**St George Girls High School**

**Trial Higher School Certificate Examination**

**2006**

# Mathematics Extension 1

**Total Marks – 84**

## General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using blue or black pen
- Attempt ALL questions.
- Begin each question on a new booklet
- Write your student number on each page
- All necessary working must be shown.
- Diagrams are not to scale.
- Board-approved calculators may be used.
- The mark allocated for each question is listed at the side of the question.

**Students are advised that this is a Trial Examination only and does not necessarily reflect the content or format of the Higher School Certificate Examination.**

**<u>Question 1</u>** – 12 marks (Start a new booklet) **Marks**

a) Differentiate $\log_e(\sin^2 x)$, writing your answer in simplest form. 2

b) Find the acute angle, to the nearest degree, between the lines $y = 2x + 3$ and $x + y = 0$ 2

c) Find the value of $k$ if $(x - 2)$ is a factor of $P(x) = x^4 - 3x^3 + kx^2 - 4$ 2

d) Evaluate $\int_0^{0.25} \frac{dx}{\sqrt{1 - 4x^2}}$ 3

e) Solve the inequality $\frac{3x - 2}{x} \geq 1$ 3

**Question 2** – 12 marks (Start a new booklet) **Marks**

a) Use the substitution $u = 3 - x^2$ to find $\int \frac{x}{\sqrt{3 - x^2}}\, dx$ 3

b) Find the coefficient of $x$ in $\left(x - \frac{2}{x^2}\right)^{10}$ 3

c) (i) Show the derivative of $x \tan^{-1} x$ is $\frac{x}{1 + x^2} + \tan^{-1} x$ 3

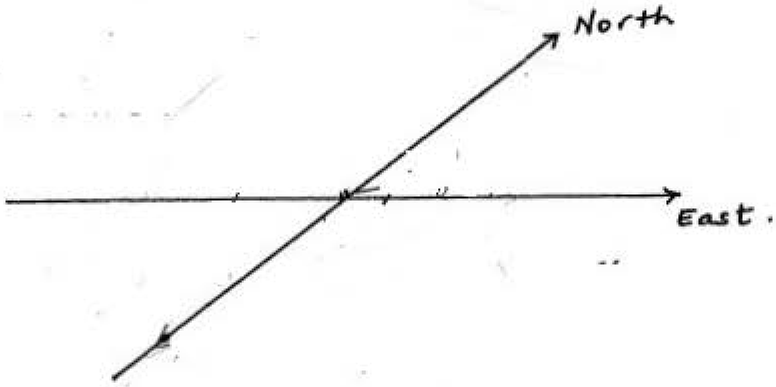
(ii) Hence, or otherwise, find $\int \tan^{-1} x\, dx$

d) Solve the equation $\cos^2 x + \sin 2x = 0,\quad 0^\circ \le x \le 360^\circ$ 3

**<u>Question 3</u>** – 12 marks (Start a new booklet) **Marks**

a) From a balloon 800 metres above a road intersection, the angle of depression of a point, $P$, on the road due south of the intersection is $45^\circ$. The angle of depression of another point, $Q$, is $35^\circ$. $Q$ is at ground level on a bearing of $080^\circ$ from the intersection. 4

(i) Copy and complete the diagram to show the above information.




(ii) Find the distance from $P$ to $Q$.

b) Consider the function $y = 2\cos^{-1}(1 - x)$ 4

(i) Find the domain and range of the function.

(ii) Sketch the graph of the function.

c) The radius of a circular oil spill ($r$ km), at a time $t$ hours after it was first observed is given by $r = \dfrac{1+3t}{1+t}$. Find the exact rate of increase of the area of the oil spill when the radius is 2 kilometres 4

**Question 4** – 12 marks (Start a new booklet) **Marks**

a) The arc of the curve $y = \sin 2x$ between the lines $x = 0$ and $x = \frac{\pi}{8}$ is rotated about the $x$-axis.

Find the volume of the solid formed. 4

b) Prove by induction: 4

$$(n+1)(n+2)(n+3)\ldots 2n = 2^n[1\times 3\times 5\times \ldots \times (2n-1)]$$

for integral values of $n \geq 1$

c) The velocity $v\ ms^{-1}$ of a particle moving in simple harmonic motion along the $x$-axis is $v^2 = 16-(x-2)^2$ 4

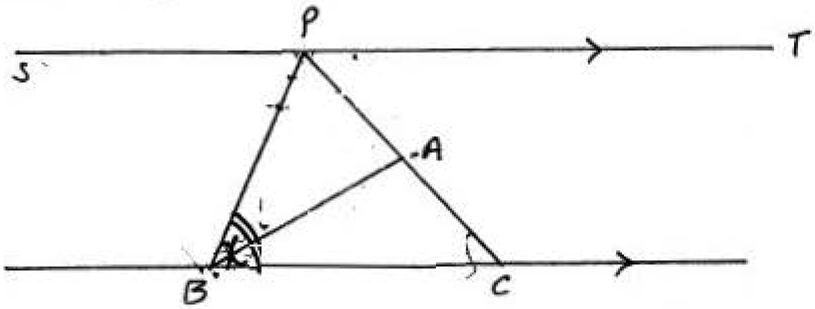
(i) Between which two points is the particle oscillating?

(ii) What is the amplitude of the motion?

(iii) Find the acceleration of the particle in terms of $x$.

**Question 5** – 12 marks (Start a new booklet) **Marks**

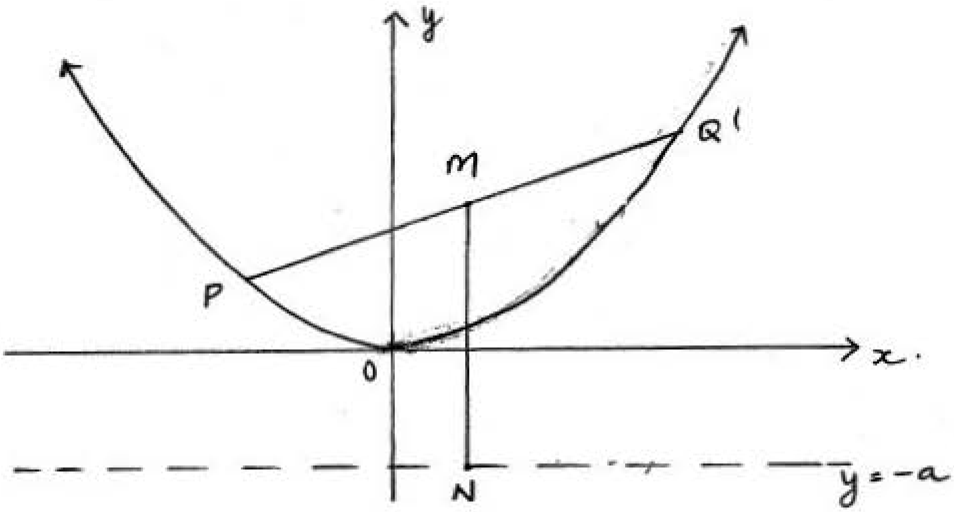
a) In the diagram $ST \parallel BC$ and $\angle SPB = \angle PAB$ 4




(i) Prove $\angle PBA = \angle PCB$

(ii) Deduce that $PB^2 = PA \times PC$

b) 5




Let $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ be points on the parabola $x^2 = 4ay$ as shown in the diagram.

(i) Show that the equation of $PQ$ is $y = \dfrac{p+q}{2}x - apq$

(ii) Show that if the chord $PQ$ passes through the focus $S(o, a)$ ~~that~~ then $pq = -1$

(iii) $M$ is the midpoint of the focal chord $PQ$ and $N$ lies on the directrix vertically below $M$. If $T$ is the midpoint of $MN$, find the locus of $T$.

**Question 5c) on next page**

## Question 5 (cont'd) **Marks**

c) The rate at which a body cools in air is proportional to the difference between the constant air temperature, $C$, and its own temperature, $T$. This can be expressed by the differential equation 3

$$\frac{dT}{dt} = -k(T - C)$$

where $t$ is time in hours and $k$ is a constant.

(i) Show that $T = C + Ae^{-kt}$ is a solution of the differential equation, where $A$ is a constant.

(ii) A heated piece of metal cools from $90^o$ to $65^o$ in 1 hour. The air temperature $C$ is $20^o$. Find the values of $A$ and $k$.

**Question 6** – 12 marks (Start a new booklet) **Marks**

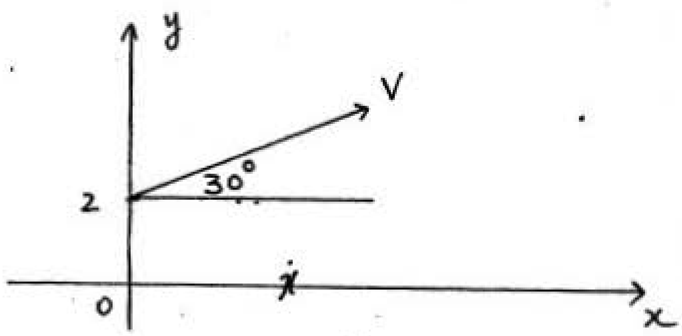
a) The polynomial $P(x) = 3x^3 + x^2 + 1$ has one real root in the interval $-1 < x < 0$ 6

(i) Sketch the graph of $y = P(x)$ for $x$ between $-1$ and 1. Clearly label any stationary points.

(ii) Let $x = -0.2$ be a first approximation to the root. Apply Newtons method once to obtain another approximation to the root.

(iii) Explain why the application of Newtons method in part (ii) was <u>not</u> effective in improving the approximation to the root.

b)




A particle is projected from the point $(0, 2)$ at an angle of $30°$ with a velocity of $V$ metres per second. The equations of motion of the particle are 6

$$\ddot{x} = 0 \text{ and } \ddot{y} = -g.$$

(i) Using calculus, derive the expressions for the position of the particle at time $t$.

Hence show the path of the particle is given by $y = 2 + \dfrac{x}{\sqrt{3}} - \dfrac{2gx^2}{3V^2}$

A soccer player 'heads' a ball with initial speed $V$ metres per second and angle of projection $30°$. At that moment the ball is 2 metres above the ground and its horizontal distance from the goal is 7.3 metres. The ball just misses the goal by scraping the top of the crossbar which is 2.5 metres high.

(ii) Find the initial speed of the ball correct to one decimal place. (take $g = 9.8ms^{-2}$)

**Question 7** – 12 marks (Start a new booklet) **Marks**

a) A particle is projected from the origin with a velocity given by $v = 2(x+1)$ 2

where $x$ is the distance from the origin in metres and $v$ is in metres per second.

Find an expression for the displacement $(x)$ as a function of the time elapsed $(t)$.

b) Given the binomial expansion 3

$$1 + {}^nC_1\,x + {}^nC_2\,x^2 + \ldots + {}^nC_n x^n = (1+x)^n$$

(i) Show that $1 - {}^nC_1 + {}^nC_2 + \ldots + (-1)^n\ {}^nC_n = 0$

(ii) Show that $1 + \frac{1}{2}{}^nC_1 + \frac{1}{3}{}^nC_2 + \ldots + \frac{1}{n+1}\ {}^nC_n = \dfrac{2^{n+1}}{n+1} - \dfrac{1}{n+1}$

c) Consider the function $f(x) = \dfrac{1}{3}\left[(x-1)^2 + 5\right]$

(i) Sketch the parabola $y = f(x)$ showing clearly any intercepts with the axes and the coordinates of its vertex. Use the same scale on both axes. 7

(ii) What is the largest domain containing the value $x = 2$, for which the function has an inverse function $f^{-1}(x)$?

(iii) Sketch the graph of $y = f^{-1}(x)$ on the same set of axes as your graph in part (i). Label the two graphs clearly.

(iv) What is the domain of the inverse function?

(v) Find the coordinates of any points of intersection of the two curves

$y = f(x)$ and $y = f^{-1}(x)$

**End of Paper**

# 2006 Extension 1 Solutions

## Question 1

(a) $y = \log_e(\sin^2 x)$ OR $y = \log_e(\sin^2 x)$

$$\frac{dy}{dx} = \frac{2\sin x \cos x}{\sin^2 x}$$

$$= \frac{2\cos x}{\sin x}$$

$$= 2\cot x$$

$$= \begin{cases} 2\log_e \sin x & \sin x > 0 \\ 2\log_e(-\sin x) & \sin x < 0 \end{cases}$$

$$\frac{dy}{dx} = \begin{cases} \frac{2.\cos x}{\sin x} & \sin x > 0 \\ \frac{2(-\cos x)}{(-\sin x)} & \sin x < 0 \end{cases}$$

$$= 2\cot x$$

(b) $y = 2x + 3$ $\qquad x + y = 0$

Let $m_1 = 2$ $\qquad y = -x$

Let $m_2 = -1$

Let $\theta$ be the acute angle between the lines

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$= \left|\frac{2 - -1}{1 + 2\times -1}\right|$$

$$= 3$$

$$\therefore \quad \theta = 71.565\ldots = 71°34'$$

$$= 72° \text{ (to the nearest degree)}$$

(d) $$\int_0^{0.25} \frac{dx}{\sqrt{1-4x^2}} = \int_0^{0.25} \frac{dx}{2\sqrt{\frac{1}{4} - x^2}}$$

$$= \left[\frac{1}{2} \cdot \sin^{-1}\left(\frac{x}{\frac{1}{2}}\right)\right]_0^{0.25}$$

$$= \frac{1}{2}\left[\sin^{-1} 2x\right]_0^{0.25}$$

$$= \frac{1}{2}\left(\sin^{-1}\tfrac{1}{2} - \sin^{-1} 0\right)$$

$$= \frac{1}{2} \times \frac{\pi}{6}$$

$$= \frac{\pi}{12}$$

(c) $P(x) = x^4 - 3x^3 + kx^2 - 4$

$P(2) = 0$ if $x-2$ is a factor of $P(x)$

$0 = 2^4 - 3\times 2^3 + k\times 2^2 - 4$

$= 16 - 24 + 4k - 4$

$4k = 12$

$k = 3$

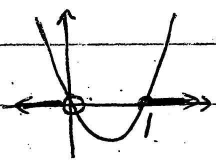
(e) $\frac{3x-2}{x} \geq 1 \qquad x \neq 0$

$\frac{3x-2}{x} - 1 \geq 0$

$\frac{3x-2-x}{x} \geq 0$

$\frac{2x-2}{x} \geq 0$

$x^2 \times \frac{(2x-2)}{x} \geq 0 \times x^2$




$x(2x-2) \geq 0 \qquad x \neq 0$

$x < 0$ or $x \geq 1$

Question 2

(a) $\int \frac{x}{\sqrt{3-x^2}}\,dx$ $\qquad u = 3-x^2$

$\qquad du = -2x\,dx$

$= -\frac{1}{2}\int \frac{-2x}{\sqrt{3-x^2}}\,dx$

$= -\frac{1}{2}\int \frac{1}{\sqrt{u}}\,du$

$= -\frac{1}{2}\,\frac{u^{1/2}}{1/2} + c$

$= -\sqrt{3-x^2} + c$

(b) $T_{k+1} = \binom{10}{k} x^{10-k}\left(\frac{-2}{x^2}\right)^k$

$= 2^k\binom{10}{k} x^{10-3k}(-1)^k$

For term in $x$ $\quad 10-3k=1$

$k = 3$

$\therefore$ Coefficient of $x = -2^3 \times \binom{10}{3}$

$= -8 \times \binom{10}{3}$

$(= -960)$

(c) (i) $\frac{d}{dx}(x\tan^{-1}x) = 1.\tan^{-1}x + x.\frac{d\tan^{-1}x}{dx}$

$= \tan^{-1}x + x.\frac{1}{1+x^2}$

$= \frac{x}{1+x^2} + \tan^{-1}x$

(ii) $\int \tan^{-1}x\,dx = \int \frac{d(x\tan^{-1}x)}{dx}\,dx - \int \frac{x}{1+x^2}\,dx$

$= x\tan^{-1}x - \frac{1}{2}\ln(1+x^2) + c$

(d) $\cos^2 x + \sin 2x = 0 \qquad 0° \le x \le 360°$

$\cos^2 x + 2\sin x \cos x = 0$

$\cos x(\cos x + 2\sin x) = 0$

$\cos x = 0$ or $2\sin x = -\cos x$ ($\cos x \neq 0$ since $\sin x \neq$ when $\cos x = 0$)

$x = 90°, 270°$

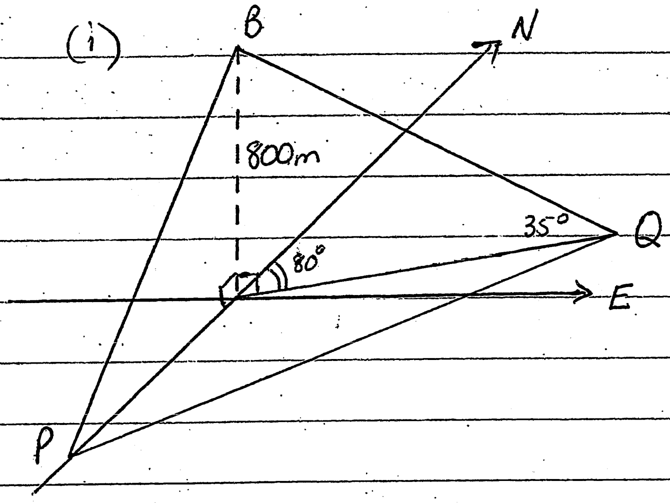
$\tan x = -\frac{1}{2}$

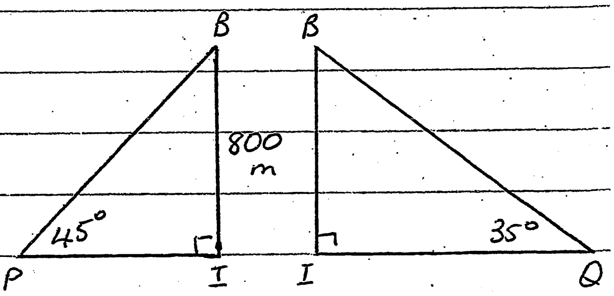
$x = 180° - 26°34', 360° - 26°34'$

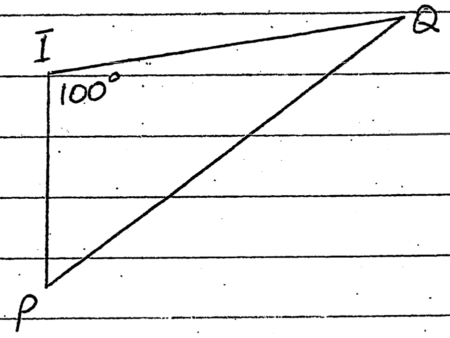
$= 153°26', 333°26'$

## Question 3

(a) (i)




In $\triangle PBI$

$PI = BI$

$= 800$

In $\triangle BIQ$

$\frac{IQ}{800} = \tan 55^\circ$

$IQ = 800 \tan 55^\circ$

$= 1142.518\ldots$

In $\triangle IPQ$

$PQ^2 = IP^2 + IQ^2 - 2 \cdot IP\, IQ \cos 100^\circ$

$= 800^2 + 1142.518\ldots^2 - 2 \times 800 \times 1142.518\ldots \cos 100^\circ$

$= 2262787.38$

$PQ = 1504.256\ldots$

Distance from P to Q is 1504 m (nearest metre)

(b) $y = 2\cos^{-1}(1-x)$

(i) Domain: $-1 \le 1-x \le 1$

$-2 \le -x \le 0$

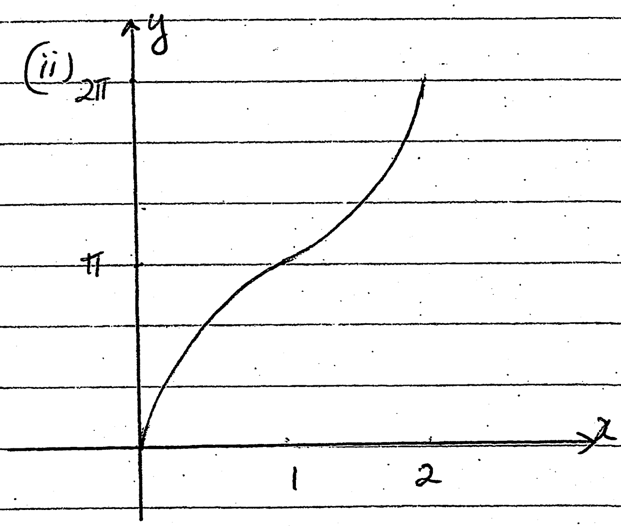
$2 \ge x \ge 0$

$0 \le x \le 2$

Range: $0 \le \frac{y}{2} \le \pi$

$0 \le y \le 2\pi$

(ii)



(c) $r = \dfrac{1+3t}{1+t}$

$$A = \pi r^2$$

$$\frac{dA}{dt} = \frac{dA}{dr}\,\frac{dr}{dt}$$

$$= 2\pi r \cdot \frac{dr}{dt}$$

$$r = \frac{1+3t}{1+t}$$

$$\frac{dr}{dt} = \frac{3(1+t) - (1+3t)\times 1}{(1+t)^2}$$

$$= \frac{3+3t-1-3t}{(1+t)^2}$$

$$= \frac{2}{(1+t)^2}$$

When $r = 2$ $\quad \dfrac{1+3t}{1+t} = 2$

$$1+3t = 2+2t$$

$$t = 1$$

$$\therefore \frac{dA}{dt} = 2\pi \times 2 \times \frac{2}{(1+1)^2}$$

$$= 2\pi$$

OR

$$A = \pi r^2$$

$$= \pi\left(\frac{1+3t}{1+t}\right)^2$$

When $r = 2$ $\quad t = 1$

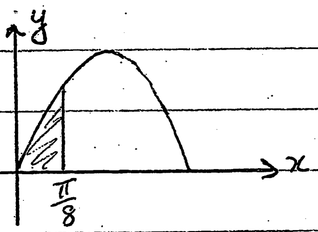
$$\frac{dA}{dt} = 2\pi\left(\frac{1+3t}{1+t}\right) \times \frac{2}{(1+t)^2}$$

$$= 2\pi\left(\frac{1+3\times 1}{1+1}\right) \times \frac{2}{2^2}$$

$$= 2\pi$$

Question 4

(a)




$$V = \pi \int_0^{\frac{\pi}{8}} \sin^2 2x \, dx$$

$$= \frac{\pi}{2} \int_0^{\pi/8} (1 - \cos 4x) \, dx$$

$$= \frac{\pi}{2} \left[ x - \frac{\sin 4x}{4} \right]_0^{\pi/8}$$

$$= \frac{\pi}{2} \left( \left( \frac{\pi}{8} - \frac{\sin \pi/2}{4} \right) - \left( 0 - \frac{\sin 0}{4} \right) \right)$$

$$= \frac{\pi^2}{16}$$

(b) RTP $(n+1)(n+2)(n+3)\dots(2n) = 2^n(1 \times 3 \times 5 \times \dots (2n-1))$
for $n \in \mathbb{Z}^+$ $(n \geq 1)$

When $n=1$ $\quad$ LHS $= 1+1 = 2$
$\quad$ RHS $= 2^1 \times 1 = 2$
$\therefore$ Proposition is true for $n=1$

Let $n=k$ be a positive integer for which proposition is t
ie $(k+1)(k+2)(k+3)\dots(2k) = 2^k(1 \times 3 \times 5 \times \dots (2k-1))$

Want to show proposition is then true for $n=k+1$
ie $(k+1+1)(k+1+2)(k+1+3)\dots(2(k+1)) = 2^{k+1}(1 \times 3 \times 5 \times \dots \times (2k-1)(2k$

$$\text{LHS} = (k+2)(k+3)(k+4)\dots(2k-1)2k(2k+1)(2(k+1))$$
$$= (k+1)(k+2)(k+3)\dots(2k) \cdot 2(2k+1)$$
$$= 2^k(1 \times 3 \times 5 \times \dots \times (2k-1)) \times 2(2k+1)$$
$$= 2^{k+1}(1 \times 3 \times 5 \times \dots \times (2k-1)(2k+1))$$
$$= \text{RHS}$$

ie If proposition is true for $n=k$ it is also true for $n=k+1$
Since proposition is true for $n=1$ it is also true for $n=2$
and hence by induction proposition is true for all positive integers

(e) $v^2 = 16-(x-2)^2$

(i) Since $v^2 \geq 0$

$$16-(x-2)^2 \geq 0$$

$$(x-2)^2 \leq 16$$

$$-4 \leq x-2 \leq 4$$

$$-2 \leq x \leq 6$$

$\therefore$ Oscillates between $x=-2$ and $x=6$

(ii) Amplitude is 4

(iii) $$\ddot{x} = \frac{d\left(\frac{1}{2}v^2\right)}{dx}$$

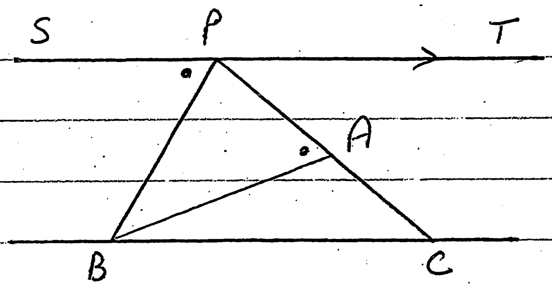
$$= \frac{d\left[8-\frac{1}{2}(x-2)^2\right]}{dx}$$

$$= -\frac{1}{2}\times 2(x-2)\times 1$$

$$= 2-x$$

Question 5

(a)




(i) Let $S\hat{P}B = \theta$

$\therefore$ $P\hat{A}B = \theta$ (given $S\hat{P}B = P\hat{A}B$)

$P\hat{B}C = S\hat{P}B$ (alternate angles equal, $ST \parallel BC$)

$= \theta$

In $\triangle PAB$, $P\hat{B}A = 180^\circ - (\theta + A\hat{P}B)$ (angle sum of $\triangle$ is $180^\circ$)

In $\triangle PBC$ $P\hat{C}B = 180^\circ - (\theta + C\hat{P}B)$ (angle sum of $\triangle$ is $180^\circ$)

$\therefore$ $P\hat{B}A = P\hat{C}B$ $(180^\circ - (\theta + A\hat{P}B))$

(ii) In $\triangle$s $PBA$, $PBC$

$A\hat{P}B$ is common

$P\hat{B}A = P\hat{C}B$ (shown above)

$\therefore$ $\triangle PBA \,|||\, \triangle PCB$ (equiangular)

$\therefore$ $\dfrac{PB}{PC} = \dfrac{PA}{PB} = \dfrac{BA}{CB}$ (corresponding sides in similar triangles in same ratio)

$\therefore$ $PB^2 = PA \times PC$

(b) $P(2ap, ap^2)$ $Q(2aq, aq^2)$

$$\text{Grad } PQ = \frac{ap^2 - aq^2}{2ap - 2aq}$$

$$= \frac{a(p-q)(p+q)}{2a(p-q)}$$

$$= \frac{p+q}{2}$$

$\therefore$ Equation of $PQ$ is

$$y - ap^2 = \frac{p+q}{2}(x - 2ap)$$

$$= \frac{p+q}{2}x - ap(p+q)$$

$$= \frac{(p+q)}{2}x - ap^2 - apq$$

$$y = \frac{p+q}{2}x - apq$$

(ii) If $PQ$ passes through $S(0, a)$ then

$$a = \frac{p+q}{2} \times 0 - apq$$

$$= -apq$$

$$\therefore pq = -1$$

(iii) Coordinates $M$ are $\left(\frac{2ap+2aq}{2}, \frac{ap^2+aq^2}{2}\right)$

$$\left(a(p+q), \frac{a(p^2+q^2)}{2}\right)$$

Coords of $N$ are $(a(p+q), -a)$

Midpt of $MN$ is $\left(a(p+q), \frac{\frac{a(p^2+q^2)}{2} + -a}{2}\right)$

$\therefore$ $T$ is $\left(a(p+q), \frac{a}{4}(p^2+q^2-2)\right)$

Locus of $T$:

$$x = a(p+q)$$

$$y = \frac{a}{4}(p^2+q^2-2)$$

$$= \frac{a}{4}(p^2+q^2+2pq) \quad \text{since } pq = -1$$

$$= \frac{a(p+q)^2}{4}$$

$$= \frac{a}{4} \cdot \left(\frac{x}{a}\right)^2$$

$$= \frac{x^2}{4a}$$

$$x^2 = 4ay$$

(e) $\dfrac{dT}{dt} = -k(T-c)$

(i) If $T = C + Ae^{-kt}$

LHS $= \dfrac{dT}{dt} = 0 + A.-ke^{-t}$
$= -kAe^{-kt}$

RHS $= -k(T-C)$
$= -k(C + Ae^{-kt} - C)$
$= -k.Ae^{-kt}$
$= LHS$

$\therefore T = C + Ae^{-kt}$ satisfies the differential equation

(ii) When $t = 0$ $T = 90$ $C = 20$
$t = 1$ $T = 65$

$T = 20 + Ae^{-kt}$
$90 = 20 + Ae^{0}$
$A = 70$

$\therefore T = 20 + 70e^{-kt}$
$65 = 20 + 70e^{-k}$
$45 = 70e^{-k}$
$e^{-k} = \dfrac{45}{70} = \dfrac{9}{14}$
$e^{k} = \dfrac{14}{9}$
$k = \log_e\left(\dfrac{14}{9}\right)$
$(\doteqdot 0.4418\ldots)$

Question 6

(a) (i) $P(x) = 3x^3 + x^2 + 1$

$P'(x) = 9x^2 + 2x$

$= x(9x+2)$

Stationary points occur when $P'(x) = 0$

$x = 0, -\frac{2}{9}$

$P(0) = 1$

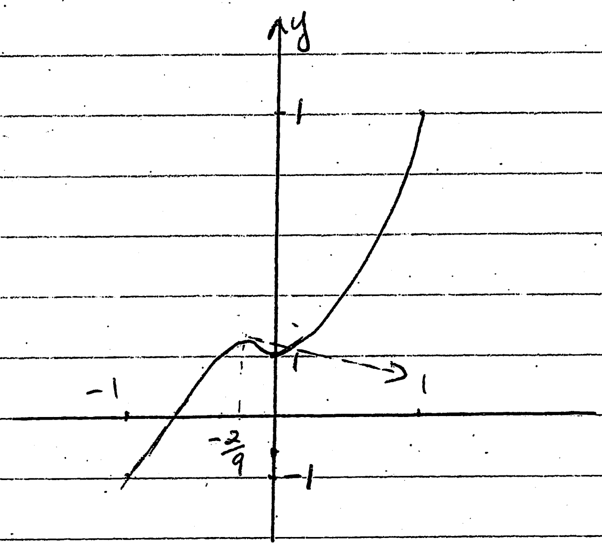
$P(-\frac{2}{9}) = 3\times\left(-\frac{2}{9}\right)^3 + \left(-\frac{2}{9}\right)^2 + 1$

$= -\frac{8}{243} + \frac{4}{81} + 1$

$= 1\frac{4}{243}$

$P(-1) = -3+1+1 = -1$

$P(1) = 3+1+1 = 5$




(ii) Newton's method does not give a better approxi[mation] to the root because the first approximation is on the opposite side of a turning point to the poi[nt] where root occurs

$\therefore$ tangent intersects $x$ axis awa

(ii) If $x_0$ is a first approximation then a second approximation ~~is~~, $x_1$, is

$x_1 = x_0 - \frac{P(x_0)}{P'(x_0)}$

$x_1 = -0.2 - \frac{P(-0.2)}{P'(-0.2)}$

$= -0.2 - \frac{1.016}{-0.04}$

$= 25.2$

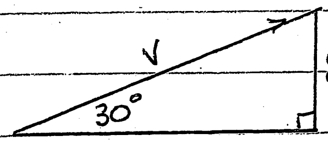
$P(-0.2) = 3(-0.2)^3 + (-0.2)^2 + 1$

$= 1.016$

$P'(-0.2) = 9\times(-0.2)^2 + 2\times(-0$

$= -0.04$

(b)(i) $t = 0 \quad x = 0 \quad y = 2$




$\dot{y} = V\sin 30° = \frac{V}{2}$

$\dot{x} = V\cos 30° = \frac{V\sqrt{3}}{2}$

$\ddot{x} = 0$

$\dot{x} = c_1$

When $t = 0$ $\dot{x} = \frac{V\sqrt{3}}{2} = c_1$

$\therefore \dot{x} = \frac{V\sqrt{3}}{2}$

$x = \frac{V\sqrt{3}}{2} t + c_2$

When $t = 0$ $x = 0 \therefore c_2 = 0$

$x = \frac{V\sqrt{3}}{2} t$

$\ddot{y} = -g$

$\dot{y} = -gt + c_3$

When $t = 0$ $\dot{y} = \frac{V}{2} = c_3$

$\dot{y} = -gt + \frac{V}{2}$

$y = -\frac{1}{2}gt^2 + \frac{V}{2}t + c_4$

When $t = 0$ $y = 2 = c_4$

$\therefore y = -\frac{1}{2}gt^2 + \frac{V}{2}t + 2$

Subst $t = \frac{2x}{V\sqrt{3}}$ into $y$

$$y = -\frac{1}{2}\cdot g \cdot \left(\frac{2x}{V\sqrt{3}}\right)^2 + \frac{V}{2}\cdot\frac{2x}{V\sqrt{3}} + 2$$

$$= -\frac{g}{2} \times \frac{4x^2}{3V^2} + \frac{x}{\sqrt{3}} + 2$$

$$= 2 + \frac{x}{\sqrt{3}} - \frac{2gx^2}{3V^2}$$

(ii) When $x = 7.3$ $y = 2.5$ $\qquad V = 9.7$ (1dp

$$2.5 = 2 + \frac{7.3}{\sqrt{3}} - \frac{2 \times 7.3^2 \times 9.8}{3V^2}$$

$$\frac{2 \times 7.3^2 \times 9.8}{3V^2} = \frac{7.3}{\sqrt{3}} - 0.5 = \frac{7.3 - 0.5\sqrt{3}}{\sqrt{3}}$$

$$3V^2 = \frac{\sqrt{3}}{7.3 - 0.5\sqrt{3}} \times 2 \times 7.3^2 \times 9.8$$

Question 7

(a) $v = 2(x+1)$

$$\frac{dx}{dt} = 2(x+1)$$

$$\frac{dt}{dx} = \frac{1}{2(x+1)}$$

$$t = \frac{1}{2}\ln(x+1) + c$$

When $t=0$ $x=0$

$$0 = \frac{1}{2}\ln 1 + c$$

$$c = 0$$

$$t = \frac{1}{2}\ln(x+1)$$

$$x+1 = e^{2t}$$

$$x = e^{2t} - 1$$

(b) $1 + {}^nC_1 x + {}^nC_2 x^2 + \dots + {}^nC_n x^n = (1+x)^n$

(i) Let $x = -1$

$$1 + {}^nC_1(-1) + {}^nC_2(-1)^2 + \dots + {}^nC_n(-1)^n = (-1)^n$$

$$1 - {}^nC_1 + {}^nC_2 + \dots + (-1)^n\, {}^nC_n = 0$$

(ii) $1 + {}^nC_1 x + {}^nC_2 x^2 + \dots + {}^nC_n x^n = (1+x)^n$

Integrate wrt $x$

$$\therefore\ x + {}^nC_1 \cdot \frac{x^2}{2} + {}^nC_2 \frac{x^3}{3} + \dots + {}^nC_n \frac{x^{n+1}}{n+1} = \frac{(1+x)^{n+1}}{n+1} + c$$

When $x=0$ $\quad 0 = \frac{1}{n+1} + c$

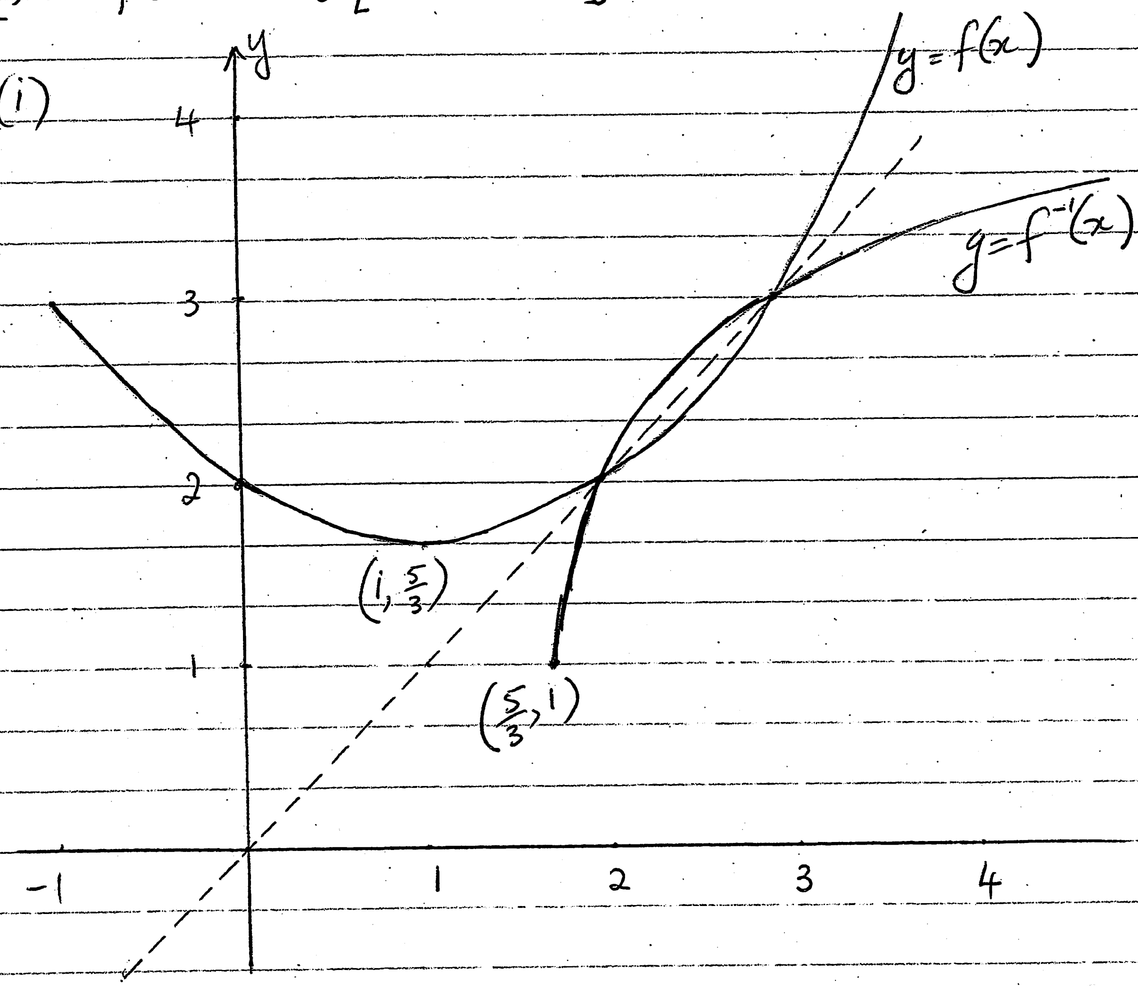
$$c = -\frac{1}{n+1}$$

$$\therefore\ x + \frac{1}{2}{}^nC_1 x^2 + \frac{1}{3}{}^nC_2 x^3 + \dots + \frac{1}{n+1}{}^nC_n = \frac{(1+x)^{n+1}}{n+1} - \frac{1}{n+1}$$

Let $x=1$ $\quad 1 + \frac{1}{2}{}^nC_1 + \frac{1}{3}{}^nC_2 + \dots + \frac{1}{n+1}{}^nC_n = \frac{2^{n+1}}{n+1} - \frac{1}{n+1}$

(c) $f(x) = \frac{1}{3}\left[(x-1)^2 + 5\right]$

(i)




(ii) $x \geq 1$

(iii) on graph

(iv) $x \geq \frac{5}{3}$

(v) $(2,2)$ and $(3,3)$

[Curve and inverse intersect when $y = x$

$$x = \frac{1}{3}(x-1)^2 + \frac{5}{3}$$
$$3x = x^2 - 2x + 1 + 5$$
$$x^2 - 5x + 6 = 0$$
$$(x-2)(x-3) = 0$$
$$x = 2, 3$$
$$y = 2, 3$$
]