Student Number:…………………..

Class Teacher:…………………….

St George Girls High School

Trial Higher School Certificate Examination

2016

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black pen.
- Write your student number and your class teacher's name on each booklet.
- Board-approved calculators may be used.
- The mark allocated for each question is listed at the side of the question.
- Marks may be deducted for careless or poorly presented work.
- A table of standard integrals is provided.
- A multiple choice answer sheet is provided for Section I.

Total marks – 70

Section I – Pages 2 to 4
10 marks

- Attempt questions 1 to 10
- Allow about 15 minutes for this section.
- Answer on the sheet provided.

Section II – Pages 5 to 10
60 marks

- Attempt questions 11 – 15.
- Allow about 1 hour 45 minutes for this section.
- Begin each question in a new booklet.
- Show all necessary working in questions 11 - 15.

Students are advised that this is a trial Examination only and does not necessarily reflect the content or format of the Higher School Certificate Examination.

Section I:

Multiple Choice (Each question is worth 1 mark)

Answer this section on the multiple choice answer sheet provided.

1. The acute angle between the lines $x - 2y + 1 = 0$ and $2x - y - 1 = 0$ is closest to:

   A. $37°$

   B. $45°$

   C. $90°$

   D. $143°$

2. The point P divides the interval from A (-2, 2) to B (8, -3) internally in the ratio 3:2. What is the x coordinate of P?

   A. 4

   B. 2

   C. 0

   D. -1

3. Which of the following is the correct expression for $\int \frac{dx}{9+25x^2}$?

   A. $\frac{1}{15}\tan^{-1}\frac{3x}{5} + C$

   B. $\frac{1}{25}\tan^{-1}\frac{3x}{5} + C$

   C. $\frac{1}{25}\tan^{-1}\frac{5x}{3} + C$

   D. $\frac{1}{15}\tan^{-1}\frac{5x}{3} + C$

4. A curve has parametric equations $x = t - 3$ and $y = t^2 + 2$. What is the Cartesian equation of this curve?

   A. $y = x^2 - x - 1$

   B. $y = x^2 + x - 1$

   C. $y = x^2 - 6x + 11$

   D. $y = x^2 + 6x + 11$

5. A particle is moving in a straight line with $v^2 = 36 - 4x^2$ and undergoing simple harmonic motion. If the particle is initially at the origin, which of the following is the correct equation for its displacement in terms of t? (v is the velocity and x is the displacement of the particle).

   A. $x = 2\sin(3t)$

   B. $x = 3\sin(2t)$

   C. $x = 2\sin(9t)$

   D. $x = 3\sin(4t)$

6. Solve the inequality $\dfrac{x^2-4}{x} \geq 0$

   A. $-2 \leq x < 0$ or $x \geq 2$

   B. $-2 \geq x > 0$ or $x \leq 2$

   C. $-4 \leq x < 0$ or $x \geq 4$

   D. $-4 \geq x > 0$ or $x \leq 4$

7. What is the value of $\int_0^1 \frac{4x}{2x+1}\,dx$? Use the substitution $= 2x + 1$.

   A. $2 - \log_e 2$

   B. $2 - \log_e 3$

   C. $4 - 2\log_e 2$

D. $4 - 2\log_e 3$

8. What is the correct expression for the indefinite integral $\int(\cos^2 x + 2\sec^2 x)dx$?

A. $\frac{1}{2}x + \frac{1}{4}\sin 2x + \tan x + C$

B. $\frac{1}{2}x - \frac{1}{4}\sin 2x + \tan x + C$

C. $\frac{1}{2}x + \frac{1}{4}\sin 2x + 2\tan x + C$

D. $\frac{1}{2}x - \frac{1}{4}\sin 2x + 2\tan x + C$

9. What is the term independent of x in the expansion of $\left(x^2 - \frac{2}{x}\right)^9$?

A. ${}^9C_3(-2)^3$

B. ${}^9C_6(-2)^6$

C. ${}^9C_3(2)^3$

D. ${}^9C_6{-}(2)^6$

10. A particle moves in a straight line with a displacement of x and velocity of v. When $t = 0$ the acceleration is $3x^2$, velocity is $-\sqrt{2}$ and displacement is 1. Which of the following is the correct equation for x as a function of t ?

A. $x = \dfrac{-2}{(t+\sqrt{2})^2}$

B. $x = \dfrac{-2}{(t-\sqrt{2})^2}$

C. $x = \dfrac{2}{(t+\sqrt{2})^2}$

D. $x = \dfrac{2}{(t-\sqrt{2})^2}$

End of Section I

Section II:

Answer **each question in a SEPARATE writing booklet.**

In Questions 11, 12, 13, 14 and 15 your responses should include relevant mathematical reasoning and/or calculations.

Marks

<u>Question 11</u> (12 marks) Start a new booklet

a) Use Newton's method to find a second approximation to the positive root of $x - 2\sin x = 0$. Take $x = 1.6$ as the first approximation. 2

b) Solve: 2

$$|2x - 1| = |x|$$

c) A particle moves in a straight line and its position (x) at any time (t) is given by:

$$x = 1 + \sqrt{3}\cos 4t + \sin 4t$$

(i) Prove the motion is simple harmonic. 2

(ii) When does the particle first reach maximum speed? $(t > 0)$ 2

d) $P\ (2p,\ p^2)$ and $Q\ (2q, q^2)$ are two points on the parabola $x^2 = 4y$. M is the midpoint of PQ.

(i) Prove the identity $(p - q)^2 = 2(p^2 + q^2) - (p + q)^2$ 1

(ii) If P and Q move on the parabola so that $p - q = 4$, show that the locus of M is the parabola $x^2 = 4y - 16$ 2

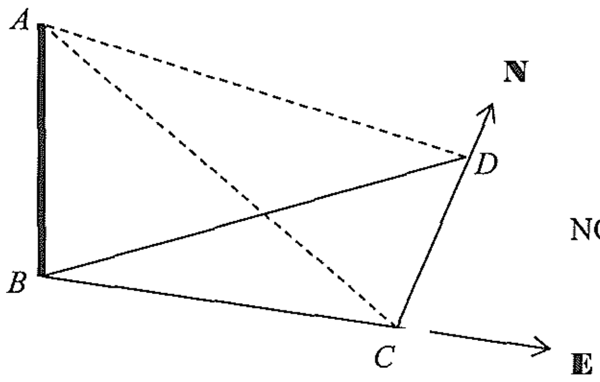
(iii) What is the focus of the locus of M? 1

Marks

Question 12 (12 marks) Start a new booklet

a) The polynomial equation $8x^3 - 36x^2 + 22x + 21 = 0$ has roots which form an arithmetic progression. Find the roots. 3

b) A is the top of a vertical radio mast AB standing on level ground. Two points C and D are on ground level such that C is due East of B and D is 500 metres due North of C.



NOT DRAWN TO SCALE

The angle of elevation of A from C is $11°13'$ and the angle of elevation of A from D is $8°14'$. Calculate the height of the tower to the nearest metre. 3

c) The velocity V of a particle decreases according to the equation:

$$\frac{dV}{dt} = -k(V - P)$$

where t is the time in seconds and k is a positive constant. The initial velocity of the particle is 0 ms^{-1} and the terminal velocity P is 60 ms^{-1}.

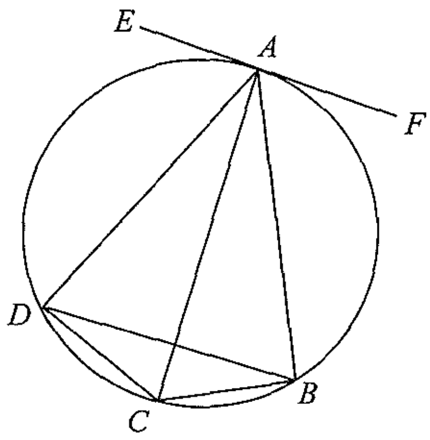
(i) Show that $V = P + Ae^{-kt}$, where A is a constant, satisfies the given differential equation. 1

(ii) Find the value of k if the velocity of the particle after 10 seconds is 35 ms^{-1}. Give your answer correct to two significant figures. 2

Marks

Question 12 continued

d) $ABCD$ is a cyclic quadrilateral. EAF is a tangent at A to the circle. CA bisects $\angle BCD$.



NOT DRAWN TO SCALE

3

Show that $EAF \parallel DB$.

Marks

Question 13 (12 marks) Start a new booklet

a) (i) Show:

$$\sin x - \cos x = \sqrt{2}\sin\left(x - \frac{\pi}{4}\right)$$

2

(ii) Hence sketch the graph of $y = \sin x - \cos x$, for $0 \leq x \leq 2\pi$. 2

(iii) Show that $x = \frac{\pi}{2}$ is a solution to $\sin x - \cos x = 1$ and hence solve $\sin x - \cos x > 1$ for $0 \leq x \leq 2\pi$. 2

b) Prove:

$$\sin^{-1}\frac{1}{\sqrt{5}} + \sin^{-1}\frac{1}{\sqrt{10}} = \frac{\pi}{4}.$$

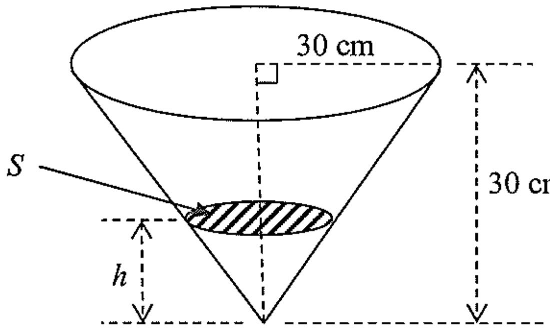
3

c) Use mathematical induction to show that the expression $7^n + 5$ is divisible by 6 for all positive integer values of n. 3

Marks

Question 14 (12 marks) Start a new booklet

a) Water is pumped into a conical vessel at a constant rate of 24 cm^3 per second. The depth of the water is 'h' cm at any time 't' seconds



NOT DRAWN TO SCALE

What is the rate of increase in the area of the surface 'S' of the water when the depth is 16 cm. **4**

b) In the expansion of $(1 + ax)^n$ in ascending powers of x, the first three terms are:

$$1 + 6x + 16x^2 + \ldots$$

(i) By comparing coefficients write two equations in a and n. **2**

(ii) Hence find the values of a and n. **2**

c) A particle is performing Simple Harmonic Motion in a straight line. At time t seconds it has displacement x metres from a fixed point O on the line, velocity v ms^{-1} and acceleration in ms^{-2} given by $\ddot{x} = -4(x - 1)$. When the particle is at the centre of its motion it has speed 6 ms^{-1}.

(i) Show that $v^2 = -4x^2 + 8x + 32$. **2**

(ii) Find the period and amplitude of the motion. **2**

Marks

Question 15 (12 marks) Start a new booklet

a) State the largest possible (natural) domain of the function: 2

$$y = \log_e(\sin^{-1} x)$$

b) A stone is projected from the top of a 80 metre high vertical cliff with an initial velocity of V ms at an angle of projection of θ. It reaches its greatest height after 3 seconds and hits the ground at a horizontal distance of 320 metres from the foot of the cliff. Assume $g = 10ms^{-2}$.

(i) Using the top of the cliff as the origin, the horizontal position of the stone is given as $x = V\cos\theta t$. Show that the vertical component of the parametric equations of the path of the stone is:

$$y = -5t^2 + V\sin\theta t$$

2

(ii) Show that $V\sin\theta = 30$. 1

(iii) Find how long it takes for the stone to reach the ground. 1

(iv) Show that $V\cos\theta = 40$ 1

(v) Find the value of V and the angle of projection. 1

c) (i) Show that for all positive integers n,

$$x[(1+x)^{n-1} + (1+x)^{n-2} + \cdots + (1+x)^2 + (1+x) + 1] = (1+x)^n - 1$$

1

(ii) Hence show that for $1 \le k \le n$

$$\binom{n-1}{k-1} + \binom{n-2}{k-1} + \binom{n-3}{k-1} + \cdots + \binom{k-1}{k-1} = \binom{n}{k}$$

1

(iii) Show that $n\binom{n-1}{k} = (k+1)\binom{n}{k+1}$

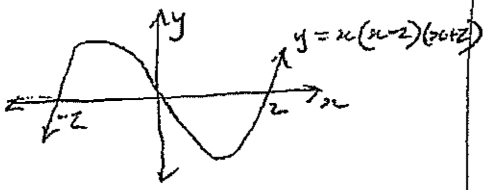
2

End of Paper

SOLUTIONS _ Extension 1 Trial 2016

SECTION 1

Question	Solution	Criteria
1.	$x-2y+1=0$ $y=\frac{1}{2}x+\frac{1}{2}$ $m_1=\frac{1}{2}$ $2x-y-1=0$ $y=2x-1$ $m_2=2$ $\tan\theta=\left\lvert\frac{m_1-m_2}{1+m_1m_2}\right\rvert$ $=\left\lvert\frac{\frac{1}{2}-2}{1+\frac{1}{2}\times 2}\right\rvert$ $=\frac{3}{4}$ $\theta=36.86989765\ldots$ $\doteqdot 37^\circ$	A
2	$\left(\frac{mx_2+nx_1}{m+n},\frac{my_2+ny_1}{m+n}\right)$ $m=3$, $n=2$, $x_1=-2$, $x_2=8$, $y_1=2$, $y_2=-3$ $=\left(\frac{3\times 8+2\times -2}{3+2},\frac{3\times -3+2\times 2}{3+2}\right)$ $=\left(\frac{24-4}{5},\frac{-9+4}{5}\right)$ $=(4,-1)$	A
3.	$\int\frac{dx}{25\left(\frac{9}{25}+x^2\right)}$ $a=\frac{3}{5}$ $=\frac{5}{3}\times\frac{1}{25}\tan^{-1}\frac{5x}{3}+c$ $=\frac{1}{15}\tan^{-1}\frac{5x}{3}+c$	D
4.	$x=t-3$ $t=x+3$ $y=t^2+2$ $y=(x+3)^2+2$ $=x^2+6x+11$	D
5	$v^2=36-4x^2$ $=2^2(9-x^2)$ $[=n^2(a^2-x^2)]$ $a^2=9$, $n=2$ and $\alpha=0$ (initially at the origin) $a=3$ $x=a\sin(nt+\alpha)$ $=3\sin 2t$	B

Question	Solution	Criteria
6	$x^2 \times \frac{x^2-4}{x} \geq 0 \times x^2 \quad x \neq 0$ $x(x^2-4) \geq 0$ $x(x-2)(x+2) \geq 0$ $-2 \leq x \leq 0$ or $x \geq 2$ 	A
7.	$u = 2x+1 \qquad x=1 \Rightarrow u=3$ $du = 2\,dx \qquad x=0 \Rightarrow u=1$ $\int_0^1 \frac{4x}{2x+1}\,dx = \int_1^3 \frac{2(u-1)}{u}\cdot\frac{1}{2}\,du$ $= \int_1^3 \left(1-\frac{1}{u}\right) du$ $= \left[u - \log_e u\right]_1^3$ $= 3 - \log_e 3 - 1 + \log_e 1$ $= 2 - \log_e 3$	B
8	$\int (\cos^2 x + 2\sec^2 x)\,dx = \int \left(\frac{1}{2}(1+\cos 2x) + 2\sec^2 x\right) dx$ $= \frac{1}{2}x + \frac{1}{4}\sin 2x + 2\tan x + c$	C
9	$T_{r+1} = {}^9C_r (x^2)^{9-r}\left(\frac{-2}{x}\right)^r$ $= {}^9C_r\, x^{18-2r}(-2)^r x^{-r}$ $= {}^9C_r\, x^{18-3r}(-2)^r$ Term independent of x $18-3r=0$ $r=6$ $T_7 = {}^9C_6(-2)^6$	B
10	$a = 3x^2$ $v^2 = 2\int(3x^2)\,dx$ $= 2x^3 + c$ when $x=1$, $v=-\sqrt{2}$ then $c=0$ $v = -\sqrt{2x^3}$ ($v<0$ when $x=1$) $\frac{dx}{dt} = -\sqrt{2x^3}$ $\frac{dt}{dx} = -\frac{1}{\sqrt{2}}x^{-\frac{3}{2}}$ $t = \frac{2}{\sqrt{2}}x^{-\frac{1}{2}} + c_1$ when $t=0$, $x=1$ then $c_1 = -\sqrt{2}$ $x^{-\frac{1}{2}} = \frac{t+\sqrt{2}}{\sqrt{2}}$ $x^{\frac{1}{2}} = \frac{\sqrt{2}}{t+\sqrt{2}}$ $x = \frac{2}{(t+\sqrt{2})^2}$	C

MATHEMATICS EXTENSION I – QUESTION 11

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
a) $f(x) = x - 2\sin x$ $\qquad x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$	1	many students
$f'(x) = 1 - 2\cos x$		forgot to use radians.
$f(1.6) = 1.6 - 2\sin 1.6$ $\qquad = 1.6 - \frac{1.6 - 2\sin 1.6}{1 - 2\cos 1.6}$ ✓		
$f'(1.6) = 1 - 2\cos 1.6$		
$= 1.977123551\ldots$		
✓ $\doteqdot 1.98$ or any other correct rounding off	1	
b) $\|2x-1\| = \|x\|$		
$x < 0$ $\qquad x \geq 0$ $\qquad$ or $(\|2x-1\|)^2 = (\|x\|)^2$		generally well done
$2x - 1 = -x$ $\qquad 2x - 1 = x$ $\qquad (2x-1)^2 = x^2$	1	only mistake
$3x = 1$ $\qquad x = 1$ ✓ $\qquad 4x^2 - 4x + 1 = x^2$		being student only
$x = \frac{1}{3}$ ✓ $\qquad 3x^2 - 4x + 1 = 0$		gave one sol'n.
$(3x-1)(x-1) = 0$		
$x = 1$ ✓ or $x = \frac{1}{3}$ ✓	1	

MATHEMATICS EXTENSION I – QUESTION 11

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
c) i) when $\ddot{x} = -n^2(x)$ SHM.		Marks lost if
$x = 1 + \sqrt{3}\cos 4t + \sin 4t$		the best
$\dot{x} = -\sqrt{3}\,4\sin 4t + 4\cos 4t$	1	statement was
$\ddot{x} = -\sqrt{3}\times 4^2\cos 4t - 4^2\sin 4t$ ✓		not written. *
$= -4^2(\sqrt{3}\cos 4t + \sin 4t)$		and transcript
$= -4^2(x-1)$ ✓ SHM. *	1	errors.
ii) Max speed when $\ddot{x}=0$ or at the centre of motion		
$\ddot{x} = -4^2(\sqrt{3}\cos 4t + \sin 4t)$		transcript errors
$0 = -4^2(\sqrt{3}\cos 4t + \sin 4t)$		caused major
$\sqrt{3}\cos 4t + \sin 4t = 0$	1	errors in method
$\sin 4t = -\sqrt{3}\cos 4t$		student used.
$\tan 4t = -\sqrt{3}$		
$4t = 2\pi/3,\ \frac{5\pi}{3}$		
$t = 2\pi/12,\ 5\pi/12.$		
$= \pi/6,\ 5\pi/12$	1	
particle first reaches max. speed when $t = \pi/6$.		

MATHEMATICS EXTENSION I – QUESTION 11

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
d) (i) $RHS = 2(p^2+q^2) - (p+q)^2$		well done
$= 2p^2+2q^2 - p^2 - 2pq - q^2$		
$= p^2+q^2-2pq$	1	
$= (p-q)^2$		
$= LHS$		
(ii) Coordinate M $\left(\frac{2p+2q}{2}, \frac{p^2+q^2}{2}\right)$		Generally mistake when made finding the coordinate of midpoint!?
$\left(p+q, \frac{p^2+q^2}{2}\right)$	1	
using part (i) and $p-q=4$		
$(p-q)^2 = 2(p^2+q^2) - (p+q)^2$		
$x = p+q, \quad y = \frac{p^2+q^2}{2}$		
$4^2 = 2\times 2y - x^2$		
$x^2 = 4y-16$	1	
locus of M is the parabola $x^2 = 4y-16$		Mark given if correct vertex indicated and focus.
(iii) $x^2 = 4y-16$		
$= 4(y-4)$	1	
focal length $=1$. Vertex $(0,4)$ Focus $(0,5)$		
		$a=1$ not good enough

Trial 2016

MATHEMATICS EXTENSION I – QUESTION 12

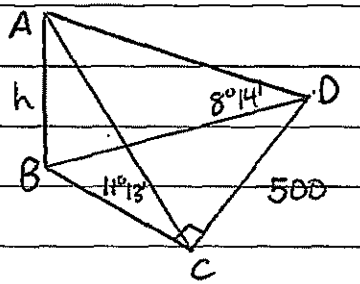
SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
(a) For $8x^3-36x^2+22x+21=0$		
Let the roots be $\alpha-d,\ \alpha,\ \alpha+d$ (where d is the common difference)		This method was the
Sum of roots one at a time: $\alpha-d+\alpha+\alpha+d=\frac{36}{8}$		better one to use to
$3\alpha=\frac{9}{2}$		find the roots and
$\alpha=\frac{3}{2}$	1	for this reason,
Product of the roots: $\alpha(\alpha-d)(\alpha+d)=-\frac{21}{8}$		most of the students
$\alpha(\alpha^2-d^2)=-\frac{21}{8}$		(who used this method)
sub $\alpha=\frac{3}{2}$, $\frac{3}{2}\left[\left(\frac{3}{2}\right)^2-d^2\right]=-\frac{21}{8}$		answered this question
$\frac{9}{4}-d^2=-\frac{7}{4}$		well.
$d^2=\frac{9}{4}+\frac{7}{4}$		
$=4$		
$d=\pm 2$ [either value will give same root]	1	
$\therefore$ roots are: $\frac{3}{2},\ \frac{3}{2}-2,\ \frac{3}{2}+2$		
$=\frac{3}{2},\ -\frac{1}{2},\ \frac{7}{2}$	1	Refer to next page
		for the alternate
Alternatively you could use sum of roots two at a time to find d.		solution where the
ie $\alpha(\alpha-d)+\alpha(\alpha+d)+(\alpha-d)(\alpha+d)=\frac{22}{8}$		roots were taken as
$\alpha^2-\alpha d+\alpha^2+\alpha d+\alpha^2-d^2=\frac{22}{8}$		$\alpha,\ \alpha+d,\ \alpha+2d$.
$3\alpha^2-d^2=\frac{22}{8}$		
$d^2=4$	3	

$d=\pm 2$

MATHEMATICS EXTENSION I – QUESTION 12

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
a) Alternate solution to 12(a)		
Let roots be α, $\alpha+d$, $\alpha+2d$		
$\Sigma\alpha$: $\alpha+\alpha+d+\alpha+2d=\frac{36}{8}$		Many students who
$3\alpha+3d=\frac{9}{2}$		used this method
$3(\alpha+d)=9/2$		struggled with
$\alpha+d=3/2$	1	the algebra
$\alpha=3/2-d$ — (1)		and the simplification
$\Sigma\alpha\beta$: $\alpha(\alpha+d)+\alpha(\alpha+2d)+(\alpha+d)(\alpha+2d)=\frac{22}{8}$		of terms.
$\alpha^2+\alpha d+\alpha^2+2\alpha d+\alpha^2+3\alpha d+2d^2=22/8$		
$3\alpha^2+6\alpha d+2d^2=22/8$ — (2)		
Sub (1) in (2): $3\left[3/2-d\right]^2+6\left[\left(\frac{3}{2}-d\right)d\right]+2d^2=22/8$		
$3\left(\frac{9}{4}-3d+d^2\right)+9d-6d^2+2d^2=22/8$		
$27/4-9d+3d^2+9d-6d^2+2d^2=22/8$		
$d^2=4$		
$d=\pm 2$ sub $d=2$ in (1)	1	
$\alpha=3/2-2$ (or $d=-2$)		
$=-1/2$		
and $\alpha+2d=-1/2+4=3\frac{1}{2}$		
and $\alpha+d=3/2$		
$\therefore$ roots are $-1/2$, $3/2$, $3\frac{1}{2}$	1	

MATHEMATICS EXTENSION I – QUESTION 12

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
12 b)  Let $AB = h$ From $\triangle ABC$, $\tan 11°13' = \frac{h}{BC}$ $\therefore BC = \frac{h}{\tan 11°13'}$ --- (1) From $\triangle ABD$ $\tan 8°14' = \frac{h}{BD}$ $BD = \frac{h}{\tan 8°14'}$ --- (2)	1	A few students were not confident in applying particular method to find the height AB.
In $\triangle BCD$, $BD^2 = BC^2 + 500^2$ and $BD^2 - BC^2 = 500^2$ --- (3) Sub (1) and (2) in (3) $\frac{h^2}{\tan^2 8°14'} - \frac{h^2}{\tan^2 11°13'} = 500^2$ --- (4)	1	Using Pythagoras Theorem was the best formula to use as $\triangle BCD$ was a right-angled $\triangle$.
$h^2\left(\frac{1}{\tan^2 8°14'} - \frac{1}{\tan^2 11°13'}\right) = 500^2$ $h^2 = \frac{500^2}{\frac{1}{\tan^2 8°14'} - \frac{1}{\tan^2 11°13'}}$ $h = \sqrt{\frac{500^2}{\frac{1}{\tan^2 8°14'} - \frac{1}{\tan^2 11°13'}}}$ $h = 105.8$	1/2	Some students used the cosine rule but did not substitute the correct angles and sides. Should be: $BD^2 = BC^2 + CD^2 - 2(BC)(DC)\cos\angle C$ $\frac{h^2}{\tan^2 8°14'} = \frac{h^2}{\tan^2 11°13'} + 500^2 - \underbrace{2(BC)(DC)\cos 90°}_{=0}$ etc.
$\therefore$ height of tower is 106 m to the nearest minute	1/2 (3)	PTO →

MATHEMATICS EXTENSION I – QUESTION 12

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
12b) Examiners Comments: Cont'd From Equation (4) students found a common denominator making the question more difficult and hence allowed for more errors.		
From (4) $\dfrac{h^2(\tan^2 11°13') - h^2(\tan^2 8°14')}{(\tan^2 11°13')(\tan^2 8°14')} = 500^2$	1	
$h^2 = \dfrac{500^2 \tan^2 11°13' \tan^2 8°14}{\tan^2 11°13' - \tan^2 8°14'}$		
$h = \sqrt{\dfrac{500^2 \tan^2 11°13' \tan^2 8°14}{\tan^2 11°13' - \tan^2 8°14'}}$		
$h = \dfrac{500 \tan 11°13' \tan 8°14}{\sqrt{\tan^2 11°13' - \tan^2 8°14'}}$		
$\doteqdot 106$		

MATHEMATICS EXTENSION I – QUESTION 12

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
Alternate solution 12 b) (But harder and NOT preferred)		
$\tan 11^\circ 13' = \frac{h}{BC}$ and $\tan 8^\circ 14' = \frac{h}{BD}$		
$h = BC\tan 11^\circ 13'$ — ① and $h = BD\tan 8^\circ 14'$ — ②		
① = ②		
$BC\tan 11^\circ 13' = BD\tan 8^\circ 14'$	1	
$BC = \frac{BD\tan 8^\circ 14}{\tan 11^\circ 13'}$		
In $\triangle BCD$, $500^2 + BC^2 = BD^2$.		
$BD^2 = 500^2 + \frac{BD^2\tan^2 8^\circ 14'}{\tan^2 11^\circ 13'}$	1	
$BD^2 - \frac{BD^2\tan^2 8^\circ 14}{\tan^2 11^\circ 13'} = 500^2$		
$BD^2(\tan^2 11^\circ 13') - BD^2(\tan^2 8^\circ 14') = 500^2(\tan^2 11^\circ 13')$		
$BD^2(\tan^2 11^\circ 13' - \tan^2 8^\circ 14) = 500^2\tan^2 11^\circ 13'$		
$BD^2 = \frac{500^2\tan^2 11^\circ 13'}{\tan^2 11^\circ 13' - \tan^2 8^\circ 14'}$		
$BD = \frac{500\tan 11^\circ 13'}{\sqrt{\tan^2 11^\circ 13' - \tan^2 8^\circ 14}}$	$\frac{1}{2}$	
Sub in ② $h = \frac{500\tan 11^\circ 13' \times \tan 8^\circ 14'}{\sqrt{\tan^2 11^\circ 13' - \tan^2 8^\circ 14'}}$		

$\doteqdot 106$ $\frac{1}{2}$

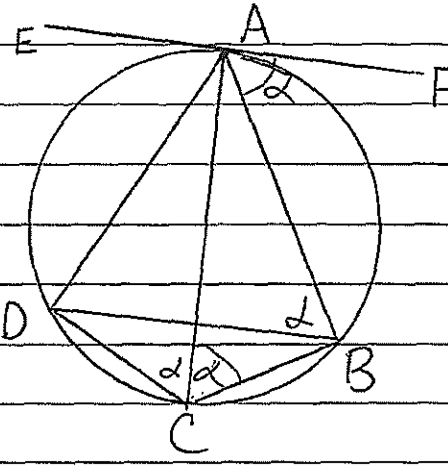
MATHEMATICS EXTENSION I – QUESTION 12

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
12 c) i) If $V = P + Ae^{-kt}$ --- (1)		
then $V - P = Ae^{-kt}$ --- (2)		This question was
From (1) $\frac{dV}{dt} = -k.Ae^{-kt}$	1	very well done.
$= -k(V-P)$ from (2)		
	(1)	
ii) Initially $t=0$, & $V=0$, $P=60$		
To find A, $V = P + Ae^{-kt}$		
$0 = 60 + Ae^{-k\times 0}$		
$A = -60$		
Also $t=10$ and $V=35$		
To find k, $35 = 60 - 60e^{-k\times 10}$	1	
$60e^{-10k} = 25$		
$e^{-10k} = \frac{25}{60}$		
$-10k = \ln\frac{25}{60}$		
$k = \frac{-1}{10}\ln\frac{25}{60}$	1	
$= \frac{1}{10}\ln\frac{60}{25}$		
$\doteq 0.88$	(2)	

MATHEMATICS EXTENSION I – QUESTION 12

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS

12d)



Solution 1.

Let $\angle BAF = \alpha$

$\angle BCA = \angle BAF$ (angle between a tangent and a chord is equal to the angle in the alternate segment).

$= \alpha$

Also

$\angle BCA = \angle DCA$ (given that CA bisects $\angle BCD$) — 1

$\therefore \angle DCA = \alpha$

Now $\angle DBA = \angle DCA$ (angles in the same segment)

ie $\angle DBA = \alpha$ — 1

$\therefore$ $\angle BAF = \angle DBA$ (both equal to α)

The alternate angles $\angle BAF$ and $\angle DBA$ are equal

$\therefore$ the two lines are parallel, — 1

$\therefore$ $EAF \parallel BD$

[Note: Nearly all students gave the incorrect reason to this final statement. They gave the converse to alternate angle theorem which is "alternate angles on parallel lines are equal"] However not penalised for this.

[Note: 1 mark was awarded when 2 angles made a connection with another angle with the correct reason in a logical manner. Max. of 2 marks for this]

(3)

MARKER'S COMMENTS:

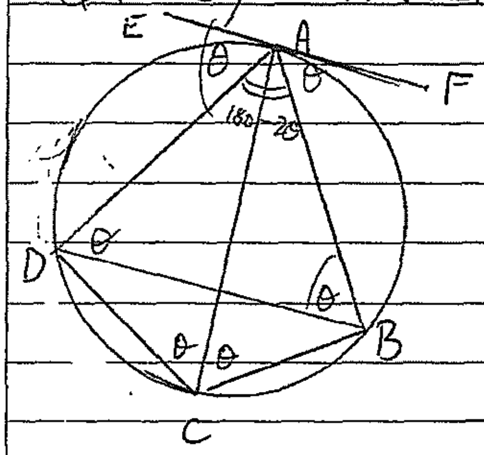
This question was not well done.

When proving in Geometry it is important to communicate in a logical sequential manner. Marks were not awarded when this did not occur.

Also, the only given facts were that AC bisects $\angle DCB$ and that ABCD was a cyclic Quadrilateral.

PTO →

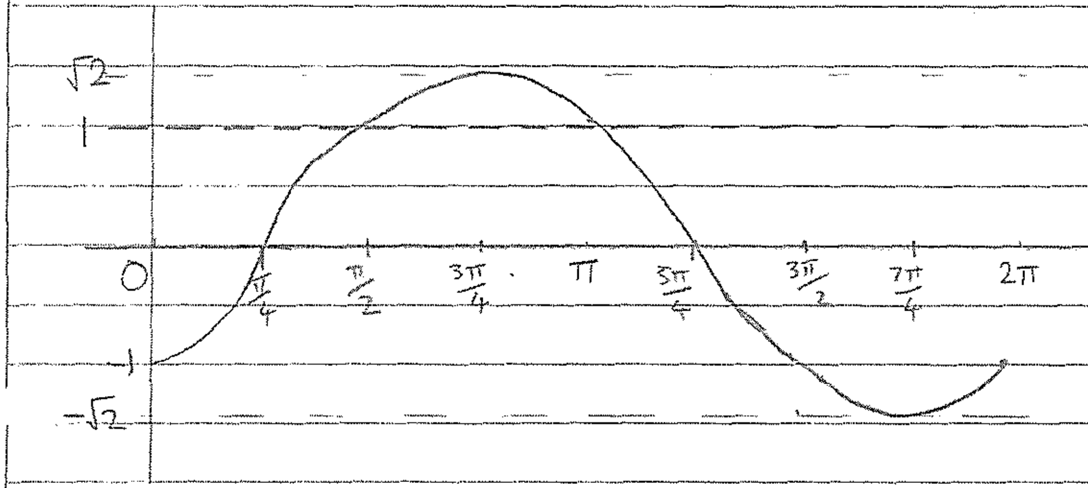
MATHEMATICS EXTENSION I – QUESTION 12

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
Q12 d) Alternate solution 2  Let $\angle EAD = \theta$. $\angle ABD = \angle EAD$ (angle between a tangent and a chord is equal to the angle in the alternate segment) $= \theta$ $\angle DCA = \angle ABD$ (angles in the same segment) $= \theta$ Also $\angle BCA = \angle DCA$ (CA bisects $\angle BCD$) $= \theta$ Now $\angle DCB = 2\theta$ Also $\angle DAB = 180 - \angle DCB$ (opposite angles in a cyclic quadrilateral) $= 180 - 2\theta$ Now $\angle BAF = 180 - \angle EAD - \angle DAB$ (angle sum of straight line) $= 180 - \theta - (180 - 2\theta)$ (shown above) $= \theta$ S.c. and $\angle ADB = \angle BAF$ (angles in the alternate segment) $= \theta$ $\therefore \angle EAD = \angle ADB$ (both equal to θ) Alternate angles $\angle EAD$ and $\angle ADB$ are equal $\therefore$ $\therefore$ two lines are paralle ie $EAF \parallel BD$.	1 1 1 3	Many students made <u>assumptions</u> from the information that CA bisects $\angle BCD$. They assumed: – AC is a diameter – $\angle EAC = 90°$ (the angle between tangent and radius is $90°$) – $AC \perp DC$ and – AC bisects $\angle DAB$ – AC bisects $\angle DB$ Although these may seem to be true, you would <u>need</u> to <u>prove</u> them first, then it would be OK to use. By making these assumptions, no marks were awarded.

MATHEMATICS EXTENSION I – QUESTION 13

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
a) i Let $\sin x - \cos x = R\sin(x-\alpha)$		One mark each
$= R\sin x\cos\alpha - R\cos x\sin\alpha$		for finding R
		and α.
$\therefore R\cos\alpha = 1$ $\quad$ $R\sin\alpha = 1$		Simply stating
$R^2\cos^2\alpha = 1$ $\quad$ $R^2\sin^2\alpha = 1$		"$R=\sqrt{2}$" or "$\alpha=\frac{\pi}{4}$"
So $R^2\cos^2\alpha + R^2\sin^2\alpha = 1+1$		earned no marks,
$R^2(\sin^2\alpha+\cos^2\alpha) = 2$		as this information
$R^2 = 2$		is provided in
$R = \sqrt{2}$ $\quad (R>0)$	1	the question
$\therefore \sqrt{2}\cos\alpha = 1$ $\quad$ $\therefore \sqrt{2}\sin\alpha = 1$		
$\cos\alpha = \frac{1}{\sqrt{2}}$ $\quad$ $\sin\alpha = \frac{1}{\sqrt{2}}$ $\quad$ $\therefore \alpha = \frac{\pi}{4}$	1	
$\therefore \sin x - \cos x = \sqrt{2}\sin\left(x-\frac{\pi}{4}\right)$		
OR $\quad$ RHS $= \sqrt{2}\sin\left(x-\frac{\pi}{4}\right)$		
$= \sqrt{2}\left(\sin x\cos\frac{\pi}{4} - \cos x\sin\frac{\pi}{4}\right)$		
$= \sqrt{2}\left(\sin x\cdot\frac{1}{\sqrt{2}} - \cos x\cdot\frac{1}{\sqrt{2}}\right)$		
$= \sqrt{2}\times\frac{1}{\sqrt{2}}(\sin x - \cos x)$		
$= \sin x - \cos x$		
$=$ RHS		

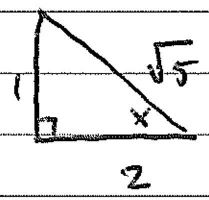
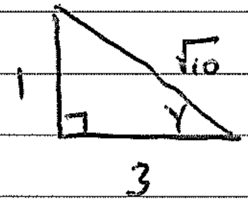
MATHEMATICS EXTENSION I – QUESTION 13 (continued)

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
a) ii		
	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	Intercepts correct and labelled Amplitude Correct shift / phase Period
iii $\sin x - \cos x = 1$ $\therefore \sqrt{2}\sin\left(x - \frac{\pi}{4}\right) = 1$ $\sin\left(x - \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$ $\therefore x - \frac{\pi}{4} = \frac{\pi}{4}, \frac{3\pi}{4} \ldots$ $\therefore x = \frac{\pi}{2}, \pi, \ldots$ $\therefore \frac{\pi}{2}$ is a solution	1	For a "show" question, anything you are given in the question is worth zero marks. Eg. simply stating $\sin\left(\frac{\pi}{2}\right) - \cos\left(\frac{\pi}{2}\right) = 1$ does not <u>show</u> the result – the truth of this statement is provided in the question itself.
<u>OR</u> when $x = \frac{\pi}{2}$, LHS $= \sin\left(\frac{\pi}{2}\right) - \cos\left(\frac{\pi}{2}\right)$ $= 1 - 0$ $= 1$ $=$ RHS $\therefore \frac{\pi}{2}$ is a solution		
$\therefore \frac{\pi}{2} < x < \pi$	1	

($\frac{1}{2}$ mark for $\frac{\pi}{2} \le x \le \pi$)

Also note that a rough sketch is insufficient to show this result – do it algebraically

MATHEMATICS EXTENSION I – QUESTION 13

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
b) Let $X = \sin^{-1}\frac{1}{\sqrt{5}}$ and $Y = \sin^{-1}\frac{1}{\sqrt{10}}$		
then		
$\sin X = \frac{1}{\sqrt{5}}$ and $\sin Y = \frac{1}{\sqrt{10}}$	1	
 		
$\sin(X+Y) = \sin X\cos Y + \cos X\sin Y$		
$= \frac{1}{\sqrt{5}} \times \frac{3}{\sqrt{10}} + \frac{2}{\sqrt{5}} \times \frac{1}{\sqrt{10}}$	1	
$= \frac{5}{5\sqrt{2}}$		
$= \frac{1}{\sqrt{2}}$		
$X+Y = \frac{\pi}{4}$	1	
$\therefore \sin^{-1}\frac{1}{\sqrt{5}} + \sin^{-1}\frac{1}{\sqrt{10}} = \pi/4$ #		

MATHEMATICS EXTENSION I – QUESTION 13 (continued)

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
c) Prove the statement true for $n=1$		
$7^1+5=12$		
$=6(2)$ which is divisible by 6	1	Correct proof for $n=1$
Let k be a positive integer for which the statement is true		
i.e. $7^k+5=6M$, where M is an integer	1	Correct assumption, including the essential requirement that M be an integer
$7^k=6M-5$		
Prove that if the statement is true for $n=k$, then it is true for $n=k+1$		
i.e. $7^{k+1}+5$ is divisible by 6		
$7^{k+1}+5 = 7(7^k)+5$		Many students made their working needlessly complicated by attempting to prove $7^{k+1}+5=6P$, without knowing what P would be (or worse, $7^{k+1}+5=6M$, which is impossible given the assumption).
$=7(6M-5)+5$, by the induction hypothesis		
$=42M-35+5$		
$=42M-30$		
$=6(7M-5)$ which is divisible by 6	1	
Therefore if the statement is true for $n=k$ it is also true for $n=k+1$. Since it is true for $n=1$, it is true for $n=2$ and so on. Therefore 7^n+5 is divisible by 6 for all positive integer values of n		

EXTENSION 1

MATHEMATICS - QUESTION NO: 14		
SUGGESTED SOLUTIONS	**MARKS**	**MARKER'S COMMENTS**
14(a) $V = \frac{1}{3}\pi r^2 h$ with $h = r$ (by similarity)	4	
$\therefore V = \frac{1}{3}\pi r^3$		There were a multitude
$\frac{dV}{dr} = \pi r^2$		of variations to the
		solution of this question
$\frac{dV}{dt} = \frac{dV}{dr}\frac{dr}{dt}$		I read carefully through
$24 = \pi r^2 \frac{dr}{dt}$		all the solutions, and
		allocated marks by
$\therefore \frac{dr}{dt} = \frac{24}{\pi r^2}$		subtracting 1 for each
		error ~~or~~ and omission of
$S = \pi r^2$		steps. No $\frac{1}{2}$ marks
$\frac{dS}{dr} = 2\pi r$		were awarded. The
$\frac{dS}{dt} = \frac{dS}{dr}\frac{dr}{dt}$		most common errors
$= 2\pi r \frac{24}{\pi r^2}$ (from above)		were of the type
		$V = \frac{1}{3}\pi r^2 h$ WRONG $V = \frac{1}{3}Sh$
$= \frac{48}{r}$		$\frac{dV}{dh} = \frac{1}{3}\pi r^2$ $\frac{dV}{dS} = \frac{h}{3}$
When $r = 16$		or $S = \pi r^2 + \pi r h$ WRONG
		$\frac{dS}{dh} = 2\pi r + \pi r$
$\frac{dS}{dt} = 3\ cm^2 s^{-1}$		(many more variations)
		STRESS TO STUDENTS that
		if h and r are variables

related to one another, not constants, must be [illegible] with differentiations

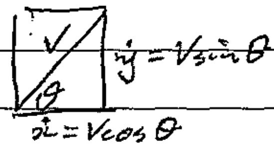
MATHEMATICS - QUESTION NO: 14

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
14 (b)(i) $(1+ax)^n = 1 + {}^nC_1 ax + {}^nC_2 a^2x^2 + \ldots$	2	A number of students thought the line marked * was the solution to part (a) but did ① and ② in part (b), no penalty. Marks were awarded by deduction for errors and/or omissions, no ½ marks were given.
$= 1 + 6x + 16x^2 + \ldots$		
${}^nC_1 a = 6$ $\quad$ ${}^nC_2 a^2 = 16$ * ←	1 mark awarded for this line	
$\therefore na = 6$ ① $\quad$ $\dfrac{n(n-1)a^2}{2} = 16$		
$n(n-1)a^2 = 32$ ②		
(ii) From (i) $a = \frac{6}{n}$	2	
$\therefore \cancel{n}(n-1)\dfrac{36}{\cancel{n^2}n} = 32$		
$\therefore (n-1).36 = 32n$		
$36n - 36 = 32n$		
$4n = 36$		
$n = 9$		
Sub in ① $a = \frac{2}{3}$		

MATHEMATICS - QUESTION NO: 14

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
14 (c) (i) $\ddot{x} = -4(x-1)$	2	
$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = -4(x-1)$		
$\frac{1}{2}v^2 = -2(x-1)^2 + c$		A lot of students had $x=0, v=6$ and found 'c' incorrectly, then manipulated to get 32 incorrectly
when $x=1, v=6$ $c=18$ Loss of 1 mark for using $x=0$ $v=6$.		
$\therefore \frac{1}{2}v^2 = -2(x-1)^2 + 18$		
$v^2 = -4(x-1)^2 + 36 = -4x^2 + 8x - 4 + 36$		
$v^2 = -4x^2 + 8x + 32$ as required		
(ii) $v^2 = -4(x^2 - 2x - 8)$	2.	Many students did this correctly, but those who got "a" incorrect lost 1 mark even if some of the working was correct. No $\frac{1}{2}$ marks were awarded. Most got $T = \pi$
$= -4[(x-1)^2 - 9]$		
$= 4[9 - (x-1)^2]$		
of the form		
$v^2 = n^2[a^2 - (x-b)^2]$ where $x=b$ is the centre of motion		
$n=2$ $T = \frac{2\pi}{n} = \pi$		
$a^2 = 9$ $a = 3$ since $a > 0$.		

MATHEMATICS EXTENSION I – QUESTION 15

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
a) $\sin^{-1}x > 0$ as $\log_e k$ defined only for $k>0$		Some did not look
$\sin^{-1}x \leq \frac{\pi}{2}$	2	at where $\log_e x$ is
		defined. Others did
$x \leq 1$		not define the
$\therefore$ Domain is $0 < x \leq 1$		domain. Overall
		well done.
b) (i) Vertical $\ddot{y} = -10$ 		
$\dot{y} = -10t + c_1$	2	Mostly done well.
when $t=0$, $\dot{y} = V\cos\theta$		Some students did
$\therefore c_1 = V\cos\theta$		not give enough
$\dot{y} = -10t + V\cos\theta$		supporting evidence.
$y = \frac{-10t^2}{2} + V\cos\theta\, t + c_2$		
when $t=0$ $y=0$ $\Rightarrow c_2 = 0$		
$y = -5t^2 + V\cos\theta\, t$		
(ii) Greatest height when $\dot{y}=0$ and $t=3$		Most student did
$\dot{y} = -10\times 3 + V\sin\theta$	1	this well.
$0 = -30 + V\sin\theta$		
$V\sin\theta = 30$		

MATHEMATICS EXTENSION I – QUESTION 15

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
b) (iii) Stone reaches the ground when $y = -80$		Some students took
$y = -5t^2 + V\sin\theta\, t$ $\quad V\sin\theta = 30$	1	ground level as $y=0$
$-80 = -5t^2 + 30t$		(They need to read the
$t^2 - 6t - 16 = 0$		question carefully)
$(t-8)(t+2) = 0$		
$\therefore t = 8$ since $t \geq 0$		
(iv) Stone reaches the ground at a range of 320m		Some students
after 8 seconds	1	had $320 \div 6 = 40$
$x = V\cos\theta\, t$ $\quad x = 320,\ t = 8$		others had
$320 = 8V\cos\theta$		$320 \div 80 = 40$
$V\cos\theta = 40$		
(v) $V^2 = x^2 + y^2$ $\quad \tan\theta = \dfrac{V\sin\theta}{V\cos\theta}$		Very well done by
$= (V\cos\theta)^2 + (V\sin\theta)^2$	1	most that attempted
$= 40^2 + 30^2$ $\quad = \dfrac{30}{40}$		it.
$= 2500$ $\quad \theta = 36°52'$		
$V = 50\ ms^{-1}$ $\quad$ (nearest minute)		

MATHEMATICS EXTENSION I – QUESTION 15

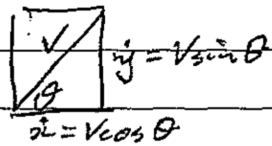
SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
c) (i) $LHS = x\left[1+(1+x)+(1+x)^2+\dots+(1+x)^{n-1}\right]$		Most students had
GP with $a=1$, $r=1+x$, n terms	1	difficulty with this
$S_n = \dfrac{1\left((1+x)^n-1\right)}{1+x-1}$		question. They did
$= \dfrac{(1+x)^n-1}{x}$		not recognise the GP
$= x \times \left[\dfrac{(1+x)^n-1}{x}\right]$		
$= (1+x)^n - 1$		
$= RHS$		
(ii) $RHS = (1+x)^n - 1$		This question was
$= \binom{n}{0}+\binom{n}{1}x+\binom{n}{2}x^2+\dots+\binom{n}{k}x^k\dots+\binom{n}{n}x^n-1$	1	quite poorly set out
coefficient of x^k is $\binom{n}{k}$		by many students.
$LHS = \left[x(1+x)^{n-1}\right]+\left[x(1+x)^{n-2}\right]+\dots+\left[x(1+x)^{k-1}\right]+\dots$		
$= \left[\binom{n-1}{0}x+\binom{n-1}{1}x^2+\dots+\binom{n-1}{k-1}x^k+\dots\right]$		
$+\left[\binom{n-2}{0}x+\binom{n-2}{1}x^2+\dots+\binom{n-2}{k-1}x^k+\dots\right]+\dots$		
$+\left[\binom{k-1}{0}x+\binom{k-1}{1}x^2+\dots+\binom{k-1}{k-1}x^k+\dots\right]$		
coefficient of x^k is:		
$\binom{n-1}{k-1}+\binom{n-2}{k-1}+\dots+\binom{k-1}{k-1}$		
$\therefore \binom{n-1}{k-1}+\binom{n-2}{k-1}+\dots+\binom{k-1}{k-1} = \binom{n}{k}$		

MATHEMATICS - QUESTION NO: 14

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
14 (b)(i) $(1+ax)^n = 1 + {}^nC_1 ax + {}^nC_2 a^2x^2 + \ldots$	2	A number of students thought the line marked * was the solution to part (a) but did ① and ② in part (b), no penalty. Marks were awarded by deduction for errors and/or omissions, no ½ marks were given.
$= 1 + 6x + 16x^2 + \ldots$		
${}^nC_1 a = 6$ ${}^nC_2 a^2 = 16$ *	← 1 mark awarded for this line	
$\therefore na = 6$ ① $\dfrac{n(n-1)a^2}{2} = 16$		
$n(n-1)a^2 = 32$ ②		
(ii) From (i) $a = \frac{6}{n}$	2	
$\therefore \cancel{n}(n-1)\dfrac{36}{\cancel{n^2}n} = 32$		
$\therefore (n-1).36 = 32n$		
$36n - 36 = 32n$		
$4n = 36$		
$n = 9$		
Sub in ① $a = \frac{2}{3}$		

MATHEMATICS - QUESTION NO: 14		
SUGGESTED SOLUTIONS	**MARKS**	**MARKER'S COMMENTS**
14 (c) (i) $\ddot{x} = -4(x-1)$	2	
$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = -4(x-1)$		
$\frac{1}{2}v^2 = -2(x-1)^2 + c$		A lot of students
when $x = 1$, $v = 6$ $c = 18$ Loss of 1 mark for using $x = 0$ $v = 6$. ⟵		had $x = 0, v = 6$
$\therefore \frac{1}{2}v^2 = -2(x-1)^2 + 18$		and found 'c' incorrectly,
$v^2 = -4(x-1)^2 + 36 = -4x^2 + 8x - 4 + 36$		then manipulated to
$v^2 = -4x^2 + 8x + 32$ as required		get 32 incorrectly
(ii) $v^2 = -4(x^2 - 2x - 8)$	2.	
$= -4[(x-1)^2 - 9]$		Many students did
		this correctly, but
$= 4[9 - (x-1)^2]$		those who got "a"
of the form		incorrect lost 1
$v^2 = n^2[a^2 - (x-b)^2]$ where $x = b$ is the		mark even if some
centre of motion		of the working was
$n = 2$ $T = \frac{2\pi}{n} = \pi$		correct. No $\frac{1}{2}$
$a^2 = 9$ $a = 3$ since $a > 0$.		marks were awarded.
		Most got $T = \pi$

MATHEMATICS EXTENSION I – QUESTION 15

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
a) $\sin^{-1}x > 0$ as $\log_e k$ defined only for $k > 0$ $\sin^{-1}x \leq \frac{\pi}{2}$ $x \leq 1$ $\therefore$ Domain is $0 < x \leq 1$	2	Some did not look at where $\log_e x$ is defined. Others did not define the domain. Overall well done.
b) (i) Vertical $\ddot{y} = -10$ $\dot{y} = -10t + c_1$ when $t=0$, $\dot{y} = V\cos\theta$ $\therefore c_1 = V\cos\theta$ $\dot{y} = -10t + V\cos\theta$ $y = \frac{-10t^2}{2} + V\cos\theta\, t + c_2$ when $t=0$ $y=0$ $\Rightarrow c_2 = 0$ $y = -5t^2 + V\cos\theta\, t$  	2	Mostly done well. Some students did not give enough supporting evidence.
(ii) Greatest height when $\dot{y} = 0$ and $t = 3$ $\dot{y} = -10 \times 3 + V\sin\theta$ $0 = -30 + V\sin\theta$ $V\sin\theta = 30$	1	Most student did this well.

MATHEMATICS EXTENSION I – QUESTION 15

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
b) (iii) Stone reaches the ground when $y=-80$		Some students took
$y=-5t^2+V\sin\theta\, t$ $\quad V\sin\theta=30$	1	ground level as $y=0$
$-80=-5t^2+30t$		(They need to read the
$t^2-6t-16=0$		question carefully)
$(t-8)(t+2)=0$		
$\therefore t=8$ since $t\geq 0$		
(iv) Stone reaches the ground at a range of 320m		Some students
after 8 seconds	1	had $320\div 6=40$
$x=V\cos\theta\, t$ $\quad x=320,\ t=8$		others had
$320=8V\cos\theta$		$320\div 80=40$
$V\cos\theta=40$		
(v) $V^2=x^2+y^2$ $\quad \tan\theta=\frac{V\sin\theta}{V\cos\theta}$		Very well done by
$=(V\cos\theta)^2+(V\sin\theta)^2$	1	most that attempted
$=40^2+30^2$ $\quad =\frac{30}{40}$		it.
$=2500$ $\quad \theta=36^\circ 52'$		
$V=50\ ms^{-1}$ $\quad$ (nearest minute)		

MATHEMATICS EXTENSION I – QUESTION 15

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
c) (iii) $n\binom{n-1}{k} = \dfrac{n(n-1)!}{k!(n-1-k)!}$		Done very well by most students that attempted it
$= \dfrac{n!}{k!(n-(k+1))!}$	2	
$(k+1)\binom{n}{k+1} = (k+1)\dfrac{n!}{(k+1)!(n-(k+1))!}$		
$= (k+1)\dfrac{n!}{(k+1)k!(n-(k+1))!}$		
$= \dfrac{n!}{k!(n-(k+1))!}$		
$\therefore n\binom{n-1}{k} = (k+1)\binom{n}{k+1}$		