SYDNEY GIRLS HIGH SCHOOL

TRIAL HIGHER SCHOOL CERTIFICATE, 1993

3/4 UNIT MATHEMATICS

TIME ALLOWED: Two hours (including reading time)

DIRECTIONS TO CANDIDATES:

* All questions may be attempted
* All questions are of equal value
* All necessary working should be shown in every question
  Marks may not be awarded for careless or badly arranged work
* Standard integrals are printed on a separate paper
* Calculators may be used
* Start a new page for each question

N.B. This is a **TRIAL PAPER ONLY** and does not necessarily reflect the content or format of the final Higher School Certificate Examination Paper for this subject

Question 1:

a) Solve $x - \frac{1}{x} < 0$

b) If $x = \sqrt{3} + \sqrt{2}$ find the value of $x + \frac{1}{x}$, hence or otherwise find the value of $x^2 + \frac{1}{x^2}$

c) The point P (6,9) divides the interval AB in the ratio -3:2. Find the point B given that A is (1,4)

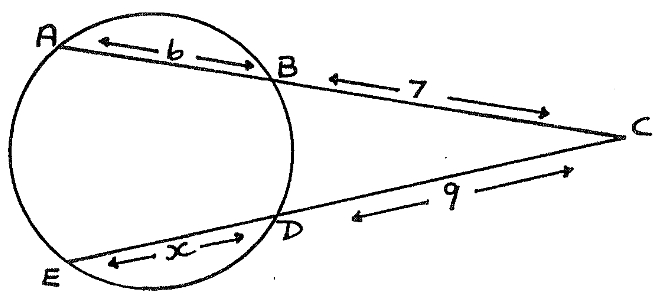
d) $\int \frac{-x}{\sqrt{1 - x^2}} dx$

e) $\int \frac{3}{16 + x^2} dx$

Question 2:

a) Solve $\sin x + \sin 2x = 0 \quad 0° \le x \le 360°$

b)



Find x, give reasons

c) The positive square root of 37 is approximately 6
Use one application of Newton's Method to find a better approximation, correct to 2 decimal places

d) A function is defined by the rules

$$f(x) = \begin{cases} \sin^{-1} x & \text{if } -1 \le x < 0 \\ \cos^{-1} x & \text{if } 0 \le x \le 1 \end{cases}$$

i) Sketch the graph of the function for $-1 \le x \le 1$

ii) For this function evaluate exactly $f(\frac{1}{2}) + 2f(0) - f(-\frac{1}{2})$

Question 3:

a) A certain particle moves along the x axis in accordance with the law

$$t = 2x^2 - 4x + 3$$

where x is measured in cms and t in seconds
Initially the particle is 1.5cms to the right of 0 and moving away from 0

i) Prove that the velocity, v cm/sec is given by

$$v = \frac{1}{4x - 4}$$

ii) Find an expression for the acceleration a cm/sec^2 in terms of x

iii) Find the velocity and acceleration of the particle when

a) $x = 2$cms
b) $t = 9$secs

iv) Describe in words the motion of the particle

b) Prove by induction that

$$3^n > 1 + 2n \text{ for } n > 1$$

Question 4:

The points P $(2ap, ap^2)$ and Q $(2aq, aq^2)$ lie on the parabola $x^2 = 4ay$

a) Derive the equation of the normal to the parabola at P

b) Find the coordinates of the point N the intersection of the normals at P and Q

c) Find the equation of the chord PQ and determine the condition necessary for PQ to be a focal chord

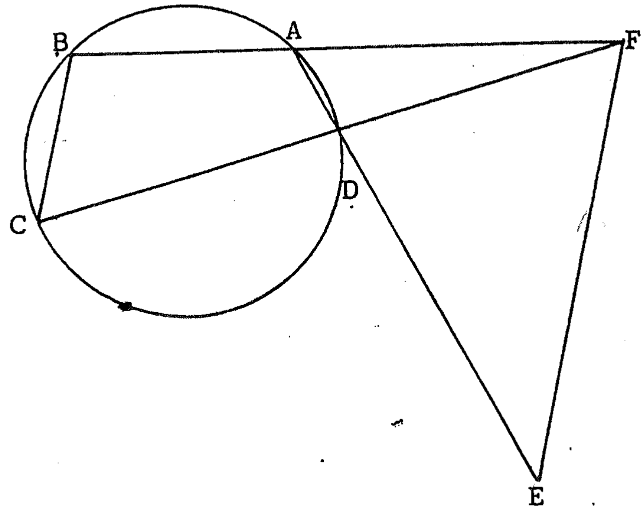
d) If PQ is a focal chord and N is the intersection of the normals at P and Q, find the equation of the locus of N. Describe the locus

Question 5:

a) Prove that

$$\frac{1 - \cos \theta}{1 + \cos \theta} = \tan^2 \frac{\theta}{2}$$

b)



BC || EF

ABCD is a cyclic quadrilateral
Prove that

$$EF^2 = EA \times ED$$

c) The product of two of the roots of the equation

$$24x^3 + 14x^2 - 11x - 6 = 0$$

is equal to the other root. Find the values of all three roots of the equation

Question 6:

a) Find the value of

$$\int_0^{\frac{\pi}{2}} \frac{\sec^2 (\sin x)\, dx}{\sec x}$$

using the substitution

$u = \sin x$

b) Express $7 \cos \alpha - \sin \alpha$ in the form $R \cos (\alpha + \beta)$ where $R > o$ and $0° < \beta < 360°$. Hence solve the equation

$7 \cos \alpha - \sin \alpha = 5$

c) Assume that the rate at which a body cools in air is proportional to the difference between its temperature T and the constant temperature A of the surrounding air. This rate can be expressed by the differential equation

$$\frac{dT}{dt} = -k (T - A)$$

where t is the time in minutes and k is a constant

i) Show that $T = A + Ce^{-kt}$ is a solution to the differential equation (C is a constant)

ii) The body of a murder victim is discovered at 2am when its temperature is 35°C. One hour later its temperature has fallen to 34°C. If the room temperature remains constant at 21°C, find the value of k

iii) Calculate the time the murder was committed. (Normal body temperature is approximately 37°C.)

Question 7:

a) Find the volume of the solid of revolution obtained by revolving the area bounded by $y = 1 + \sin x$ and the x-axis between $x = 0$ and $x = \pi$ about the x axis

b) A particle undergoes simple harmonic motion about the origin 0. Its displacement x cm from 0 at time t seconds is given by

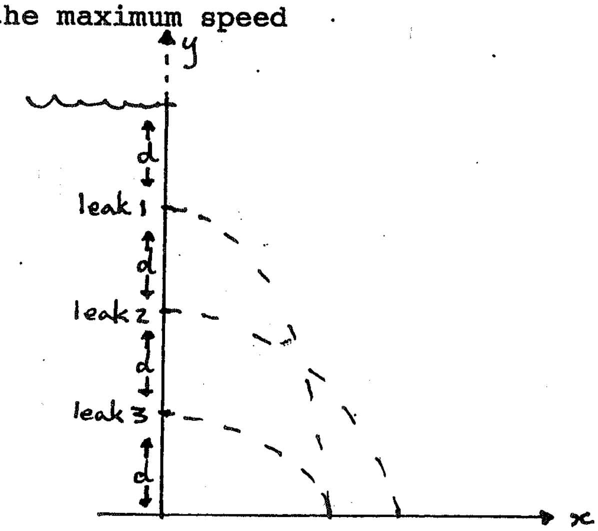
$$x = 5 \cos\left(4t + \frac{\pi}{2}\right)$$

i) Find the acceleration in terms of t and hence or otherwise as a function of displacement

ii) Write down the amplitude of the motion

iii) Find the maximum speed

c)



A dam wall is 4d units high. Small leaks occur at 3 evenly spaced points 3d, 2d and 1d units down from the water level

i) Using axes as shown, show that the parametric equations for the water from leak 1 are:

$$x = Vt \qquad\qquad y = 3d - \tfrac{1}{2}gt^2$$

ii) Find the parametric equations for leak 2 and leak 3

iii) Show that the jets of water from leak 1 and leak 3 reach the ground level at the same point given that the velocity of water issuing from a leak is $V^2 = kh$ where h is the height of water above the leak and k is a constant

iv) Find the velocity of impact of the water from leak 2 in terms of g, k and d

v) A leak occurs at a distance ℓ units above ground. Show that it hits the ground a maximum distance from the dam if $\ell = 2d$