

ST CATHERINE'S SCHOOL

YEAR 12 HSC TRIAL EXAMINATIONS
3/4 UNIT MATHEMATICS

TIME ALLOWED: 2 HOURS

DATE: AUGUST 1997

Examination Number: _____

INSTRUCTIONS:

- All questions are to be attempted.
- All questions are of equal value.
- All necessary *working* should be shown in every question.
- Full marks may not be awarded for careless or badly arranged work.
- Each question attempted should be started on a **NEW PAGE**.
- Approved calculators and geometrical instruments are required.
- Standard Integrals are printed on the last page.
- Hand in your work in 3 bundles:
 - Section A: Questions 1 and 2
 - Section B: Questions 3, 4 and 5
 - Section C: Questions 6 and 7.

TOTAL MARKS	
A	
B	
C	
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SECTION A

Question 1. Use a separate Writing Booklet. Marks

- a) Differentiate $x \sin x$ 2
- b) Find the exact value of $\cos 23^\circ \cos 22^\circ - \sin 23^\circ \sin 22^\circ$ 2
- c) Find the co-ordinates of the point P which divides the interval AB with endpoints A(3,5) and B(4,-6) internally in the ratio 4 : 9. 2
- d) Solve for x : $\frac{x^2 - 5x}{x - 4} \leq 3$ 3
- e) If $\sin \alpha = \frac{4}{5}$ and $\cos \beta = \frac{5}{13}$ find $\sin(\alpha + \beta)$ in exact form 3

Question 2. Use a separate Writing Booklet

- a) i) Show that $x = -1$ is a root of the polynomial $P(x) = x^3 - 7x - 6$. 4
- ii) Find the values of the other roots.
- iii) Sketch the curve $y = P(x)$ (Do not find the turning points)
- b) i) Evaluate $\int_0^3 x\sqrt{1+x^2} dx$ using the substitution $u = 1 + x^2$ 6
- ii) Find $\int (\cos x + \sin x)^2 dx$
- c) Find $\lim_{x \rightarrow 0} \frac{3x}{\sin 5x}$ 2

SECTION B

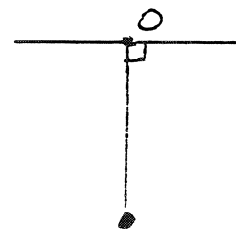
Question 3. Use a separate Writing Booklet.

Marks

- a) The points $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ lie on the parabola $x^2 = 4ay$. 5
- i) Find the equation of the chord PQ
- ii) If PQ is a focal chord, prove that the normals to the parabola at P and Q are perpendicular to each other.
- b) Prove that $\frac{1 + \sin \theta - \cos \theta}{1 + \sin \theta + \cos \theta} = t$ where $t = \tan \frac{\theta}{2}$ 3
- c) Find the acute angle between $y = 2x - 1$ and $x + 3y - 1 = 0$ 2
- d) How many odd numbers between 5000 and 6000 can be made from the digits 4, 5, 6, 7 if each digit is to be used once only in any order? 2

Question 4 Use a separate Writing Booklet.

- a) The population of a town has its rate of growth proportional to the population. If the annual growth rate is 0.02 and the original population is 2400 5
- i) what would the population be in 4 years?
- ii) how many years would it take for the population to reach 3400?
- b) The approximate root of $\cos x = \frac{x}{3}$ was found to be $x = 1.1$ 2
Use one application of Newton's method to find a closer approximation.
- c) I have forgotten the combination of my lock for my locker. It is a combination lock with four rollers each having the numerals 0, 1, 2, 3, . . . , 9. What is the chance of guessing the correct combination in 3 tries? 2
- d) I attached a small stone to the end of a piece of elastic and tied the other end onto a handrail. If I throw the stone vertically down from the handrail (the point O) with a velocity of $\sqrt{14} \text{ cm/sec}$ and if it has an acceleration of $(6 - 2x) \text{ cm/sec}^2$ after it has travelled $x \text{ cm}$, find its greatest velocity. 3



Question 5 Use a separate Writing Booklet

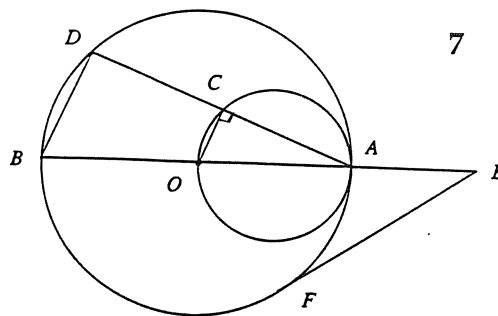
Marks

- a) The polynomial equation $P(x) = 0$ has a triple root at $x = a$.
Form an expression for this polynomial and show that $P(a) = P'(a) = 0$. 3
- b) If $x = 2$ is a double root of $ax^4 - 2x^3 - 8x + 16 = 0$
find the value of a and the sum of the other two roots. 3
- c) Using Mathematical Induction, prove $4^n \geq 3n + 7$ for all integers $n > 1$. 3
- d) A thin circular plate is put into a refrigerator so that it contracts uniformly. When the radius of the plate is 30 cm, it is decreasing at a rate of 4mm/sec. Find the rate of decrease of the area at this point. 3

SECTION C

Question 6 Use a separate Writing Booklet

- a) Given $AE = OA$
O is the centre of the larger circle and EF is the tangent to the circle
- i) Copy this diagram into your writing booklet
 - ii) Prove $OC \parallel BD$
 - iii) Show $EF = \sqrt{3} AE$
 - iv) If $OB = 8\text{cm}$ and $BD = 7\text{cm}$, find the length of OC

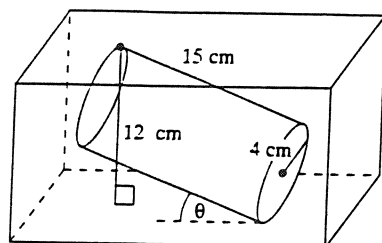


- b) If there is a 70% chance that Susan can hit a target and an 80% chance that Mei can hit the target, what is the chance that
- i) both will hit it
 - ii) at most one will hit the target?
- c) Ulug Beg (1393 - 1449) used the relation $\sin^3 \theta = \frac{1}{4}(3\sin \theta - \sin 3\theta)$ to draw up a table of values. Prove this relation. 3

Question 7 Use a separate Writing Booklet

Marks

- a) A cylinder with radius 4 cm and perpendicular height 15 cm is tilted so that it will just fit inside a 12 cm high box. At what angle to the horizontal must it be tilted? 3



- b) Assume that over several days of constant weather the cycle of temperatures each day is simple harmonic between 13° at 4 am and 23° at 4 pm. At what time of the day would the temperature be 15° ? 4
- c) The acceleration after t sec of a particle is given in metres per second per second by $a = 3t^2 + 1$. If you were to draw an acceleration-time graph, what would the area under the curve from $t = 0$ to $t = 1$ measure? (You do not have to sketch the graph) 1
- d) Two particles P and Q move along a given line, their displacement at $t > 0$ from the origin being $x(t)$ and $X(t)$ respectively. 4
- i) Given that $\frac{d^2x}{dt^2} = 6 + e^{-t}$, and $\frac{dx}{dt} = -1$ and $x = 0$ when $t = 0$, find an expression for $x(t)$.
 - ii) If $X(t) = 2\sin 5t + 3t^2 + 2$ prove that $X(t) > x(t)$ for all $t > 0$. Explain this result in terms of the motion of the particles P and Q.