SYDNEY HIGH SCHOOL

Moore Park, Surry Hills

Trial Higher School Certificate Examination

1994

MATHEMATICS

3/4 UNIT

Time allowed - Two hours
(Plus 5 minutes reading time)

Examiner: R. Boros

DIRECTIONS TO CANDIDATES

* *ALL* questions may be attempted.
* *ALL* questions are of equal value.
* All necessary working should be shown in every question. Full marks may not be awarded for careless or badly arranged work.
* Standard integrals are printed at the back. Approved calculators may be used.
* *Each* section attempted is to be returned in a *separate* bundle, clearly marked Section A (Q1, Q2), Section B (Q3, Q4), or Section C (Q5, Q6, Q7). Each bundle must also show your name. Start each question on a new page.
* If required, additional paper may be obtained from the Examination Supervisor upon request.

NOTE: This is a trial paper only and does not necessarily reflect the content or format of the Higher School Certificate Examination Paper for this subject.

Section A (Hand up separately)

Question 1 (Start a new page)

a) Solve $\frac{x+3}{x-2} \leq 3$

b) The sum of an infinite geometric series is 30. If the common ratio is doubled the sum of the resulting infinite geometric series is 90. Find the first term and common ratio of the original series.

c) State the domain and range of the function $y = \sin^{-1} 2x$ and hence sketch its graph.

d) Find the coordinates of the vertex and focus of the parabola $y = x^2 - 6x + 7$. Sketch the parabola.

e) Using the substitution $u = 9 - x^2$ find $\int x\sqrt{9 - x^2}\ dx$.

Question 2 (Start a new page).

a) Write down a primitive function of

(i) $\frac{1}{4 + x^2}$

(ii) $\frac{x}{4 + x^2}$

b) Differentiate with respect to x

(i) $y = \log_e 2x(x - 1)^3$

(ii) $y = \cos^{-1} 5x$

c) A and B are the points (-2, -1) and (2, 1) respectively.
Find the coordinates of the point P(x,y) dividing AB externally in the ratio 5:2

d) Prove that $\dfrac{\text{cosec}B - \cot B}{\text{cosec}B + \cot B} = \tan^2 \dfrac{B}{2}$

e) Find $\lim\limits_{x \Rightarrow 0} \dfrac{\sin 3x}{2x}$

Section B (Hand up separately)

Question 3 (Start a new page)

a) Solve $5\sin\theta - 12\cos\theta = 13$ given that $0° \leq \theta \leq 360°$

b) Find all solutions to the equation
$\cos 2x = -1$ (where x is in radian measure).

c) Find $\int \sin^2 x \, dx$

d) By using one step of Newton's Method, find an approximation to that root of

$$\frac{1}{3}x - \log_e x = 0 \text{ near } x = 1$$

e) The perimeter of a circle is increasing at 3cm/s. Leaving your answer in terms of π, find the rate at which the area is increasing when the perimeter is 1m.

<u>Question 4</u> (Start a new page)

a) The arc of the parabola $y = 2x^2$ between the points (0, 0) and (2, 8) is rotated about the y axis. Calculate the volume of the solid of revolution.

b) If α, β and γ are the roots of the equation $x^3 - 4x + 1 = 0$ evaluate

(i) $\alpha + \beta + \gamma$

(ii) $\alpha\beta + \beta\gamma + \alpha\gamma$

(iii) $\alpha^2 + \beta^2 + \gamma^2$

c) (i) Find the value of 'a' such that the polynomial

$P(x) = x^4 + 2x^3 - x^2 - 8x - a$ is divisible by $Q(x) = x^2 - 4$.

(ii) Hence or otherwise find all real roots of the polynomial P(x) with that particular value of 'a'.

Section C (Hand up separately)

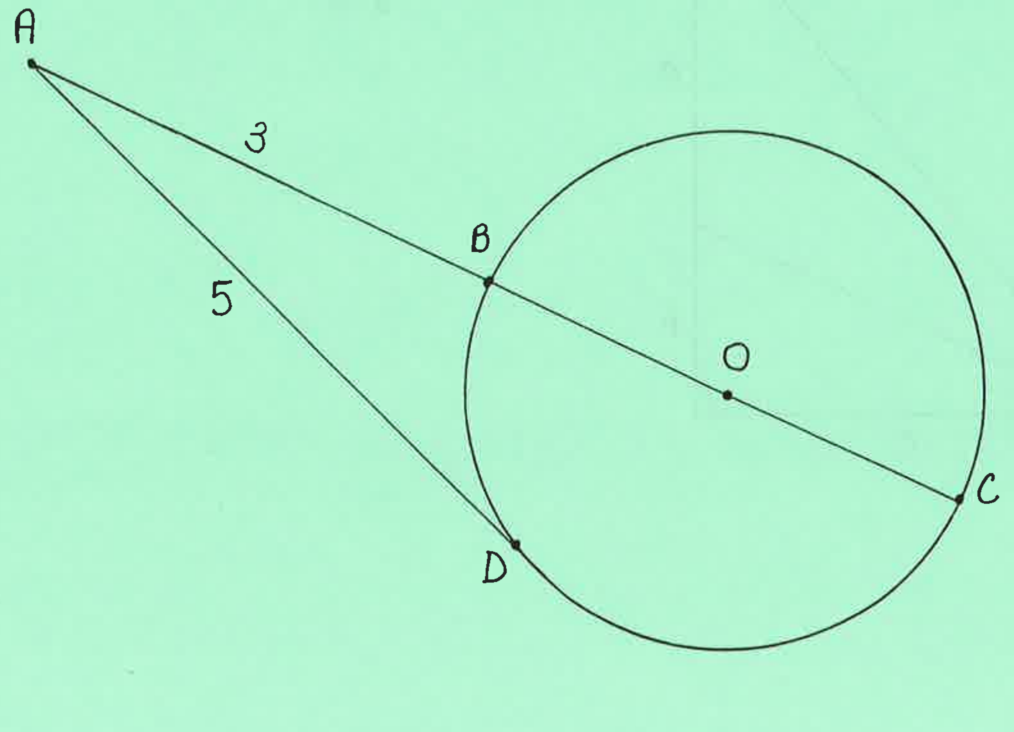
Question 5 (Start a new page)

a) (i) Determine the equation of the tangent to the curve $C : y = 2x^2$ at the point $P(t,\ 2t^2)$

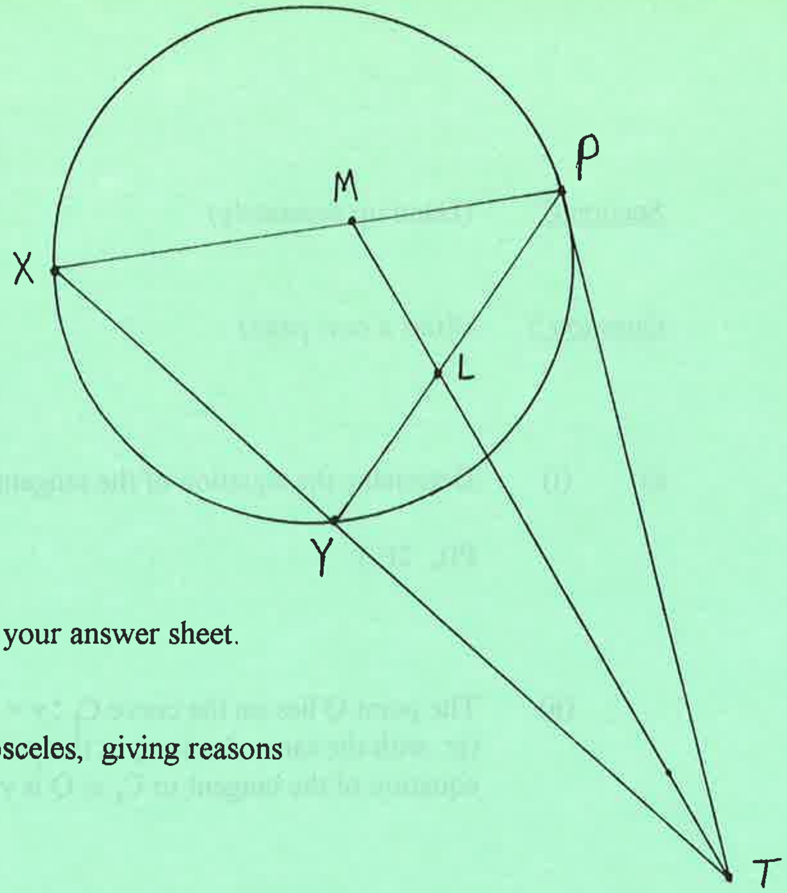
(ii) The point Q lies on the curve $C_1 : y = x^2 + 1$, on the same vertical line (ie: with the same abscissa) as the point P of part (a)(i). Show that the equation of the tangent to C_1 at Q is $y = 2tx + (1 - t^2)$

(iii) Find the precise locus of the points of intersection of these two tangents, as the common abscissa t of the points P and Q assume all positive values. Indicate this locus on a sketch.

(b) O is the centre of the circle and AD is a tangent.
AB = 3cm
AD = 5cm
Find $B\hat{A}D$ to the nearest minute giving reasons for your answer.



Question 6 (Start a new page)



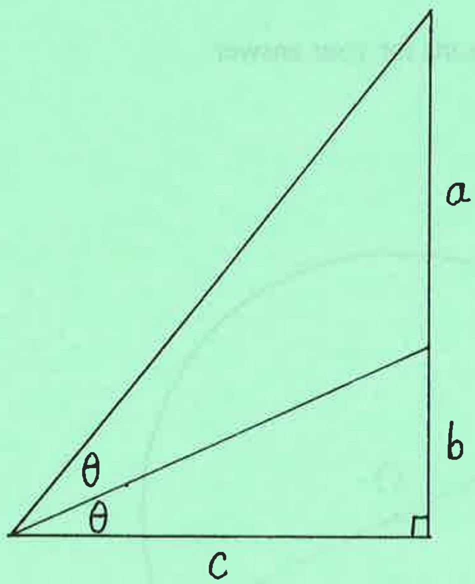
a) XY is any chord of a circle. XY is produced to T and TP is a tangent to the circle. The bisector of $P\hat{T}X$ meets XP at M and cuts PY at L.

(i) Copy the diagram onto your answer sheet.

(ii) Show that Δ MPL is isosceles, giving reasons for your answer.

b) A sequence, (known as the Fibonacci sequence) is defined by $T_1 = 1$, $T_2 = 2$ and $T_n = T_{n-1} + T_{n-2}$ for $n > 2$. Use the Principle of Mathematical Induction to prove that $T_1 + T_2 + T_3 + \dots\dots\dots + T_n = T_{n+2} - 2$.

c)



In the given triangle, prove that $c^2 = \dfrac{(a+b)\,b^2}{a-b}$

Question 7 (Start a new page)

a) An object passes through the point x = 2 at time t = 0, and has velocity V at time t given by $V = \frac{1}{2}(1 + 3t)^{-\frac{1}{4}}$

(i) Find the position x of the object at time t = 5.

(ii) How long does this object take to move a distance of 14 units from its starting point?

(iii) Find the acceleration of the object with the velocity given at the beginning of this question. Show that this acceleration is proportional to a power of the velocity and give the precise relationship between the acceleration and velocity.

b) Due to particular circumstances, e.g. food, water, space it is found that the rate of increase of the population of a species of bird, is given by the equation

$$\frac{d}{dt} N(t) = K\{N(t) - 50\} \text{ where K is a negative constant and } t \geq 0.$$

(i) Verify that for any constant C, the expression $N(t) = 50 + Ce^{Kt}$ satisfies the equation.

(ii) The initial population was 250 birds and the population halved after 3 years. Find the growth rate (as a % to the nearest %) and estimate the bird population after 5 years.

END OF PAPER

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$