

## Student Number

## St. Catherine's School, Waverley

2014
TRIAL HIGHER SCHOOL CERTIFICATE
EXAMINATION

## Mathematics Extension 1

## General Instructions

- Reading Time - 5 minutes
- Working Time - 2 hours
- Write using black or blue pen Black pen is preferred
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- In Questions 11-14, show relevant mathematical reasoning and/or calculations
- Task weighting $-40 \%$


## Section I <br> Pages 3-6

## 10 marks

- Attempt Questions 1 - 10
- Allow about 15 minutes for this section
- Answer on the multiple choice answer sheet provided.


## Section II

Pages 7-13

## 60 marks

- Attempt Questions 11-14
- Allow about 1 hour and 45 minutes for this section
- Answer each question in the booklet provided.

Extension 1 trials.
1 If $y=\sin ^{-1}\left(x^{2}\right)$, then $\frac{d y}{d x}=$
A $\frac{2 x}{\sqrt{1-x^{4}}}$
B $\frac{2 x}{\sqrt{1-x^{2}}}$
C $2 x \cos ^{-1}\left(x^{2}\right)$
D $\frac{2 x}{1-x^{4}}$
$2 \int \sin ^{2} 3 x d x$ is

A $\cos ^{2} 3 x+C$

B $2 \sin 3 x \cos 3 x+C$

C $\frac{1}{2}\left(x-\frac{\cos 6 x}{6}\right)+C$

D $\frac{1}{2}\left(x-\frac{\sin 6 x}{6}\right)+C$

3 The primitive of $\sqrt{e^{3 x}}$ is given by

A $\frac{3}{2 \sqrt{e^{3 x}}}+C$

B $\frac{3}{2} e^{\frac{3 x}{2}}+C$

C $\frac{2}{3} e^{\frac{3 x}{2}}+C$

D $\frac{1}{2} e^{3 x}+C$
$\int \frac{d x}{\sqrt{1-4 x^{2}}}$ is given by

A $\sin ^{-1} 2 x+C$

B $\sin ^{-1} \frac{x}{2}+C$

C $4 \sin ^{-1} 2 x+C$

D $\frac{1}{2} \sin ^{-1} 2 x+C$

5 The equation of motion of a particle moving in Simple harmonic Motion is given by $\ddot{x}=1-3 x$, which of the following statements is true?

A The period of motion is $\frac{2 \pi}{3}$ and the centre is $x=\frac{1}{3}$

B The period of motion is $\frac{-2 \pi}{3}$ and the centre is $x=3$

C The period of motion is $\frac{2 \pi}{3}$ and the centre is $x=3$

D The period of motion is $\frac{2 \pi}{\sqrt{3}}$ and the centre is $x=\frac{1}{3}$

6
Given that $\frac{f^{\prime}(x)}{f(x)}=1$, which of the following statements is true?
( note: C is a constant in each case)

A $\quad f(x)=\ln x+C$

B $f(x)=e^{x}+C$

C $f(x)=C e^{x}$

D $f(x)=C \ln x$

7 Consider the graph of the function $y=\frac{x^{2}+1}{x}$. The equations of the asymptotes are

A $\quad x=0$ and $x=1$

B $\quad x=0$ and $y=x$

C $x=-1$ and $x=1$

D No asymptotes

8 The solution to the inequality $x\left(x^{2}-4\right)>0$ is

A $-2<x<0$ or $x>2$

B $-2<x<2$

C $x<-2$ or $x>2$

D $x<-2$ or $0<x<2$

9 Surface area $S$ of a spherical balloon is given by the formula $S=4 \pi r^{2}$,
where $r$ is the radius.
A spherical balloon is being inflated so that, $\frac{d r}{d t}=2 \mathrm{~cm} / \mathrm{sec}$.
The value of $\frac{d S}{d t}$, when the surface area is $16 \pi \mathrm{~cm}^{2}$ is given by
A $32 \pi$

B $\frac{16}{\pi}$

C $256 \pi^{2}$

D No sufficient information

10

$$
\cos ^{-1}\left(\cos \frac{11 \pi}{6}\right) \text { is }
$$

A $\frac{11 \pi}{6}$

B $\frac{\sqrt{3}}{2}$

C $\frac{7 \pi}{6}$

D $\frac{\pi}{6}$

## Question 11 Start a new page

a) Solve for $\mathrm{x}: \frac{1}{x-1} \geq 5$
b) Find the value of $\mathrm{k}: x^{3 k+4}=e^{8 \ln x}$
c) Find the ratio in which $P$ divides the interval $A B$, where $P$ is $\left(2, \frac{20}{3}\right) A$ is $(1,5)$ and $B$ is $(4,10)$.
d The lines $y=3 m x+1$ and $y=m x$ are inclined at an angle of $\alpha$, where $\tan \alpha=\frac{1}{2}$.
(i) Show that $3 m^{2}-4 m+1=0$
(ii) Hence find the possible values of $m$.
e
If $x^{2}-x-2$ is a factor of the polynomial $x^{4}+3 x^{3}+a x^{2}-2 x-b$, find the values 4 of $a$ and $b$.

## Question 12 Start a new page

a $\quad$ Show that $\tan ^{-1} \frac{1}{3}+\tan ^{-1} \frac{1}{5}=\tan ^{-1} \frac{4}{7}$
c Find the general solution to the equation $\sin 2 \theta=\cos \theta$
d (i) Show that $\frac{1-x^{2}}{1+x^{2}}=-1+\frac{2}{x^{2}+1}$
(ii) Hence or otherwise clearly sketch the graph of the function $y=\frac{1-x^{2}}{1+x^{2}}$, locating any stationary points and equations of asymptotes.
(Use at least one third of a page)

## Question 13 Start a new page

a
Find $\int \frac{5}{1+16 x^{2}} d x$
b Find the constant term in the expansion of $\left(2 x-\frac{1}{x^{2}}\right)^{9}$

C
Consider the expansion of $(3+4 x)^{12}$ in ascending powers of $x$.
(i) Show that $\frac{\text { coefficient of } t_{r+1}}{\text { coefficient of } t_{r}}=\frac{4(13-r)}{3 r}$, where $t_{r}$ is the $r^{\text {th }}$ term.
(ii) Hence or otherwise find the greatest coefficient in the expansion of

$$
(3+4 x)^{12}
$$

d A particle is projected with velocity 20 metres per second. This hits a target at a horizontal distance of 20 metres and a vertical height of 10 metres. Take the acceleration due to gravity as $10 \mathrm{~m} \mathrm{per} \mathrm{sec}{ }^{2}$
(i) Show that the equations of motion are given by
$x=20 \cos \alpha t \quad$ and $y=-5 t^{2}+20 \sin \alpha t$, where $\alpha$, is the angle of projection.
(ii) Show that the Cartesian equation of the motion is

$$
y=x \tan \propto-\frac{x^{2}}{80} \sec ^{2} \propto
$$

(iii) Hence find the values of $\propto$
a A particle moves with an acceleration given by the expression $a=7 x$ Initially the particle start from the origin with a velocity of -3 metres pe second. Find an expression for the velocity in terms of the displacement
b The displacement of a particle (in cm ) from a point $O$ on a line after $t$ seconds is given by $x=3 \sin (2 t+\alpha)$. Initially the particle is at $x=1.5$
(i) Find the value of $\propto$ ?
(ii) Find the acceleration $\ddot{x}$ in terms of the displacement $x$
(iii) Find the time the particle takes to reach the point $x=0$, for the first time
(iii) Find the time it takes to reach an acceleration of $-12 \mathrm{~cm} \mathrm{per} \mathrm{sec}{ }^{2}$, for the first time.
c
A function is defined as $f(x)=1-\cos \frac{x}{2}$
(i) Stae the period of this function
(ii) Find the largest domain of the function for which the inverse function $f^{-1}(x)$ exists. 1 Include $x=0$ in the domain.
(iii) Find the equation of $y=f^{-1}(x)$
(iv) Sketch $y=f^{-1}(x)$

The following list of standard integrals may be used:

$$
\begin{aligned}
& \int x^{n} d x=\frac{1}{n+1} x^{n+1}, n \neq-1 ; x \neq 0 \text {, if } n<0 \\
& \int \frac{1}{x} d x=\ln x, x>0 \\
& \int e^{a x} d x=\frac{1}{a} e^{a x}, a \neq 0 \\
& \int \cos a x d x=\frac{1}{a} \sin a x, a \neq 0 \\
& \int \sin a x d x=-\frac{1}{a} \cos a x, a \neq 0 \\
& \int \sec ^{2} a x d x=\frac{1}{a} \tan a x, a \neq 0 \\
& \int \sec a x \tan a x d x=\frac{1}{a} \sec a x, a \neq 0 \\
& \int \frac{1}{a^{2}+x^{2}} d x=\frac{1}{a} \tan ^{-1} \frac{x}{a}, a \neq 0 \\
& \int \frac{1}{\sqrt{a^{2}-x^{2}}} d x=\sin ^{-1} \frac{x}{a}, a>0,-a<x<a \\
& \int \frac{1}{\sqrt{x^{2}-a^{2}}} d x=\ln \left(x+\sqrt{x^{2}-a^{2}}\right) x>a>0 \\
& \int \frac{1}{\sqrt{x^{2}+a^{2}}} d x=\ln \left(x+\sqrt{x^{2}+a^{2}}\right)
\end{aligned}
$$






| $\mathbf{Q n}$ |
| :--- |

$$
\begin{aligned}
30 & =-36 \\
0 & =-12 \quad b=-12
\end{aligned}
$$

Q. $12 \quad \tan ^{-1} \frac{1}{3}+\tan ^{-1} \frac{1}{5}=\tan ^{-1} \frac{4}{7}$.

Ler.

$$
\begin{array}{ll}
\alpha=\operatorname{ran}^{-1} \frac{1}{3} & \beta=\tan ^{-1} \frac{1}{5} \\
\tan \alpha=\frac{1}{3} & \operatorname{ran} \beta=\frac{1}{5}
\end{array}
$$

$$
\begin{aligned}
& \text { Conside } \operatorname{ran}(\alpha+\beta) \\
&= \frac{\operatorname{ran} \alpha+\operatorname{ran} \beta}{1-\operatorname{ran} \alpha \operatorname{ran} \beta} \\
&= \frac{\frac{1}{3}+\frac{1}{5}}{1-\frac{1}{15}}=\frac{8}{14}=\frac{4}{7} \\
& \therefore \tan ^{-1} \frac{4}{7}=\alpha+\beta \\
&=\operatorname{ran}^{-1} \frac{1}{3}+\operatorname{ran}^{-1} \frac{1}{5}
\end{aligned}
$$

b).

$$
\begin{aligned}
& x=2 \sin \theta \\
& d x=2 \cos \theta d \theta \\
& x=0: \quad \theta=0 \\
& x=2 \quad \sin \theta=1 \\
& \theta=\frac{\pi}{2} \pi / 2 \\
& \therefore \int_{0}^{2} \sqrt{4-x^{2}} d x=\int_{0} \sqrt{4-4 \sin ^{2} \theta} 2 \cos \theta \\
&=4 \int_{0}^{\pi / 2} \cos 2 \theta d \theta \\
&=2 \int_{0}^{\pi / 2}(1+\cos 20) d \theta
\end{aligned}
$$










