Shore School

2014

Year 12 Trial HSC Examination

Exam Number:

Set:

Mathematics Extension 1

General Instructions

- Reading time − 5 minutes
- Working time – 2 hours
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- In Questions 11–14, show relevant mathematical reasoning and/or calculations
- Write your examination number on the front cover of each booklet
- If you do not attempt a question, submit a blank booklet marked with your examination number and “N/A” on the front cover

Total marks – 70

Section I Pages 2–5

10 marks

- Attempt questions 1–10
- Allow about 15 minutes for this section

Section II Pages 6–11

60 marks

- Attempt Questions 11–14
- Allow about 1 hour 45 minutes for this section

Note: Any time you have remaining should be spent revising your answers.

DO NOT REMOVE THIS PAPER FROM THE EXAMINATION ROOM

Section I

10 marks
Attempt Questions 1−10
Allow about 15 minutes for this section

Use the Multiple Choice Answer Sheet for Questions 1−10.

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1 Which of the following polynomials has a factor of $(x+3)$?

(A) $P(x)=(x+3)^3-7$

(B) $P(x)=x^3-4x^2+9$

(C) $P(x)=x^3+5x^2+6x+3$

(D) $P(x)=x^3+5x^2+4x-6$

2 Which of the following has an inverse which is a function?

(A) $(y-3)^2=16(x-2)$

(B) $x^2+y^2=5$

(C) $y=x(x-2)(x-4)$

(D) $y=\log_e x^2$

3 What is $\lim\limits_{x\to 0}\dfrac{\sin\frac{2x}{3}}{4x}$?

(A) $\frac{1}{6}$

(B) $\frac{1}{2}$

(C) $\frac{8}{3}$

(D) 6

4 A particle is moving in simple harmonic motion according to the formula

$$x = -3\sin 4t + 5,$$

where x is its distance from O in metres and t is measured in seconds.

Which of the following is the maximum speed of the particle?

(A) -12 m/s

(B) -7 m/s

(C) 12 m/s

(D) 17 m/s

5 Which expression below shows the number of possible arrangements of the letters in the word PERSEVERANCE?

(A) $\dfrac{12!}{6!}$

(B) $\dfrac{12!}{4!2!}$

(C) $\dfrac{12!}{4\times 2}$

(D) $12!$

6 If n and k are integers with $n > k > 1$, which of the following statements about binomial coefficients is FALSE?

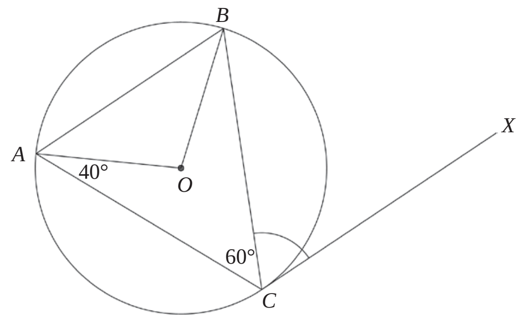
(A) ${}^nC_k = {}^nC_{n-k}$

(B) ${}^nC_n = 1$

(C) ${}^nC_k = {}^{n-1}C_{k-1} + {}^{n-1}C_k$

(D) ${}^nC_k = \dfrac{n!}{(n-k)!}$

7 In the diagram below A, B and C are points on the circumference of circle centre O, while tangent XC meets the circle at C. As shown, $\angle ACB = 60°$ and $\angle OAC = 40°$.



NOT TO SCALE

What is the size of $\angle BCX$?

(A) 30°

(B) 50°

(C) 70°

(D) 120°

8 A security lock has six buttons labelled A, B, C, D, E, and F.
The lock was initially on a setting where repetition was allowed for the 3 letter code and the order in which the buttons were pushed was important.
However, it was changed to a setting where the order in which buttons were pushed was not important and repetition was also not allowed.
Which of the following expressions gives the reduction in the number of possible 3 letter codes?

(A) $6^3 - {}^6P_3$

(B) ${}^6P_3 - {}^6C_3$

(C) $6^3 - {}^6C_3$

(D) $6^3 - 3!$

9 The line $y = 2x - 1$ intersects with the cubic $y = x^3$ at the point $(1, 1)$.
Which of the following is closest to the acute angle between them at this point?

(A) 8°

(B) 18°

(C) 72°

(D) 82°

10 Two balls of the same type are thrown horizontally off a 100 metre high cliff.
Ball A is thrown at 20 m/s, while Ball B is thrown at 10 m/s.

Ignoring wind-resistance, which of the following is FALSE?

(A) Ball A and Ball B have the same vertical velocity on impact.

(B) Ball A is moving at a velocity double that of Ball B when they strike the ground.

(C) Ball A and Ball B are in the air for the same amount of time.

(D) Ball A hits the ground twice as far from the base of the cliff as Ball B.

Section II

60 marks
Attempt Questions 11–14
Allow about 1 hour 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11–14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet

(a) Consider the polynomial $P(x) = 5x^3 - 9x^2 + 3x - 11$, with roots α, β and γ. **2**

Determine the value of $\alpha^2\beta\gamma + \alpha\beta^2\gamma + \alpha\beta\gamma^2$.

(b) The point P divides the interval EF externally in the ratio $5:2$, where $E = (7, 3)$ and $F = (1, 6)$. **2**

Determine the coordinates of the point P.

(c) Differentiate $\dfrac{x}{5}\sin^{-1} 5x$ with respect to x. **3**
Leave your answer in simplified form.

(d) Solve the inequality $\dfrac{|x+35|}{x} > 6$. **3**

(e) Find $\displaystyle\int \cos^2 3x \, dx$. **2**

(f) Using the substitution $u = e^x$, find $\displaystyle\int_0^{\ln\left(\frac{\sqrt{3}}{2}\right)} \frac{e^x}{\sqrt{1 - e^{2x}}} \, dx$. **3**

Question 12 (15 marks) Use a SEPARATE writing booklet.

(a) Use the binomial theorem to find the constant term in the expansion of $\left(2x-\frac{1}{4x^2}\right)^9$. **3**

(b) A steak is removed from a freezer and left on the kitchen bench to thaw. The temperature in the kitchen is a constant $20°C$. At time t minutes after being taken out of the freezer, the steak's temperature T increases according to the rule $\frac{dT}{dt} = -k(T-20)$.

(i) Verify that $T = 20 - Ae^{-kt}$ satisfies this equation, where A is a constant. **1**

(ii) The steak which was initially at $-10°C$ when taken out of the freezer, takes 30 minutes to reach $0°C$. **3**

How much longer will it take for the steak to reach $10°C$?
Give your answer correct to the nearest minute.

(c) Consider the function $f(x)=\frac{3x}{x+2}$. **2**

Find its inverse function $f^{-1}(x)$ and state the domain over which $f^{-1}(x)$ is defined.

(d) A particle's displacement x metres from O at time t seconds is given by $x=\sqrt{15}\sin 2t+\sqrt{5}\cos 2t$.

(i) Prove that the particle is in simple harmonic motion, by showing that $\ddot{x}=-n^2x$. **2**

(ii) Express the displacement in the form $x = R\sin(2t+\alpha)$, where $R>0$ and $0<\alpha<\frac{\pi}{2}$. **2**

(iii) Determine when the particle is first $\sqrt{15}$ metres to the right of O and its speed at this time. **2**

Question 13 (15 marks) Use a SEPARATE writing booklet.

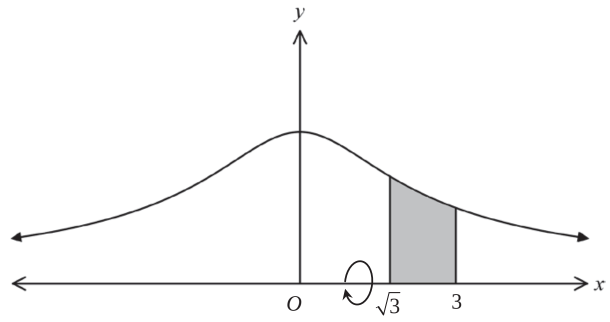
(a) Five fair dice are rolled. Determine the probability that exactly three of the dice land with six showing on the uppermost face. **2**

(b) Use mathematical induction to prove that for all integers $n \geq 4$, **3**

$$2^n < n!.$$

(c) Find the general solution(s) of $\sec^2\theta+\sqrt{3}\tan\theta=1$. **2**
Leave your answer in radians.

(d) The region bounded by the curve $y=\frac{1}{\sqrt{3+x^2}}$, the lines $x=\sqrt{3}$, $x=3$ and the x-axis is rotated about the x-axis to form a solid. **3**



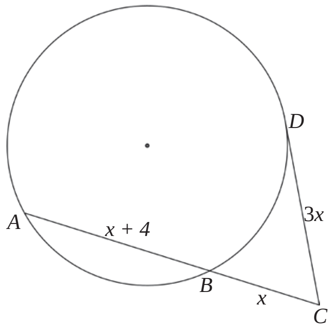
Find the exact volume of the solid.

(e) The equation $e^x-2=\cos\frac{x}{2}$ has only one solution which is near $x=1$. **3**

Use ONE application of Newton's method to find a better approximation of this solution. Give your answer correct to two decimal places.

Question 13 continues on the following page

(f) In the diagram, the chord AB is produced to C, while CD is tangent to the circle at D. **2**



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Find the value of x, stating clearly any geometric theorem(s) used.

End of Question 13

Question 14 (15 marks) Use a SEPARATE writing booklet.

(a) A projectile is fired from ground level with initial velocity v ms^{-1} at an angle of $\alpha°$ to the horizontal.

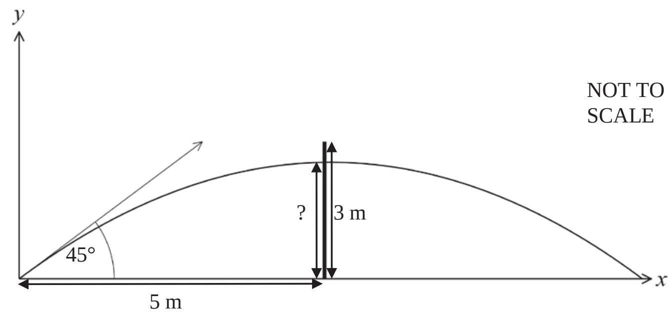
(i) Starting from $\ddot{x} = 0$ and $\ddot{y} = -g$, show that: **2**

$$x = vt\cos\alpha \quad \text{and} \quad y = -\frac{1}{2}gt^2 + vt\sin\alpha .$$

(ii) Prove that the flight path of the projectile is given by: **2**

$$y = -\frac{gx^2}{2v^2}(1+\tan^2\alpha) + x\tan\alpha .$$

(iii) Tom fires a projectile at 10 ms^{-1} from ground level at 45° to the horizontal. It strikes a 3 metre high wall which is 5 metres away. Determine how high up the wall the projectile hits. Use $g = 9.8$ ms^{-2}. **2**



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(iv) Tom fired a second projectile at the same angle but this time it cleared the wall. Determine the minimum velocity at which the second projectile could have been fired. Leave your answer correct to 2 decimal places. **2**

(b) The helium gas from a tank is flowing into a large spherical balloon at the rate of 15 litres per second. Find the instantaneous rate of increase of the radius when it is 10 centimetres. Give the exact answer in simplified form. **2**

Question 14 continues on the following page

(c) King Arthur, Sir Lancelot and the 10 other knights of the round table are about to sit down.

(i) Find the number of possible seating arrangements. **1**

(ii) If they randomly choose their seats, what is the probability that King Arthur and Sir Lancelot sit next to each other? **1**

(d) Use the binomial theorem $(1+x)^n = \sum_{k=0}^{n} \binom{n}{k} x^k$ to prove that **3**

$$\sum_{k=0}^{n} \frac{2^{k+1}}{k+1}\binom{n}{k} = \frac{1}{n+1}\left(3^{n+1}-1\right).$$

☺ **END OF PAPER** ☻

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1;\ x \neq 0,\ \text{if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x\ , \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0,\ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

Note $\ln x = \log_e x, \quad x > 0$

Extension 1 Trial 2014

(1) D $P(-3) = (-3)^3 + 5\times(-3)^2 + 4\times(-3) - 6$
$= 0$

(2) A  Only 1 point hit in 'horizontal line test'

(3) A $\dfrac{\frac{2}{3}}{4} \times \lim_{x\to 0} \dfrac{\sin\frac{2x}{3}}{\frac{2x}{3}} = \frac{1}{6} \times 1$
$= \frac{1}{6}$

(4) C $\dot{x} = -12\cos 4t$
$\therefore -12 \le \dot{x} \le 12$
$\therefore$ max speed $= 12$ m/s

(5) B 12 letters, 4 E's, 2 R's
$\dfrac{12!}{4!\,2!}$
Repetition of E $\rightarrow$ 4! 2! $\leftarrow$ Repetition of R

(6) D $^nC_k = \dfrac{n!}{k!(n-k)!}$ $^nP_k = \dfrac{n!}{(n-k)!}$

(7) C $\angle AOB = 120°$
$\angle OAB = \angle OBA = 30°$
$\angle XCB = 40 + 30$
$= 70°$

(8) C $6^3 - {}^6C_3$

(9) A $y = 2x - 1 \rightarrow m_2 = 2$
$y = x^3 \rightarrow \frac{dy}{dx} = 3x^2$
when $x = 1$ $\frac{dy}{dx} = 3 \therefore m_1 = 3$
$\tan\theta = \left|\dfrac{3-2}{1+3\times 2}\right|$
$= \frac{1}{7}$
$\theta = \tan^{-1}(\frac{1}{7})$
$\doteqdot 8°$

(10) B Same height, Same time $\rightarrow$ Same vertical velocity
Velocity double $\rightarrow$ distance double
BUT velocity has both vertical & horizontal component, different on impact

11 a) $\alpha\beta\gamma(\alpha+\beta+\gamma) = \frac{-d}{a} \times \frac{-b}{a}$

$= \frac{11}{5} \times \frac{9}{5}$

$= \frac{99}{25}$

b) $(7,3)$ $(1,6)$

$5 : -2$

$P = \left(\frac{5(1)-2(7)}{5-2}, \frac{5(6)-2(3)}{5-2} \right)$

$= (-3, 8)$

d) Critical Points

Equality — Zero Denom

$\frac{|x+35|}{x} = 6$ — $x = 0$

$-$: $\frac{x+35}{x} = -6$ — $+$: $\frac{x+35}{x} = 6$

$x + 35 = -6x$ — $x+35 = 6x$

$7x = -35$ — $5x = 35$

$x = -5$ — $x = 7$

$0 < x < 7$

11 c) $u = \frac{x}{5}$ $\quad v = \sin^{-1}5x$

$u' = \frac{1}{5}$ $\quad v' = \frac{5}{\sqrt{1-25x^2}}$

$\frac{d}{dx}\left(\frac{x}{5}\sin^{-1}5x\right) = \frac{1}{5}\sin^{-1}5x + \frac{x}{\cancel{5}} \times \frac{\cancel{5}}{\sqrt{1-25x^2}}$

$= \frac{1}{5}\sin^{-1}5x + \frac{x}{\sqrt{1-25x^2}}$

e) $\cos 2\theta = 2\cos^2\theta - 1$

$\cos^2\theta = \frac{1}{2}\cos 2\theta + \frac{1}{2}$

$\int \cos^2 3x \, dx = \int \left(\frac{1}{2}\cos 6x + \frac{1}{2}\right) dx$

$= \frac{1}{12}\sin 6x + \frac{x}{2} + C$

f) $x = \ln\frac{\sqrt{3}}{2} \rightarrow u = \frac{\sqrt{3}}{2}$

$x = 0 \rightarrow u = 1$

$u = e^x$

$du = e^x dx$

$\int_0^{\ln\frac{\sqrt{3}}{2}} \frac{e^x}{\sqrt{1-e^{2x}}} dx = \int_1^{\frac{\sqrt{3}}{2}} \frac{1}{\sqrt{1-u^2}} du$

$= \left[\sin^{-1}u\right]_1^{\frac{\sqrt{3}}{2}}$

$= \frac{\pi}{3} - \frac{\pi}{2}$

$= -\frac{\pi}{6}$

⑫ a) For constant $x^{9-k}\times(x^{-2})^k = x^0$

$9-k-2k=0$

$3k=9$

$k=3$

$\therefore\ {}^9C_3(2x)^6\left(-\frac{1}{4}x^{-2}\right)^3 = 84\times 64\times\frac{-1}{64}$

$=-84$

b) i) $T=20-Ae^{-kt} \rightarrow T-20=-Ae^{-kt}$ ①

$\frac{dT}{dt}=kAe^{-kt}$ ②

$\frac{dT}{dt}=-k(-Ae^{-kt})$

$=-k(T-20)$

ii) When $t=0$, $T=-10$

$-10=20-Ae^0$

$\therefore A=30$

$T=20-30e^{-kt}$

When $t=30$, $T=0$

$0=20-30e^{-30k}$

$30e^{-30k}=20$

$e^{-30k}=\frac{2}{3}$

$e^{30k}=\frac{3}{2}$

$k=\frac{1}{30}\ln\left(\frac{3}{2}\right)$

$T=20-30e^{-\frac{1}{30}\ln\left(\frac{3}{2}\right)t}$

$10=20-30e^{-\frac{1}{30}\ln\left(\frac{3}{2}\right)t}$

⑫ b) ii) (ctd)

$-10=-30e^{-\frac{1}{30}\ln\left(\frac{3}{2}\right)t}$

$\frac{1}{3}=e^{-\frac{1}{30}\ln\left(\frac{3}{2}\right)t}$

$e^{\frac{1}{30}\ln\left(\frac{3}{2}\right)t}=3$

$\frac{1}{30}\ln\left(\frac{3}{2}\right)t=\ln 3$

$t=\frac{30\ln 3}{\ln\left(\frac{3}{2}\right)}$

$=81.2853\ldots$

$\doteqdot$ 81 min 17 sec

$\therefore$ Approx 51 min more

c) $y=\frac{3x}{x+2}$ Inv: $x=\frac{3y}{y+2}$

$x(y+2)=3y$

$xy+2x=3y$

$xy-3y=-2x$

$y(x-3)=-2x$

$y=\frac{-2x}{x-3}$

D: $x\neq 3,\ x\in\mathbb{R}$

d) i) $x=\sqrt{15}\sin 2t+\sqrt{5}\cos 2t$

$\dot{x}=2\sqrt{15}\cos 2t-2\sqrt{5}\sin 2t$

$\ddot{x}=-4\sqrt{15}\sin 2t - 4\sqrt{5}\cos 2t$

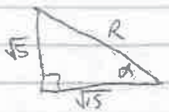
$=-4(\sqrt{15}\sin 2t+\sqrt{5}\cos 2t)$

$=-2^2x$

(12) d) ii) $R\sin(2t+\alpha) = R\sin 2t\cos\alpha + R\cos 2t\sin\alpha$

$\therefore R\cos\alpha = \sqrt{15}$ $\quad R\sin\alpha = \sqrt{5}$

$\cos\alpha = \frac{\sqrt{15}}{R}$ $\quad \sin\alpha = \frac{\sqrt{5}}{R}$



$R = \sqrt{(\sqrt{5})^2 + (\sqrt{15})^2}$

$= 2\sqrt{5}$

$\tan\alpha = \frac{\sqrt{5}}{\sqrt{15}}$

$= \frac{1}{\sqrt{3}}$

$\alpha = \frac{\pi}{6}$

$\therefore x = 2\sqrt{5}\sin\left(2t + \frac{\pi}{6}\right)$

iii) $\sqrt{5} = 2\sqrt{5}\sin\left(2t + \frac{\pi}{6}\right)$

$\frac{\sqrt{3}}{2} = \sin\left(2t + \frac{\pi}{6}\right)$

$2t + \frac{\pi}{6} = \frac{\pi}{3}$

$2t = \frac{\pi}{6}$

$t = \frac{\pi}{12}$ sec

$\dot{x} = 4\sqrt{5}\cos\left(2t + \frac{\pi}{6}\right)$

$= 4\sqrt{5}\cos\left(\frac{\pi}{6} + \frac{\pi}{6}\right)$

$= 2\sqrt{5}$ m/s

(13) d) $V = \pi\int_{\sqrt{3}}^{3}\left(\frac{1}{\sqrt{3+x^2}}\right)^2 dx$

$= \pi\int_{\sqrt{3}}^{3}\frac{1}{3+x^2}\,dx$

$= \pi\left[\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)\right]_{\sqrt{3}}^{3}$

$= \pi\left(\frac{1}{\sqrt{3}}\tan^{-1}\sqrt{3} - \frac{1}{\sqrt{3}}\tan^{-1}1\right)$

$= \frac{\pi}{\sqrt{3}}\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$

$= \frac{\pi^2}{12\sqrt{3}}$ units^3

OR $\frac{\pi^2\sqrt{3}}{36}$ units^3

e) $e^x - 2 = \cos\frac{x}{2}$

$f(x) = e^x - 2 - \cos\frac{x}{2}$

$f'(x) = e^x + \frac{1}{2}\sin\frac{x}{2}$

$x_2 = 1 - \frac{e^1 - 2 - \cos\frac{1}{2}}{e^1 + \frac{1}{2}\sin\frac{1}{2}}$

$\doteqdot 1.05$

f) $(3x)^2 = x(2x+4)$ (Square of tangent equals product of intercepts of chord)

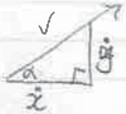
$9x^2 = 2x^2 + 4x$

$7x^2 - 4x = 0$

$x(7x-4) = 0$

$\therefore x = \frac{4}{7}$

(14) a) i) Initially



$\dot{x} = v\cos\alpha$

$\dot{y} = v\sin\alpha$

$\ddot{x} = 0$

$\dot{x} = C_1$

$= v\cos\alpha$

$x = vt\cos\alpha + C_3$

when $t = 0, x = 0$

$\therefore C_3 = 0$

$x = vt\cos\alpha$

$\ddot{y} = -g$

$\dot{y} = -gt + C_2$

$C_2 = v\sin\alpha$

$\therefore \dot{y} = -gt + v\sin\alpha$

$y = -\frac{1}{2}gt^2 + vt\sin\alpha + C_4$

when $t = 0, y = 0 \therefore C_4 = 0$

$y = -\frac{1}{2}gt^2 + vt\sin\alpha$

ii) $x = vt\cos\alpha$ $\quad y = -\frac{1}{2}gt^2 + vt\sin\alpha$ ②

$t = \frac{x}{v\cos\alpha}$ ①

sub ① into ②

$$y = -\frac{1}{2}g\left(\frac{x}{v\cos\alpha}\right)^2 + v\left(\frac{x}{v\cos\alpha}\right)\sin\alpha$$

$$= \frac{-gx^2}{2v^2\cos^2\alpha} + x\tan\alpha$$

$$= \frac{-gx^2}{2v^2}\sec^2\alpha + x\tan\alpha$$

$$= \frac{-gx^2}{2v^2}(1+\tan^2\alpha) + x\tan\alpha$$

iii) $x = 5$, $\alpha = 45$, $g = 9.8$

$$y = \frac{-9.8\times5^2}{2\times10^2}(1+\tan^2 45°) + 5\times\tan 45°$$

$= 2.55$

$\therefore$ 2.55 m up the wall

(13) (a) $P(6) = \frac{1}{6}$

$P(\text{Not } 6) = \frac{5}{6}$

$$P = {}^5C_3\left(\frac{5}{6}\right)^2\left(\frac{1}{6}\right)^3$$

$$= \frac{125}{3888}$$

(b) Prove for $n = 4$

LHS $= 2^4$ $\quad$ RHS $= 4!$

$= 16$ $\quad$ $= 24$

$\therefore$ RHS > LHS for $n = 4$

Assume for $n = k$

ie assume $2^k < k!$

Prove for $n = k+1$

$\therefore$ RTP $2^{k+1} < (k+1)!$

RHS $= (k+1)!$ $\quad$ LHS $= 2^{k+1}$

$= (k+1)k!$ $\quad$ $= 2\times2^k$

But $(k+1)k! > (k+1)2^k$ by assumption

and $(k+1)2^k > 2\times2^k$ as $k \geq 4$

$\therefore$ RHS > LHS

$\therefore$ true by induction for $n \geq 4$

(c) $1 + \tan^2\theta + \sqrt{3}\tan\theta = 1$

$\tan^2\theta + \sqrt{3}\tan\theta = 0$

$\tan\theta(\tan\theta + \sqrt{3}) = 0$

$\tan\theta = 0$, $\quad \tan\theta = -\sqrt{3}$

$\theta = \pi n$, $\quad -\frac{\pi}{3} + \pi n$ where n is an integer

14 a) iv) $3 = \frac{-9.8 \times 5^2}{2 \times v^2}(1+\tan^2 45°) + 5$

$3 = \frac{-245}{v^2} + 5$

$-2 = \frac{-245}{v^2}$

$\sqrt{v^2} = \sqrt{\frac{245}{2}}$

$v \doteqdot 11.07 \text{ m/s}$

b) i) No. arrangements $= (12-1)!$

$= 11!$

ii) $P = \frac{2! \times 10!}{11!}$

$= \frac{2}{11}$

c) $V = \frac{4}{3}\pi r^3$

$\frac{dV}{dr} = 4\pi r^2$ $\frac{dV}{dt} = 15000 \text{ cm}^3/\text{s}$

When $r = 10$

$\frac{dV}{dr} = 400\pi$

$\frac{dr}{dt} = \frac{dr}{dV} \times \frac{dV}{dt}$

$= \frac{1}{400\pi} \times 15000$

$= \frac{75}{2\pi}$ cm/s

14 d) $(1+x)^n = \sum_{k=0}^{n} \binom{n}{k} x^k$

$(1+x)^n = \binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \binom{n}{3}x^3 + \dots + \binom{n}{n}x^n$

$\therefore \int (1+x)^n dx = \int \left(\binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \binom{n}{3}x^3 + \dots + \binom{n}{n}x^n\right) dx$

$\frac{1}{n+1}(1+x)^{n+1} + C = \binom{n}{0}x + \frac{1}{2}\binom{n}{1}x^2 + \frac{1}{3}\binom{n}{2}x^3 + \dots + \frac{1}{n+1}\binom{n}{n}x^{n+1}$

When $x = 0$

$\frac{1}{n+1} + C = 0$

$\therefore C = -\frac{1}{n+1}$

$\therefore \frac{1}{n+1}(1+x)^{n+1} - \frac{1}{n+1} = \binom{n}{0}x + \frac{1}{2}\binom{n}{1}x^2 + \frac{1}{3}\binom{n}{2}x^3 + \dots + \frac{1}{n+1}\binom{n}{n}x^{n+1}$

Let $x = 2$

$\frac{1}{n+1}(3^{n+1} - 1) = \binom{n}{0}2 + \frac{1}{2}\binom{n}{1}2^2 + \frac{1}{3}\binom{n}{2}2^3 + \dots + \frac{1}{n+1}\binom{n}{n}2^{n+1}$

$= \sum_{k=0}^{n} \frac{2^{k+1}}{k+1}\binom{n}{k}$