St Catherine's School
Waverley, Sydney

Student Number: ....

Year 12
Trial
2007

Extension I Mathematics

Time allowed: 2 hrs

Reading time: 5 min

General Instructions

- Attempt ALL questions
- Write your Student NUMBER at the top of this page and on each writing booklet. Calculators may be used

Sections	Marks
Q1-3 in Booklet 1 Q4-7 in Booklet 2	12 marks per Question

Total marks
84

Question 1

a) Find $\lim_{x \to 0} \frac{\sin 5x}{3x}$ (2m)

b) (i) Show that $(\frac{\pi}{4}, \frac{1}{\sqrt{2}})$ is a point of intersection of the curves $y = \sin x$ and $y = \cos x$ for $0 \le x \le \frac{\pi}{2}$ (1m)

(ii) Hence find the acute angle between the curves $y = \sin x$ and $y = \cos x$ for $0 \le x \le \frac{\pi}{2}$ (3m)

c) The point P *(1,2)* divides the join of A *(3,6)* and B *(x,y)* internally in the ratio 3 : 2. Find *x* and *y*. (2m)

d) P$(4p, 2p^2)$ and Q $(4q, 2q^2)$ are two points on the parabola $x^2 = 8y$.

PQ subtends a right angle at O, the origin and the vertex of the parabola.

(i) Show that $pq = -4$ (1m)

(ii) It is given that the equation of the tangent at the point $(4p, 2p^2)$ is

$$y = px - 2p^2.$$

The tangents at P and Q meet at a point R.
Find the coordinates of R. (2m)

(iii) Hence find the locus of R. (1m)

Question 2 (Start a new page)

a) (i) Sketch without calculus the polynomial $y = x(x^2 - 4)$ (1m)
(No need to find stationary points)

(ii) Hence or otherwise solve the inequality $\dfrac{x^2 - 4}{x} > 0$ (2m)

b) Show using Mathematical Induction that:

$2 \times 1! + 5 \times 2! + 10 \times 3! + \ldots\ldots\ldots\ldots + (n^2 + 1)n! = n(n+1)!$, $n \geq 1$ (3m)

c) The polynomial : $P(x): x^3 + ax^2 - 6x + 7$, when divided by $(x-1)$, leaves a remainder of 2. Find *a*. (2m)

d) If $g(\theta) = \sin\theta - \theta\cos\theta$. By considering $g'(\theta)$, explain why $g(\theta) > 0$ for $0 < \theta < \pi$ (4m)

Question 3 (Start a new page)

a) State the Domain and Range of $y = \cos^{-1}\sqrt{x}$ (2m)

b) (i) State the Domain and Range of $y = \log_e(\sin^{-1} x)$ (2m)

(ii) Find $\frac{dy}{dx}$ (1m)

(iii) Explain why $y = \log_e(\sin^{-1} x)$ has an inverse function (1m)

(iv) Find the equation of this inverse function (2m)

c) (i) If $y = \tan^{-1} x + \tan^{-1}\frac{1}{x}$, show that $\frac{dy}{dx} = 0$ (2m)

(ii) Hence sketch $y = \tan^{-1} x + \tan^{-1}\frac{1}{x}$ in its natural domain (2m)

Question 4 (Start a new booklet)

a) Solve for *x*: $0.5^x < 2$ (2m)

b) A die is biased so that in any single throw the probability of an even score is *p*, where $p \neq 0.5$. The die is thrown six times.
Find in terms of *p* the probability of scoring 4 even scores in the six throws. (2m)

c) Consider the letters: D E L E T E D

(i) How many ways can the letters be rearranged? (1m)

(ii) In how many arrangements are the three **E's** together? (2m)

(iii) How many arrangements start with the letter **L?** (1m)

d) (i) 5 girls and 5 boys line up. In how many ways can they line up if two particular boys need to be among the first five and two particular girls need to be among the last 5? (2m)

(ii) What is the probability that the 5 boys are seated together if 5 boys and 5 girls sit around a round table? (2m)

Question 5 (Start a new page)

a) Solve for *n*: ${}^{n}C_2 + {}^{n-1}C_1 = 0$ (2m)

b) Using the fact that $(1+x)^4(1+x)^8 = (1+x)^{12}$, show that:

$${}^{4}C_0\,{}^{8}C_5 + {}^{4}C_1\,{}^{8}C_4 + {}^{4}C_2\,{}^{8}C_3 + {}^{4}C_3\,{}^{8}C_2 + {}^{4}C_4\,{}^{8}C_1 = {}^{12}C_5$$ (2m)

c) Use the binomial theorem and state which term is independent of *x* in the expansion of $(2x^2 - \frac{3}{x})^{12}$ (3m)

d) Expand $(2+5x)^{10}$ in ascending powers of *x* and consider it in the form:

$$(2+5x)^{10} = \sum_{k=0}^{k=10} t_k x^k$$

(i) Show that: $\dfrac{t_{k+1}}{t_k} = \dfrac{5(10-k)}{2(k+1)}$ (3m)

(ii) Hence find the term with the greatest coefficient in the expansion of : $(2+5x)^{10}$ (2m)

Question 6 (Start a new page)

a) Find $\int \frac{dx}{\sqrt{4-9x^2}}$ (2m)

b) A particle moves in Simple Harmonic Motion about a fixed point *O* and its equation of motion is given by $x = \cos 2t - \sqrt{3}\sin 2t$ where $x=0$

(i) Show that: $x = 2\cos(2t + \frac{\pi}{3})$ (2m)

(ii) Determine the initial position, initial velocity and initial acceleration of the particle and state with reasons if initially the particle is moving towards *O* or away from *O* and whether it is speeding or slowing down. (4m)

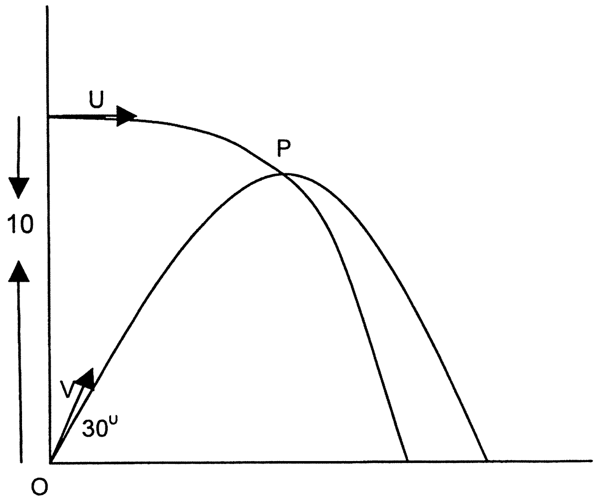
(iii) Find the time at which the particle first reaches an end point of motion. (2m)

c) A particle is moving along the x axis. Its velocity v at position x is given by: $v = \sqrt{x - 2x^2}$. Find the acceleration of the particle in terms of x . (2m)

Question 7 (Start a new page)

a) Use $x = 5\sin\theta$ to find the integral $\int \sqrt{25 - x^2}\,dx$ (4m)

b) A **ball** is projected from a point *O* with velocity *V* metres per second at an angle of 30^0 to the horizontal. At the same time a **stone** is dropped from a height of 10 metres above the point *O* with a velocity of *U* metres per second. The stone meets the ball when the **ball** reaches its highest point *P*.



(i) Write down the equations of motion of the two particles (3m)
(Take the acceleration due to gravity as 10 metres per second per second).

(ii) Show that $U = 10\sqrt{3}\ m/\sec$ and $V = 20m/\sec$ (3m)

(iii) Derive the Cartesian equation of the motion of the ~~**ball**~~. Stone (2m)

End of Paper

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1} + C, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x + C, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax} + C, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax + C, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax + C, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax + C, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax + C, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + C, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a} + C, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right) + C, \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right) + C$$

NOTE : $\ln x = \log_e x, \quad x > 0$