Student Number:
Teacher:

St George Girls High School

Mathematics Extension 1

2020 Trial HSC Examination

General Instructions

- Reading time – 10 minutes
- Working Time – 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- For questions in **Section I,** use the multiple-choice answer sheet provided
- For questions in **Section II:**
  - Answer the questions in the writing booklets provided
  - Extra writing booklets are provided if needed
  - Start each question in a new writing booklet
  - Show relevant mathematical reasoning and/or calculations
  - Marks may not be awarded for incomplete or poorly presented solutions, or where multiple solutions are provided

Total marks: 70

Section I – 10 marks (pages 3 – 7)

- Attempt Questions 1 - 10
- Allow about 15 minutes for this section

Section II – 60 marks (pages 8 – 14)

- Attempt Questions 11-16
- Allow about 1 hour and 45 minutes for this section

Q1 – Q10	/10
Q11	/10
Q12	/10
Q13	/10
Q14	/10
Q15	/10
Q16	/10
Total	**/70**
	%

Section I - Multiple Choice

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section.
Use the multiple-choice answer sheet provided for Questions 1 - 10.

1. Consider the vectors $\underset{\sim}{a} = 2\underset{\sim}{i} + 3\underset{\sim}{j}$, $\underset{\sim}{b} = -3\underset{\sim}{i} + 2\underset{\sim}{j}$ and $\underset{\sim}{c} = 2\underset{\sim}{i} - \underset{\sim}{j}$.
Which of the following vectors is parallel to $\underset{\sim}{a} + \underset{\sim}{b} + \underset{\sim}{c}$?

(A) $-2\underset{\sim}{i} - 6\underset{\sim}{j}$

(B) $2\underset{\sim}{i} - 8\underset{\sim}{j}$

(C) $2\underset{\sim}{i} - 6\underset{\sim}{j}$

(D) $2\underset{\sim}{i} + 8\underset{\sim}{j}$

2. Which of the following is the coefficient of x^4 in the expansion $\left(x + \frac{3}{x}\right)^8$?

(A) 28

(B) 56

(C) 84

(D) 252

3. What is the derivative of $\cos^{-1} 3x$?

(A) $-\frac{1}{\sqrt{1 - 9x^2}}$

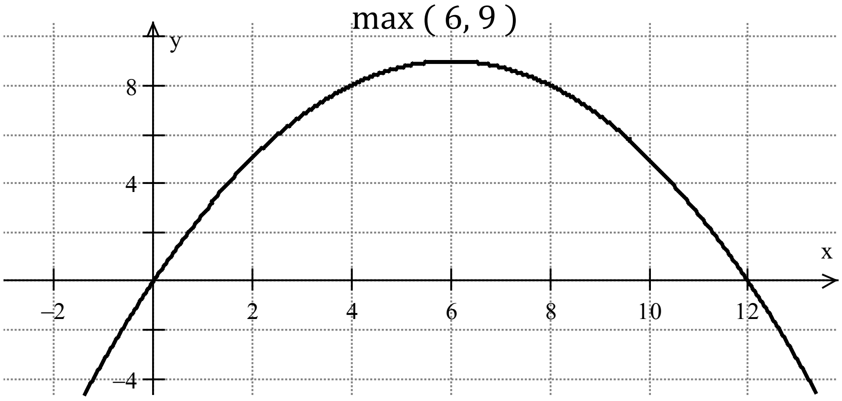
(B) $-\frac{3}{\sqrt{1 - 9x^2}}$

(C) $-\frac{1}{\sqrt{1 - 3x^2}}$

(D) $-\frac{3}{\sqrt{1 - 3x^2}}$

(Section I continued)

4.



Which of the parametric equations below represents the parabola above?

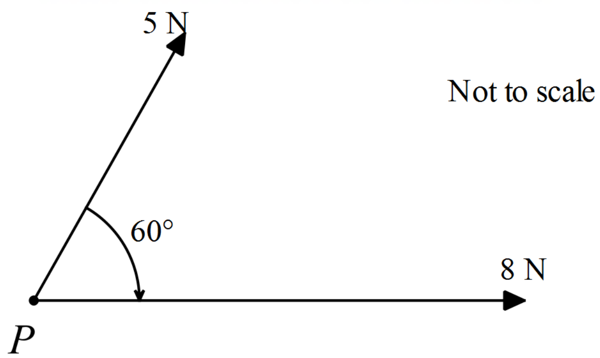
(A) $x = 12t,\ y = 9t$

(B) $x = 12t,\ y = 9 - t^2$

(C) $x = 6 - 2t,\ y = 9 - t^2$

(D) $x = 2t - 6,\ y = 9t - t^2$

5. Forces of magnitude 8 N and 5 N act on a particle P. The angle between the directions of the two forces is $60°$ as shown in the diagram.



Which of the following is the correct magnitude and direction of the resultant force acting on P ?

(A) 11.36 N, 22°25′ to the horizontal

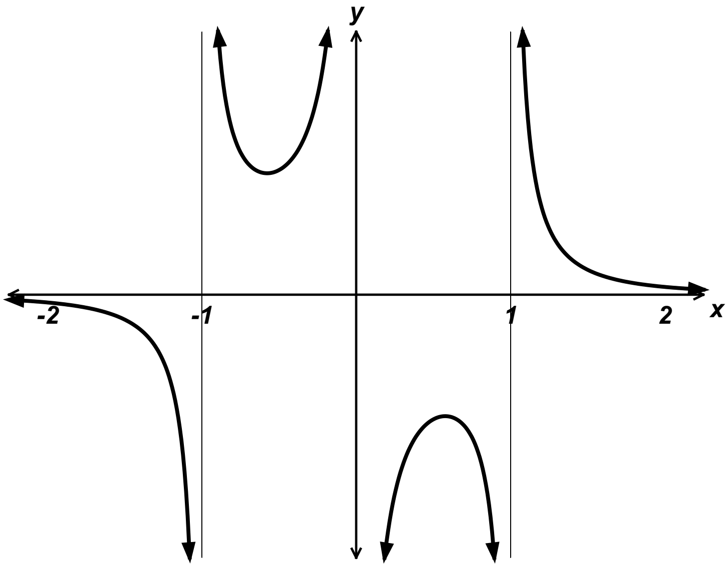
(B) 11.36 N, 67°35′ to the horizontal

(C) 12.58 N, 22°25′ to the horizontal

(D) 12.58 N, 67°35′ to the horizontal

(Section I continued)

6.



Not to scale

The graph above shows $y = \dfrac{1}{f(x)}$.

Which of the equations below best represents $y = f(x)$?

(A) $f(x) = x^2 - 1$

(B) $f(x) = x(x^2 - 1)$

(C) $f(x) = x^2(x^2 - 1)$

(D) $f(x) = x^2(x^2 - 1)^2$

7. Which of the following is the primitive of $\dfrac{3}{\sqrt{4 - 9x^2}}\, dx$?

(A) $\frac{1}{2}\sin^{-1} 3x + c$

(B) $\frac{3}{2}\sin^{-1}\frac{3x}{2} + c$

(C) $\sin^{-1}\frac{3x}{2} + c$

(D) $\sin^{-1}\frac{2x}{3} + c$

(Section I continued)

8. Which expression is equivalent to $\cos 5x \cos 2x - \sin 6x \sin 3x$?

(A) $\cos 7x - \sin 9x$

(B) $\cos 3x - \sin 3x$

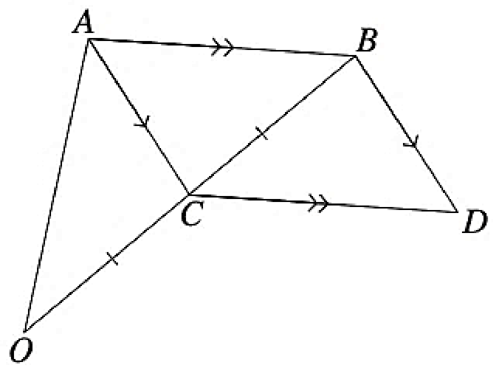
(C) $\sin 8x \sin x$

(D) $\cos 8x \cos x$

9. The position vectors of the points A and B are $\underset{\sim}{a}$ and $\underset{\sim}{b}$ respectively.

Point C is the midpoint of OB and point D is such that ABDC is a parallelogram.

O is the origin.



Not to scale

Which of the following is the position vector of D?

(A) $\frac{3}{2}\underset{\sim}{b} + \underset{\sim}{a}$

(B) $\frac{3}{2}\underset{\sim}{b} - \underset{\sim}{a}$

(C) $\frac{1}{2}\underset{\sim}{b} - \frac{1}{2}\underset{\sim}{a}$

(D) $\frac{1}{2}\underset{\sim}{b} - \underset{\sim}{a}$

(Section I continued)

10. The graph of the function $y = \tan^{-1}\frac{1}{2}(x - 2)$ is to be transformed by a translation left by 1 unit, then a horizontal dilation with a scale factor of 2.

The equation of the transformed graph is:

(A) $y = \tan^{-1}\left(\frac{x-3}{4}\right)$

(B) $y = \tan^{-1}\left(\frac{x-2}{4}\right)$

(C) $y = \tan^{-1}(x - 2)$

(D) $y = \tan^{-1}(x - \frac{1}{2})$

END OF SECTION I

Section II

60 marks
Attempt Questions 11 – 16
Allow about 1 hour and 45 minutes for this section.
Answer each question in the appropriate writing booklet. Extra writing booklets are available.
In Questions 11-16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (10 marks) Start a NEW Writing Booklet. **Marks**

(a) The polynomial $2x^3 - 4x^2 + 3x - 6 = 0$ has roots α, β and γ.

Calculate the value of $\dfrac{1}{\alpha} + \dfrac{1}{\beta} + \dfrac{1}{\gamma}$. **2**

(b) Solve $\dfrac{1}{x+1} \leq -1$. **3**

(c) Consider the word STATISTICS.

(i) How many arrangements of the letters are there? **1**

(ii) How many arrangements of the letters are there where the A and C are next to each other? **2**

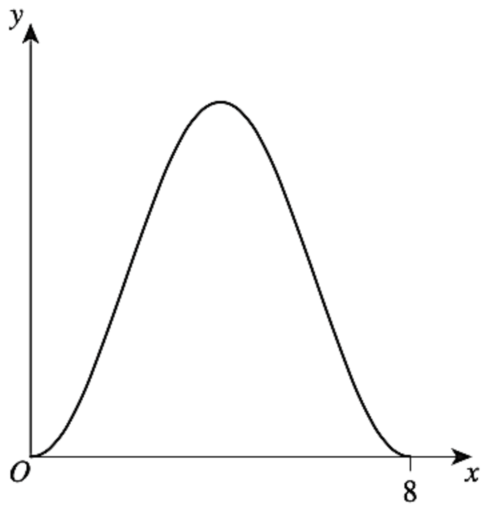
(d) In a barrel there are 50 marbles of various colours. Of these, 5 are green, 17 are blue, 12 are yellow, 12 are purple and 4 are orange.

What is the least number of marbles that can be selected from the barrel to ensure that 7 of the selected marbles are of the same colour? **2**

Question 12 (10 marks) Start a NEW Writing Booklet. **Marks**

(a) Express $2\sqrt{3}\sin x - 2\cos x$ in the form $R\cos(x+\alpha)$, where $R > 0$ and $[0, 2\pi]$. **3**

(b) A proposed plan for a garden is shown in the diagram. The curved boundary of the garden is modelled by the function $f(x) = 6\sin^2\left(\frac{\pi x}{8}\right)$, where $0 \le x \le 8$.



Not to scale

(i) Use the identity $\sin A \sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$ **2**
to show that $\sin^2\left(\frac{\pi x}{8}\right) = \frac{1}{2}\left(1 - \cos\frac{\pi x}{4}\right)$.

(ii) Use the result from part (i) to find the area A of the garden. **3**

Question 12 continues on page 10

Question 12 (continued)

(c) Consider the statement $P(n)$:

$$2^0 + 2^1 + 2^2 + \cdots + 2^{n-1} = 2^n - 1 \text{ for integers } n \geq 1.$$

An attempted proof of this statement by induction is given below.

Proof:
Assume the statement is true for $n = k + 1$.
That is, $2^0 + 2^1 + 2^2 + \cdots + 2^{k-1} + 2^k = 2^{k+1} - 1$ (1)

Next, we shall show it is true for $n = k$ by noting that if

$$2^0 + 2^1 + 2^2 + \cdots + 2^{k-1} + 2^k = 2^{k+1} - 1$$

is true, then

$$2^0 + 2^1 + 2^2 + \cdots + 2^{k-1} + 2^k = 2 \times 2^k - 1$$
$$2^0 + 2^1 + 2^2 + \cdots + 2^{k-1} + 2^k = 2^k + 2^k - 1$$

Now subtracting 2^k from both sides of this equation, we have

$$2^0 + 2^1 + 2^2 + \cdots + 2^{k-1} = 2^k - 1$$

Which is true by statement (1). Therefore, by the principle of induction, the statement $P(n)$ is true.

Give two reasons why the given proof is incorrect and does not prove $P(n)$. **2**

End of Question 12

Question 13 (10 marks) Start a NEW Writing Booklet. **Marks**

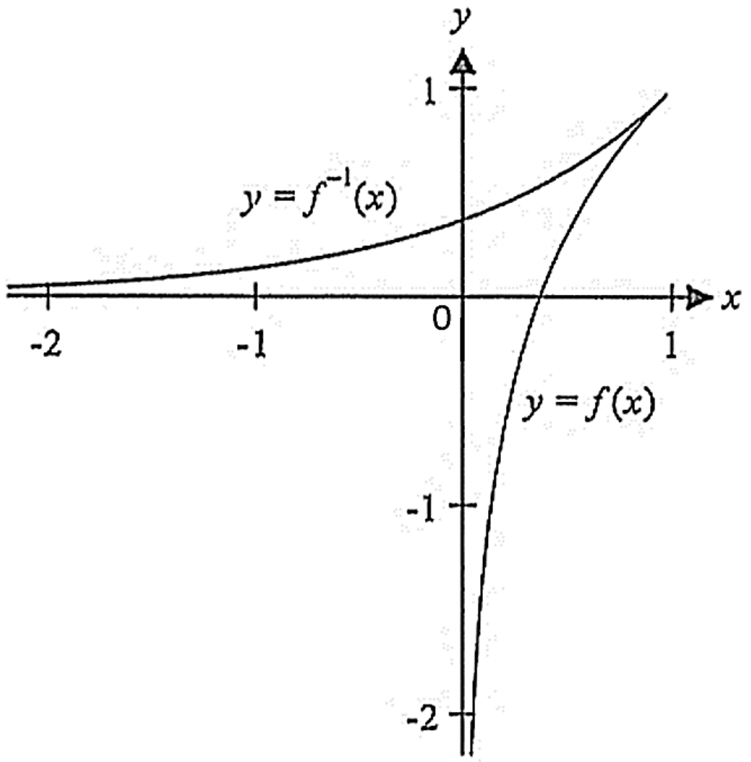
(a) Find $\int_0^{\ln 2} \frac{e^{2x}}{1+e^{4x}}\,dx$ by using the substitution $u = e^{2x}$, to two decimal places. **3**

(b) Use the t-formulae to solve the equation $\cos x - \sin x = 1$ where $0 \le x \le 2\pi$. **3**

(c) The function $f(x) = 1 + \ln x$ is defined in the domain $(0,1]$.

(i) Show that $\frac{d}{dx}(x \ln x) = 1 + \ln x$. **1**

(ii) The diagram shows the graphs of the function $y = f(x)$ and the inverse function $y = f^{-1}(x)$.



Not to scale

Find in simplest exact form the area of the region in the first quadrant bounded by the curves $y = f(x)$, $y = f^{-1}(x)$ and the coordinate axes. **3**

Question 14 (10 marks) Start a NEW Writing Booklet. **Marks**

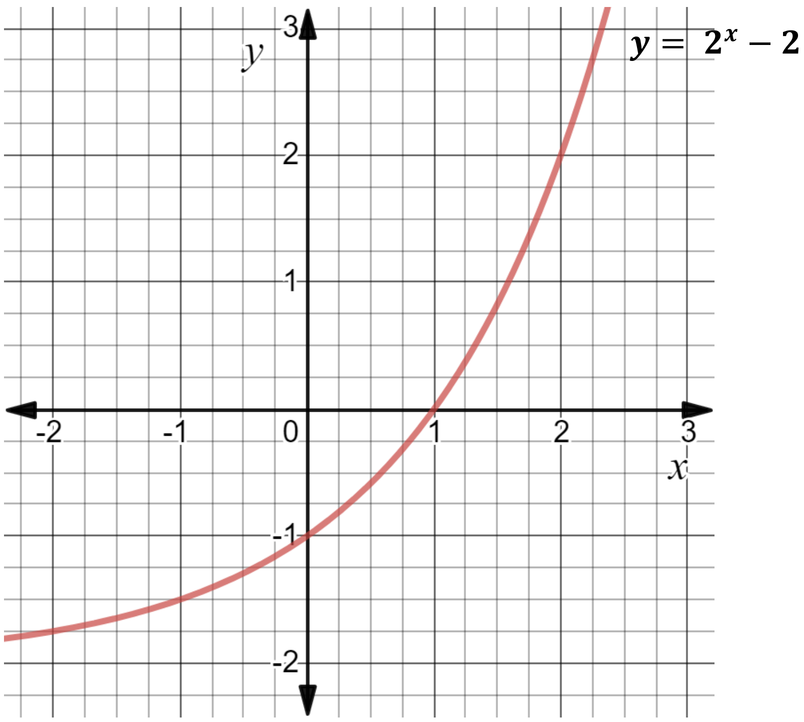
(a) Use mathematical induction to prove that $3^{3n} + 2^{n+2}$ is divisible by 5 for all positive integers $n \geq 1$. **3**

(b) Consider the points $P(a, 2a)$, $Q(-a, 5a)$, $R(3a, 4a)$ and $S(9a, 12a)$, where a is a positive real number.

(i) Express $\overrightarrow{PQ}$ in component form. **1**

(ii) Given the length of the projection of $\overrightarrow{PQ}$ onto $\overrightarrow{RS}$ is 12, find the value of a. **3**

(c) In the diagram, the region bounded by the curve $y = 2^x - 2$ and the x-axis between $x = -1$ and $x = 2$ is rotated through one revolution about the x- axis. Find the volume of the solid formed, correct to two decimal places. **3**



Question 15 (10 marks) Start a NEW Writing Booklet. **Marks**

(a) Find $\int_0^{0.125} \frac{2}{\sqrt{1-4x^2}}\, dx$. Write your answer correct to 3 significant figures. **2**

(b) (i) Show that $2\sin x\cos(2k+1)x = \sin 2(k+1)x - \sin 2kx$. **1**

(ii) Using the result from part (i), prove by mathematical induction that **3**

$$\cos x + \cos 3x + \cos 5x + \cdots + \cos(2n-1)x = \frac{\sin 2nx}{2\sin x}$$

for all integers n, $n \geq 1$.

(c) An acute-angled triangle XYZ has an area of 40 square units. **4**

The vector $\overrightarrow{YX} = \begin{bmatrix} 6 \\ 2 \end{bmatrix}$ and $\overrightarrow{YZ} = \begin{bmatrix} p \\ q \end{bmatrix}$. Given $\left|\overrightarrow{YZ}\right| = 8\sqrt{5}$, find the possible values of p and q.

Question 16 (10 marks) Start a NEW Writing Booklet. **Marks**

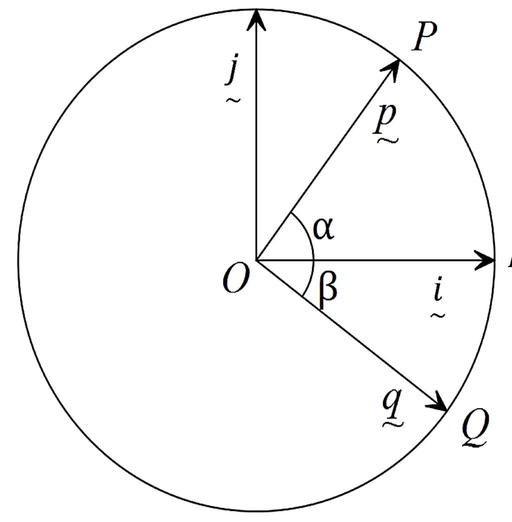
(a) (i) Prove the trigonometric identity $\sin 3A = 3\sin A - 4\sin^3 A$. **3**

(ii) Hence, show that the equation $6x - 8x^3 = 1$ has the roots **3**

$\sin\frac{\pi}{18}$, $\sin\frac{5\pi}{18}$ and $\sin\frac{25\pi}{18}$. Hint: Let $x = \sin A$.

(iii) Hence show that $\sin\frac{\pi}{18} \times \sin\frac{5\pi}{18} \times \sin\frac{25\pi}{18} = -\frac{1}{8}$. **1**

(b) For the **unit circle, centre *0*,** $\overrightarrow{OP} = \underset{\sim}{p}$, $\overrightarrow{OQ} = \underset{\sim}{q}$, $\angle POR = \alpha$ and $\angle QOR = \beta$.



Not to scale

(i) Show that $\underset{\sim}{p} \cdot \underset{\sim}{q} = \cos(\alpha + \beta)$. **1**

(ii) By expressing $\underset{\sim}{p}$ and $\underset{\sim}{q}$ as vectors in component form, and using your result in (i), derive the expansion of $\cos(\alpha + \beta)$. **2**

END OF PAPER

MATHEMATICS EXTENSION 1 – 2020 Trial HSC Examination SOLUTIONS

SECTION I – MULTIPLE CHOICE SOLUTIONS

1 $\underset{\sim}{a} + \underset{\sim}{b} + \underset{\sim}{c}$

$= 2\underset{\sim}{i} + 3\underset{\sim}{j} - 3\underset{\sim}{i} + 2\underset{\sim}{j} + 2\underset{\sim}{i} - \underset{\sim}{j}$

$= i + 4j$

$2\underset{\sim}{i} + 8\underset{\sim}{j}$

$= 2(\underset{\sim}{i} + 4\underset{\sim}{j})$

$= 2(\underset{\sim}{a} + \underset{\sim}{b} + \underset{\sim}{c}) \quad \therefore$ D

2 $T_{k+1} = {}^8C_k\, x^{8-k} \left(\frac{3}{x}\right)^k$

$= {}^8C_k\, x^{8-k} \cdot 3^k \cdot x^{-k}$

$= {}^8C_k\, 3^k\, x^{8-2k}$

ie $8 - 2k = 4$

$2k = 4$

$k = 2$

${}^8C_2\, 3^2 = 252 \quad \therefore$ D

3 $y = \cos^{-1} 3x \qquad f(x) = 3x$

$\frac{dy}{dx} = \frac{-3}{\sqrt{1-(3x)^2}} \qquad f'(x) = 3$

$= \frac{-3}{\sqrt{1-9x^2}} \quad \therefore$ B

4 test $(0,0)$ & $(12,0)$ in each one $\therefore$ C

OR $y = k(x-6)^2 + 9$

substitute $(0,0)$: $0 = k(0-6)^2 + 9$

$0 = 36k + 9$

$36k = -9$

$k = -\frac{1}{4}$

$y = -\frac{1}{4}(x-6)^2 + 9$

$-4(y-9) = (x-6)^2$

of the form

$4a(y-q) = (x-p)^2$

$4a = -4$ $\quad x - p = 2at \quad y - q = at^2$

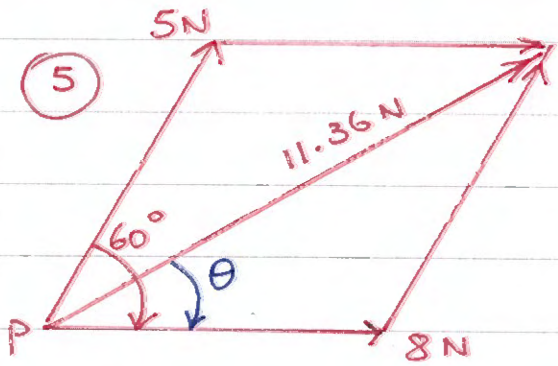
$a = -1$ $\quad x - 6 = -2t \quad y - 9 = -t^2$

$x = -2t + 6 \quad y = 9 - t^2$

$x = 6 - 2t$

$\therefore$ C

5



Horizontal components:

$x = 5\cos 60^\circ + 8$

Vertical components:

$y = 5\sin 60^\circ + 0$

$$\text{Magnitude} = \sqrt{x^2+y^2}$$
$$= \sqrt{(5\cos 60^\circ + 8)^2 + (5\sin 60^\circ)^2}$$
$$= 11.35781669$$
$$= 11.36 \text{ N}$$

$$\text{Direction: } \tan\theta = \frac{y}{x}$$
$$= \frac{5\sin 60^\circ}{5\cos 60^\circ + 8}$$
$$\therefore \theta = 22^\circ 24' 39.28''$$
$$= 22^\circ 25'$$

• this step is not necessary as θ must be less than 60°, but you can use it as a check!

∴ A

6 Asymptotes at $x = -1$

$x = 0$

$x = 1$

$$\therefore f(x) = x(x+1)(x-1)$$
$$= x(x^2-1)$$

∴ B

7 $$\int \frac{3}{\sqrt{4-9x^2}}\,.dx = 3\int \frac{1}{\sqrt{9\left(\frac{4}{9}-x^2\right)}}\,.dx$$
$$= \frac{3}{3}\int \frac{1}{\sqrt{\left(\frac{2}{3}\right)^2 - x^2}}\,.dx$$
$$= \sin^{-1}\left(\frac{x}{\frac{2}{3}}\right) + C$$
$$= \sin^{-1}\left(\frac{3x}{2}\right) + C$$

∴ C

(8) $\cos 5x \cos 2x - \sin 6x \sin 3x$

$= \frac{1}{2}\left[\cos(5x-2x) + \cos(5x+2x)\right] - \frac{1}{2}\left[\cos(6x-3x) - \cos(6x+3x)\right]$

$= \frac{1}{2}\left[\cos 3x + \cos 7x - \cos 3x + \cos 9x\right]$

$= \frac{1}{2}\left[\cos 7x + \cos 9x\right]$

$= \cos 8x \cos x$

$A - B = 7x$
$A + B = 9x$
$B = 9x - A$
$A - (9x - A) = 7x$
$2A = 16x$
$\therefore \boxed{A = 8x}$
$B = 9x - 8x$
$\therefore \boxed{B = x}$

(9) $\vec{OD} = \vec{OC} + \vec{CD}$

$= \frac{\underset{\sim}{b}}{2} + \vec{AB}$

$= \frac{\underset{\sim}{b}}{2} + \vec{AO} + \vec{OB}$

$= \frac{\underset{\sim}{b}}{2} - \underset{\sim}{a} + \underset{\sim}{b}$

$= \frac{3}{2}\underset{\sim}{b} - \underset{\sim}{a}$

$\therefore$ (B)

(10) $y = \tan^{-1} \frac{1}{2}(x-2)$

$x \to x+1: \quad y = \tan^{-1} \frac{1}{2}(x+1-2)$
$= \tan^{-1} \frac{1}{2}(x-1)$

$x \to \frac{1}{2}x: \quad y = \tan^{-1} \frac{1}{2}\left(\frac{1}{2}x - 1\right)$
$= \tan^{-1}\left(\frac{1}{4}x - \frac{1}{2}\right)$
$= \tan^{-1}\left(\frac{x-2}{4}\right)$

$\therefore$ (B)

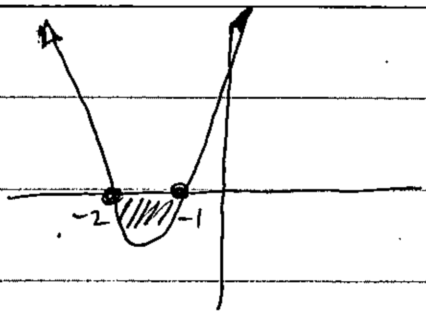
Summary:

1.	D	6.	B
2.	D	7.	C
3.	B	8.	D
4.	C	9.	B
5.	A	10.	B

NOTE: Use your Reference Sheet!!!

MATHEMATICS EXTENSION 1 – QUESTION 11 TRIAL 2020

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
a) $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}$ $a=2$, $b=-4$, $c=3$, $d=-6$		Well done by most. Some students need to revise theory.
$=\frac{\beta\gamma}{\alpha\beta\gamma}+\frac{\alpha\gamma}{\alpha\beta\gamma}+\frac{\alpha\beta}{\alpha\beta\gamma}$		
$=\frac{\alpha\beta+\beta\gamma+\gamma\alpha}{\alpha\beta\gamma}$	1	1 MARK.
$\alpha+\beta+\gamma=-\frac{b}{a}=2$	1	1 MARK (all need last 2)
$\alpha\beta+\beta\gamma+\gamma\alpha=\frac{c}{a}=\frac{3}{2}$		
$\alpha\beta\gamma=-\frac{d}{a}=3$		
$\therefore \frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}=\frac{3/2}{3}$		Some students believe
$=\frac{1}{2}$		$\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}=(\alpha+\beta+\gamma)^{-1}$ WHICH IS WRONG.
b) $\frac{1}{x+1}(x+1)^2 \le -1\,(x+1)^2$		Can also be done using
$x+1 \le -(x+1)^2$		(1) critical values
$(x+1)^2+(x+1) \le 0$		(2) graphing
$(x+1)(x+1+1) \le 0$		
$(x+1)(x+2) \le 0$	1	$y=\frac{1}{x+1}$ and $y=-1$

MATHEMATICS EXTENSION 1 – QUESTION 11		
SUGGESTED SOLUTIONS	**MARKS**	**MARKER'S COMMENTS**
		Many students drew small untidy graphs that led to errors.
$-2 \le x \le -1$ but $x \neq -1$	1	
$\therefore -2 \le x < -1$	1	Many did not mention $x \neq -1$
c) STATISTICS		
10 letters 3 S ; 3 T ; 2 I		
i) $\dfrac{10!}{3!\,3!\,2!} = 50\,400$	1	
ii) Block the A and C together		Also can use
This can be done 2! ways	1	${}^9C_1 \times 2! \times \dfrac{8!}{3!\,3!\,2!}$
So you have $\boxed{AC}$ _ _ _ _ _ _ _ _		
$\dfrac{2!\,9!}{3!\,3!\,2!} = 10080$	1	

MATHEMATICS EXTENSION 1 – QUESTION 11

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
d) 5G ; 17B ; 12Y ; 12P ; 4O.		
∴ 50 marbles.		
Need 7 of one colour		
∴ this can not be green or orange		
To ~~NOT~~ have 7 of any colour		
the maximum (worst case) is		
5G ; 6B ; 6Y ; 6P ; 4O		
27 marbles		
The 28^{th} marble (either B, Y, P)	1 for +1	
gives 7 of a colour.		
So if you select 28	1 Mark correct answer.	
marbles you ensure 7 of		
a colour.		
OR		
As only B, Y, P can give 7		
leave out G and O [∴ 41 marbles]		
$\frac{x}{3} > 6 \quad x > 18$	1	
$19 + 5 + 4 = 28$	1	

∴ 28 marbles.

MATHEMATICS – QUESTION 11

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
OR		
G B Y P O G B Y P O G B Y P O G B Y P O G B Y P	1	Or similar
B Y P B Y P B Y P		diagram
↑		
28^{th}	1	
OR		
Worst Case		
5G ; 6B ; 6Y ; 6P ; 4O		
has no 7 of any colour.		
So the next marble of B, Y, P		
will give 7 of a colour		
$\therefore\ 27 + 1 = 28$	1	+1 is 1 mark.
28 marbles	1	

MATHEMATICS EXTENSION 1 TRIAL HSC 2020 – QUESTION 12 (10 marks)		
SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS

(a) Students should learn the process for Auxiliary angles and not just quote the formulas. So many students got R wrong & α in the wrong quadrant. Show working, so you can be awarded some marks! Use your Reference Sheet ... wrong formula ... no marks!

$$2\sqrt{3}\sin x - 2\cos x = R\cos(x+\alpha)$$
$$= R\cos x\cos\alpha - R\sin x\sin\alpha$$

Equating coefficients:

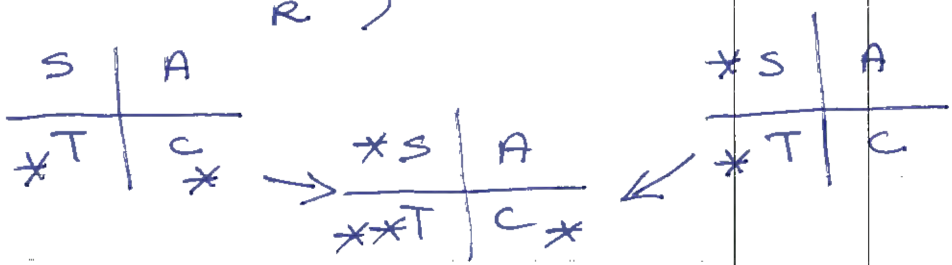
$2\sqrt{3} = -R\sin\alpha$

$R\sin\alpha = -2\sqrt{3}$

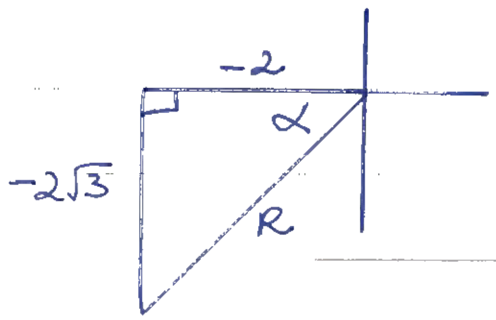
$\sin\alpha = \dfrac{-2\sqrt{3}}{R}$ — $\frac{1}{2}$ mark

$-2 = R\cos\alpha$

$\cos\alpha = \dfrac{-2}{R}$ — $\frac{1}{2}$ mark



$\therefore$ "α" is in the 3RD QUADRANT



$R = \sqrt{(-2)^2 + (-2\sqrt{3})^2}$

$= \sqrt{4+12}$

$= \sqrt{16}$ where $R > 0$

$= 4$ — $\frac{1}{2}$ mark

MATHEMATICS EXTENSION 1 TRIAL HSC 2020 – QUESTION 12 (10 marks)

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
OR $R^2(\sin^2\alpha + \cos^2\alpha) = (-2)^2 + (-2\sqrt{3})^2$ $R^2 = 4 + 12$ $R^2 = 16$ $R = 4$ (where $R > 0$)		
From triangle in 3rd quadrant, use any trig ratio to find α, once R is calculated. →		
OR $\dfrac{R\sin\alpha}{R\cos\alpha} = \dfrac{-2\sqrt{3}}{-2}$		
$\tan\alpha = \sqrt{3}$ $\alpha = 60^\circ$ $= \frac{\pi}{3}$ $= 1.047197551$		$\left(\frac{1}{2}\right)$ mark for related angle
$\therefore \alpha = 180^\circ + 60^\circ$ $= 240^\circ$ $= \frac{4\pi}{3}$ $= 4.188790205$		$\left(\frac{1}{2}\right)$ mark for correct angle in radians.
$\therefore 2\sqrt{3}\sin x - 2\cos x = 4\cos\left(x + \frac{4\pi}{3}\right)$ $= 4\cos(x + 4.19)$		$\left(\frac{1}{2}\right)$ mark for correct form.
$\left(-\frac{1}{2}\right)$ mark for each error.		

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SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
Answers as • $4\cos\left(x+\frac{\pi}{3}\right)$ OR $4\cos\left(x+\frac{4\pi}{3}\right)$ received $2\frac{1}{2}$ marks with correct working. • Note, there is only <u>one</u> correct answer not both !!! • From 3rd quadrant: $\tan\alpha = \frac{-2\sqrt{3}}{-2}$, $\tan\alpha=\sqrt{3}$, $\alpha = 60^\circ = \frac{\pi}{3}$ $\sin\alpha = \frac{-2\sqrt{3}}{4}$, $\alpha = -60^\circ = -\frac{\pi}{3}$ $\cos\alpha = \frac{-2}{4}$, $\cos\alpha = -\frac{1}{2}$, $\alpha = 120^\circ = \frac{2\pi}{3}$ • all 3 will need to be converted to an angle in the 3rd quadrant $[\pi + \theta]$. ie $\pi + 60^\circ = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$ for any ratio used • noting $60^\circ = \frac{\pi}{3}$ is the related angle in the 1st quadrant for all 3 ratios.		

MATHEMATICS EXTENSION 1 TRIAL HSC 2020 – QUESTION 12 (10 marks)

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS

(b) (i) using $\sin A \sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$

NOTE: $A = B = \frac{\pi x}{8}$

$$\begin{aligned}
LHS &= \sin^2\left(\frac{\pi x}{8}\right) \\
&= \sin\left(\frac{\pi x}{8}\right)\sin\left(\frac{\pi x}{8}\right) && -\tfrac{1}{2} \\
&= \frac{1}{2}\left[\cos\left(\frac{\pi x}{8} - \frac{\pi x}{8}\right) + \cos\left(\frac{\pi x}{8} + \frac{\pi x}{8}\right)\right] && -\tfrac{1}{2} \\
&= \frac{1}{2}\left[\cos 0 - \cos\left(\frac{2\pi x}{8}\right)\right] && -\tfrac{1}{2} \\
&= \frac{1}{2}\left[1 - \cos\left(\frac{\pi x}{4}\right)\right] && -\tfrac{1}{2} \\
&= RHS
\end{aligned}$$

Areas for students to improve include: avoiding the omission of too many steps of the proof, and communicating clearly about how they went from one step to the next.
In a 'show' question it must be clear how one line is obtained from another.

- You cannot work on both sides at the same time
- $(-\frac{1}{2})$ mark for every line that was missing.
- SHOW ALL STEPS!

MATHEMATICS EXTENSION 1 TRIAL HSC 2020 – QUESTION 12 (10 marks)

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
(b) (ii) Students should show all relevant working in responses involving calculations. This ensures that marks can be allocated for working even if the student's final answer is incorrect.		
$A = \int_0^8 6\sin^2\left(\frac{\pi x}{8}\right).dx$		
$= 6 \times \frac{1}{2}\int_0^8 1 - \cos\left(\frac{\pi x}{4}\right).dx$		– ① for correct substitution
$= 3\left[x - \frac{\sin\left(\frac{\pi x}{4}\right)}{\frac{\pi}{4}}\right]_0^8$		Note: derivative of $\frac{\pi x}{4}$ is $\frac{\pi}{4}$.
$= 3\left[x - \frac{4}{\pi}\sin\left(\frac{\pi x}{4}\right)\right]_0^8$		– ① for correct integration
$= 3\left[8 - \frac{4}{\pi}\sin\left(\frac{8\pi}{4}\right) - (0 - \sin 0)\right]$		
$= 3\left[8 - \frac{4}{\pi}\sin 2\pi - 0\right]$		– ① for correct answer with working.
$= 3\left[8 - \frac{4}{\pi}\times 0\right]$		
$= 3 \times 8$		
$= 24$ square units		

- Many students forgot the 6 in the original question ($2\frac{1}{2}$) marks were awarded if 4 square units was obtained with all correct steps of working).

MATHEMATICS EXTENSION 1 TRIAL HSC 2020 – QUESTION 12 (10 marks)

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
• the students that assumed $\cos\left(\frac{\pi x}{4}\right) = x\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}x$ and continued were not awarded any marks. • no need for absolute value sign, area is above the x-axis. (c)		

Paying attention to the mark value of the question and using it as a guide to the complexity of solution required.

Give 2 reasons: 1 mark – "show true" for base case where $n=1$ has been omitted. 1 mark – wrong assumption and inductive step. ie assume the statement is true for $n=k$ not $n=k+1$, then using the assumption, prove true for $n=k+1$.		

MATHEMATICS EXTENSION 1 – QUESTION 13

a) SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
$\int_0^{\ln 2} \frac{e^{2x}}{1+e^{4x}}\,dx$ $u = e^{2x}$ $\frac{du}{dx} = 2e^{2x}$ $du = 2e^{2x}\,dx$ when $x = \ln 2$, $u = e^{2\ln 2}$, $u = e^{\ln 2^2}$, $u = 4$ when $x = 0$, $u = e^0 = 1$	½ ½	½ if they did one of these
$= \frac{1}{2}\int_0^{\ln 2} \frac{2e^{2x}}{1+e^{4x}}\,dx$		
$= \frac{1}{2}\int_1^4 \frac{1}{1+u^2}\,du$	1	
$= \frac{1}{2}\left[\tan^{-1} u\right]_1^4$	1	
$= \frac{1}{2}\left[\tan^{-1} 4 - \tan^{-1} 1\right]$	½	Calculator must be in radians.
$= 0.27020\ldots$		
$= 0.27$ (to 2dp)	½	Many students did not realise this.

MATHEMATICS EXTENSION 1 – QUESTION 13

b) SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
$\cos x - \sin x = 1$ $t = \tan\frac{x}{2}$ $0 \le x \le 2\pi$ $0 \le \frac{x}{2} \le \pi$ $\frac{1-t^2}{1+t^2} - \frac{2t}{+t^2} = 1$ $1 - t^2 - 2t = 1 + t^2$ $2t^2 + 2t = 0$ $2t(t+1) = 0$ $t = 0$ or $t = -1$	1	1 mark for t-values
$\tan\frac{x}{2} = 0$ $\tan\frac{x}{2} = 0$ $\frac{x}{2} = 0, \pi$ $\frac{x}{2} = \frac{3\pi}{4}$ $\therefore \frac{x}{2} = 0, \frac{3\pi}{4}, \pi$ $x = 0, \frac{3\pi}{2}, 2\pi$	1	Some ½ marks if solutions were missing.
Please don't forget to test $x = \pi$ $LHS = \cos x - \sin x$ $= \cos\pi - \sin\pi$ $= -1 - 0$ $= -1$ $\neq RHS$ $\therefore$ not a solution	Not many students checked. No marks lost, as it was not a solution.	−½ mark if missing 2π in the solution.
$\therefore x = 0, \frac{3\pi}{2}, 2\pi$	1	

MATHEMATICS EXTENSION 1 – QUESTION 13

c) SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS

$f(x) = 1 + \ln x$

(i) $\text{LHS} = \frac{d}{dx}(x + \ln x)$

$= \frac{d}{dx}(uv)$

$u = x \quad v = \ln x$

$u' = 1 \quad v' = \frac{1}{x}$

Not enough to just do this.

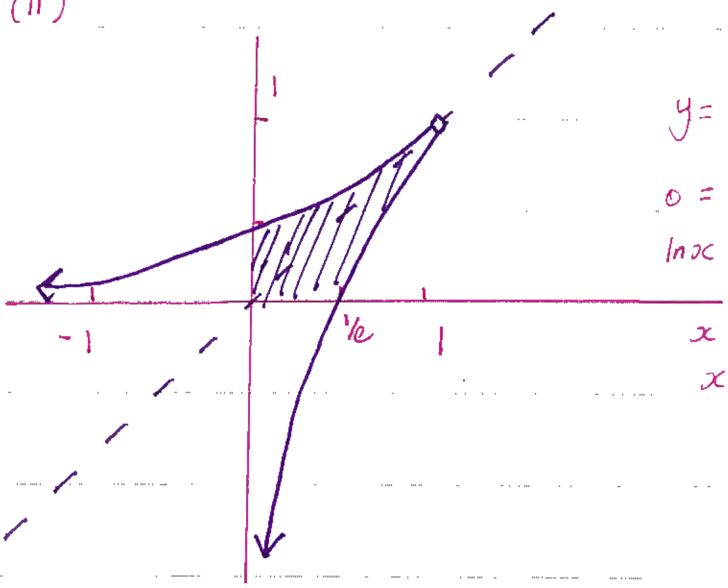
$= vu' + uv'$

$= 1 \times \ln x + \underbrace{x\left(\frac{1}{x}\right)}$

they needed this at the very least.

$= \ln x + 1$

(ii)



$y = 1 + \ln x$

$0 = 1 + \ln x$

$\ln x = -1$

$x = e^{-1}$

$x = 1/e$

From (i) $y = x \ln x$

Diff ↓ $\frac{dy}{dx} = 1 + \ln x$ ↑ Integration

Students need to improve setting out of a "show that" question. They needed to show that they applied the product rule. It was not enough just to do the differentiation at the side.

MATHEMATICS EXTENSION 1 – QUESTION 13		
Method 1 SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
$A = \left[\frac{1}{2}bh - \int_{1/e}^{1}(1+\ln x)\,dx\right] \times 2$	1	
$= \left[\frac{1}{2}\times 1\times 1 - [x\ln x]_{1/e}^{1}\right] \times 2$	1	
$= \left[\frac{1}{2} - \left(1\ln 1 - \frac{1}{e}\ln\frac{1}{e}\right)\right]\times 2$		
$= \left(\frac{1}{2} - \left(0 + \frac{1}{e}\right)\right) \times 2$		
$= \left(1 - \frac{2}{e}\right)\ u^2$	1	
		Each method: Most papers 2/3 correct first step / second step but with no $x\ln x$. 2/3 correct first step/second step but on integral $\int_0^1$ instead of $\int_{1/e}^{1}$ 1/3 if correct method but no $x\ln x$ or $1/e$.

MATHEMATICS EXTENSION 1 – QUESTION 13

Method 2 SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
$y = 1 + \ln x$		
interchange x and y		
$x = 1 + \ln y$		
$\ln y = x - 1$		
$y = e^{x-1}$		
$A = \int_0^1 e^{x-1}\,dx - \int_{1/e}^1 (1 + \ln x)\,dx$	1	
$= \left[e^{x-1}\right]_0^1 - \left[x\ln x\right]_{1/e}^1$	1	
$= e^0 - \frac{1}{e} - \left[1\ln 1 - \frac{1}{e}\ln\frac{1}{e}\right]$		
$= 1 - \frac{1}{e} - \left(0 - \frac{1}{e}\ln e^{-1}\right)$		
$= 1 - \frac{1}{e} - \frac{1}{e}$		
$= \left(1 - \frac{2}{e}\right)u^2$	1	

MATHEMATICS EXTENSION 1 – QUESTION 13

METHOD 3 — SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
$A = 1 \times 1 - 2\int_{1/e}^{1} (1 + \ln x)\,dx$	1	
$= 1 - 2\left[x \ln x\right]_{1/e}^{1}$	1	
$= 1 - 2\left(0 - \frac{1}{e}\ln e^{-1}\right)$		
$= 1 - 2\left(\frac{1}{e}\right)$		
$= \left(1 - \frac{2}{e}\right) u^2$	1	

MATHEMATICS EXTENSION 1 – QUESTION 14

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
a) Step 1 – [First case or base case]		
Prove that the statement is true	½ mk	Generally
for $n=1$		well done
LHS		
$3^{3(1)} + 2^{1+2}$		
$27 + 8$		
$= 35$		
$= 5(7)$ which is divisible by 5.		
$\therefore$ true for $n=1$		
Step 2 [Assumption]		A few
Assume the statement is true	½ mk	students
for $n=k$.		wrote down
That is,		Prove the
$\frac{3^{3k} + 2^{k+2}}{5} = M$, for		statement is
some integer M.		true for $n=k$
$3^{3k} + 2^{k+2} = 5M$		Also, ~~th~~ is
$\therefore 2^{k+2} = 5M - 3^{3k}$		statement
		applies to
		any integer
		and not a only positive integers.

MATHEMATICS EXTENSION 1 – QUESTION 14

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS

Step 3 – [Inductive Step]

Prove the statement is true for $n = k+1$

$$3^{3(k+1)} + 2^{(k+1)+2}$$

$$= 3^{3k+3} + 2^{k+3}$$

$$= 3^3 . 3^{3k} + 2^{k+2} . 2^1$$

$$= 3^3 . 3^{3k} + 2(5M - 3^{3k})$$

(from our assumption)

$$= 27 . 3^{3k} + 10M - 2 . 3^{3k}$$

$$= 27 \cdot 3^{3k} - 2 \cdot 3^{3k} + 10M$$

$$= 25 . 3^{3k} + 10M$$

$$= 5(5 . 3^{3k} + 2M)$$

$= 5N$, where $N = (5 . 3^{3k} + 2M)$

True for any integer N

Therefore, divisible by 5.

Marks:

1mk for correct substitution of the assumption

1/2 mk for appropriate simplification

Marker's comments:

students kept on making mistakes

$3^{3(k+1)} = 3^{3k+1}$ instead of 3

A few students attempted to substitute the assumption twice, thus resulting in weird answers involving fractions.

You substitute the assumption once only

MATHEMATICS EXTENSION 1 – QUESTION 4		(5)
SUGGESTED SOLUTIONS	**MARKS**	**MARKER'S COMMENTS**
OR		
From the assumption		
$3^{3k} = 5M - 2^{k+2}$		
Step (3) : Prove the statement		
is true for $n = k+1$		
$= 3^{3(k+1)} + 2^{k+1+2}$		
$= 3^{3k}.3^3 + 2 . 2^{k+2}$		
$= 3^3 \left[5M - 2^{k+2}\right] + 2 . 2^{k+2}$	1mk for using the assumption	
(using our assumption)		
$= 27.5M - 27.2^{k+2} + 2.2^{k+2}$		
$= 27.5M - 25.2^{k+2}$		
$= 5\left(27M - 5.2^{k+2}\right)$	½ mk for appropriate simplification	
Which is divisible by 5		
Concluding statement		
Therefore true for $n=k+1$ when it is true for $n=k$.	½	
Since it is true for $n=1$ then it is also true for $n=1+1=2$, $n=2+1=3$ and so on. Therefore, it is true for all integers n by the principle of mathematical induction.		

MATHEMATICS EXTENSION 1 – QUESTION 14

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
b)(i) $\overrightarrow{PQ} = (-a-a)i + (5a-2a)j$	½	Not attempted
$= -2ai + 3aj$	½	well.
or $(-2a, 3a)$		
Method 1 (easiest way – shortest method)		
ii) Scalar projection of $\overrightarrow{PQ}$ onto $\overrightarrow{RS}$		Poorly
$= \dfrac{\overrightarrow{PQ} \cdot \overrightarrow{RS}}{\lvert\overrightarrow{RS}\rvert}$		attempted.
$\overrightarrow{RS} = (9a-3a)i + (12a-4a)j$		
$= 6a\underset{\sim}{i} + 8a\underset{\sim}{j}$	½	
Dot product of $\overrightarrow{PQ} \cdot \overrightarrow{RS}$	½ mk	
$\overrightarrow{PQ}$ $(-2a, 3a)$ $\overrightarrow{RS}$ $(6a, 8a)$	for	
	correct	
$\overrightarrow{PQ} \cdot \overrightarrow{RS} = -2a \times 6a + 3a \times 8a$	substitution	
$= -12a^2 + 24a^2$	into the	
	right	
$\lvert\overrightarrow{RS}\rvert = \sqrt{(6a)^2 + (8a)^2}$	formula	
$= \sqrt{36a^2 + 64a^2}$	as many different formulae	
$= \sqrt{100a^2}$	could have	
	been used.	
$\therefore \dfrac{\overrightarrow{PQ} \cdot \overrightarrow{RS}}{\lvert\overrightarrow{RS}\rvert} = \dfrac{-12a^2 + 24a^2}{10a} = 12$ →	strictly	
	required	
$\dfrac{12a^2}{10a} = 12$	to attain all the	
	components	
$\dfrac{6a}{5} = 12$	~~for~~ 1 mk	

$\therefore a = 10$ 1 mk for the final answer.

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
Method 2 (Harder way) – longer method		
Vector projection of $\overrightarrow{PQ}$ onto $\overrightarrow{RS}$		
$\dfrac{\overrightarrow{PQ}\cdot\overrightarrow{RS}}{\lvert\overrightarrow{RS}\rvert^2}\cdot\overrightarrow{RS}$		
Vector $\overrightarrow{RS}$	½	(See previous working)
$= \dfrac{\begin{pmatrix}-2a\\3a\end{pmatrix}\begin{pmatrix}6a\\8a\end{pmatrix}}{(6a)^2+(8a)^2}(6a\underset{\sim}{i}+8a\underset{\sim}{j})$ ½		
$= \dfrac{-12a^2+24a^2}{36a^2+64a^2}(6a\underset{\sim}{i}+8a\underset{\sim}{j})$		½ mk for appropriate substitution into a valid formula
$= \dfrac{12a^2}{100a^2}(6a\underset{\sim}{i}+8a\underset{\sim}{j})$		
$= \dfrac{3}{25}(6a\underset{\sim}{i}+8a\underset{\sim}{j})$		
$= \dfrac{18a}{25}\underset{\sim}{i}+\dfrac{24a}{25}\underset{\sim}{j}$	1mk	– strictly required this statement to attain 1mk.
Length of this projection vector $= 12$		
or the scalar projection $= 12$		(previous method)
$\sqrt{\left(\dfrac{18a}{25}\right)^2+\left(\dfrac{24a}{25}\right)^2} = 12$		
$\sqrt{\dfrac{900a^2}{625}} = 12$		
$\dfrac{30a}{25} = 12$		
$\dfrac{6a}{5} = 12$		

$6a = 60$

$\therefore a = 10$ 1mk

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
Method 3		
Could have used the formula		
$\dfrac{\overrightarrow{PQ} \cdot \overrightarrow{RS}}{\lvert\overrightarrow{RS}\rvert}$		
(similar working to method 1)		
Method 4		
Could have used the formula		
$\overrightarrow{PQ} \cdot \widehat{RS}$		
as in method 3		
$\dfrac{\overrightarrow{RS}}{\lvert\overrightarrow{RS}\rvert}$ is the unit vector for $\overrightarrow{RS}$ or $\widehat{RS}$		
In this question, students failed to realise that the question was actually dealing with scalar projection and 12 represented this.		
A number of students used the vector projection formula but did not know		

what to do afterwards; that is, equate the magnitude of this vector to 12.

MATHEMATICS EXTENSION 1 – QUESTION 14

(7)

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
Some students forgot to use $\vec{RS}$ when using $\frac{3}{25}(6a\underset{\sim}{i} + 8a\underset{\sim}{j})$ when using the vector projection formula, thereby failing to get a vector quantity. As a result, their working did not make sense (as the magnitude of this vector had to be equated to 12). A number of students failed to include a's, $\underset{\sim}{i}$'s and $\underset{\sim}{j}$'s in their relevant working. Lack of substitution into the appropriate formula was heavily penalised. At this stage, we expect students to understand the different formulae and how they could be used in vector problems?		

MATHEMATICS EXTENSION 1 – QUESTION 14

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
c) $V = \pi \int_{-1}^{2} (2^x - 2)^2 \, dx$	½	→ establishing this working with π included.
$= \pi \int_{-1}^{2} (2^x - 2)(2^x - 2)\, dx$		
$= \pi \int_{-1}^{2} (2^{2x} - 2\cdot 2^x - 2\cdot 2^x + 4)\, dx$		
$= \pi \int_{-1}^{2} (2^{2x} - 4\cdot 2^x + 4)\, dx$	½ mk	for correct expansion
$= \pi \int_{-1}^{2} (e^{2x\ln 2} - 4\cdot e^{x \ln 2} + 4)\, dx$		
$= \pi \left[\frac{e^{2x\ln 2}}{2\ln 2} - \frac{4e^{x\ln 2}}{\ln 2} + 4x \right]_{-1}^{2}$		
or 1mk		plus $4x$ = 1mk for displaying appropriate integration skills without simplifying the question
$= \pi \left[\frac{2^{2x}}{2\ln 2} - \frac{4\cdot 2^x}{\ln 2} + 4x \right]_{-1}^{2}$		
$= \pi \left[\frac{2^4}{2\ln 2} - \frac{4\cdot 2^2}{\ln 2} + 4(2) \right.$		
$\left. - \left(\frac{2^{-2}}{2\ln 2} - 4\cdot 2^{-1} + 4(-1) \right) \right]$		
$= 9.938405974\ldots$		

$= 9.94$ (2 dp) — 1mk – final answer.

MATHEMATICS EXTENSION 1 – QUESTION 14		
SUGGESTED SOLUTIONS	**MARKS**	**MARKER'S COMMENTS**
This question was poorly attempted. Students need to re-visit expansion skills. Some students used absolute value which was not required as when you square something, it is always positive anyway. Others made the question harder by splitting up the integration into 2 parts.		
$V = \pi \int_{-1}^{0} y^2 \, dx + \int_{0}^{2} y^2 \, dx$		
This is a valid method but very long + tedious especially the calculations at the end.		

MATHEMATICS EXTENS.ON 1 – QUESTION 15

a) $$\int_0^{0.125} \frac{2}{\sqrt{1-4x^2}}\,dx = \int_0^{0.125} \frac{2}{\sqrt{1-(2x)^2}}\,dx$$

$$= \left[\sin^{-1} 2x\right]_0^{0.125}$$

1 correct integration

$$= \sin^{-1} 0.25 - \sin^{-1} 0$$

$$= 0.25268\ldots$$

$$= 0.253$$

1 correct evaluation

Calculus works for trigonometric functions in radians only – most students seemed unaware of this fact.
The average mark for this question was less than 50%.

Using the wrong formula (eg. some other trig function) earned no marks.
Please use your formula sheet.

b) i LHS $= 2\sin x \cos(2k+1)x$
$= \sin[x + (2k+1)x] + \sin[x - (2k+1)x]$
$= \sin(2k+2x) + \sin(x - 2kx - x)$
$= \sin 2(k+1)x + \sin(-2kx)$
$= \sin 2(k+1)x - \sin 2kx$ (since $\sin(-\theta) = -\sin\theta$)
$=$ RHS 1 full solution

This is a show question; don't leave anything out!

b) ii

Prove true for $n=1$

$LHS = \cos x$

$RHS = \dfrac{\sin 2(1)x}{2\sin x}$

$= \dfrac{\sin 2x}{2\sin x}$

$= \dfrac{2\sin x \cos x}{2\sin x}$

① correctly proving true for $n=1$

$= \cos x$

$= LHS$

$\therefore$ the statement is true for $n=1$

Assume true for $n=k$

That is,

$$\cos x + \cos 3x + \ldots + \cos(2k-1)x = \frac{\sin 2kx}{2\sin x}$$

Prove true for $n=k+1$

That is,

$$\cos x + \cos 3x + \ldots + \cos(2k-1)x + \cos(2k+1)x = \frac{\sin 2(k+1)x}{2\sin x}$$

$LHS = \cos x + \cos 3x + \ldots + \cos(2k-1)x + \cos(2k+1)x$

$= \dfrac{\sin 2kx}{2\sin x} + \cos(2k+1)x$ (by assumption)

① correctly using the induction hypothesis

$= \dfrac{\sin 2kx}{2\sin x} + \dfrac{2\sin x \cos(2k+1)x}{2\sin x}$

$$= \frac{\sin 2kx + 2\sin x \cos(2k+1)x}{2\sin x}$$

$$= \frac{\sin 2kx + \sin 2(k+1)x - \sin 2kx}{2\sin x}$$ (from part i)

$$= \frac{\sin 2(k+1)x}{2\sin x}$$ ① complete solution.

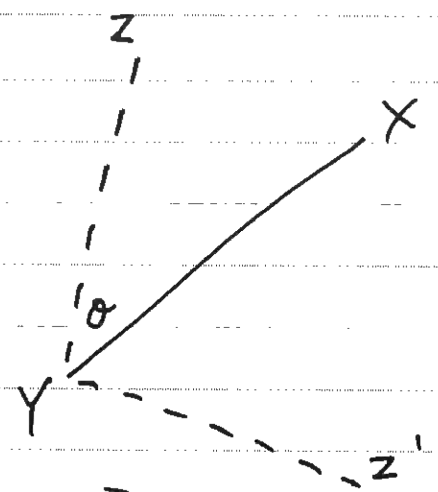
$= RHS$

$\therefore$ the statement is true for $n=k+1$ if it is true for $n=k$.
Since it is true for $n=1$, it is also true for $n=2$ and so on.
$\therefore$ The statement is true for all integers n, $n \geq 1$

The process of proof by Mathematical Induction is prescriptive, with very little flexibility or room for "creativity". This is not the place to take short cuts or invent your own method; follow the steps exactly.

A common problem was failing to start the proof with the given LHS; if we'd wanted you to prove a different statement, we would have asked for it.

c)



$|\vec{YX}| = \sqrt{6^2+2^2}$
$= \sqrt{40}$

$A = \frac{1}{2} \times \sqrt{40} \times 8\sqrt{5} \times \sin\theta$

$40 = 4\sqrt{200} \times \sin\theta$

$\sin\theta = \frac{1}{\sqrt{2}}$

$\theta = 45°$ (since θ is acute)   ① finding the angle between $\vec{YX}$ and $\vec{YZ}$

$\therefore \begin{bmatrix} 6 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} p \\ q \end{bmatrix} = \sqrt{40} \times 8\sqrt{5} \times \cos 45°$

$6p + 2q = 80\sqrt{2} \times \frac{1}{\sqrt{2}}$ ① Correct, complete dot product statement

$6p + 2q = 80$

$3p + q = 40$

$\therefore q = 40 - 3p$ ①

Also, given $|\vec{YZ}| = 8\sqrt{5}$

$$\sqrt{p^2+q^2} = 8\sqrt{5}$$
$$p^2+q^2 = 320 \quad (2)$$

Sub (1) into (2)

$$p^2 + (40-3p)^2 = 320$$
$$p^2 + 1600 - 240p + 9p^2 = 320$$
$$10p^2 - 240p + 1280 = 0$$
$$p^2 - 24p + 128 = 0$$
$$(p-8)(p-16) = 0$$

(1) Correct equation in p alone.

$$\therefore p = 8, \quad p = 16$$

when $p=8$, $q = 40 - 3(8)$
$= 16$

when $p = 16$, $q = 40 - 3(16)$
$= -8$

$$\therefore \begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} 8 \\ 16 \end{bmatrix} \text{ or } \begin{bmatrix} 16 \\ -8 \end{bmatrix}$$

(1) Full solution.

Very few students produced a coherent, logical solution to this problem. Marks were awarded only for clear, unambiguous statements that followed a logical sequence and maintained the correctness of the solution.

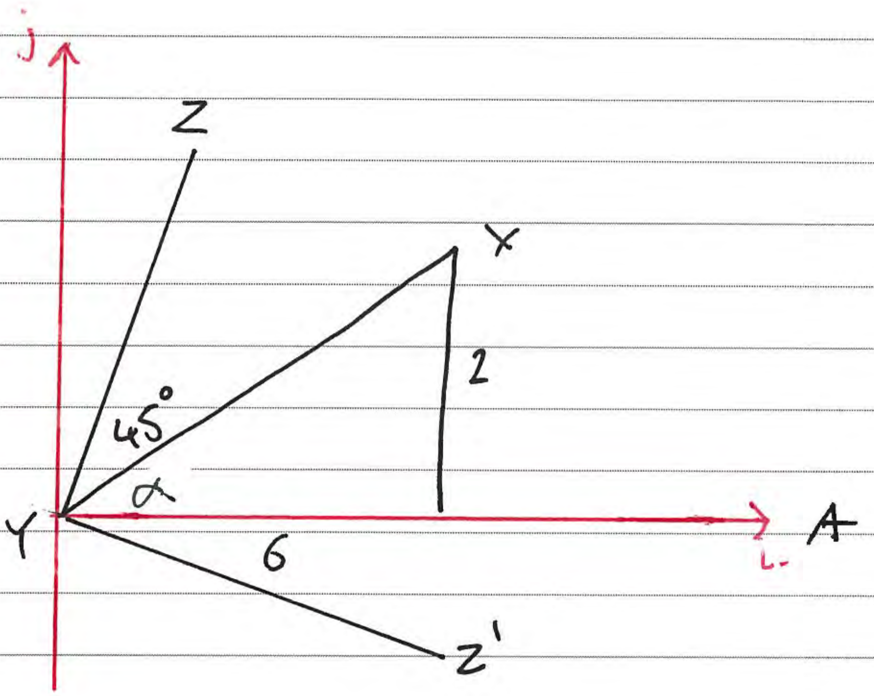
Alternative solution

$$40 = \frac{1}{2} \times 8\sqrt{5} \times \sqrt{40} \times \sin\theta$$

$$\sin\theta = \frac{80}{8\sqrt{5} \times \sqrt{40}}$$

$$= \frac{1}{\sqrt{2}}$$

$$\therefore \theta = 45^\circ$$ (1)



$$\tan\alpha = \frac{2}{6}$$

$$\alpha = 18.4349^\circ$$

$$\therefore \angle AYZ = 18.4349 + 45$$
$$= 63.43^\circ$$ (1)

$$\therefore p = 8\sqrt{5}\cos 63.43^\circ$$
$$= 8$$

$$q = 8\sqrt{5}\sin 63.43^\circ$$
$$= 16$$ (1)

$$\angle AYZ' = 18.43 - 45$$
$$= -26.56^\circ$$

$$p = 8\sqrt{5}\cos(-26.56^\circ)$$
$$= 16$$

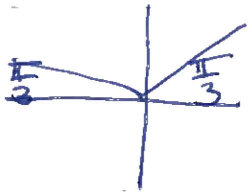
$$q = 8\sqrt{5}\sin(-26.56^\circ)$$
$$= -8$$

$$\therefore \begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} 8 \\ 16 \end{bmatrix} \text{ or } \begin{bmatrix} 16 \\ -8 \end{bmatrix}$$ (1)

MATHEMATICS EXTENSION 1 – QUESTION 16 2020 trial

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
Question 16		
(a) Prove		
$\sin 3A = 3\sin A - 4\sin^3 A$	3	Well done generally.
LHS $= \sin 3A$		
$= \sin(2A + A)$		← $\frac{1}{2}$ mk.
$= \sin 2A \cos A + \cos 2A \sin A$		← $\frac{1}{2}$ mk
$= 2\sin A \cos A \cos A + (1 - 2\sin^2 A)\sin A$		Some students left out essential
$= 2\sin A \cos^2 A + \sin A - 2\sin^3 A$		
$= 2\sin A(1 - \sin^2 A) + \sin A - 2\sin^3 A$		
$= \underbrace{2\sin A} - \underbrace{2\sin^3 A} + \underbrace{\sin A} - \underbrace{2\sin^3 A}$		← 1 mark.
$= 3\sin A - 4\sin^3 A$		
$=$ RHS		
$\therefore \sin 3A = 3\sin A - 4\sin^3 A$ — (1)		
(ii) Hence show $6x^3 - 8x^3 = 1$ has roots	3	
$\sin\frac{\pi}{18}$, $\sin\frac{5\pi}{18}$, $\sin\frac{25\pi}{18}$		
a cubic has at most 3 real solutions	1	
So if we find 3 distinct solutions		Only a handful
we have found them all (as degree 3)		of students achieved
let $x = \sin A$ — (2)		3 marks. Most
		received 2
sub (2) in (1) $6\sin A - 8\sin A = 1$		as didn't show
$3\sin A - 4\sin A = \frac{1}{2}$		at most 3 solutions

MATHEMATICS EXTENSION 1 – QUESTION

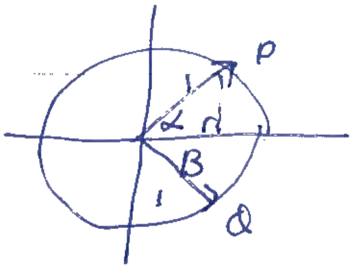
SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
∴ using part (a) $\sin 3A = 3\sin A - 4\sin^3 A$.		
$\sin 3A = \frac{1}{2}$.		
Sine is positive in Q1 & Q2	1	Many students just substituted in the answers given...
  $\sin\frac{\pi}{6} = \frac{1}{2}$ ∴ $R = \frac{\pi}{6}$		
∴ $3A = \frac{\pi}{6} + 2n(\pi)$ or $3A = \left(\pi - \frac{\pi}{6}\right) + 2n\pi$.		This is a hence question
$A = \frac{\pi}{18} + \frac{2n\pi}{3}$ or $A = \frac{5\pi}{18} + \frac{2n\pi}{3}$		
∴ $A = \frac{\pi}{18}, \frac{5\pi}{18}, \frac{13\pi}{18}, \frac{17\pi}{18}, \ldots \frac{19\pi}{18}, \ldots \frac{25\pi}{18}, \ldots$		
There will be repeats but only 3 distinct solutions...		
∴ the solutions are.		
$\sin\frac{\pi}{18}$, $\sin\frac{5\pi}{18}$ and $\sin\frac{25\pi}{18}$,	1	
(b) From formula sheets $\underset{\sim}{u}\cdot\underset{\sim}{v} = \lvert u\rvert\lvert v\rvert\cos\theta$.		
∴ $p\cdot q = \lvert p\rvert\lvert q\rvert\cos\theta$ $= 1 \times 1 \times \cos(\alpha + \beta)$		
∴ $p\cdot q = \cos(\alpha+\beta)$		

MATHEMATICS EXTENSION 1 – QUESTION

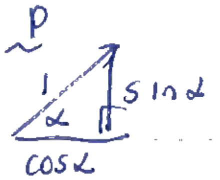
SUGGESTED SOLUTIONS | MARKS | MARKER'S COMMENTS

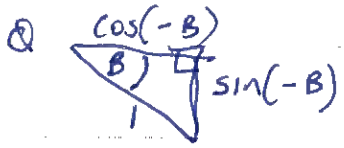
(ii) $\underset{\sim}{p}\cdot\underset{\sim}{q} = x_1x_2 + y_1y_2$ is used. — 2 marks

now



$\beta > 0$





$\therefore \underset{\sim}{p} = \cos\alpha\,\underset{\sim}{i} + \sin\alpha\,\underset{\sim}{j}$

$\underset{\sim}{q} = \cos(\beta)\,\underset{\sim}{i} + \sin(\beta)\,\underset{\sim}{j}$ — 1

$\underset{\sim}{p}\cdot\underset{\sim}{q} = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(-\beta)$

$= \cos\alpha\cos\beta - \sin\alpha\sin\beta.$ — 1

Marker's comments:

students were trying to prove $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

many students used the identity to be proved in the proof, which made no sense.

If students used the working from question (ii) in question (i) they needed to use the working shown in (i) for the proof.