THE SCOTS COLLEGE
Sydney

2006
HSC TRIAL
EXAMINATION

Extension One Mathematics

General Instructions

- Working time – 120 Minutes
  + 5 minutes reading time
- Write using black or blue pen
- Board-approved calculators may be used
- Start a new booklet for each question
- A table of Standard Integrals is attached
- All necessary working should be shown in every question

- **Total marks - 84**
- Attempt all questions
- Start a new booklet for every question

QUESTION ONE [12 MARKS] START A NEW BOOKLET

a. Solve the inequality $\dfrac{x^2-4}{x} \geq 3$. 3

b. Let A(-3,6) and B(1,10) be the points on the number plane. Find the coordinates of the point C which divides the interval AB externally in the ratio 5:3. 2

c. If α, β and γ are the roots of $3x^3+5x^2-7x+4=0$ find the value of $\dfrac{1}{\alpha}+\dfrac{1}{\beta}+\dfrac{1}{\gamma}$. 2

d. Use the substitution $u=1-x^3$ to evaluate $\int_0^1 x^2\left(1-x^3\right)^4 dx$. 2

e. Find the acute angle between the lines $x+3y-4=0$ and $2x-5y=0$. 3
Answer to the nearest degree.

QUESTION TWO [12 MARKS] START A NEW BOOKLET

a. Find the exact value of $\displaystyle\int_0^{\frac{\pi}{4}} \cos^2\frac{x}{2}\,dx$. 3

b. Find $\displaystyle\int_0^{\frac{1}{5}} \frac{2dx}{\sqrt{16-100x^2}}$ 3

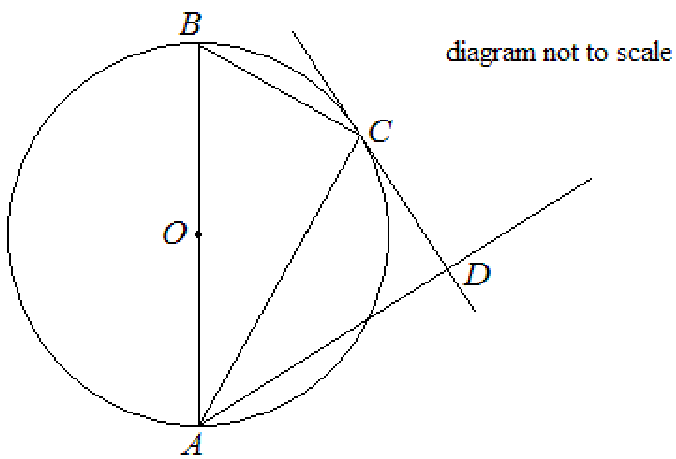
c. Consider the function $y=\dfrac{\pi}{2}+\cos^{-1}\dfrac{x}{2}$.

(i) State the domain of this function. 1

(ii) State the range of this function. 1

(iii) Sketch the graph of this function. 1

d.



In the diagram AOB is the diameter of a circle centre O, and C is the point of contact of the tangent DC such that AC bisects $\angle DAB$. 3
Prove that AD is perpendicular to DC.

QUESTION THREE **[12 MARKS]** **START A NEW BOOKLET**

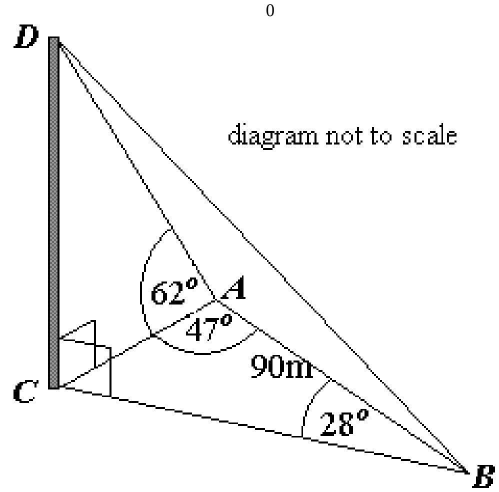
a. (i) Using the identities for the expansion of $\sin(A+B)$, $\sin 2A$ and $\cos 2A$, prove that $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$. 2

(ii) Hence solve the equation $3\sin\theta - 4\sin^3\theta = -1$ for $0 \le \theta \le 2\pi$. 2

b. (i) Find $\frac{d}{dx}(x\tan^{-1}x)$. 1

(ii) Hence find the exact value of $\int_0^1 \tan^{-1}x\,dx$. 3

c.



A person sees the top of a pole from 2 points A and B both on flat ground. The angle of elevation from point A to the top of the pole D is 62°.
If $\angle ABC = 28°$, $\angle BAC = 47°$ and A and B are 90 metres apart, find;

i) how far A is from the base of the pole to the nearest metre. 2

ii) the height of the pole, correct to 3 significant figures. 2

QUESTION FOUR **[12 MARKS]** **START A NEW BOOKLET**

a. Rewrite $\sqrt{3}\sin x - \cos x$ in the form $R\sin(x-\alpha)$. Hence solve $\sqrt{3}\sin x - \cos x = 1$ for $0 \le x \le 2\pi$ 3

b. Find the constant term of the expansion of $\left(2x - \frac{1}{x^3}\right)^{20}$ 2

c. (i) Show that the normal to the parabola $x^2 = 4ay$ at the point $(2at, at^2)$ has the equation $x + ty = 2at + at^3$ 2

(ii) Hence show that there is only one normal which passes through the focus. 2

d. It is known that the equation $e^{-x^2} - 5x^2 - 0.99 = 0$ has a positive root close to the origin. Attempt to find the root using one application of Newton's method, starting with $x = 0$ as the first approximation and hence explain why Newton's method fails with $x = 0$ as the first approximation. 3

QUESTION FIVE [12 MARKS] START A NEW BOOKLET

a. Prove by Mathematical Induction that $5^{2n}-1$ is divisible by 6 when *n* is a positive integer. 4

b. A spherical balloon is being inflated at a constant rate of $1000\text{cm}^3\text{s}^{-1}$
Given that a sphere has volume $V=\frac{4}{3}\pi r^3$ and surface area $A=4\pi r^2$ find:

(i) An expression for the instantaneous rate of change of the radius in terms of *r*. (find $\frac{dr}{dt}$) 3

(ii) The rate of change of the surface area of the balloon when the radius is 10cm. 2

c. If ${}^nC_r={}^nC_{r+1}$, where *n* and *r* are positive integers, show that *n* is odd. 3

QUESTION SIX [12 MARKS] START A NEW BOOKLET

a. A particle's motion is defined by the equation $v^2=12+4x-x^2$ where *x* is the displacement in metres of the particle from the origin and *v* is the velocity in ms^{-1}. Initially the particle is 6 metres to the right of the origin.

(i) Show that the particle is moving in Simple Harmonic Motion. 1

(ii) Find the centre, the period and the amplitude of the motion. 3

(iii) The displacement of the particle at any time *t* is given by the equation $x=a\sin(nt+\theta)+b$. Find the values of θ and *b*, given $0\le\theta\le 2\pi$. 2

b. Newton's Law of Cooling states that the rate of change in the temperature, *T* of a body is proportional to the difference between the temperature of the body and the surrounding temperature, *P*.

(i) If *A* and *k* are constants, show that the equation $T=P+Ae^{kt}$ satisfies Newton's Law of Cooling. 2

(ii) On a day when the air temperature was 18°C, water in a container is heated to 100°C. Three minutes after being taken off the heat the temperature has dropped to 87°C, calculate *A* and *k* (to 4 d.p). 2

(iii) If the temperature has to be below 70°C to wash your hands in the container how long, to the nearest minute, will you have to wait? 2

QUESTION SEVEN **[12 MARKS]** **START A NEW BOOKLET**

a. A projectile is fired horizontally with speed v ms^{-1} from a point h m above the horizontal ground. Let acceleration due to gravity be g

(i) Prove that it will reach the ground after $\sqrt{\frac{2h}{g}}$ seconds. 2

(ii) If it does so at an angle of 60° to the horizontal, prove that $3v^2 = 2gh$. 3

Hint: $\frac{dy}{dx} = \tan 120^\circ$

b. Given $y = \log_e(e^x \sin^2 x)$

(i) Show that $\frac{dy}{dx} = 1 + 2\cot x$. 1

(ii) Prove that the equation of the normal at $x = \frac{\pi}{2}$ is given by $x + y = \pi$ 3

c. Find $\int \frac{x\,dx}{\left(25 + x^2\right)^{\frac{3}{2}}}$ using the substitution $x = 5\tan\theta$ 3

$\boxed{\text{Question One}}$

a) $\dfrac{x^2-4}{x} \geqslant 3$

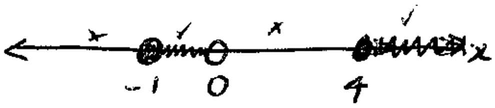
$x(x^2-4) \geqslant 3x^2$

$x^3-4x-3x^2 \geqslant 0$

$x(x^2-3x-4) \geqslant 0$

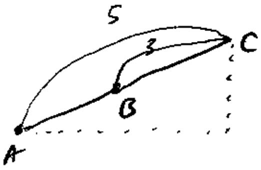
$x(x-4)(x+1) \geqslant 0$

$x \neq 0$



$-1 \leqslant x < 0$ and $x \geqslant 4$

b) $\overset{x_1\ y_1}{A(-3,6)}$ $\overset{x_2\ y_2}{B(1,10)}$ $\overset{m\ \ n}{-5:3}$



$C = \left(\dfrac{mx_2+nx_1}{m+n}, \dfrac{my_2+ny_1}{m+n}\right)$

$= \left(\dfrac{-5.1+3.-3}{-5+3}, \dfrac{-5.10+3.+6}{-5+3}\right)$

$= (7, 16)$

c) $\overset{a}{3x^3} + \overset{b}{5x^2} - \overset{c}{7x} + \overset{d}{4} = 0$

now $\dfrac{1}{\alpha}+\dfrac{1}{\beta}+\dfrac{1}{\gamma} = \dfrac{\alpha\beta+\alpha\gamma+\beta\gamma}{\alpha\beta\gamma}$

$= \dfrac{c/a}{-d/a}$

$= \dfrac{-c}{d}$

$= \dfrac{+7}{4}$

d) $\int_0^1 x^2(1-x^3)^4\,dx$

let $u = 1-x^3$

$\dfrac{du}{dx} = -3x^2$

$du = -3x^2dx$

$-\frac{1}{3}du = x^2dx$

when $x=0$, $x=1$
$u=1$, $u=0$

$\int_0^1 x^2(1-x^3)^4\,dx = -\frac{1}{3}\int_1^0 u^4\,du$

$= -\frac{1}{3}\left[\frac{u^5}{5}\right]_1^0$

$= -\frac{1}{3}\left(0-\frac{1}{5}\right)$

$= \frac{1}{15}$

e)

$x+3y-4=0$

$3y = -x+4$

$y = -\frac{1}{3}x+\frac{4}{3}$

$2x-5y=0$

$5y = 2x$

$y = \frac{2}{5}x$

$\tan\theta = \left|\dfrac{m_1-m_2}{1+m_1m_2}\right|$

$= \left|\dfrac{-\frac{1}{3}-\frac{2}{5}}{1+-\frac{1}{3}.\frac{2}{5}}\right|$

$= \dfrac{-11/15}{13/15}$

$\tan\theta = 11/13$

$\theta = 40.236^\circ$

$= 40^\circ$ to nearest degree

Question Two

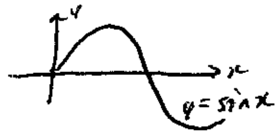
a) $\int_0^{\pi/4} \cos^2 \frac{x}{2}\, dx = \frac{1}{2}\int_0^{\pi/4} 1 + \cos x \, dx$

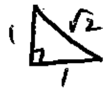
$= \frac{1}{2}\left[x + \sin x \right]_0^{\pi/4}$

$= \frac{1}{2}\left[\left(\pi/4 + \sin \pi/4\right) - 0 \right]$

$= \pi/8 + 1/2\sqrt{2}$

$= \frac{\pi + 2\sqrt{2}}{8}$





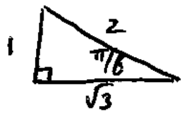
b) $\int_0^{1/5} \frac{2dx}{\sqrt{16 - 100x^2}} = \int_0^{1/5} \frac{2dx}{2\sqrt{4-25x^2}}$

$= \int_0^{1/5} \frac{dx}{\sqrt{4/25 - x^2}}$

$= \left[\sin^{-1} \frac{x}{2/5} \right]_0^{1/5}$

$= \sin^{-1}\frac{1}{2} - \sin^{-1} 0$

$= \pi/6$



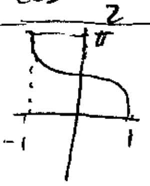
c) $y = \frac{\pi}{2} + \cos^{-1}\frac{x}{2}$

Base

$y = \cos^{-1} x$

$D: -1 \le x \le 1$

$0 \le y \le \pi$



i) ∴ $\cos^{-1}\frac{x}{2}$ has domain.

$-1 \le \frac{x}{2} \le 1$

$D: -2 \le x \le 2$

ii) $\frac{\pi}{2} + \cos^{-1}\frac{x}{2}$ range

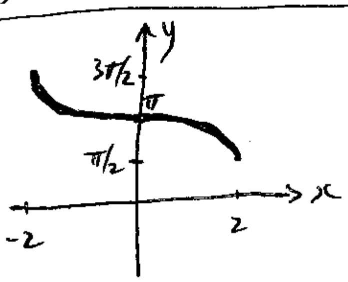
$y = \cos^{-1} x/2$ has range

$0 \le y \le \pi$.

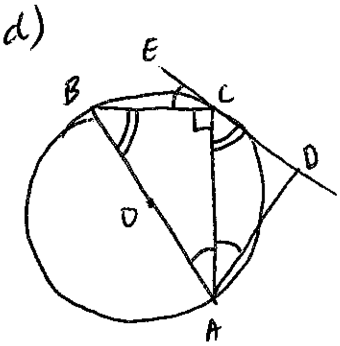
∴ $\cos^{-1} x/2 + \frac{\pi}{2}$ shifts up $\pi/2$.

$R: \pi/2 \le y \le \frac{3\pi}{2}$

iii)



d)



$\angle BCA = 90°$ angle in a semicircle.

$\angle ACD = \angle ABC$ (angle between tangent and chord at point of contact equals angle in alternate segment.)

$\angle BCE = \angle CAB$

$\angle CAD = \angle BAC$ given AC bisects $\angle OAB$.

Question three

a i) $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

LHS $\sin 3\theta = \sin(\theta + 2\theta)$

$= \sin\theta\cos 2\theta + \cos\theta \sin 2\theta$

$= \sin\theta(1 - 2\sin^2\theta) + \cos\theta(2\sin\theta\cos\theta)$

$= \sin\theta - 2\sin^3\theta + 2\sin\theta\cos^2\theta$

$= \sin\theta - 2\sin^3\theta + 2\sin\theta(1 - \sin^2\theta)$

$= \sin\theta - 2\sin^3\theta + 2\sin\theta - 2\sin^3\theta$

$= 3\sin\theta - 4\sin^3\theta$

$=$ RHS.

a ii) $3\sin\theta - 4\sin^3\theta = -1$.

$\sin 3\theta = -1$.

now if $0 \le \theta \le 2\pi$

then $0 \le 3\theta \le 6\pi$

$3\theta = \frac{3\pi}{2}, \frac{7\pi}{2}, \frac{11\pi}{2}, \cancel{\frac{15\pi}{2}}$

$\theta = \pi/2, 7\pi/6, \frac{11\pi}{6}$

b) i) $\frac{d}{dx} x\tan^{-1}x = \tan^{-1}x \cdot 1 + x \cdot \frac{1}{1+x^2}$

$= \tan^{-1}x + \frac{x}{1+x^2}$

ii) $\tan^{-1}x = \frac{d}{dx}(x\tan^{-1}x) - \frac{x}{1+x^2}$

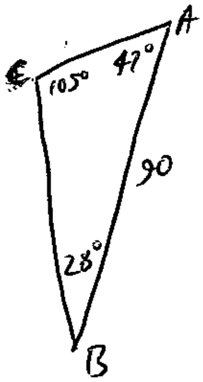
$\therefore \int_0^1 \tan^{-1}x\,dx = \int_0^1 \frac{d}{dx}(x\tan^{-1}x) - \int_0^1 \frac{x}{1+x^2}dx$.

$= \left[x\tan^{-1}x\right]_0^1 - \left[\frac{1}{2}\ln(1+x^2)\right]_0^1$

$= \left(1 \cdot \frac{\pi}{4} - 0\right) - \left(\frac{1}{2}\ln 2 - \frac{1}{2}\ln 1\right)$

$= \frac{\pi}{4} - \frac{\ln 2}{2}$

c) i)



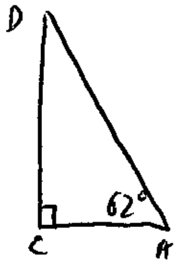
$\frac{AC}{\sin 28} = \frac{90}{\sin 105}$.

$AC = \frac{90 \times \sin 28}{\sin 105}$

$= 43.74$

$= 44$m to nearest metre.

ii)



$\tan 62 = \frac{DC}{AC}$

$DC = AC\tan 62$

$= 82.26\ldots$

$= 82.3$ to 3 sig. fig.

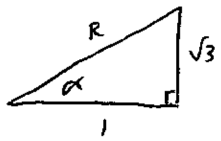
QUESTION 4

4a) i) $\sqrt{3}\sin x - \cos x = R\sin(x-\alpha)$

$= R\sin\alpha\sin x - R\cos\alpha\cos x$

$\therefore R\sin\alpha = \sqrt{3}$ and $R\cos\alpha = 1$

$\sin\alpha = \frac{\sqrt{3}}{R}$ $\qquad \cos\alpha = \frac{1}{R}$.



$R = \sqrt{3+1}$

$= 2$

$\therefore \cos\alpha = \frac{1}{2}$

$\alpha = \frac{\pi}{6}$

$\therefore \sqrt{3}\sin x - \cos x = 2\sin\left(x - \frac{\pi}{6}\right)$

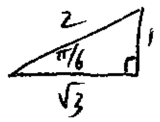
ii) $\sqrt{3}\sin x - \cos x = 1$

$\therefore 2\sin\left(x - \frac{\pi}{6}\right) = 1$

$\sin\left(x - \frac{\pi}{6}\right) = \frac{1}{2}$.

now $0 \leq x \leq 2\pi$

$\therefore -\frac{\pi}{6} \leq x - \frac{\pi}{6} \leq \frac{11\pi}{6}$



$x - \frac{\pi}{6} = \frac{\pi}{6}, \frac{5\pi}{6}, \cancel{\frac{13\pi}{6}}$

$x = \frac{\pi}{3}, \pi$

4b) $\left(2x - \frac{1}{x^3}\right)^{20} = \left(2x - x^{-3}\right)^{20}$

$T_{r+1} = {}^nC_r\, a^{n-r} b^r$

$= {}^{20}C_r\, 2x^{20-r}\left(-x^{-3}\right)^r$

$= {}^{20}C_r\, 2^{20-r}\, x^{20-r}\, (-1)^r\, x^{-3r}$

$= {}^{20}C_r (-1)^r\, 2^{20-r}\, x^{20-4r}$.

for independent term. $20 - 4r = 0$ ie $r = 5$.

$\therefore T_6 = {}^{20}C_5 (-1)^5\, 2^{15}$.

$= -\,{}^{20}C_5\, 2^{15}$

$= -\frac{20!}{15!\,5!}\, 2^{15}$.

$= -15504 \times 2^{15}$

$= -508035072$

4c) i) $x^2 = 4ay$.

$y = \frac{x^2}{4a}$

$y' = \frac{x}{2a}$

when $x = 2at$.

$y' = t$

$\therefore M_{normal} = \frac{-1}{t}$

so $y - at^2 = -\frac{1}{t}(x - 2at)$

$ty - at^3 = -x + 2at$

$x + ty = 2at + at^3$

ii) to pass through the focus means it will pass through $(0,a)$

ie $\cancel{0} + at = 2at + at^3$

$0 = at(1 + t^2)$. this has only one solution ie $t = 0$ because $t^2 \neq -1$

d) let $f(x) = e^{-x^2} - 5x^2 - 0 \cdot 99$

$f'(x) = -2xe^{-x^2} - 10x$

Newtons method $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

NOTE: $f'(0) = 0$ which will make this undefined.

$f'(0) = 0$ cannot have gradient as it at $x = 0$ ie. turning point at $x = 0$.

now $f(0) = e^0 - 0 \cdot 99$

$= 0 \cdot 01$ so not a root either.

Question 5

a) Step 1 prove true for $n=1$ ie $5^2-1 = 24$ which is divisible by 6.
$\therefore$ true for $n=1$

step 2 assume true for $n=k$ ie $5^{2k}-1 = 6M$ where M is an integer.

step 3 prove true for $n=k+1$ ie $5^{2k+2}-1 = 5^2 5^{2k}-1$

from step 2 $5^{2k} = 6M+1$.

$= 25(6M+1)-1$

$= 25.6M + 24$.

both terms are divisible by 6.

$= 6(25M+4)$

$\therefore$ true for $n=k+1$.

Step 4 if true for $n=k$ then true for $n=k+1$.
since true for $n=1$. so true for $n=1+1 = 2$ then $n=3$ and so on for all integer values of n.

b) i) $V = \frac{4}{3}\pi r^3$

$\frac{dV}{dr} = 4\pi r^2$ now $\frac{dV}{dt} = 1000$

$\frac{dr}{dt} \times \frac{dV}{dr} = \frac{dV}{dt}$

$\frac{dr}{dt} \times 4\pi r^2 = 1000$

$\frac{dr}{dt} = \frac{1000}{4\pi r^2}$

$= \frac{250}{\pi r^2}$ cm/s

ii) aim $\frac{dA}{dt}$ when $r=10$

$A = 4\pi r^2$

$\frac{dA}{dr} = 8\pi r$

$\therefore \frac{dA}{dt} = \frac{dA}{dr} \cdot \frac{dr}{dt}$

$= 8\cancel{\pi}\cancel{r} \times \frac{250}{\cancel{\pi} r^{\cancel{2}}}$

$= \frac{2000}{r}$

when $r=10$ $\frac{dA}{dt} = 200 cm^2/s$

c) ${}^nC_r = {}^nC_{r+1}$

NOTE: $(n-r)! = (n-r-1)!(n-r)$
$(r+1)! = r!(r+1)$.

$\frac{n!}{(n-r)!\,r!} = \frac{n!}{(n-r-1)!(r+1)!}$

$\frac{n!}{(n-r-1)!(n-r)r!} = \frac{n!}{(n-r-1)!\,r!(r+1)}$

$\frac{n!(r+1)}{(n-r-1)!(n-r)r!(r+1)} = \frac{n!(n-r)}{(n-r-1)!(n-r)r!(r+1)}$

$\therefore r+1 = n-r$.

$n = 2r+1$ which is odd for all positive integer values of r as $2r$ is even

$\boxed{\text{Question } 6\ a)}$

i) $a = \dfrac{d\ \frac{1}{2}v^2}{dx}$ so $a = \dfrac{d\left(6+2x-\frac{x^2}{2}\right)}{dx}$ for SHM $\ddot{x} = -n^2x + k$

$= 2 - x$

$= -1^2x + 2 \therefore$ SHM $n^2 = 1$

ii) at ends $v=0$ ie $v^2=0$ so $12+4x-x^2=0$

$x^2-4x-12=0$

$(x-6)(x+2)=0$

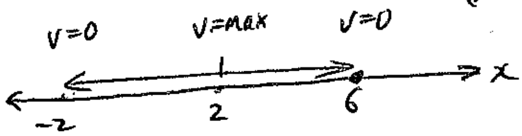
ends $x=-2$ and $x=6$

so centre = halfway

$x = 2m$

amplitude = centre to end

$= 4m$



Period $T = \dfrac{2\pi}{n}$ and $n=1 \therefore T = \dfrac{2\pi}{1}$

$= 2\pi$ seconds.

iii) $x = a\sin(nt+\theta)$ is SHM about the centre $\therefore b=2$

$x = a\sin(nt+\theta)+b$ is SHM about pt. b Now $a=4$, $n=1$, $b=2$

$x = 4\sin(1t+\theta)+2$

θ unknown but when $t=0$ $x=6$ (GIVEN)

$6 = 4\sin\theta + 2$

$4 = 4\sin\theta$

$\sin\theta = 1 \quad \therefore \theta = \pi/2$

6b) i) $T = P + Ae^{kt}$ ——— (1)

$\frac{dT}{dt} = kAe^{kt}$ now $Ae^{kt} = T - P$ from (1).

$= k(T-P)$

ii) $P = 18$ $t = 0$ $T = 100$

$100 = 18 + Ae^{0}$

$A = 82$

$\therefore T = 18 + 82e^{kt}$

when $t = 3$ $T = 87$

$87 = 18 + 82e^{3k}$

$e^{3k} = \frac{69}{82}$

$k = \frac{1}{3} \ln \frac{69}{82}$

$= -0.05753758$

iii) $70 = 18 + 82e^{-0.0575t}$

$\ln \frac{52}{82} = -0.0575t$.

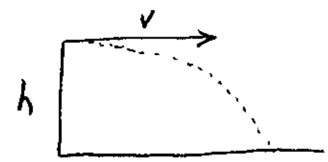
$t = \frac{\ln \frac{52}{82}}{-0.0575}$.

$= 7.916$

$= 8$ min to nearest minute.

QUESTION 7 a

i)



$\ddot{x} = 0$

$\dot{x} = c_1$

$= v$ since all horizontal

$x = vt + c_2$

when $t=0$ $x=0$ $\therefore c_2 = 0$

$\therefore x = vt$

$\ddot{y} = -g$

$\dot{y} = -gt + c_3$

when $t=0$ vert. velocity $=0$ so $c=0$

$\therefore \dot{y} = -gt$

$y = -\frac{1}{2}gt^2 + c_4$

when $t=0$ $y=h$ $\therefore c_4 = h$.

$\therefore y = -\frac{1}{2}gt^2 + h$

to be on the ground $y=0$

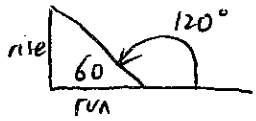
$\therefore 0 = -\frac{1}{2}gt^2 + h$

$\frac{1}{2}gt^2 = h.$

$t^2 = \frac{2h}{g}$

$t = \sqrt{\frac{2h}{g}}$ note: no negative time

ii) to strike ground at 60° ie 120°



hint given: $\frac{dy}{dx} = \tan 120^\circ$

$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$ now $\frac{dy}{dt} = -gt$ and $\frac{dx}{dt} = v$ $\therefore \frac{dt}{dx} = \frac{1}{v}$

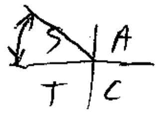
$= -gt \cdot \frac{1}{v}$

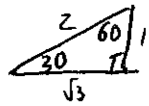
strikes the ground at time $t = \sqrt{\frac{2h}{g}}$

$\frac{dy}{dx} = -\frac{g\sqrt{\frac{2h}{g}}}{v}$

now $\tan 120^\circ = -\tan 60$

$= -\sqrt{3}$





so $-\sqrt{3} = -\frac{g\sqrt{\frac{2h}{g}}}{v}$

$-\sqrt{3}v = -g\sqrt{\frac{2h}{g}}.$

$3v^2 = g^2 \cdot \frac{2h}{g}$

$3v^2 = 2hg.$ #

7b) i) $y = \log_e(e^x \sin^2 x)$

$= \ln e^x + \ln \sin^2 x.$

$= x + 2\ln \sin x$

$\frac{dy}{dx} = 1 + 2\frac{\cos x}{\sin x}$

$= 1 + 2\cot x.$ ✓

ii) $y = x + 2\ln \sin x.$

$\frac{dy}{dx} = 1 + 2\cot x.$

when $x = \frac{\pi}{2}$

$y = \frac{\pi}{2} + 2\ln \sin\frac{\pi}{2}$

$= \frac{\pi}{2}$

$\frac{dy}{dx} = 1 + 2\cot\frac{\pi}{2}$ so $M_{normal} = -1.$

$= 1$

$\therefore$ eq^n of normal $y - \frac{\pi}{2} = -1\left(x - \frac{\pi}{2}\right)$

$y - \frac{\pi}{2} = -x + \frac{\pi}{2}.$

$\boxed{x + y = \pi}$

$\boxed{7c}$

$x = 5\tan\theta$

$x^2 = 25\tan^2\theta$

$\frac{dx}{d\theta} = 5\sec^2\theta$

$dx = 5\sec^2\theta\, d\theta.$

$$\int \frac{x\,dx}{(25+x^2)^{3/2}} = \int \frac{5\tan\theta.\ 5\sec^2\theta\, d\theta}{(25 + 25\tan^2\theta)^{3/2}}$$

$$= \int \frac{25\tan\theta\sec^2\theta}{25^{3/2}(1+\tan^2\theta)^{3/2}}\, d\theta$$

$$= \int \frac{25\tan\theta\sec^2\theta}{125\sec^3\theta}\, d\theta$$

$$= \frac{1}{5}\int \frac{\tan\theta}{\sec\theta}\, d\theta$$

$$= \frac{1}{5}\int \sin\theta\, d\theta.$$

$$= -\frac{1}{5}\cos\theta + C$$

$$= -\frac{1}{\cancel{5}}\cos\left[\cos^{-1}\frac{\cancel{5}}{\sqrt{x^2+25}}\right] + C.$$

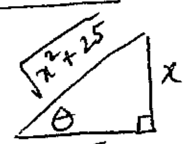
$$= \frac{-1}{\sqrt{x^2+25}} + C.$$

$$\boxed{\begin{aligned}&(1+\tan^2\theta)^{3/2}\\ &= (\sec^2\theta)^{3/2}\\ &= \sec^3\theta.\end{aligned}}$$

$$\boxed{\begin{aligned}\frac{\tan\theta}{\sec\theta} &= \frac{\sin\theta}{\cancel{\cos\theta}} \times \frac{\cancel{\cos\theta}}{1}\\ &= \sin\theta.\end{aligned}}$$

$$\boxed{\begin{aligned}\text{now } x &= 5\tan\theta.\\ \tan\theta &= \frac{x}{5}.\\ \theta &= \tan^{-1}\frac{x}{5}.\end{aligned}}$$

$\tan\theta = \frac{x}{5}$



$\cos\theta = \frac{5}{\sqrt{x^2+25}}$ so $\theta = \cos^{-1}\frac{5}{\sqrt{x^2+25}}.$