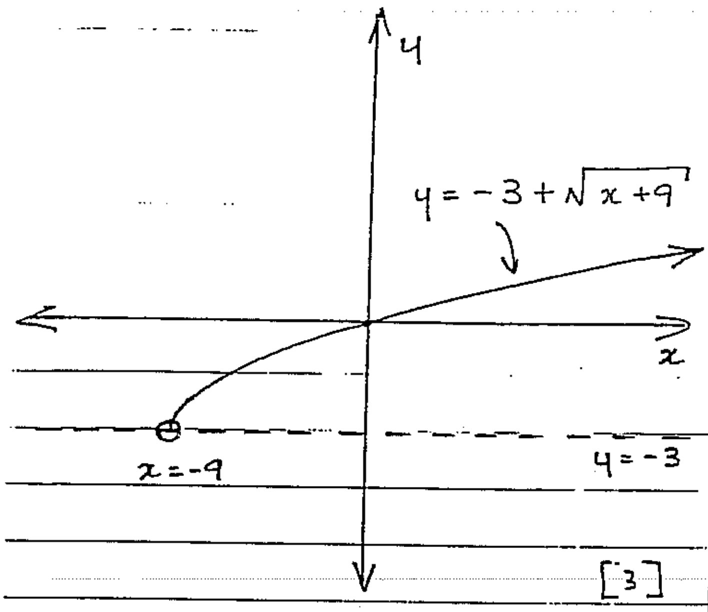
$$x + 9 = y^2 + 6y + 9$$
$$= (y+3)^2$$
$$y + 3 = \pm\sqrt{x+9}$$
$$y = -3 \pm \sqrt{x+9}$$

but the range will be $y > -3$

$\therefore$ $y = -3 + \sqrt{x+9}$ // is the inverse function.

domain : $x \geq -9$

range : $y > -3$



[3]

iii $\cos\left[\tan^{-1}\left(\frac{-1}{\sqrt{3}}\right)\right]$

$= \cos\left(-\frac{\pi}{6}\right)$

$= \cos\left(\frac{\pi}{6}\right)$

$= \frac{\sqrt{3}}{2}$ // [1]

iv. $y = 3\sin^{-1}\left(\frac{x}{2}-1\right)$

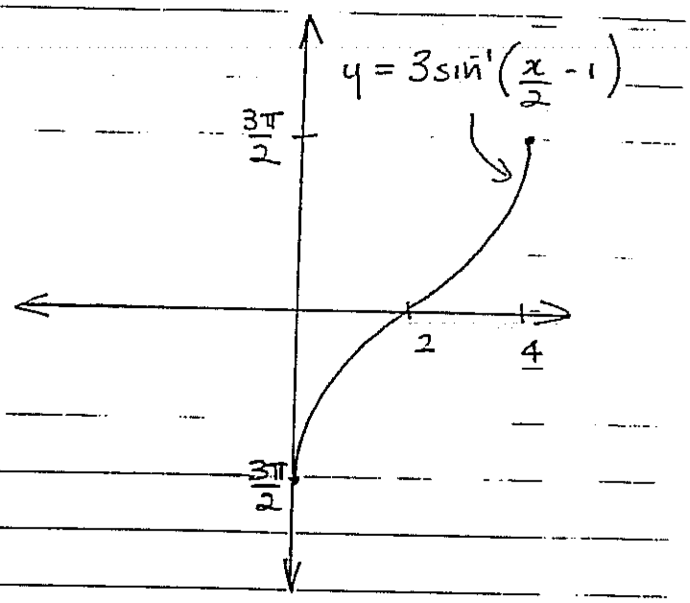
$-1 \leq \frac{x}{2}-1 \leq 1$

$-2 \leq x-2 \leq 2$

$0 \leq x \leq 4$

$\therefore$ domain : $0 \leq x \leq 4$

range : $-\frac{3\pi}{2} \leq y \leq \frac{3\pi}{2}$



[3]

Question 6:

i $\frac{dT}{dt} = -k(T-S)$

let $T = S + Ae^{-kt}$

$\frac{dT}{dt} = -Ake^{-kt}$

$-k(T-S) = -k(S+Ae^{-kt}-S)$

$= -Ake^{-kt}$

$= \frac{dT}{dt}$

$\therefore T = S + Ae^{-kt}$ is a soln. of the differential equation.

[2]

ii Initial conditions:

$t=0$, $S=15$, $\frac{dT}{dt}=-1$, $T=65$

$T = S + Ae^{-kt}$

$65 = 15 + A \cdot 1$

$\therefore A = 50$

$\frac{dT}{dt} = -k(T-S)$

$-1 = -k(65-15)$

$\therefore k = \frac{1}{50}$ //

[2]

iii $T = 15 + 50e^{-\frac{t}{50}}$

$= 15 + 50e^{-\frac{20}{50}}$

$= 49^{\circ}$ (to nearest degree)

[2]

iv $35 = 15 + 50e^{-\frac{t}{50}}$

$20 = 50e^{-t/50}$

$e^{-t/50} = 0.4$

$-\frac{t}{50} = \ln(0.4)$

$t = -50\ln(0.4)$

$= 45.8$ minutes. //

[2]

b.

i

$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = -4\left(x + 16x^{-3}\right)$

$\therefore \frac{1}{2}v^2 = \int -4x - 64x^{-3} \cdot dx$

$= -2x^2 + 32x^{-2} + C$

$\therefore v^2 = -4x^2 + \frac{64}{x^2} + D$

now $v=0$ when $x=2$

$\therefore 0 = -16 + 16 + D$

$\therefore D = 0$

so $v^2 = \frac{64}{x^2} - 4x^2$

$= 4\left(\frac{16}{x^2} - x^2\right)$

$= 4\left(\frac{16-x^4}{x^2}\right)$ //

[3]

ii when P is halfway to the origin $x=1$

$v^2 = 4\left(\frac{16-1}{1}\right)$

$= 4 \times 15$

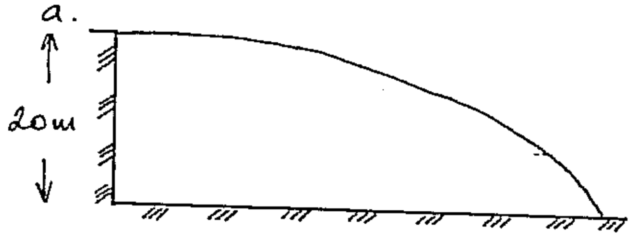
$\therefore v = \pm 2 \times \sqrt{15}$

hence speed is $2\sqrt{15}$ m/s.

[1]

Question 7:

a.



i horizontal motion:

$\ddot{x} = 0$

$\dot{x} = \int 0 \cdot dt$

$= C$

since $\dot{x} = 60$ when $t = 0$

$C = 60$

$\therefore \dot{x} = 60$ and is constant.

$x = \int 60 \cdot dt$

$= 60t + D$

$x = 0$ when $t = 0$

$\therefore D = 0$

$\therefore x = 60t$ //

vertical motion:

$\ddot{y} = -10$

$\dot{y} = \int -10 \cdot dt$

$= -10t + E$

$= 0$ when $t = 0$

$\therefore E = 0$

$y = \int -10t \cdot dt$

$= -5t^2 + F$

$= 20$ when $t = 0$

$\therefore F = 20$

$\therefore y = -5t^2 + 20$ // [3]

ii $0 = -5t^2 + 20$

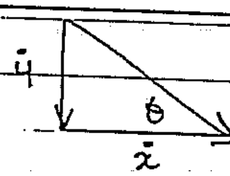
$5t^2 = 20$

$t^2 = 4$

$t = 2$ s. // [2]

iii when $t = 2$, $x = 120$ m. [1]

iv.



$\tan\theta = \dot{y} / \dot{x}$

when $t = 2$, $\dot{y} = -20$

$\dot{x} = 60$

$\therefore \tan\theta = -1/3$

$\theta = \tan^{-1}(1/3)$

$= 18.4^\circ$ with the horizontal.

[2]

b.

i $\#\text{ways} = {}^{10}C_5 \times {}^{10}C_2 \times {}^{10}C_1$

$= 113400$ // [2]

ii #ways of choosing Joe and Fred $= {}^{9}C_4 \times {}^{9}C_1 \times {}^{10}C_1$

$\therefore P(\text{Joe, Fred}) = 11340 / 113400$ [2]

$= 0.1$ //