Student Number

St. Catherine's School Waverley
August 2012

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Extension I Mathematics

General Instructions

- Reading Time – 5 minutes
- Working Time – 2 hours
- Write using black or blue pen Black pen is preferred
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- Show all necessary working in Questions 11-14
- Task weighting – 40%

Total Marks – 70

Section I Pages 3-6

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section
- Answer on the multiple choice answer sheet provided.

Section II Pages 7-13

60 marks

- Attempt Questions 11-14
- Allow about 1 hours and 45 minutes for this section
- Answer each question in the booklet provided.

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

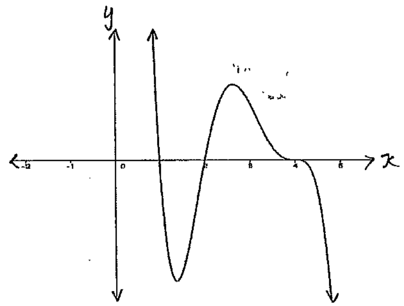
Section I

Total marks - 10
Attempt Questions 1-10
All questions are of equal value.

Answer either *A,B,C* or *D* on the multiple choice answer sheet provided.

---

1)



A possible equation for the graph above is:

a) $y = (x-1)(x-2)(x-4)^2$

b) $y = (x-1)(2-x)(x-4)^3$

c) $y = (x-1)(x-2)(x-4)$

d) $y = (x-1)^2(x-2)^2(4-x)$

2) The Cartesian equation of the curve with parameters p, q where

$$x = p + q$$

$y = p^2 + q^2 + 4pq$ and $pq = -1$ is given by:

a) $y = x^2 - 4$

b) $y = x^2 + 2$

c) $y = (x-1)^2$

d) $y = x^2 - 2$

3) A particle moves in a straight line and its position at time t (in seconds) is given by

$$x = 3\sin\left(4t + \frac{\pi}{4}\right) + 1$$ where x is measured in metres.

Its maximum speed will be:

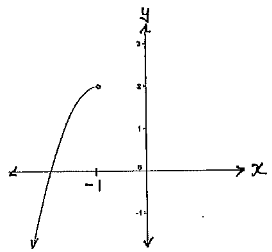
a) 4 m/s

b) 12 m/s

c) 13 m/s

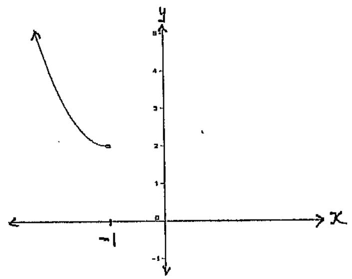
d) unable to be determined.

4)

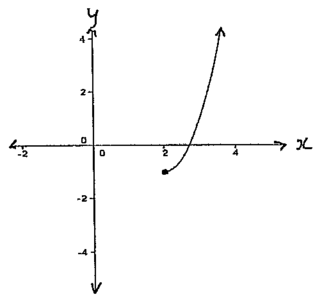


A possible inverse function for the graph shown above is:

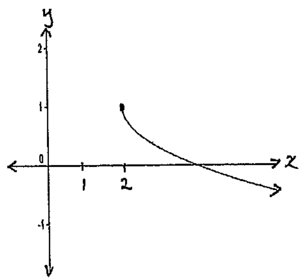
a)



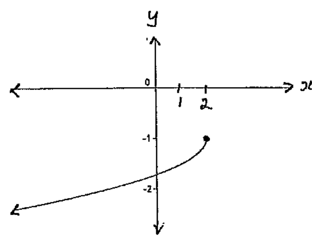
b)



c)



d)



5) How many solutions does $sin2\theta = cos\theta$ have in the domain $0 \leq \theta \leq 2\pi$?

a) 2 b) 3

c) 4 d) 5

6) The sides of an ice-cube are melting at the rate of $0.5cm/\min$ (Assume that it remains a cube as it melts). At what rate is the volume decreasing in $\text{cm}^3/\min$ when its side length is 4cm?

a) 24 b) 48

c) 240 d) 96

7) The letters of the word MONOTONIC are arranged in a row. The number of different arrangements that are possible if the two N's remain together are:

a) 2.9! b) $\frac{4.3!2!}{3!}$

c) $\frac{8!}{3!}$ d) $\frac{9!}{3!2!}$

8) For the polynomial $2x^3 + 8x^2 - 5x - 2 = 0$ with roots α, β and γ, the value of $\alpha^2 + \beta^2 + \gamma^2$ is:

a) 21 b) 16

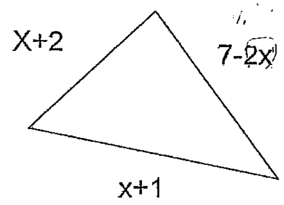
c) 18 d) 24

9) Which of the following expressions is <u>not</u> equivalent to $\ddot{x}$?

a) $\frac{dv}{dt}$ b) $\frac{dv}{dx}$

c) $v\frac{dv}{dx}$ d) $\frac{d\left(\frac{1}{2}v^2\right)}{dx}$

10)



The domain of x for the triangle above to exist, is given by:

a) $1 < x < 3$ b) $-2 < x < 3\frac{1}{2}$

c) $0 < x < 4$ d) $-1 < x < 3\frac{1}{2}$

End of Section I

Section II

Total marks - 60
Attempt Questions 11-14
All questions are of equal value

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

Marks

Question 11 (15 marks) Start a new booklet

a) Find the gradient of the tangent to $y = 3\cos^{-1}\frac{x}{2}$ at the point where $x = 1$. **2**

b) Use the substitution $u = x^2$ to find $\int \frac{x}{\sqrt{1-x^4}}\,dx$ **2**

c) A class consists of 9 girls and 6 boys. How many ways are there of selecting a committee of 3 girls and 2 boys from this class? **1**

d) Calculate $\int_0^{\frac{3}{2}} \frac{2}{\sqrt{9-4x^2}}\,dx$ **3**

e) The interval AB has endpoints $A(-4,6)$ and $B(8,14)$. Find the coordinates of the point P which divides the interval AB internally in the ratio $1:3$. **2**

Question 11 continues on page 8

Marks

f) Find the greatest coefficient in the expansion of $(2x+3)^9$. **3**

g) How many ways can 12 people be seated in a circle if 2 particular people must sit apart from each other? **2**

Question 12 (15 marks) Start a new booklet Marks

a) Find the coefficient of x^2 in the expansion of $\left(x^2 - \frac{3}{x}\right)^7$ **2**

b) Solve the inequality $\frac{4}{x-2} \leq 1$. **2**

c) Use mathematical induction to prove that $7^n - 3^n$ is divisible by 4 for all positive integers $n \geq 1$. **3**

d) i) Show that $2\sin x = x$ has a root between $x = 1$ and $x = 2$. **1**

ii) Taking $x = 1 \cdot 8$ as an approximation for the solution of $2\sin x = x$, use Newton's Method once to give a better approximation (1 decimal place). **2**

e) An archer hits a target on average 3 out of every 5 times she shoots. Find the probability that in 10 shots at the target:

i) she hits it exactly once (3significant figures) **1**

ii) She hits it at least 2 times (3significant figures) **2**

f) Find the value of the constants a and b if $x^2 + x - 6$ is a factor of the polynomial $x^3 + 5x^2 + ax + b$. **2**

Question 13 (15 marks) Start a new booklet Marks

a) When a particle is x metres from the origin, its velocity, $v\ m/s$, is given by **2**

$$v = \sqrt{8x - 2x^2}$$

Find the acceleration when the particle is $2\ m$ to the right of the origin.

b) The points $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ lie on the parabola $x^2 = 4ay$. It is given that the chord PQ has equation $y = \left(\frac{p+q}{2}\right)x - apq$.

i) Show that the gradient of the tangent at P is p. **1**

ii) Prove that if PQ passes through the focus, then the tangent at P is parallel to the normal at Q. **2**

c) i) State the domain and range of $y = 4\sin^{-1}(1 - x)$ **2**

ii) Hence sketch $y = 4\sin^{-1}(1 - x)$, clearly showing all essential features. **1**

Question 13 continues on page 11

d) For the graph of $f(x) = \frac{x+1}{x^2+4}$

(1 DECIMAL PLACE)

i) Find the coordinates of any stationary points and determine their nature. **2**

ii) Find the horizontal asymptotes of $f(x) = \frac{x+1}{x^2+4}$ **1**

iii) Sketch the graph showing all essential features. **1**

iv) By using the fact that $\frac{x+1}{x^2+4} = \frac{x}{x^2+4} + \frac{1}{x^2+4}$, or otherwise, show that the area bounded by $f(x) = \frac{x+1}{x^2+4}$, the x-axis and the lines $x = 0$ and $x = 2$ is equal to $\frac{1}{2}\left(Ln2 + \frac{\pi}{4}\right)\ units^2$. **3**

Question 14 (15 marks) Start a new booklet

a) Prove that $sin^{-1}\frac{1}{\sqrt{5}} + sin^{-1}\frac{1}{\sqrt{10}} = \frac{\pi}{4}$ **2**

b) A particle is projected from a point O with an initial velocity $v\ m/s$ and with an angle of projection α, where $0 \le \alpha \le 90°$ and where $g\ m/s^2$ is the acceleration due to gravity. Under these conditions you may assume that the equations for the horizontal and vertical displacements at time t are given by: $x = vtcos\alpha$ $\quad y = vtsin\alpha - \frac{1}{2}gt^2$

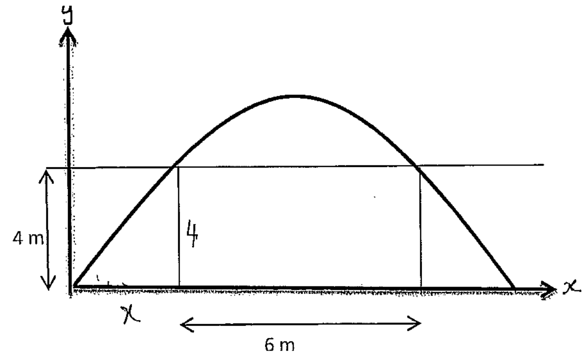
i) Prove that $y = xtan\alpha - \frac{gx^2}{2v^2}sec^2\alpha$. **2**

ii) Find the time of flight and the range in terms of v, α *and* g. **2**

iii) If R is the range of the projectile on the horizontal plane, prove that: **2**

$$y = x\left(1 - \frac{x}{R}\right)tan\alpha$$

iv) If $\alpha = 45°$ and the particle just clears two walls $6\ m$ apart, both at a height of $4\ m$, find the range of the projectile, R. **3**



Question 14 continues on page 13

Marks

c) i) Write down the expansion of $(1+x)^{2n}$ 1

ii) Prove that $2^{2n} = \sum_{k=0}^{2n} \binom{2n}{k}$ 3

iii) Prove that $\sum_{k=0}^{n} \binom{2n}{k} = 2^{2n-1} + \frac{(2n)!}{2(n!)^2}$

End of examination

SECTION 1

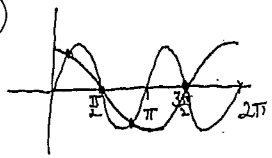
(1) "b" (2) $y=(p+q)^2+2pq$

$y=x^2-2$

"d"

(3) "b" (4) "d"

5)



"c"

(6) $V=x^3$

$\frac{dV}{dx}=3x^2$

$\frac{dV}{dt}=\frac{dV}{dx}\cdot\frac{dx}{dt}$

$=3(4)^2.(-0.5)$

$=-24\,cm^3/min$

"a"

(7) NO. OF WAYS $=\frac{8!}{3!}$ "c"

(8) $\alpha+\beta+\gamma=-\frac{8}{2}=-4$ $\quad\Sigma\alpha\beta=-5/2$

$\alpha^2+\beta^2+\gamma^2=(\Sigma\alpha)^2-2(\Sigma\alpha\beta)$

$=(-4)^2-2\left(\frac{-5}{2}\right)$

$=21$

"a"

(9) "b"

(10) $x+1+x+2>7-2x$

$4x>4$

$x>1$

$x+1+7-2x>x+2$

$2x<6$

$x<3$

$x+2+7-2x>x+1$

$2x<8$

$x<4$

$x+1>0$

$x>-1$

$x+2>0$

$x>-2$

$7-2x>0$

$2x<7$

$x<3\frac{1}{2}$

$\therefore\ 1<x<3$ "a"

SECTION 2

(11) a) $y=3\cos^{-1}\frac{x}{2}$

$y'=\frac{-3}{\sqrt{1-x^2/4}}\cdot\frac{1}{2}$

$=\frac{-3}{\sqrt{4-x^2}}$

$x=1$

$m=\frac{-3}{\sqrt{3}}$

$=-\sqrt{3}$

b) $I=\int\frac{x}{\sqrt{1-x^4}}\,dx$

$u=x^2$

$du=2x\,dx$

$I=\frac{1}{2}\int\frac{1}{\sqrt{1-u^2}}\,du$

$=\frac{1}{2}\sin^{-1}(u)+c$

$=\frac{1}{2}\sin^{-1}(x^2)+c$

c) NO. OF SELECTIONS $={}^9C_3\ {}^6C_2$

$=1260$

d) $I=\int_0^{\frac{3}{2}}\frac{2}{\sqrt{9-4x^2}}\,dx$

$=\frac{2}{2}\int_0^{\frac{3}{2}}\frac{1}{\sqrt{\frac{9}{4}-x^2}}\,dx$

$=\left[\sin^{-1}\frac{2x}{3}\right]_0^{3/2}$

$=\sin^{-1}1-\sin^{-1}0$

$=\pi/2$

e) $P(x,y)=\left(\frac{mx_2+nx_1}{m+n},\frac{my_2+ny_1}{m+n}\right)$

$=\left(\frac{1.8+3.(-4)}{1+3},\frac{1(14)+3(6)}{1+3}\right)$

$=(-1,8)$

f) $\frac{T_{k+1}}{T_k}=\frac{n-k+1}{k}\cdot\frac{b}{a}$

$=\frac{9-k+1}{k}\cdot\frac{3}{2}$

$=\frac{30-3k}{2k}$

GREATEST COEFF $\frac{T_{k+1}}{T_k}\geq 1$

$\frac{30-3k}{2k}\geq 1$

$5k\leq 30$

$k\leq 6$

$\therefore$ T_6 AND T_7 HAVE GREATEST COEFF.

$T_7={}^9C_6(2x)^3(3)^6$

$=489\,888x^3$

$\therefore$ GREATEST COEFF $=489\,888$

g) NO OF WAYS TOGETHER $=10!\,2!$

NO. OF WAYS APART $=11!-10!\,2!$

$=32\,659\,200$

a) $T_{k+1}={}^nC_k\,a^{n-k}\,b^k$

$={}^7C_k\,(x^2)^{7-k}(-3x^{-1})^k$

TERM IN x^2 $=x^{14-2k}.x^{-k}$

$=x^{14-3k}$

$\therefore\ 14-3k=2$

$3k=12$

$k=4$

$\therefore$ COEFF $={}^7C_4(-3)^4$

$=2835$

b) $\frac{4}{x-2}\leq 1$

$4(x-2)\leq(x-2)^2$

$(x-2)^2-4(x-2)\geq 0$

$(x-2)[x-2-4]\geq 0$

$(x-2)(x-6)\geq 0$

$x<2,\ x\geq 6$

c) PROVE TRUE FOR $n=1$

$7^1-3^1=4$

$=4\times 1$

$\therefore$ DIVISIBLE BY 4 AND TRUE FOR $n=1$

ASSUME TRUE FOR $n=k$

$7^k-3^k=4Q$ (Q SOME INTEGER)

PROVE TRUE FOR $n=k+1$

$7^{k+1}-3^{k+1}=4m$ (m SOME INTEGER)

$LHS=7(7^k)-3(3^k)$

$=7(4Q+3^k)-3(3^k)$

$= 28Q + 4(3^k)$

$= 4(7Q + 3^k)$

$= 4M$ (WHERE $M = 7Q + 3^k$, SOME INTEGER)

$=$ RHS

$\therefore$ IF TRUE FOR $n = k$, THEN PROVED TRUE FOR $n = k+1$. BUT TRUE FOR $n = 1$, $\therefore$ TRUE FOR $n = 2$, AND BY PRINCIPLES OF INDUCTION, TRUE FOR ALL $n \geq 1$.

d) i) $2\sin x = x \qquad f(x) = 2\sin x - x$

$2\sin x - x = 0$

$f(1) = 2\sin 1 - 1 = 0.6829\ldots \qquad f(2) = 2\sin 2 - 2 = -0.1814\ldots$

SINCE $f(1)$ AND $f(2)$ CHANGE SIGN, THERE IS A ROOT OF $f(x) = 0$ BETWEEN $x = 1$ AND $x = 2$.

ii) $f'(x) = 2\cos x - 1$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_2 = 1.8 - \frac{2\sin 1.8 - 1.8}{2\cos 1.8 - 1}$$

$= 1.90155\ldots$

$= 1.9$ (1 DECIMAL PLACE)

e) i) $P(X = r) = {}^nC_r\, q^{n-r} p^r \qquad p = \frac{3}{5} \quad q = \frac{2}{5}$

i) $P(X = 1) = {}^{10}C_1 \left(\frac{2}{5}\right)^9 \left(\frac{3}{5}\right)^1$

$= 0.00157$ (3 SIG FIG)

ii) $P(X \geq 2) = 1 - P(X = 0) - P(X = 1)$

$= 1 - \left(\frac{2}{5}\right)^{10} - 0.00157$

$= 0.998$

f) $(x+3)(x-2)$ IS A FACTOR

$\therefore P(-3) = -27 + 45 - 3a + b = 0$ (1)

$P(2) = 8 + 20 + 2a + b = 0$ (2)

$3a - b = 18$ (1)

$2a + b = -28$ (2)

$5a = -10$

$a = -2$

IN (1)

$-6 - b = 18$

$b = -24$

$\therefore a = -2, b = -24$

13) a) $V = \sqrt{8x - 2x^2}$

$V^2 = 8x - 2x^2$

$\frac{1}{2}V^2 = 4x - x^2$

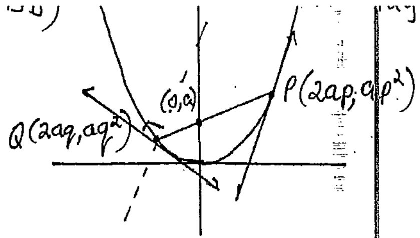
$\frac{d\left(\frac{1}{2}V^2\right)}{dx} = 4 - 2x$

$x = 2$

$\ddot{x} = 4 - 4$

$= 0 \text{ ms}^{-2}$

ii)



i) $y = \frac{x^2}{4a}$

$y' = \frac{2x}{4a}$

$= \frac{x}{2a}$

at $P(2ap, ap^2)$

$m = \frac{2ap}{2a}$

$= p$

ii) $(0,a)$ SATISFIES $y = \left(\frac{p+q}{2}\right)x - apq$

$\therefore a = \left(\frac{p+q}{2}\right).0 - apq$

$a = -apq$

$pq = -1$

$q = -\frac{1}{p}$

BUT q IS THE GRADIENT OF THE TANGENT AT Q

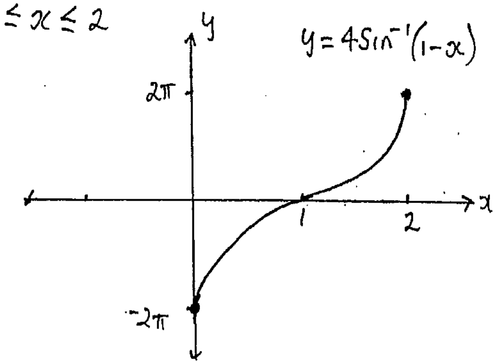
$\therefore m_{\text{NORMAL AT } Q} = \frac{-1}{-\frac{1}{p}}$

$= p$

$\therefore$ NORMAL AT Q IS $\parallel$ TO TANGENT AT P.

$y = 4\sin^{-1}(1-x)$

DOMAIN	RANGE
$-1 \leq 1-x \leq 1$	$-2\pi \leq y \leq 2\pi$
$-2 \leq -x \leq 0$	
$0 \leq x \leq 2$	



d) $f(x) = \frac{x+1}{x^2+4}$

$$f'(x) = \frac{(x^2+4)1 - (x+1)(2x)}{(x^2+4)^2}$$

$$= \frac{x^2 + 4 - 2x^2 - 2x}{(x^2+4)^2}$$

$$= \frac{-x^2 - 2x + 4}{(x^2+4)^2}$$

STAT PTS $f'(x) = 0$

$$\frac{-x^2 - 2x + 4}{(x^2+4)^2} = 0$$

$x^2 + 2x - 4 = 0$

$x = \frac{-2 \pm \sqrt{20}}{2}$

$= -1 + \sqrt{5}, -1 - \sqrt{5}$

$= 1.2, -3.2$

ii) VERTICAL ASYMPTOTES WHEN $x^2+4=0$

$\therefore$ NO VERTICAL ASYMPTOTES

x	1	1·2	2
$f'(x)$	+ve	0	-ve

$\therefore$ MAXIMUM TURNING POINT AT $(1{\cdot}2, 0{\cdot}4)$

x	-4	-3·2	-3
$f'(x)$	-ve	0	+ve

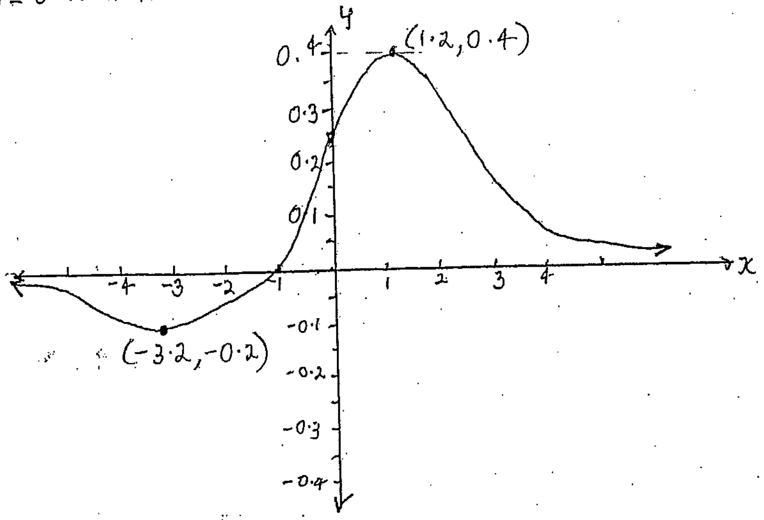
$\therefore$ MINIMUM TURNING POINT AT $(-3{\cdot}2, -0{\cdot}2)$

$$\lim_{x\to\infty} \frac{x+1}{x^2+4} = 0^+$$

$$\lim_{x\to-\infty} \frac{x+1}{x^2+4} = 0^-$$

$\therefore$ $y=0$ IS A HORIZONTAL ASYMPTOTE

iii)



iv) $$A = \int_0^2 \frac{x+1}{x^2+4}\,dx$$

$$= \int_0^2 \frac{x}{x^2+4}\,dx + \int_0^2 \frac{1}{x^2+4}\,dx$$

$$= \frac{1}{2}\Big[\ln(x^2+4)\Big]_0^2 + \frac{1}{2}\Big[\tan^{-1}\frac{x}{2}\Big]_0^2$$

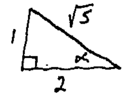
$$= \frac{1}{2}\big[\ln 8 - \ln 4\big] + \frac{1}{2}\big[\tan^{-1}1 - \tan^{-1}0\big]$$

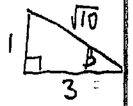
$$= \frac{1}{2}\Big[\ln 2 + \frac{\pi}{4}\Big] \; u^2$$

a) $$\sin^{-1}\frac{1}{\sqrt5} + \sin^{-1}\frac{1}{\sqrt{10}} = \frac{\pi}{4}$$

$\sin^{-1}\frac{1}{\sqrt5} = \alpha$ $\qquad$ $\sin^{-1}\frac{1}{\sqrt{10}} = \beta$

$\sin\alpha = \frac{1}{\sqrt5}$ $\qquad$ $\sin\beta = \frac{1}{\sqrt{10}}$





$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta}$$

$$= \frac{\frac12 + \frac13}{1-\frac12\cdot\frac13}$$

ie $\tan(\alpha+\beta) = 1$

$\therefore \alpha+\beta = \frac{\pi}{4}$

b) i) $x = vt\cos\alpha$ $\qquad$ $y = vt\sin\alpha - \frac12 gt^2$

$t = \frac{x}{v\cos\alpha}$

$$y = v\sin\alpha\left(\frac{x}{v\cos\alpha}\right) - \frac12 g\left(\frac{x^2}{v^2\cos^2\alpha}\right)$$

$$y = x\tan\alpha - \frac{gx^2\sec^2\alpha}{2v^2}$$

ii) t_{FLIGHT}, $y=0$

$vt\sin\alpha - \frac12 gt^2 = 0$

$t\left(v\sin\alpha - \frac{gt}{2}\right) = 0$

$t = 0, \frac{2v\sin\alpha}{g}$

$\therefore t_{FLIGHT} = \frac{2v\sin\alpha}{g}$

$$x_{RANGE} = v\cos\alpha\left(\frac{2v\sin\alpha}{g}\right)$$

$$= \frac{v^2}{g}\sin 2\alpha$$

iii) $$R = \frac{v^2\sin2\alpha}{g}$$

$$\therefore v^2 = \frac{gR}{\sin 2\alpha}$$

SUBST IN $y = x\tan\alpha - \frac{gx^2\sec^2\alpha}{2v^2}$

$$y = x\tan\alpha - \frac{gx^2\sin2\alpha}{2gR\cos^2\alpha}$$

$$= x\tan\alpha - \frac{x^2\,2\sin\alpha\cos\alpha}{2R\cos\alpha\cos\alpha}$$

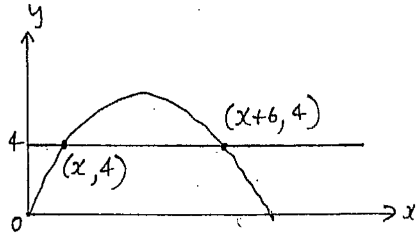
$$= x\tan\alpha - \frac{x^2\tan\alpha}{R}$$

$$\therefore y = x\left(1-\frac{x}{R}\right)\tan\alpha$$

iv)

14 b)

iv)



SUB $(x,4)$ IN $y = x\left(1-\frac{x}{R}\right)\tan\alpha$

$$4 = x\left(1-\frac{x}{R}\right)\tan 45^\circ$$

$$4 = x\left(1-\frac{x}{R}\right) \quad ①$$

SUB $(x+6,4)$ IN $y = x\left(1-\frac{x}{R}\right)\tan\alpha$

$$4 = (x+6)\left(1-\frac{x+6}{R}\right)\tan 45^\circ$$

$$4 = (x+6)\left(1-\frac{x+6}{R}\right)$$

FROM 1

MULTIPLY ① BY R

$$4R = xR - x^2$$
$$x^2 = R(x-4)$$
$$R = \frac{x^2}{x-4}$$

INTO ②

$$4 = (x+6)\left(1-\frac{(x+6)(x-4)}{x^2}\right)$$

$$\frac{4}{x+6} = \frac{x^2-(x^2+2x-24)}{x^2}$$

$$\frac{4}{x+6} = \frac{24-2x}{x^2}$$

$$4x^2 = 24x - 2x^2 + 144 - 12x$$
$$6x^2 - 12x - 144 = 0$$
$$x^2 - 2x - 24 = 0$$

$$(x-8)(x+6) = 0$$
$$x = 8, -6$$
$$\therefore x = 8 \quad (x \geq 0)$$
$$\therefore R = \frac{8^2}{8-4}$$
$$= 16\text{ m}$$

c)

i) $(1+x)^{2n} = \binom{2n}{0} + \binom{2n}{1}x + \binom{2n}{2}x^2 + \ldots + \binom{2n}{n}x^n + \ldots + \binom{2n}{2n}x^{2n}$

ii) IN THE EXPANSION IN i)

LET $x = 1$

$$2^{2n} = \binom{2n}{0} + \binom{2n}{1}1 + \binom{2n}{2}1^2 + \ldots + \binom{2n}{2n}1^{2n}$$

$$= \binom{2n}{0} + \binom{2n}{1} + \binom{2n}{2} + \ldots + \binom{2n}{2n}$$

$$= \sum_{k=0}^{2n}\binom{2n}{k}$$

iii)

GIVEN

$$2^{2n} = \underbrace{\binom{2n}{0} + \binom{2n}{1} + \ldots + \binom{2n}{n-1}}_{S_n} + \binom{2n}{n} + \underbrace{\binom{2n}{n+1} + \ldots + \binom{2n}{2n}}_{S_{\text{LAST } n \text{ TERMS}}}$$

SINCE ${}^nC_k = {}^nC_{n-k}$, THEN DUE TO SYMMETRY

$$\therefore S_n = S_{\text{LAST } n}$$

So $2^{2n} = 2S_n + \binom{2n}{n}$

$$2S_n = 2^{2n} - \frac{(2n)!}{n!\,n!}$$

$$\therefore S_n = 2^{2n-1} - \frac{(2n)!}{2(n!)^2}$$

$$\therefore \sum_{k=0}^{n}\binom{2n}{k} = 2^{2n-1} - \frac{(2n)!}{2(n!)^2} + \frac{(2n)!}{(n!)^2}$$

$$= 2^{2n-1} + \frac{(2n)!}{2(n!)^2}$$