St George Girls High School

Trial Higher School Certificate Examination

2012

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using blue or black pen.
- Write your student number on each booklet.
- Board-approved calculators may be used.
- The mark allocated for each question is listed at the side of the question.
- Marks may be deducted for careless or poorly presented work.

Total Marks – 100

Section I – Pages 2 – 4
10 marks

- Attempt Questions 1 – 10.
- Allow about 15 minutes for this section.
- Answer on the sheet provided.

Section II – Pages 5 – 12
60 marks

- Attempt Questions 11 – 14
- Allow about 2 hours 45 minutes for this section.
- Begin each question in a new booklet.
- Show all necessary working in Questions 11 – 14.

Students are advised that this is a Trial Examination only and does not necessarily reflect the content or format of the Higher School Certificate Examination.

Section I – (10 marks)

Marks

Answer this section on the answer sheet provided at the back of this paper.
Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

1. The value of $\lim\limits_{x \to 0} \dfrac{\sin 4x}{9x}$ is:

   A. $2\frac{1}{4}$

   B. 1

   C. $\frac{4}{9}$

   D. 0

2. For the function $f(x) = 3\sin^{-1}\left(\frac{x}{4}\right)$ the domain and range of $y = f(x)$ are:

   A. domain $\left\{x:\ -\frac{3\pi}{2} \le x \le \frac{3\pi}{2},\ x \in \mathbb{R}\right\}$
   range $\{y:\ -4 \le y \le 4,\ y \in \mathbb{R}\}$

   B. domain $\{x:\ -1 \le x \le 1,\ x \in \mathbb{R}\}$
   range $\{y:\ -3 \le y \le 3,\ y \in \mathbb{R}\}$

   C. domain $\{x:\ -3 \le x \le 3,\ x \in \mathbb{R}\}$
   range $\left\{y:\ -\frac{\pi}{2} \le y \le \frac{\pi}{2},\ y \in \mathbb{R}\right\}$

   D. domain $\{x:\ -4 \le x \le 4,\ x \in \mathbb{R}\}$
   range $\left\{y:\ -\frac{3\pi}{2} \le y \le \frac{3\pi}{2},\ y \in \mathbb{R}\right\}$

3. Solve for x, $\dfrac{2x+1}{1-x} \ge 1$

   A. $0 \le x < 1$

   B. $x \le 0$ or $x > 1$

   C. $x > 0$ or $x > 1$

   D. $0 < x \le 1$

Section I (cont'd)

Marks

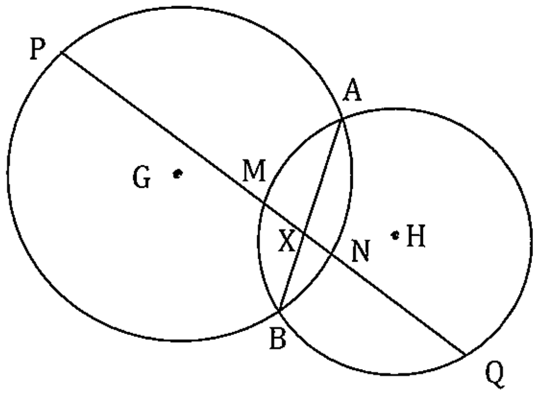
4. A particle is oscillating in Simple Harmonic Motion where its position x metres from a fixed point O on the same line as its motion after t seconds is given by $x = 2\cos\left(3t + \frac{\pi}{6}\right)$. What is the maximum speed of the particle?

   A. 2 m/s

   B. 6 m/s

   C. 0 m/s

   D. $\frac{\pi}{9}$ m/s

5. 

   AB is a common chord to the circles with centres G and H.

   PQ is drawn intersecting circle centre G at P and N, intersecting circle centre H at M and Q and intersecting AB at X as shown in the diagram.

   If $PM = 18$, $MX = 6$ and $NQ = 15$ then the length NX is:

   A. 5

   B. 4

   C. 3

   D. 2

6. The derivative of $\tan^{-1}\frac{2x}{3}$ is:

   A. $\frac{1}{3+4x^2}$

   B. $\frac{1}{\frac{9}{4}+x^2}$

   C. $\frac{6}{9+4x^2}$

   D. $\frac{3}{4+x^2}$

Section I (cont'd) **Marks**

7. The exact value of $\sin^{-1}\left(\cos\frac{2\pi}{3}\right)$ is:

A. $\frac{\pi}{6}$

B. $-\frac{\pi}{6}$

C. $\frac{\pi}{3}$

D. $-\frac{\pi}{3}$

8. Consider $(1+2x)^n$. If the ratio of the coefficient of x^4 to the coefficient of x^6 is $5:8$ then the value of n is:

A. 5

B. 6

C. 7

D. 8

9. The polynomial $P(x) = x^4 - 2x^3 - 7x^2 + 20x - 12$ has a zero of multiplicity 2 at $x =$:

A. 1

B. -3

C. 2

D. -2

10. A particle moves in a straight line. At time t seconds, where $t \geq 0$, its displacement x metres from the origin and its velocity v metres per second are such that $v = 25 + x^2$.

If $x = 5$ initially, then t is equal to:

A. $25x + \frac{x^3}{3}$

B. $25x + \frac{x^3}{3} + \frac{500}{3}$

C. $\tan^{-1}\left(\frac{x}{5}\right) - \frac{\pi}{4}$

D. $\frac{1}{5}\tan^{-1}\left(\frac{x}{5}\right) - \frac{\pi}{20}$

Section II – Show all working

Question 11 – Start A New Booklet – (15 marks) Marks

a) (i) Find the derivative of $\log_e(\cos^2 x)$ 1

(ii) $\int_0^1 \frac{x^2}{x^3+1} \quad dx$ 1

b) In the expression of $\left(x^2+\frac{2}{x}\right)^{10}$ find the coefficient of x^2 1

c) The quadratic polynomial $ax^2+bx+14$ leaves a remainder of -12 when divided by $(x-1)$, and has $(x+2)$ as a factor. Find the values of a and b. 2

d) Find the acute angle between the lines $y=5-x$ and $\sqrt{3}y=x+1$ 1

e) (i) Show that the area of an equilateral triangle of side length x is given by 1

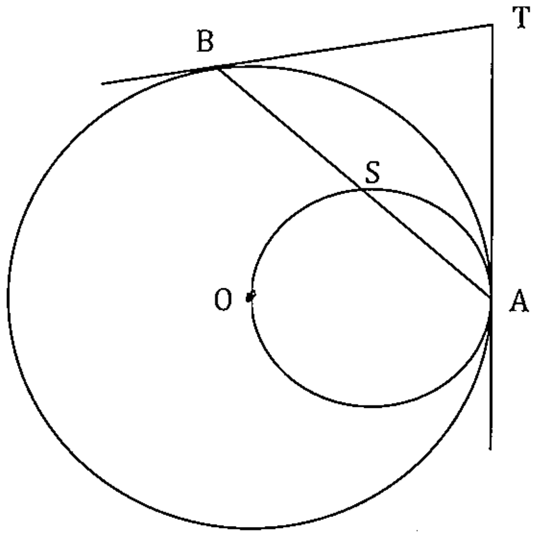
$$A=\frac{\sqrt{3}}{4}x^2$$

(ii) The sides of an equilateral triangle are increasing at the rate of 5 mm/s. At what rate is the area of the triangle increasing at the instant the sides are 10 cm long. 2

Question 11 continued on next page

Question 11 (cont'd) Marks

f)



Two circles touch internally at a point A and the smaller of the two circles passes through O, the centre of the larger circle.

AB is any chord of the larger circle at S. The tangents to the larger circle at A and B meet at the point T

Prove:

(i) AB is bisected at S. 4

(ii) O, S and T are collinear. 2

Question 12 – Start A New Booklet – (15 marks)

Marks

a) Use the principle of Mathematical Induction to prove that $7^n + 2$ is divisible by 3 for all positive integers n 2

b) (i) Show that $\sin(A + B) + \sin(A - B) = 2\sin A\cos B$ 1

(ii) Hence or otherwise find the exact value of: 2

$$\int_0^{\frac{\pi}{6}} \sin 4x \ \cos 2x \ dx$$

c) (i) Show that $\frac{d}{dx}\ (x - \tan^{-1}x) = \frac{x^2}{1+x^2}$ 1

(ii) Hence or otherwise find the exact value of 1

$$\int_0^1 \frac{x^2}{1+x^2}\ dx$$

d) Given $A(-2, 3)$ and $B(4, 7)$ find the coordinates of the point which divides the interval AB externally in the ratio $3:1$ 1

e) If α, β and γ are the roots of $x^3 - 2x^2 + 4x - 7 = 0$ evaluate $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$ 2

Question 12 continued on next page

Question 12 (cont'd)

Marks

f) A particle moving in a straight line has an acceleration given by $\ddot{x} = x^2$ where its displacement is x metres from the origin. If initially the particle is at rest 2 metres from the origin, find its velocity when it is 4 metres from the origin. 2

g) The normal at $P(2ap,\ ap^2)$ to the parabola $x^2 = 4ay$ meets the curve again at $Q(2aq,\ aq^2)$

(i) Given that the equation of the normal at P is $x + py = ap^3 + 2ap$ 1

show that $q = -\frac{(2+p^2)}{p}$

(ii) Find a value for p so that the lines OP and OQ are at right angles, where O is the origin. 2

Question 13 – Start A New Booklet – (15 marks)

Marks

a) If $3n^2 - 7n + 5 \equiv An(n-1) + Bn + C$ find A, B and C 2

b) Evaluate, leaving your answer in exact form 3

$$\int_{\frac{1}{8}}^{\frac{\sqrt{3}}{8}} \frac{dx}{\sqrt{1-16x^2}}$$

c) If α and β are the roots of $x^2 + bx + c = 0$, form the equation, in general form, whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ 2

d) A curve is defined by the parametric equations $x = t - 3,\ y = t^2 - 9$

(i) Find $\frac{dy}{dx}$ in terms of t 1

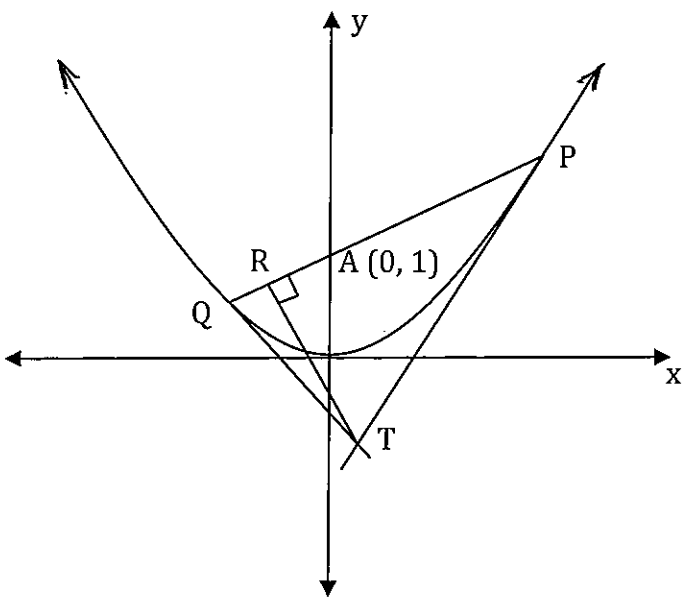
(ii) Find the equation of the tangent to the curve at the point where $t = -3$ 2

Question 13 continued on next page

Question 13 (cont'd)

Marks

e)



PQ is a chord of the parabola $x^2 = 8y$ passing through the point $A(0,1)$ where P is $(4p, 2p^2)$ and Q is $(4q, 2p^2)$

The tangents to the parabola at P and Q meet at the point T.

R is a point on the chord PQ with $RT \perp PQ$

(i) Write down the equations of the tangents at P and Q and hence find the coordinates of T **2**

(ii) Show that the equation of the chord PQ is given by **1**

$$2y = (p+q)\,x - 4pq$$

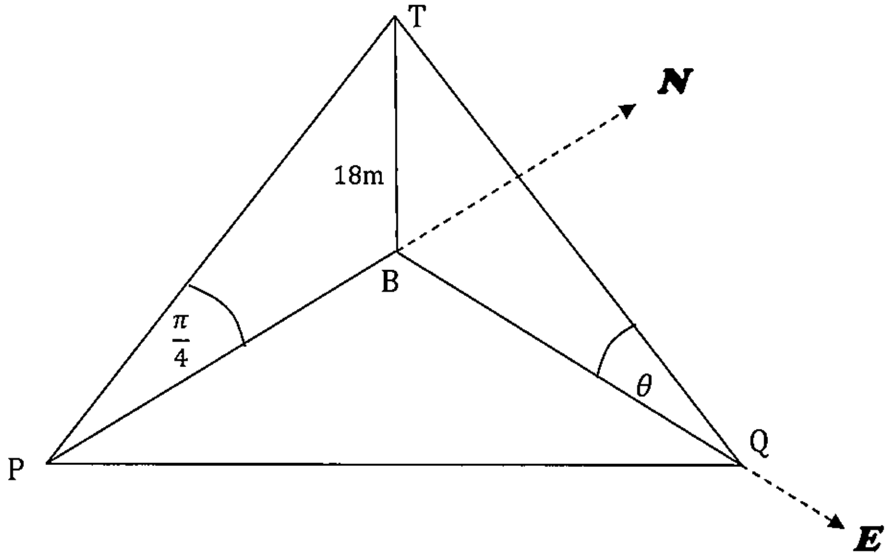
(iii) Show that $pq = -\dfrac{1}{2}$ **1**

(iv) Find the equation of RT **1**

Question 14 – Start A New Booklet – (15 marks)

Marks

a)



A vertical tower BT of height 18 metres stands with its base B on horizontal grounds. B is due North of a fixed point P and the angle of elevation from P to the top of the tower T is $\frac{\pi}{4}$ radians. Q is a moving point on the ground due East of B and the angle of elevation from Q to T is θ radians where $0 < \theta < \frac{\pi}{2}$. The size of the angle θ is increasing at a constant rate of 0.02 radians per minute.

(i) Show that $PQ = 18 \operatorname{cosec} \theta$ — 2

(ii) Find the rate at which the length PQ is changing when $\theta = \frac{\pi}{3}$ — 2

b) A person hits a ball off the ground with a bat, projecting the ball at a velocity of 50 m/s at an angle of projection θ such that $\tan \theta = \frac{3}{4}$

(i) Taking the origin as the point of projection and $g = 10$ m/s^2 show that $\dot{x} = 40$ and $\dot{y} = -10t + 30$ and then find x and y in terms of t — 3

(ii) A tall building is 100 m from where the ball is hit on horizontal ground. If the ball passes through a small open window in the building find the height of the window above the ground. — 2

(iii) Find the velocity and angle that the ball makes with the horizontal as it passes through the window. — 2

Question 14 continued on next page

Question 14 (cont'd) Marks

c) Find the general solution in radians of the equation $\sin 2x = \cos x$ 2

d) By considering the expansion of both sides of the identity

$(1+x)^{m+n} = (1+x)^m(1+x)^n$, where m and n are positive integers, show that

$$\binom{m+n}{3} = \binom{m}{3} + \binom{m}{2}\binom{n}{1} + \binom{m}{1}\binom{n}{2} + \binom{n}{3}$$

End of Paper

Student Number: ______________________________ Teacher: ____________________

Year 12 Mathematics Extension 1 Trial HSC Examination 2012

Section I

Multiple-choice Answer Sheet – Questions 1 – 10

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ~~●~~ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and drawing an arrow as follows:

correct

A ~~●~~ B ~~●~~ C ○ D ○

1.	A ○	B ○	C ○	D ○
2.	A ○	B ○	C ○	D ○
3.	A ○	B ○	C ○	D ○
4.	A ○	B ○	C ○	D ○
5.	A ○	B ○	C ○	D ○
6.	A ○	B ○	C ○	D ○
7.	A ○	B ○	C ○	D ○
8.	A ○	B ○	C ○	D ○
9.	A ○	B ○	C ○	D ○
10.	A ○	B ○	C ○	D ○

TRIAL HSC EXTENSION 1 2012 ST GEORGE GIRLS HIGH

Question 11

(a) (i) $\frac{d}{dx}\log_e(\cos^2 x) = \frac{2\cos x(-\sin x)}{\cos^2 x}$ $(\cos x \neq 0)$

$= -2\tan x$

(ii) $\int_0^1 \frac{x^2}{x^3+1}\,dx = \frac{1}{3}\int_0^1 \frac{3x^2}{x^3+1}\,dx$

$= \frac{1}{3}\left[\log_e(x^3+1)\right]_0^1$

$= \frac{1}{3}\left(\log_e 2 - \log_e 1\right)$

$= \frac{1}{3}\log_e 2$

(b) $T_{k+1} = {}^{10}C_k\,(x^2)^k\left(\frac{2}{x}\right)^{10-k}$

$= {}^{10}C_k\, x^{2k}\, 2^{10-k}\, x^{k-10}$

$= {}^{10}C_k\, 2^{10-k}\, x^{3k-10}$

If $3k-10 = 2$

$k = 4$

Coeff of $x^2 = {}^{10}C_4 \times 2^6$

(c) Let $P(x) = ax^2 + bx + 14$

$P(1) = -12$

$a + b + 14 = -12$

$a + b = -26$ ①

$P(-2) = 0$

$4a - 2b + 14 = 0$

$4a - 2b = -14$

$2a - b = -7$ ②

① + ② $3a = -33$

$a = -11$

Subst in ①

$-11 + b = -26$

$b = -15$

$a = -11 \quad b = -15$

(d) $y = 5 - x \qquad \sqrt{3}y = x + 1$

$m_1 = -1 \qquad m_2 = \frac{1}{\sqrt{3}}$

Let θ be the acute angle between the lines

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

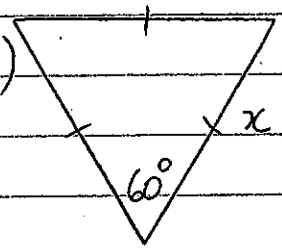
$$= \left|\frac{-1 - \frac{1}{\sqrt{3}}}{1 + -1 \times \frac{1}{\sqrt{3}}}\right|$$

$$= \left|\frac{-\sqrt{3} - 1}{\sqrt{3} - 1}\right|$$

$$= \frac{\sqrt{3} + 1}{\sqrt{3} - 1}$$

$$\therefore \quad \theta = 75^\circ$$

(e) (i)



$$A = \frac{1}{2} x \times x \sin 60^\circ$$

$$= \frac{1}{2} x^2 \times \frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{3}}{4} x^2$$

(ii) $\frac{dx}{dt} = 0.5 \text{ cm/s} \qquad \frac{dA}{dt} = \frac{dA}{dx}\frac{dx}{dt}$

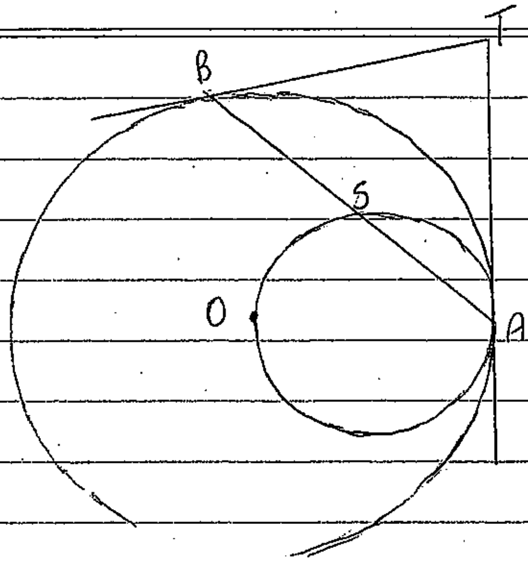
$$= \frac{\sqrt{3}}{2} x \,.\, 0.5$$

When $x = 10 \qquad \frac{dA}{dt} = \frac{\sqrt{3}}{2} \times 10 \times 0.5$

$$= \frac{5\sqrt{3}}{2}$$

$\therefore$ Area is increasing at a rate of $\frac{5\sqrt{3}}{2}$ cm^2/s when

$x = 10$



Join OA

OA is a radius of larger circle

$\angle OAT = 90^\circ$ (angle between radii and tangent at point of contact)

AT is a tangent to smaller circle

Since $\angle TAO = 90^\circ$ AO must pass through centre of smaller circle

$\therefore$ AO is a diameter of smaller circle.

Join OS $\therefore$ $O\hat{S}A = 90^\circ$ (angle in a semicircle)

Join OB

OB = OA (radii of larger circle)

$\therefore$ $\triangle OBA$ is isosceles

Since $OS \perp AB$ then OS bisects AB (perp to base of isosceles $\triangle$ bisects base)

ie S is the midpoint of AB

(ii) $\triangle ABT$ is isosceles BT = AT (tangents from external point equal in length)

Since S is midpt of AB then $TS \perp AB$

$\therefore$ $T\hat{S}O = 90^\circ + 90^\circ$

$= 180^\circ$

$\therefore$ TSO is a straight angle

ie O, S, T are collinear

Question 12

(a) Aim: To prove 7^n+2 is divisible by 3

ie $7^n+2 = 3A$ where A is an integer

For $n=1$ $\quad 7^1+2 = 9 = 3\times 3$

$\therefore$ Proposition true for $n=1$

Assume proposition is true for $n=k$ where k is a positive integer

ie $\quad 7^k+2 = 3B$ where B is an integer

Aim to show that proposition is then true for $n=k+1$

$$\begin{aligned}7^{k+1}+2 &= 7\times 7^k+2\\ &= 7\times(3B-2)+2 \quad \text{(by inductive hypothesis)}\\ &= 7\times 3B-14+2\\ &= 3\times 7B-12\\ &= 3(7B-4)\\ &= 3C \quad \text{(C is an integer since integers closed under mult}^n \text{ and subtraction)}\end{aligned}$$

ie proposition is true for $n=k+1$ if true for $n=k$.

Hence by induction proposition is true for all positive integers n.

(b) (i) $\sin(A+B)+\sin(A-B) = \sin A\cos B+\cos A\sin B + \sin A\cos B - \cos A\sin B$

$= 2\sin A\cos B$

(ii) $\sin 4x\cos 2x = \frac{1}{2}\left(\sin(4x+2x)+\sin(4x-2x)\right)$

$= \frac{1}{2}(\sin 6x + \sin 2x)$

$\therefore \int_0^{\frac{\pi}{6}} \sin 4x \cos 2x \, dx = \frac{1}{2} \int_0^{\frac{\pi}{6}} \sin 6x + \sin 2x \, dx$

$= \frac{1}{2} \left[-\frac{1}{6} \cos 6x - \frac{1}{2} \cos 2x \right]_0^{\pi/6}$

$= \left(-\frac{1}{12} \cos \pi - \frac{1}{4} \cos \frac{\pi}{3} \right) - \left(-\frac{1}{12} \cos 0 - \frac{1}{4} \cos 0 \right)$

$= \left(-\frac{1}{12} \times -1 - \frac{1}{4} \times \frac{1}{2} \right) - \left(-\frac{1}{12} \times 1 - \frac{1}{4} \times 1 \right)$

$= \frac{1}{12} - \frac{1}{8} + \frac{1}{12} + \frac{1}{4}$

$= \frac{7}{24}$

(c) (i) $\frac{d}{dx}(x - \tan^{-1} x) = 1 - \frac{1}{1+x^2}$

$= \frac{1+x^2-1}{1+x^2}$

$= \frac{x^2}{1+x^2}$

(ii) $\int_0^1 \frac{x^2}{1+x^2} \, dx = \left[x - \tan^{-1} x \right]_0^1$

$= (1 - \tan^{-1} 1) - (0 - \tan^{-1} 0)$

$= 1 - \frac{\pi}{4} - 0$

$= \frac{4-\pi}{4}$

(d) $(-2, 3)$ $(4, 7)$

$-3 : 1$

$\left(\frac{1 \times -2 + -3 \times 4}{-3+1}, \frac{1 \times 3 + -3 \times 7}{-3+1} \right) = \left(\frac{-14}{-2}, \frac{-18}{-2} \right)$

Point is $(7, 9)$

(e) $x^3 - 2x^2 + 4x - 7 = 0$ has roots α, β, γ

$$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\beta\gamma + \alpha\gamma + \alpha\beta}{\alpha\beta\gamma} \qquad \frac{4}{1}, \; \frac{--7}{1}$$

$$= \frac{4}{7}$$

(f)
$$\ddot{x} = x^2$$

$$\frac{d\left(\frac{1}{2}v^2\right)}{dx} = x^2$$

$$\frac{1}{2}v^2 = \frac{x^3}{3} + c$$

When $t=0$ $v=0$ $x=2$

$$0 = \frac{8}{3} + c_1$$

$$c_1 = -\frac{8}{3}$$

$$\frac{1}{2}v^2 = \frac{x^3}{3} - \frac{8}{3}$$

$$v^2 = \frac{2x^3}{3} - \frac{16}{3}$$

When $x = 4$

$$v^2 = \frac{2 \times 64}{3} - \frac{16}{3}$$

$$= \frac{112}{3}$$

$$v = \pm\sqrt{\frac{112}{3}}$$

But $v \geq 0$

$$\therefore v = \sqrt{\frac{112}{3}}$$

OR When $t=0$ $v=0$ $x=-2$

$$0 = -\frac{8}{3} + c_2$$

$$c_2 = \frac{8}{3}$$

$$\frac{1}{2}v^2 = \frac{x^3}{3} + \frac{8}{3}$$

$$v^2 = \frac{2x^3}{3} + \frac{16}{3}$$

Since $\ddot{x} \geq 0$ $(\ddot{x} = x^2)$ and $v=0$ when $t=0$ particle will always move in a positive direction

$\therefore x \neq -4$

ie When $x = 4$

$$v^2 = \frac{2 \times 64}{3} + \frac{16}{3}$$

$$= \frac{144}{3}$$

$$v = \sqrt{\frac{144}{3}}$$

$$= \sqrt{48} = 4\sqrt{3}$$

(g) (i) $x + py = ap^3 + 2ap$

meets $x^2 = 4ay$

$$x + p.\frac{x^2}{4a} = ap^3 + 2ap$$

$$\frac{p}{4a}.x^2 + x - (ap^3 + 2ap) = 0$$

has roots $2ap$ and $2aq$

$$2ap + 2aq = \frac{-1}{p/4a}$$

$$= \frac{-4a}{p}$$

$$2aq = -\frac{4a}{p} - 2ap$$

$$q = -\frac{2}{p} - p$$

$$= \frac{-2-p^2}{p}$$

$$= \frac{-(2+p^2)}{p}$$

(ii) Grad $OP = \dfrac{ap^2 - 0}{2ap - 0}$

$$= \frac{p}{2}$$

Grad $OQ = \dfrac{q}{2}$

If $OP \perp OQ$ then $\dfrac{p}{2} \times \dfrac{q}{2} = -1$

$$pq = -4$$

$$\therefore \quad q = -\frac{4}{p}$$

$$\therefore \quad \frac{-4}{p} = \frac{-(2+p^2)}{p}$$

$$2+p^2 = 4$$

$$p^2 = 2$$

$$p = \pm\sqrt{2}$$

Question 13

(a) $3n^2 - 7n + 5 \equiv An(n-1) + Bn + C$ 2 marks

Let $n=0$ $\quad 5 = 0 + 0 + C \qquad C = 5$

Coeff n^2 $\quad 3 = A \qquad A = 3$

Let $n=1$ $\quad 3-7+5 = 0 + B + C$

$1 = B + 5$

$B = -4$

(b) $$\int_{\frac{1}{8}}^{\sqrt{3}/8} \frac{dx}{\sqrt{1-16x^2}} = \int_{\frac{1}{8}}^{\sqrt{3}/8} \frac{dx}{4\sqrt{\frac{1}{16} - x^2}}$$ 3 marks

$$= \frac{1}{4}\left[\sin^{-1}\frac{x}{1/4}\right]_{\frac{1}{8}}^{\sqrt{3}/8}$$

$$= \frac{1}{4}\left[\sin^{-1} 4x\right]_{\frac{1}{8}}^{\sqrt{3}/8}$$

$$= \frac{1}{4}\left(\sin^{-1}\frac{\sqrt{3}}{2} - \sin^{-1}\frac{1}{2}\right)$$

$$= \frac{1}{4}\left(\frac{\pi}{3} - \frac{\pi}{6}\right)$$

$$= \frac{\pi}{24}$$

(c) $x^2 + bx + c = 0$ 2 marks

equation of quad roots $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$

$\alpha + \beta = -b$

$\alpha\beta = \frac{c}{1}$

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta} \qquad \frac{\alpha}{\beta} \cdot \frac{\beta}{\alpha} = 1$$

$$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$$

$$= \frac{b^2 - 2c}{c}$$

$$Eq^n \text{ is } \left(x - \frac{\alpha}{\beta}\right)\left(x - \frac{\beta}{\alpha}\right) = 0$$

$$x^2 - \left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right)x + \frac{\alpha}{\beta}\cdot\frac{\beta}{\alpha} = 0$$

$$x^2 - \frac{(b^2-2c)}{c}x + 1 = 0$$

$$cx^2 - (b^2-2c)x + c = 0$$

(d) Eq^n of tangents at P and Q

$$y = px - 2p^2 \quad (1)$$
$$y = qx - 2q^2 \quad (2)$$

2 marks.
Write equations of tangent wrt P, Q

Subst (1) in (2)

$$px - 2p^2 = qx - 2q^2$$
$$(p-q)x = 2(p^2-q^2)$$
$$x = \frac{2(p-q)(p+q)}{p-q} \quad (p \neq q)$$
$$= 2(p+q)$$

$$y = p.2(p+q) - 2p^2$$
$$= 2pq$$

$\therefore$ T is point $(2(p+q), 2pq)$

(ii) Grad PQ $= \dfrac{2p^2-2q^2}{4p-4q}$

$= \dfrac{2(p-q)(p+q)}{4(p-q)}$

$= \dfrac{p+q}{2}$

Eqⁿ of chord PQ is

$y - 2p^2 = \dfrac{p+q}{2}(x-4p)$

$= \dfrac{p+q}{2}\cdot x - 2p(p+q)$ 1 mark

$= \dfrac{p+q}{2}x - 2p^2 - 2pq$

$y = \dfrac{p+q}{2}x - 2pq$

$2y = (p+q)x - 4pq$

(iii) Since PQ passes through A(0,1)

$2 = 0 - 4pq$

$pq = -\dfrac{1}{2}$ 1 mark

(iv) RT ⊥ PQ 1 mark

∴ Grad RT $= \dfrac{-2}{p+q}$ equation RT

∴ Eqⁿ of RT is

$y - 2pq = -\dfrac{2}{p+q}(x - 2(p+q))$

$y - 2\times -\dfrac{1}{2} = -\dfrac{2}{p+q}x + 4$

$y = -\dfrac{2}{p+q}x + 3$

(d) (i) $x = t - 3 \qquad y = t^2 - 9$ 1 mark

$\frac{dx}{dt} = 1 \qquad \frac{dy}{dt} = 2t$

$\frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt}$

$= \frac{2t}{1}$

(ii) When $t = -3$ 2 marks

$x = -6$

$y = 0$

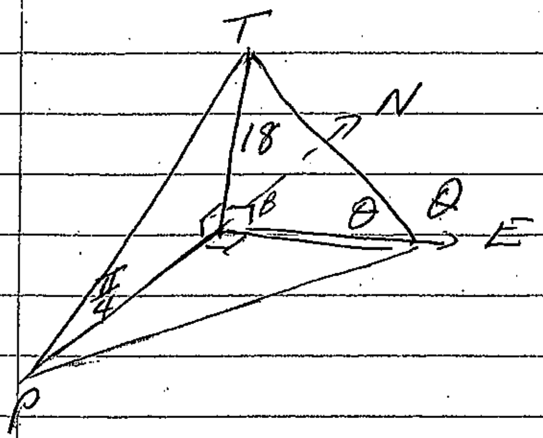
$\frac{dy}{dx} = -6$

Eq^n of tangent is

$y - 0 = -6(x - -6)$

$y = -6x + 36$

Question 14



(i) $\frac{BQ}{18} = \cot\theta$

$BQ = 18\cot\theta$

$\frac{BP}{18} = \tan\frac{\pi}{4}$

$= 1$

$BP = 18$

In $\triangle PBQ$ $\quad PQ^2 = BP^2 + BQ^2$

$= 18^2 + 18^2\cot^2\theta$

$= 18^2(1+\cot^2\theta)$

$= 18^2\operatorname{cosec}^2\theta$

$PQ = 18\operatorname{cosec}\theta$

(ii) $\frac{d\,PQ}{dt} = \frac{d\,PQ}{d\theta} \cdot \frac{d\theta}{dt}$

$= -18\operatorname{cosec}\theta\cot\theta \cdot \frac{d\theta}{dt}$

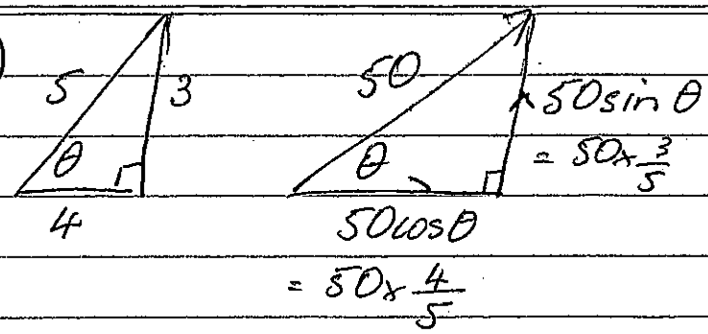
$= -18\operatorname{cosec}\theta\cot\theta \cdot 0.02$

$= -0{\cdot}36 \times \operatorname{cosec}\frac{\pi}{3} \cdot \cot\frac{\pi}{3}$

$= -0{\cdot}36 \times \frac{2}{\sqrt{3}} \times \frac{1}{\sqrt{3}}$

$= -0{\cdot}24$

(b) (i)



$\ddot{x} = 0$ $\qquad$ $\ddot{y} = -10$

$\dot{x} = c_1$ $\qquad$ $\dot{y} = -10t + c_2$

When $t=0$ $\dot{x} = 50\cos\theta$ $\qquad$ When $t=0$ $\dot{y} = 50\sin\theta$

$= 40$ $\qquad$ $= 30$

$\therefore$ $= c_1$ $\qquad$ $= c_2$

$\dot{x} = 40$ $\qquad$ $\dot{y} = -10t + 30$

$x = 40t + c_3$ $\qquad$ $y = -5t^2 + 30t + c_4$

When $t=0$ $x=0$ $\qquad$ When $t=0$ $y=0$ $\therefore$

$\therefore c_3 = 0$ $\qquad$ $\therefore c_4 = 0$

$x = 40t$ $\qquad$ $y = -5t^2 + 30t$

(ii) If $x = 100$ then

$100 = 40t$

$t = \frac{5}{2}$

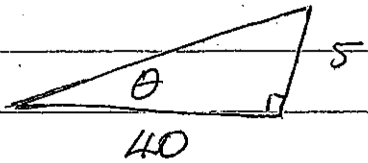
$y = -5 \times \left(\frac{5}{2}\right)^2 + 30 \times \frac{5}{2}$

$= \frac{175}{4}$

$\therefore$ Height of window is 43.75m

(iii) When $t = \frac{5}{2}$ $\quad \dot{x} = 40$

$\dot{y} = -10 \times \frac{5}{2} + 30 = 5$



$$v^2 = 40^2 + 5^2$$
$$= 1600 + 25$$
$$= 1625$$
$$v = \sqrt{1625}$$
$$= 5\sqrt{65}$$

$$\tan\theta = \frac{5}{40}$$
$$= \frac{1}{8}$$
$$\theta = \tan^{-1}\left(\frac{1}{8}\right)$$
$$= 7^\circ 8'$$

Velocity is $5\sqrt{65}$ m/s and angle ball's path makes with the horizontal is $7^\circ 8'$

(c) $\sin 2x = \cos x$

$$2\sin x\cos x - \cos x = 0$$
$$\cos x(2\sin x - 1) = 0$$
$$\cos x = 0 \quad \text{or} \quad \sin x = \frac{1}{2}$$

$$x = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \ldots \quad x = \frac{\pi}{6} + 2k\pi, \ \pi - \frac{\pi}{6} + 2k\pi$$

$$x = \frac{(2k+1)\pi}{2}, \ \frac{\pi}{6} + 2k\pi, \ \frac{5\pi}{6} + 2k\pi \quad (k \in \mathbb{Z})$$

(d) $(1+x)^{m+n} = (1+x)^m (1+x)^n$

On LHS coeff $x^3 = \binom{m+n}{3}$

$$\text{RHS} = \left(1 + \binom{m}{1}x + \binom{m}{2}x^2 + \binom{m}{3}x^3 + \ldots\right)$$
$$\times \left(1 + \binom{n}{1}x + \binom{n}{2}x^2 + \binom{n}{3}x^3 + \ldots\right)$$

Term in $x^3 = 1 \times \binom{n}{3}x^3 + \binom{m}{1}x \times \binom{n}{2}x^2 + \binom{m}{2}x^2 \times \binom{n}{1}x + 1 \times \binom{m}{3}x^3$

$$\therefore \text{Coeff } x^3 = \binom{n}{3} + \binom{m}{1} \times \binom{n}{2} + \binom{m}{2} \times \binom{n}{1} + 1 \times \binom{m}{3}$$
$$= \binom{m+n}{3}$$