Student Number



St. Catherine's School, Waverley

2014

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

General Instructions

- Reading Time 5 minutes
- Working Time 2 hours
- Write using black or blue pen Black pen is preferred
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- In Questions 11-14, show relevant mathematical reasoning and/or calculations
- Task weighting 40%

Section I Pages 3-6

10 marks

- Attempt Questions 1 10
- Allow about 15 minutes for this section
- Answer on the multiple choice answer sheet provided.

Section II Pages 7-13

60 marks

- Attempt Questions 11-14
- Allow about 1hour and 45 minutes for this section
- Answer each question in the booklet provided.

Total Marks – 70

Extension 1 trials.

1 If
$$y = sin^{-1}(x^2)$$
, then $\frac{dy}{dx} =$
A $\frac{2x}{\sqrt{1-x^4}}$
B $\frac{2x}{\sqrt{1-x^2}}$
C $2xcos^{-1}(x^2)$
D $\frac{2x}{1-x^4}$

2 $\int \sin^2 3x \, dx$ is

- A $cos^2 3x + C$
- B $2\sin 3x\cos 3x + C$

$$C \quad \frac{1}{2}\left(x - \frac{\cos 6x}{6}\right) + C$$

$$\mathsf{D} \quad \frac{1}{2} \left(x - \frac{\sin 6x}{6} \right) + \mathsf{C}$$

3 The primitive of $\sqrt{e^{3x}}$ is given by

A
$$\frac{3}{2\sqrt{e^{3x}}} + C$$

B
$$\frac{3}{2}e^{\frac{3x}{2}} + C$$

C
$$\frac{2}{3}e^{\frac{3x}{2}} + C$$

D
$$\frac{1}{2}e^{3x} + C$$

4
$$\int \frac{dx}{\sqrt{1-4x^2}}$$
 is given by
A $sin^{-1}2x + C$
B $sin^{-1}\frac{x}{2} + C$
C $4sin^{-1}2x + C$

$$D \quad \frac{1}{2}sin^{-1}2x + C$$

- 5 The equation of motion of a particle moving in Simple harmonic Motion is given by $\ddot{x} = 1 - 3x$, which of the following statements is true?
 - A The period of motion is $\frac{2\pi}{3}$ and the centre is $x=\frac{1}{3}$
 - B The period of motion is $\frac{-2\pi}{3}$ and the centre is x=3
 - C The period of motion is $\frac{2\pi}{3}$ and the centre is x=3
 - D The period of motion is $\frac{2\pi}{\sqrt{3}}$ and the centre is $x=\frac{1}{3}$

6 Given that $\frac{f'(x)}{f(x)} = 1$, which of the following statements is true? (note: C is a constant in each case)

- A f(x) = lnx + C
- $B \quad f(x) = e^x + C$
- C $f(x) = C e^x$
- $D \quad f(x) = C \ lnx$

⁷ Consider the graph of the function
$$y = \frac{x^2+1}{x}$$
. The equations of the asymptotes are

- A x = 0 and x = 1
- B x = 0 and y = x
- $C \quad x = -1 \text{ and } x = 1$
- D No asymptotes
- 8 The solution to the inequality $x(x^2 4) > 0$ is
 - A -2 < x < 0 or x > 2
 - B -2 < x < 2
 - C x < -2 or x > 2
 - D x < -2 or 0 < x < 2

9

Surface area S of a spherical balloon is given by the formula $S = 4\pi r^2$,

where r is the radius.A spherical balloon is being inflated so that, $\frac{dr}{dt} = 2$ cm/sec.The value of $\frac{ds}{dt}$, when the surface area is 16π cm² is given byA 32π

 $\mathsf{B} \quad \frac{16}{\pi}$

C 256 π^2

D No sufficient information

$$\begin{array}{rcl} 10 & \cos^{-1}\left(\cos\frac{11\pi}{6}\right)is \\ A & \frac{11\pi}{6} \\ B & \frac{\sqrt{3}}{2} \\ C & \frac{7\pi}{6} \\ D & \frac{\pi}{6} \end{array}$$

Question 11 Start a new page

a) Solve for x:
$$\frac{1}{x-1} \ge 5$$
 3

b) Find the value of k:
$$x^{3k+4} = e^{8 \ln x}$$
 2

- c) Find the ratio in which P divides the interval AB, where 3 P is $(2, \frac{20}{3}) A$ is (1, 5) and B is (4, 10).
- d The lines y = 3mx + 1 and y = mx are inclined at an angle of α , where $tan\alpha = \frac{1}{2}$.
 - (i) Show that $3m^2 4m + 1 = 0$ 2
 - (ii) Hence find the possible values of *m*. 1
- e If $x^2 x 2$ is a factor of the polynomial $x^4 + 3x^3 + ax^2 2x b$, find the values 4 of a and b.

Question 12 Start a new page

a Show that
$$tan^{-1}\frac{1}{3} + tan^{-1}\frac{1}{5} = tan^{-1}\frac{4}{7}$$
 3

b Use the substitution
$$x = 2 \sin \theta$$
, to evaluate $\int_0^2 \sqrt{4 - x^2} dx$ 4

c Find the general solution to the equation
$$\sin 2\theta = \cos \theta$$
 3

d (i) Show that
$$\frac{1-x^2}{1+x^2} = -1 + \frac{2}{x^2+1}$$

 $y = \frac{1-x^2}{1+x^2}$, locating any stationary points and equations of asymptotes.

(Use at least one third of a page)

Question 13 Start a new page

a Find
$$\int \frac{5}{1+16x^2} dx$$
 2

b Find the constant term in the expansion of
$$(2x - \frac{1}{x^2})^9$$
 3

c Consider the expansion of $(3 + 4x)^{12}$ in ascending powers of x.

(i) Show that
$$\frac{coefficient of t_{r+1}}{coefficient of t_r} = \frac{4(13-r)}{3r}$$
, where t_r is the r^{th} term. 2

(ii) Hence or otherwise find the greatest coefficient in the expansion of
$$(3 + 4x)^{12}$$

A particle is projected with velocity 20 metres per second. This hits a target at a horizontal distance of 20 metres and a vertical height of 10 metres. Take the acceleration due to gravity as $10 m per sec^2$

 $x = 20 \cos \alpha t$ and $y = -5t^2 + 20 \sin \alpha t$, where α , is the angle of projection.

(ii) Show that the Cartesian equation of the motion is 2

$$y = x \tan \propto -\frac{x^2}{80} \sec^2 \propto$$

2

(iii) Hence find the values of \propto

d

Question 14 Start a new page

- a A particle moves with an acceleration given by the expression a = 7x Initially the 3 particle start from the origin with a velocity of -3 metres pe second. Find an expression for the velocity in terms of the displacement
- b The displacement of a particle (in cm) from a point O on a line after t seconds is given by $x = 3\sin(2t + \alpha)$. Initially the particle is at x = 1.5

(i)	Find the value of $\propto 2$	1
(ii)	Find the acceleration \ddot{x} in terms of the displacement x	1
(iii)	Find the time the particle takes to reach the point $x = 0$, for the first time	2
(iii)	Find the time it takes to reach an acceleration of $-12 \ cm \ ner \ sec^2$ for the first	2

(iii) Find the time it takes to reach an acceleration of $-12 \ cm \ per \ sec^2$, for the first 2 time.

c A function is defined as $f(x) = 1 - \cos \frac{x}{2}$

(i) Stae the period of this function

(ii) Find the largest domain of the function for which the inverse function $f^{-1}(x)$ exists. 1 Include x = 0 in the domain.

1

- (iii) Find the equation of $y = f^{-1}(x)$ 2
- (iv) Sketch $y = f^{-1}(x)$ 2

End of Task

The following list of standard integrals may be used:

$$\int x^n dx = \frac{1}{n+1} x^{n+1}, n \neq -1; x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, x > 0$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}, a \neq 0$$

$$\int \cos ax \ dx = \frac{1}{a} \sin ax, a \neq 0$$

$$\int \sin ax \ dx = -\frac{1}{a} \cos ax, a \neq 0$$

$$\int \sec^2 ax \ dx = \frac{1}{a} \tan ax, a \neq 0$$

$$\int \sec ax \tan ax \ dx = \frac{1}{a} \sec ax, a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \ dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \ dx = \sin^{-1} \frac{x}{a}, a > 0, -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \ dx = \ln \left(x + \sqrt{x^2 - a^2}\right) x > a > 0$$

Qn	Solutions	Marks	Comments: Criteria
•	Multiple choice.		
,	$y = \sin^{-1}(n^2)$		
2.	$y' = \frac{(2x)}{\sqrt{1-x^{4}}} \qquad A$ $\int S_{10}^{2} 3x dx = \frac{1}{2} \int (1 - \cos bx) dx$ $= \frac{1}{2} \left(x - \frac{\sin bx}{6} \right) + c$ D_{10}		
3	$\int e^{\frac{3x}{2}} dx = \frac{2}{3}e^{\frac{3x}{2}} + C.$		
	dr dr		
4	$\int \sqrt{1-4x^2} \int \sqrt{4(\frac{1}{4}-x^2)}$		
	$= \frac{1}{2} \cdot \frac{\sin^2 2x + C}{2}$		
5	$\dot{x} = 1 - 3x$	c	
	= - ol 3. centi A=2 ()		
	211 V3		
6	$\left(\begin{array}{c} f'(n) \\ f(n) \end{array}\right) dn = \int dn$		
	$\left 1 + f_{n} \right = \lambda + C.$		
	$f(x) = e^{x+L}$ = A e^{x} or C e^{x}		

I

Qn	Solutions	Marks	Comments: Criteria
7	$\begin{array}{rcl} y &=& \frac{2}{\chi} \\ &=& \chi \\ &=& \chi$		
8	B. H(x-2)(x+2) > 0		
	$= \frac{1}{2} $	2	
9.	$S = 4\pi r^{2}$ $S = 4\pi r^{2}$ $\frac{36\pi}{4} = r$ $\frac{4}{2} = r$ $\frac{2}{4} = r$	2	
10	$= 3177$ G_{0} $= G_{0} = 1 (G_{0} = 177)$ $= G_{0} = 1 (G_{0} = 177)$ $= G_{0} = 1 (G_{0} = 177)$ $= G_{0} = 177$ $G_{0} = 177$ $G_{0} = 177$		

On	Solutions	Marks	Comments: Criteria
	Nore: works for y. condicate		
	na need to do this as well.		
	NO RECT		
	10x + 5 = 20		
	K+1 3		
	10×+15 = 20×+20		
	5 = 10k		
	$K = \frac{1}{2}$		
al)	$y = 3\pi^2 + 1$ $y = \pi^2 \pi^2$		
18	grea 12		
	graam, Jam-ml		
	land = 1+3m2		
	32		
	1 = 1+3m2		
	$3m^2 \pm 1 = 4m$		
	$2m^2 - 4m + 1 = b$		
	$\sum_{i=1}^{n} (m-i) = 0$		
	(3m-1)(1)m=1.		
	$m=\frac{1}{3}$		
0	(a) (n + 1) p a fastri		
19	(a-c) (at of P(x))		
	(d-I) is a facture		
	P(2) = D		
	24 + 49 - 4 - 01 - 0		
	10T 40-5= - 30		
	(at) is a factor .		
	1 - 3 + a + 2 - b = 0		
	Q-5=0 -13		



8-

Comments: Criteria Marks Solutions $\frac{Cooff}{Cooff} \frac{oftr+1}{ftr} = \frac{12c_r}{\frac{12c_r}{r+1}} \cdot \frac{3^{12-r}}{\frac{13-r}{3}} \cdot \frac{4^r}{4^{r+1}}$ Qn 1 for 12ch 4 12ch 3 $= \frac{124!}{r!(12-r)!} \frac{(r-1)!(13-r)!}{124!} \frac{24}{3}$ (2) $= \frac{13-r}{5} \cdot \frac{4}{3}$ coeff of tr+1 2 weft of tr 4 (13-r) = 3r 52-4r 23r $7r \leq 52 \quad (52, 73)$ $r \leq 7 \qquad \ddagger$:. [coeff of tr+1] $\ll \text{ coeff of tr}$ for $\sigma \geq 8$. Coff & b8 > Coeff of by > --. Cooff of t 2 (also. Cooff of . tiz < Coeff of tiz --. & coeff of ts. $T_{g} = \frac{12}{c_{7}} \cdot 3^{5} \cdot (4x)^{7} \cdot \frac{1}{2}$ Coeff. of T8. = 12 35.47. Dr greatour CONTA. Or greator coeff: = 3153199104

Marks **Comments:** Criteria **Solutions** Qn 130. The only acceleration on the posticle 2 - 1 PEREDEDA is gravity au = D $-\frac{1}{2}$ if didn't this from $\dot{x} = 0$, $\dot{y} = -10$ y = - 10 Initially $\dot{\chi} = 20\cos x$ $\chi = 0$ f=0 $\dot{y} = 20\sin x$. Y=0-LIF NO +C. 7=0 n'=0 x = Const. y = -10 à=200051 $\hat{y} = -10t + C$ x = 20 t losa + C 205in A = -1000 + Cy' = -10t + 205ing $y = -5t^2 + (205ing)t + C$ 620 : x =0 :. C=0 x=20+6064.0 t=0 -- C=D climinare t in @ a @ to get to y=0 Carrosian equation. t = X nowsa O $y = -5 \times^2 + 20.2$ $y = -5 \times^2 + 20.2$ $y = -5 \times^2 + 20.2$ $y = -5 \times^2 + 20.2$ Sub in Q = - 80 seid x2 + × land = - 80 (1+10,22) x2 + 2 rond. z xland - X seid PO

