ST CATHERINE'S SCHOOL

YEAR 12 - 3 UNIT MATHEMATICS

TIME ALLOWED: 2 HOURS *(plus 5 mins reading time).*

DATE: AUGUST 1996

Student Number: _____

INSTRUCTIONS:

- All questions are to be attempted.
- All questions are of equal value.
- All necessary working should be shown in every question.
- Full marks may not be awarded for careless or badly arranged work.
- Approved calculators and geometrical instruments are required.
- Standard Integrals are printed on the last page.
- Each question should be started in a *separate* Writing Booklet, clearly marked with the question number and your student number on the cover.
- You may ask for extra Writing Booklets if you need them.
- Tie your Booklets in ~~2~~ 3 bundles:
  - Section A: Questions 1, 2 and 3.
  - Section B: Questions 4 and 5.
  - Section C: Questions 6 and 7.
- Hand in Section A, Section B and Section C and this examination paper separately

TEACHERS USE ONLY TOTAL MARKS
A
B
TOTAL

SECTION A

Question 1. *Use separate Writing Booklet.* **Marks**

a) Solve for x: $(x^2-1)(x+5)>0$ 1

b) Differentiate $y=\log\sqrt{x+1}$ 2

c) Find $\int \frac{t}{\sqrt{1+t}}\,dt$. use $u=1+t$ 3

d) Find the area enclosed by the x axis, $y = \sin x$ and $y = \cos x$ in the first quadrant. 3

e) Find the exact value of $\int_0^{\sqrt{3}} \frac{1}{9+x^2}\,dx$ 3

Question 2. *Use separate Writing Booklet.*

a) $\int x e^{x^2}\,dx$ 2

b) Use the principle of mathematical induction to show that $2^{3n}-1$ is divisible by 7. 4

c) i) Show that the equation of the normal to the parabola $x^2 = 4ay$ at $(2ap, ap^2)$ is $x + py = ap^3 + 2ap$ 6

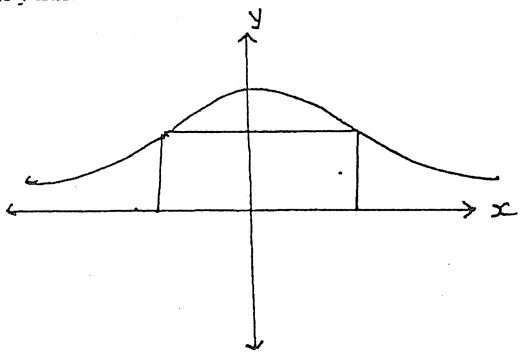
ii) Derive the equation of the line which passes through the focus $S(0, a)$ and is perpendicular to the normal. If this line meets the normal at N, then

iii) Find the coordinates of N

iv) Find the locus of N

Marks

c) A rectangle is inscribed under the curve $y = \frac{1}{x^2+1}$ as shown, such that the rectangle is symmetrical about the y axis. 5



i) Find the expression for the area of the rectangle in terms of x.

ii) Find the maximum area.

SECTION C

Question 6. *Use separate Writing Booklet.*

a) For the curve $y = 1 + 2\cos x - 2\cos^2 x$. 5

i) Show that $\frac{dy}{dx} = 2\sin x(2\cos x - 1)$.

ii) Hence, find the stationary point(s) in the interval $o \le x \le \frac{\pi}{2}$

iii) Sketch the curve and find the greatest and least value of y in $0 \le x \le \frac{\pi}{2}$.

b) Show that $\frac{1-\cos\theta}{\sin\theta} + \frac{\sin\theta}{1+\cos\theta} = 2\tan\frac{\theta}{2}$. 3

Marks

c) Evaluate $\cos\left(\sin^{-1}\left(-\frac{1}{2}\right)\right)$. 1

d) Find the domain and range of $y = 3\sin^{-1}\sqrt{1-x^2}$. 3

Question 7. *Use separate Writing Booklet.*

a) Solve $4x^3 - 12x^2 + 11x - 3 = 0$ if the roots are in Arithmetic progression. 3

b) i) A particle is projected from a point O with a velocity V at an angle θ to the horizontal. Taking the coordinate axes at the point of projection. find parametric expression for velocity and the position of the particle at any time t. 9

ii) After 1 second the position of the particle is $\left(6\sqrt{3}, 1\right)$.

Show that the initial velocity and the angle of projection are respectively 12cm/sec and 30°. (Take g = 10cm/s^2).

iii) Find the range of the motion.

iv) Find the maximum height reached.

END OF EXAMINATION