St George Girls High School

Trial Higher School Certificate Examination

2004

Mathematics
Extension 1

Total Marks – 84

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using blue or black pen
- Attempt ALL questions.
- Begin each question on a new page
- Write your student number on each page
- All necessary working must be shown.
- Diagrams are not to scale.
- Board-approved calculators may be used.
- The mark allocated for each question is listed at the side of the question.

Question	Mark
Q1	/12
Q2	/12
Q3	/12
Q4	/12
Q5	/12
Q6	/12
Q7	/12
Total	/84

Students are advised that this is a Trial Examination only and does not necessarily reflect the content or format of the Higher School Certificate Examination.

Question 1 – (12 marks) – Start a new page **Marks**

a) Differentiate the following: 3

(i) $f(x) = \tan^{-1} 3x$

(ii) $y = \log\left(\dfrac{6-x^2}{1+2x}\right)$

$u = \dfrac{6-x^2}{(1+2x)}$

$\dfrac{du}{dx} = \dfrac{-2x(1+2x) - (6-x^2) \times 2}{(1+2x)^2}$

$= -2x - 4x^2 - 12 + 2x^2$

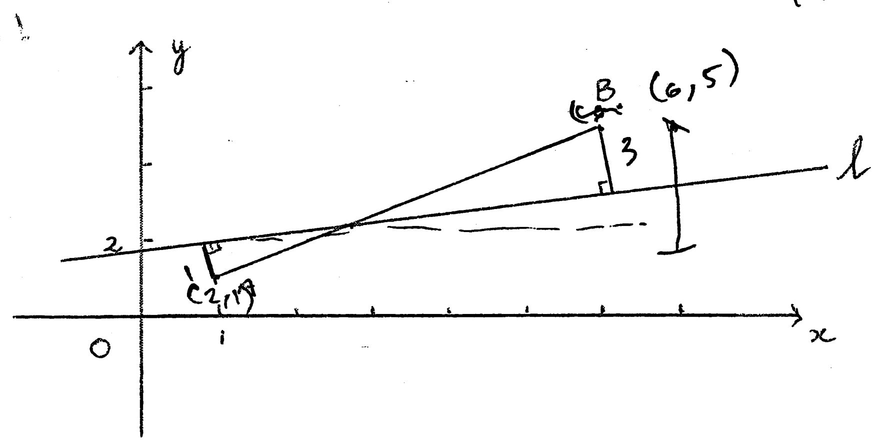
$= -2x - 12 - 2x^2$

$= \dfrac{\dfrac{-2(x^2+x+6)}{(1+2x)^{\cancel{2}}}}{\dfrac{(6-x^2)}{\cancel{(1+2x)}}}$

$= \dfrac{-2(x^2+x+6)}{(1+2x)(6-x^2)}$

b) Solve $\dfrac{3}{1-x} \geq 2$

c)



$\dfrac{y-y_1}{y_2-y_1} = \dfrac{x-x_1}{x_2-x_1}$

$\dfrac{y-1}{5-1} = \dfrac{x-2}{6-2}$

$\dfrac{y-1}{4} = \dfrac{x-2}{4}$

$y-1 = x-2$

The points $A(2, 1)$ and $B(6, 5)$ are 1 unit and 3 units respectively from the line l and are on opposite sides of l.

$y = x-1$

Find the coordinates of the point where the interval AB crosses the line l. 2

$\therefore$ when $y = 2$

$\therefore 2 = x-1$

$\therefore x = 3$

d) Using the substitution $u = 2x+1$ evaluate $\displaystyle\int_0^2 \frac{2x}{(2x+1)^2}\,dx$ 4

Question 2 – (12 marks) – Start a new page **Marks**

a) Find the value of $\lim_{x \to 0} \dfrac{x^2}{1-\cos 2x}$ 3

b) The graphs of $y = \dfrac{1}{x}$ and $y = x^3$ intersect at $x = 1$. Find the size of the acute angle between these curves at $x = 1$. 4

c) Find the exact value of $\displaystyle\int_1^{\sqrt{3}} \frac{1}{\sqrt{4-x^2}}\,dx$ 2

d) Using the identity $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$ solve the equation 3

$\sin 3\theta = 2\sin\theta \qquad 0 \le \theta \le 2\pi$

Question 3 – (12 marks) – Start a new page **Marks**

a) Find the term independent of x in the expansion of $\left(2x^2 - \frac{3}{x}\right)^{12}$ 3

b) Taking $x = 0.5$ as the first approximation use Newton's method to find a second approximation to the root of $x - e^{-x} = 0$ 3

c) Two points $P\left(2ap,\, ap^2\right)$ and $Q\left(2aq,\, aq^2\right)$ lie on the parabola $x^2 = 4ay$. 6

(i) Show that the equation of the tangent to the parabola at P is $y = px - ap^2$

(ii) The tangent at P and the line through Q parallel to the y-axis intersect at T. Find the coordinators of T.

(iii) Write down the coordinates of M, the midpoint of PT.

(iv) Determine the locus of M if $pq = -1$.

<u>Question 4</u> – (12 marks) – Start a new page **Marks**

a) Find the gradient of the tangent to $y = \cos^{-1}\frac{x}{3}$ at the point where $x = 0$. 2

b) The acceleration of a particle moving in a straight line is given by $\ddot{x} = -\dfrac{900}{x^3}$
where x metres is the displacement from the origin after t seconds.
Initially the particle is 10m to the right of the origin with velocity $3ms^{-1}$. 6

(i) Show that the velocity is given by $\dot{x} = \dfrac{30}{x}$

(ii) Find an expression for the time (t) as a function of x.

c) Prove by mathematical induction that 4

$$\frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + \ldots + \frac{1}{(3n-2)(3n+1)} = \frac{n}{3n+1}$$ for all positive integral values of n greater than or equal to 1.

Question 5 – (12 marks) – Start a new page **Marks**

a) The polynomial $P(x) = x^3 + ax^2 + bx + c$ has real roots $\sqrt{k}, -\sqrt{k}$ and γ. 4

(i) Explain why $\gamma + a = 0$

(ii) Show that $k\gamma = c$

(iii) Show $ab = c$

b) Consider the function $f(x) = \dfrac{x}{x-3}$

(i) Show that $f'(x) < 0$ for all x in the domain. 2

(ii) State the equation of the horizontal asymptote. 1

(iii) Without using any further calculus sketch the graph of $y = f(x)$.
(You should show relevant intercepts and asymptotes). 2

(iv) Explain why $f(x)$ has an inverse function $f^{-1}(x)$

(v) Find an expression for $f^{-1}(x)$ 1

(vi) Write down the domain of $f^{-1}(x)$ 1

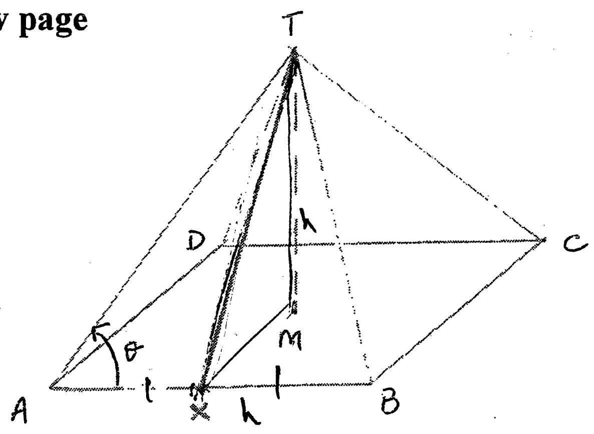
$\frac{x}{x-3}$

$= \frac{x-3+3}{x-3}$

$= 1 + \frac{3}{x-3}$

Question 6 – (12 marks) – Start a new page **Marks**

a) 3



The diagram shows a right square pyramid with base *ABCD*, vertex *T* and height *TM*. It is given that $TM = AB = h$ units. *X* is the midpoint of *AB*.

(i) Show the length of *TX* is $\frac{h}{2}\sqrt{5}$

(ii) Hence show that if $T\hat{A}B = \theta$, then $\cos\theta = \dfrac{1}{\sqrt{6}}$

b) A saucepan of water at temperature $T°C$ loses heat when placed in a cooler environment. It cools according to the law $\dfrac{dT}{dt} = k(T - T_\circ)$ where t is the time elapsed in minutes and $T_\circ$ is the temperature of the environment in degrees Celsius.

It is given that $T = T_\circ + Ae^{kt}$.

(i) A saucepan of water at 100°C is placed in an environment at $-10°$C for 8 minutes, and cools to 70°C. Find k. 3

(ii) The saucepan of water is left in this environment for a further 8 minutes. Find its temperature after this time. 2

$0{\cdot}5 = \dfrac{\pi h^2}{9}$ $\therefore$ $0{\cdot}5 = \pi 16 \dfrac{dh}{dt}$

$\therefore \dfrac{dh}{dt} = \dfrac{1}{32\pi}$

c) A cone-shaped candle whose height is three times its radius is melting at the constant rate of $0.5\text{cm}^3 s^{-1}$. 4

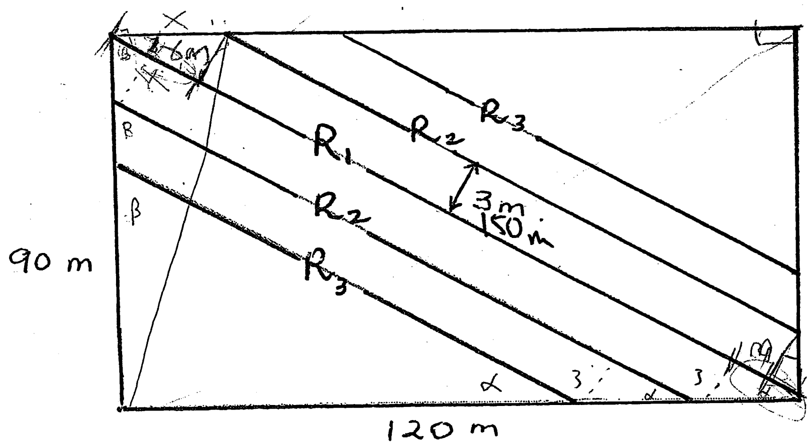
If the proportion of radius to height is preserved as the candle burns:

(i) Show that the volume of the candle is given by $V = \dfrac{\pi h^3}{27}$

(ii) Find the rate at which the height of the candle is decreasing when the candle height is 12cm.

Question 7 – (12 marks) – Start a new page **Marks**

a) A particular paddock in a vineyard measures 90m by 120m. In order to make best use of the sun the grape vines are planted in diagonal rows as shown, with a 3 metre gap between adjacent rows. 6



(i) Find the length of R_1, the diagonal of the field.

(ii) Show that the length of the equal rows, R_2 is 143.75m.

(iii) Given that the rows $R_1 + R_2 + R_3 + \ldots$ form an arithmetic series find the total number of rows of vines in the paddock.

$S = \frac{n}{2}[2a+(n-1)d]$

$S_n = \frac{n}{2}(2\times150 + (n-1)\times -6.25)$

$0 = 300n - n(n-1)\times 6.25$

$= 1200n - 25n(n-1)$

$25n^2 - 25n - 1200n = 0$

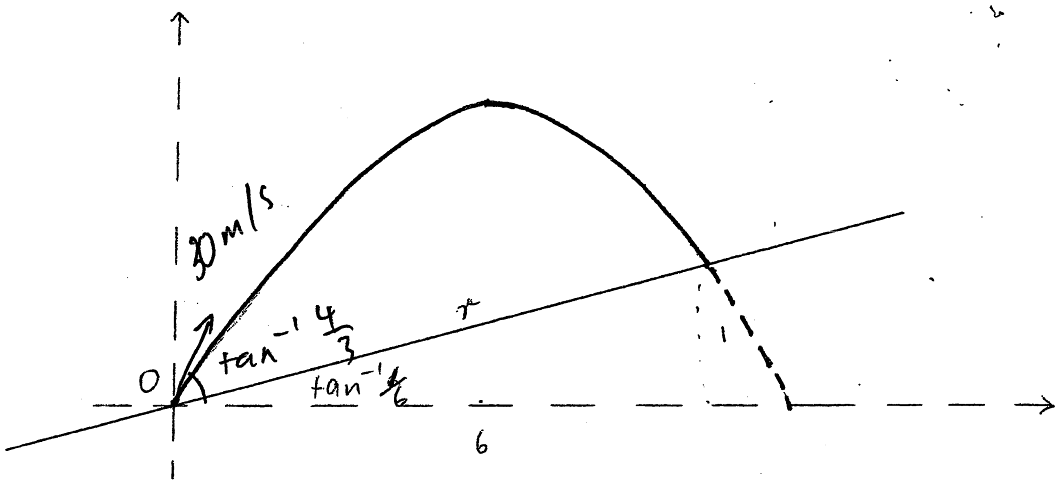
$25n^2 - 1225n = 0$

$n = \frac{1225}{25}$

Question 7 continued on page 9

Question 7 (continued) **Marks**

b) A golf ball is lying at point O on an inclined fairway as shown.



The golf ball is hit with an initial velocity of 30m/s at an angle of elevation of $\tan^{-1}\frac{4}{3}$ (You may assume that the acceleration due to gravity is 10m/s²).

The golf ball's trajectory at time t seconds after being hit may be defined by the equations $x = 18t$ and $y = 24t - 5t^2$, where x and y are the horizontal and vertical displacements, in metres, of the ball from the origin O shown in the diagram.

(i) Find the horizontal range of the ball and its greatest height if it had been hit on a horizontal part ~~of the golf~~ course. 3

(ii) If the fairway is as shown, inclined at an angle of $\tan^{-1}\frac{1}{6}$, show that the time of flight is 4.2 seconds and calculate the distance (r) the ball has been hit up the fairway (correct to 1 decimal place). 3

End of Paper

2004 3U SOLUTIONS TRIAL

2.

Jenny Q3.

QUESTION 1:

(a) (i) $y = \tan^{-1} 3x$

$$\frac{dy}{dx} = \frac{1}{1+9x^2} \cdot 3$$

1 $$= \frac{3}{1+9x^2}$$ (1)

(ii) $y = \log(6-x^2) - \log(1+2x)$ (1)

$$\frac{dy}{dx} = \frac{-2x}{6-x^2} - \frac{2}{1+2x}$$ (1)

$$= \frac{-2x-4x^2-12+2x^2}{(6-x^2)(1+2x)}$$

2 $$= \frac{-12-2x-2x^2}{(6-x^2)(1+2x)}$$

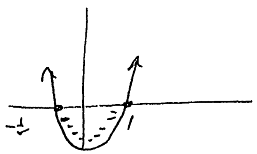
(b) $\frac{3}{1-x} \geq 2 \qquad x \neq 1$

$$\Rightarrow 3(1-x) \geq 2(1-x)^2$$

$$0 \geq 2(1-x)^2 - 3(1-x)$$

$$0 \geq (1-x)[2(1-x)-3]$$

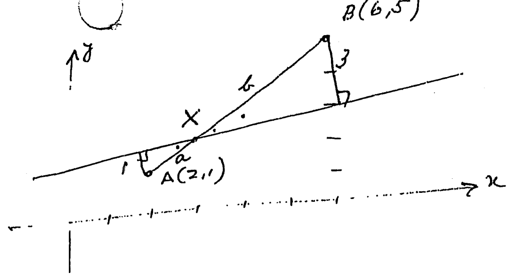
3 $$0 \geq (1-x)(-1-2x)$$



$$\Rightarrow -\tfrac{1}{2} \leq x \leq 1$$

but $x \neq 1 \Rightarrow -\frac{1}{2} \leq x < 1$

(c)



By similar triangles $\frac{a}{b} = \frac{1}{3}$ ie $a:b = 1:3$

2 $\therefore$ Point X is $\left(\frac{1\times 6 + 3\times 2}{1+3}, \frac{1\times 5 + 3\times 1}{1+3}\right)$

$$= (3, 2)$$

(d) $\displaystyle\int_0^2 \frac{2x}{(2x+1)^2}\,dx$ $\qquad u = 2x+1$, $du = 2\,dx$

$$= \int_1^5 \frac{u-1}{u^2} \times \frac{du}{2}$$

4 $$= \frac{1}{2}\int_1^5 \left(\frac{1}{u} - \frac{1}{u^2}\right) du$$ $\qquad \int u^{-2}\,du = \frac{u^{-1}}{-1}$

$$= \frac{1}{2}\left[\ln u + \frac{1}{u}\right]_1^5$$

$$= \frac{1}{2}\left[\left(\ln 5 + \frac{1}{5}\right) - (\ln 1 + 1)\right]$$

$$= \frac{1}{2}\left[\ln 5 - \frac{4}{5}\right]$$

QUESTION 2:

(a) $\lim_{x\to 0} \dfrac{x^2}{1-\cos 2x}$ $\qquad \cos 2x = 1 - 2\sin^2 x$

$= \lim_{x\to 0} \dfrac{x^2}{2\sin^2 x}$

3 $= \lim_{x\to 0} \frac{1}{2}\cdot\left(\dfrac{x}{\sin x}\right)^2$

$= \frac{1}{2}$

(b) $y = \frac{1}{x}$

$\Rightarrow y' = -\frac{1}{x^2}$

at $x=1$, $y' = -1$
$= m_1$

$y = x^3$

$y' = 3x^2$

at $x=1$, $y' = 3$
$= m_2$

If θ is the acute angle then

$\tan\theta = \left|\dfrac{m_1 - m_2}{1+m_1 m_2}\right|$

$= \left|\dfrac{-1-3}{1+(-1)(3)}\right|$

4 $= \left|\dfrac{-4}{-2}\right|$

$= 2$

$\therefore \theta = 63^\circ 26'$

(c) $\displaystyle\int_1^{\sqrt{3}} \frac{dx}{\sqrt{4-x^2}} = \sin^{-1}\frac{x}{2}\Big]_1^{\sqrt{3}}$

$= \sin^{-1}\frac{\sqrt{3}}{2} - \sin^{-1}\frac{1}{2}$

$= \frac{\pi}{3} - \frac{\pi}{6}$

2 $= \frac{\pi}{6}$

(d) $\sin 3\theta = 2\sin\theta$

$\Rightarrow 3\sin\theta - 4\sin^3\theta = 2\sin\theta$

$0 = 4\sin^3\theta - \sin\theta$

$= \sin\theta(4\sin^2\theta - 1)$

$\Rightarrow \sin\theta = 0$ OR $4\sin^2\theta - 1 = 0$

3 $\theta = 0, \pi, 2\pi$ $\qquad \sin\theta = \pm\frac{1}{2}$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

$\therefore \theta = 0, \frac{\pi}{6}, \frac{5\pi}{6}, \pi, \frac{7\pi}{6}, \frac{11\pi}{6}, 2\pi$

5.

QUESTION 3:

(a) $\left(2x^2 - \frac{3}{x}\right)^{12}$

$T_{r+1} = {}^{12}C_r\,(2x^2)^{12-r} \cdot \left(-\frac{3}{x}\right)^r$

$= {}^{12}C_r \cdot 2^{12-r} \cdot (-3)^r \cdot x^{24-3r}$ 1

Independent of $x \Rightarrow 24 - 3r = 0$
$\therefore r = 8$ 1

3

Hence term is $T_9 = {}^{12}C_8 \cdot 2^4 \cdot (-3)^8$

$= 51\,963\,120$ 1

(b) Let $f(x) = x - e^{-x}$ ~~½~~ $\Rightarrow f'(x) = 1 + e^{-x}$ ~~½~~ 1

If $x_1 = 0.5$ is first approximation then

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$ ½

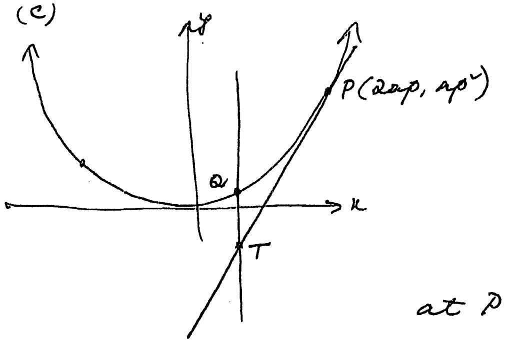
$= 0.5 - \frac{0.5 - e^{-0.5}}{1 + e^{-0.5}}$ ½

3

$= 0.566\ldots$ ½

$= 0.57$ (correct to 2 dec. places) ½

(c)



(i) $x^2 = 4ay$

$\Rightarrow y = \frac{x^2}{4a}$

$\frac{dy}{dx} = \frac{2x}{4a}$

$= \frac{x}{2a}$ ½

at $P(2ap, ap^2)$ $\frac{dy}{dx} = \frac{2ap}{2a}$

$= p$ ½

6.

$\therefore$ Tangent at P is

$y - ap^2 = p(x - 2ap)$ ½

$= px - 2ap^2$

2 ie $y = px - ap^2$ ½

(ii) Vertical through Q is $x = 2aq$ ½ 1

$\therefore$ $y = px - ap^2$ —— (1)

$x = 2aq$. —— (2)

sub x in (1)

$\Rightarrow y = p(2aq) - ap^2$ ½

$= 2apq - ap^2$ 1

2

$\therefore T \equiv (2aq,\ 2apq - ap^2)$

(iii) $M \equiv \left(\frac{2ap + 2aq}{2},\ \frac{2apq - ap^2 + ap^2}{2}\right)$

1 $= (a(p+q),\ apq)$

(iv) If $pq = -1$ then the locus of M is the line $y = -a$ since M will

1 then be $(a(p+q), -a)$ Must state locus is

~~½ for showing substitution~~

QUESTION 4.

(a) $y = \cos^{-1}\frac{x}{3}$

$$\frac{dy}{dx} = \frac{-1}{\sqrt{1-\frac{x^2}{9}}}\cdot\frac{1}{3}$$

2 at $x=0$ $\frac{dy}{dx} = \frac{-1}{1}\cdot\frac{1}{3}$

$$= -\frac{1}{3}$$

(b) $\ddot{x} = -\frac{900}{x^3}$

(i) $\frac{d}{dx}\left(\frac{1}{2}v^2\right) = -\frac{900}{x^3}$

$$\therefore \frac{1}{2}v^2 = \int -\frac{900}{x^3}\,dx$$

$$= -\frac{900x^{-2}}{-2} + c$$

$$= \frac{450}{x^2} + c$$

$$\therefore v^2 = \frac{900}{x^2} + c$$

at $x=10$ $v=3$

$\therefore 9 = 9 + c \Rightarrow c = 0$

$$\therefore v^2 = \frac{900}{x^2}$$

ie $v = \pm\frac{30}{x}$

3

but initially $v > 0$ and clearly v cannot be zero.

$$\therefore v = \dot{x} = \frac{30}{x}$$

(ii) $\dot{x} = \frac{30}{x}$

$$\Rightarrow \frac{dx}{dt} = \frac{30}{x}$$

$$\therefore \frac{dt}{dx} = \frac{x}{30}$$

$$t = \frac{x^2}{60} + c$$

at $t=0$, $x=10$

$\therefore 0 = \frac{100}{60} + c$

$\therefore c = -\frac{5}{3}$

3 $\therefore t = \frac{x^2}{60} - \frac{5}{3}$

when $x = 100$

$t = \frac{100^2}{60} - \frac{5}{3}$

$= 165$ s

$\therefore$ at 165 seconds

(c) (i) Test for $n=1$

LHS $= \frac{1}{1.4}$ RHS $= \frac{1}{4}$

$= \frac{1}{4}$

$\therefore$ True for $n=1$

(ii) Assume assertion is true for some integer $n=k$

$$\frac{1}{1.4} + \frac{1}{4.7} + \cdots + \frac{1}{(3k-2)(3k+1)} = \frac{k}{3k+1}$$

[And we aim to then prove it true for $n = k+1$

ie $\frac{1}{1\cdot4} + \frac{1}{4\cdot7} + \cdots + \frac{1}{(3k-2)(3k+1)} + \frac{1}{(3k+1)(3k+4)} = \frac{k+1}{3(k+1)+1}$]

now $\frac{1}{1\cdot4} + \frac{1}{4\cdot7} + \cdots + \frac{1}{(3k-2)(3k+1)} + \frac{1}{(3k+1)(3k+4)}$

$= \frac{k}{3k+1} + \frac{1}{(3k+1)(3k+4)}$ using ①

$= \frac{k(3k+4)+1}{(3k+1)(3k+4)}$

$= \frac{3k^2+4k+1}{(3k+1)(3k+4)}$

$= \frac{(k+1)\cancel{(3k+1)}}{\cancel{(3k+1)}(3k+4)}$

$= \frac{k+1}{3k+4}$

$= \frac{n}{3n+1}$ where $n = k+1$

Hence, if the assertion is true for $n=k$ then it is also true for $n = k+1$

But since true for $n=1$, it must be true for $n=2$ and then by the principle of mathematical Induction it is true for all integers $n \geq 1$

QUESTION 5

(a) $P(x) = x^3 + ax^2 + bx + c$ Roots are $\sqrt{k}, -\sqrt{k}, \gamma$

(i) Sum of roots $\Rightarrow \sqrt{k} + (-\sqrt{k}) + \gamma = -a$

$\therefore \gamma + a = 0.$ ——①

1

(ii) Product of roots $\Rightarrow \sqrt{k} \times (-\sqrt{k}) \times \gamma = -c$

$\therefore k\gamma = c$ ——②

1

(iii) sum of roots 2 at time

$\Rightarrow -k + \gamma\sqrt{k} - \gamma\sqrt{k} = b$

$\therefore -k = b$

2 from ①: $\gamma = -a$

$\therefore$ ② $\Rightarrow -ka = c$

but $-k = b$

$\Rightarrow ab = c$

(b) $f(x) = \frac{x}{x-3}$ $x \neq 3$

(i) $f'(x) = \frac{(x-3).1 - x.1}{(x-3)^2}$

2 $= \frac{-3}{(x-3)^2}$

< 0 for all x since $(x-3)^2 > 0$ for all $x \neq 3$

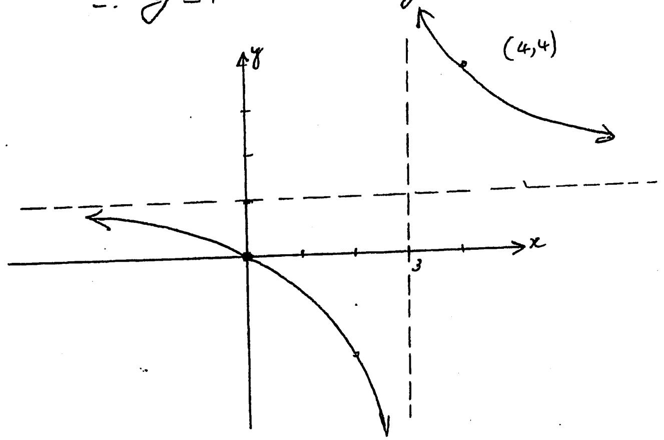
11.

(ii) as $x \to \infty \quad f(x) \to 1$

$\therefore y = 1$ is horizontal asymptote

1

(iii)



2

(iv) Inverse exists because for any y-value there is at most 1 x-value.

1

(v) $f: \quad y = \dfrac{x}{x-3}$ D: all real x, $x \neq 3$ R: all real y, $y \neq 1$

$f^{-1}: \quad x = \dfrac{y}{y-3}$

$\Rightarrow xy - 3x = y$

$xy - y = 3x$

$y(x-1) = 3x$

$y = \dfrac{3x}{x-1}$

1

ie $f^{-1}(x) = \dfrac{3x}{x-1}$

(vi) Domain: all real x, $x \neq 1$

1

a) i) In ΔTMX

$TX^2 = XM^2 + TM^2$

$= \left(\frac{h}{2}\right)^2 + h^2$

1

$= \frac{h^2}{4} + h^2$

$= \frac{5h^2}{4}$

$\therefore T = \frac{h}{2}\sqrt{5}$

ii) In ΔTXA

$AT^2 = TX^2 + AX^2$

$= \frac{5h^2}{4} + \left(\frac{h}{2}\right)^2$

2

$= \frac{5h^2}{4} + \frac{h^2}{4}$

$= \frac{6h^2}{4}$

$AT = \frac{h\sqrt{6}}{2}$

$\cos\theta = \frac{AX}{AT}$

$= \frac{h}{2} \div \frac{h\sqrt{6}}{2}$

$= \frac{h}{2} \times \frac{2}{h\sqrt{6}}$

$= \frac{1}{\sqrt{6}}$.

b) $\frac{dT}{dt} = k(T - T_0)$

(i) $\Rightarrow T = T_0 + Ae^{kt}$

$T = -10 + Ae^{kt}$

at $t = 0$, $T = 100$

$\therefore 100 = -10 + Ae^0$

$\therefore A = 110$

3

ie $T = -10 + 110e^{kt}$

at $t = 8$, $T = 70$

$\therefore 70 = -10 + 110e^{8k}$

$80 = 110e^{8k}$

$e^{8k} = \frac{8}{11}$

$8k = \ln\left(\frac{8}{11}\right)$

(ii) at $t = 16$ $\quad T = -10 + 110e^{16k}$

$= -10 + 110\left(e^{8k}\right)^2$

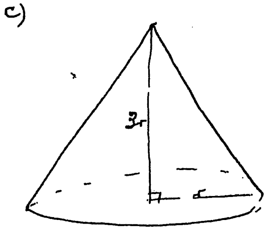
$= -10 + 110\left(\frac{8}{11}\right)^2$

2

$= 48.18\ldots$

$= 48.2^\circ C$ (correct to 1 decpl)

(c)



$\frac{dV}{dt} = 0.5 \text{ cm}^3/s$

(i) $V = \frac{1}{3} \cdot \pi r^2 \cdot h$

$= \frac{1}{3} \cdot \pi \cdot r^2 \cdot 3r$

$= \pi r^3 \qquad r = \frac{h}{3}$

$= \pi\left(\frac{h}{3}\right)^3$

2 $\quad = \frac{\pi h^3}{27}$

(ii) $\frac{dV}{dh} = \frac{\pi h^2}{9} \qquad \frac{dV}{dt} = 0.5 \text{ cm}^3/s$

$\frac{dh}{dt} = \frac{dh}{dV} \cdot \frac{dV}{dt}$ cm/s

$= \frac{9}{\pi h^2} \cdot 0.5$ cm/s

2 at $h = 12$

$\frac{dh}{dt} = \frac{9}{\pi \cdot 144} \times 0.5$ cm/s

$= \frac{1}{32\pi}$ cm/s

QUESTION 7:

(a) (i) $d^2 = 120^2 + 90^2$ where d is diagonal

1 $\quad \therefore d = 150$ m

(ii) $\tan\alpha = \frac{3}{4} \Rightarrow x = 4$

$\tan\beta = \frac{4}{3} \Rightarrow \frac{4}{3} = \frac{3}{y}$

$\therefore 4y = 9$

$y = \frac{9}{4}$

2

$\therefore$ Diagonal $= 150 - 4 - \frac{9}{4}$

$= 143\frac{3}{4}$ m

(iii) next line $= 143\frac{3}{4} - 4 - \frac{9}{4}$

$= 137.5$ m

$\therefore 150 + 2\Big[\underbrace{143\tfrac{3}{4} + 137\tfrac{1}{2} + \cdots}_{\text{arithmetic series}}\Big]$

$a = 143\frac{3}{4}, \quad d = -6\frac{1}{4}$

3 $\quad T_n = 143\frac{3}{4} + (n-1)\cdot\left(-6\frac{1}{4}\right)$

$= 150 - \left(6\frac{1}{4}\right) \cdot n$

want $T_n > 0$

$\therefore 150 - \frac{25n}{4} > 0$

$150 > \frac{25n}{4}$

$600 > 25n$

$n < 24$

$\therefore$ There are 23 terms.

(b) $x = 18t$ $\qquad y = 24t - 5t^2$

(i) $\dot{y} = 24 - 10t$

max. ht at $\dot{y} = 0$

ie $24 - 10t = 0$

$t = 2.4$

$\therefore y_{max} = 24(2.4) - 5(2.4)^2$

$= \underline{28.8\,m}$

Hits ground at $y = 0$

3 ie $24t - 5t^2 = 0$

$t(24 - 5t) = 0$

$t = \frac{24}{5}$

at $t = \frac{24}{5}$ $\qquad x = 18 \times \frac{24}{5}$

$= \underline{86.4\,m}$

(ii) Let ball be at $P(x, y)$

Then $\tan\alpha = \frac{1}{6} \Rightarrow \frac{y}{x} = \frac{1}{6}$

$\frac{24t - 5t^2}{18t} = \frac{1}{6}$

$144t - 30t^2 = 18t$

$0 = 30t^2 - 126t$

$0 = 3t(10t - 42)$

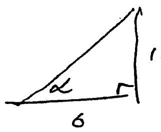
$\therefore$ at P, $t = \frac{42}{10}$

$= 4.2.$

at $t = 4.2$, $\qquad x = 18 \times 4.2$

$= 75.6$

now $\tan\alpha = \frac{1}{6}$



$\Rightarrow \cos\alpha = \frac{6}{\sqrt{37}}$

Then $\frac{6}{\sqrt{37}} = \frac{75.6}{r}$

$\therefore r = \frac{75.6 \times \sqrt{37}}{6}$

$= \underline{76.6\,m}$