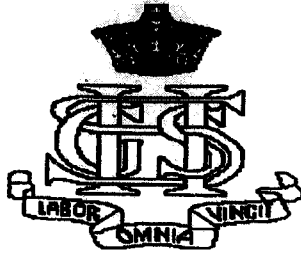


**SYDNEY GIRLS HIGH SCHOOL
TRIAL HIGHER SCHOOL CERTIFICATE**



1999

MATHEMATICS

**3 UNIT (Additional)
and
3/4 UNIT (Common)**

Time allowed - 2 hours
(Plus 5 minutes' reading time)

DIRECTIONS TO CANDIDATES

NAME _____

- Attempt ALL questions.
 - ALL questions are of equal value.
 - All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
 - Board-approved calculators may be used.
 - Each question attempted should be started on a new sheet. Write on one side of the paper only.
-

QUESTION ONE

a) If $3 \cot x = 4$, find the value of

$$\frac{6 \sin x - 4 \cos x}{\operatorname{cosec} x + \sec x} \quad (x \text{ is acute}) \quad [2]$$

b) Evaluate $\int_0^2 x e^{x^2} dx$ [2]

c) Differentiate $x^3 \sin^{-1} 4x$ [2]

d) Given $\log_a b = 0.3$ and $\log_a c = 0.4$, find $\log_a \left(\frac{b}{c}\right) + \log_a ac$ [2]

e) Find the exact value of $\cos 2x$ if $\sin x = \sqrt{3} - 1$ [2]

f) A cosine curve has an amplitude of 5 and a period of 3π . It has a minimum turning point at $(0, 5)$. Find its equation. [2]

QUESTION TWO

a) Write down the domain of the function

$$y = \frac{1}{x^2 + 5x + 6} \quad [1]$$

b) The roots of $x^3 + 5x^2 + 8x + 2 = 0$ are $\alpha, \beta,$ and γ [4]

i) Find $(\alpha + 1) + (\beta + 1) + (\gamma + 1)$

ii) Find $(\alpha + 1)(\beta + 1)(\gamma + 1)$

c) The half life of a radioactive substance is 24 hrs. How long will it take for only 15% of the substance to remain. (Assume $M = M_0 e^{-kt}$ and give your answer to the nearest hour) [2]

d) Find the equation of the tangent to the curve $y = e^{\tan^{-1}x}$ at the point where it cuts the y-axis. [2]

e) The area of the region below the curve $y = e^{-x}$ and above the x-axis, between $x = 0.5$ and $x = 1.5$ is rotated about the x-axis. Find the volume of the solid generated. (Answer correct to 2 decimal places) [3]

QUESTION THREE

a) If $\frac{dx}{dt} = 5(x-3)$ [3]

i) Show that $x = 3 + A e^{5t}$ is a solution, where A is a constant.

ii) Find A if $x = 20$ when $t = 0$.

b) [5]
The points $P(2ap, ap^2)$, $Q(2aq, aq^2)$ lie on the parabola $x^2 = 4ay$.

i) If PQ passes through $(4a, 0)$ show that $pq = 2(p+q)$

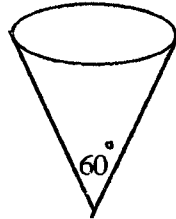
ii) Hence find the locus of M, the mid point of PQ.

c) Find the size of the acute angle between the lines [2]
 $y = -x$ and $\sqrt{3}y = 2x$ (Answer to the nearest minute)

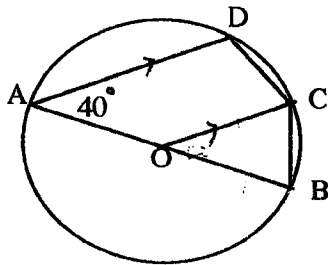
d) Differentiate $\log_e \left(\frac{3+x}{3-x} \right)$ [2]

QUESTION FOUR

- a) A right circular cone of vertical angle 60° is being filled with liquid. The depth of liquid in the cone is increasing at a rate of 4cm/s . Find the rate of increase of the volume of the liquid in the cone when the depth is 9cm . [3]



- b) A projectile is fired at an angle of $\tan^{-1}\left(\frac{5}{12}\right)$ to the horizontal with initial velocity 130 m/s . Using $g=10\text{ m/s}^2$ [6]
- i) Derive equations for the horizontal and vertical position of the projectile at time t .
- ii) What is the horizontal range of this projectile?
- c) AB is the diameter of the circle centre O . AD is parallel to OC , and angle $BAD = 40^\circ$. Find the size of angle DCO , giving reasons. [3]



(figure not to scale)

QUESTION FIVE

- a) [9]
- i) Find the remainder when $P(x) = x^3 - (k+1)x^2 + kx + 12$ is divided by $A(x) = x - 3$
- ii) Find k if $P(x)$ is divisible by $A(x)$
- iii) Find the zeros of $P(x)$, for this value of k
- iv) Solve $P(x) > 0$
- b) It is known that $\log_e x + \sin x = 0$ has one root close to $x = 0.5$.
Use one application of Newton's method to obtain a better approximation of the root correct to 3 decimal places. [3]

QUESTION SIX

- a) Show that $7^n + 2$ is divisible by 3, for all positive integral n . [3]
- b) Find the general solution of $\cos 2x = \sin x$ [3]
- c) Find the area bounded by the curve $y = \frac{1}{\sqrt{25-x^2}}$, the x axis and the ordinates at $x = -2$ and $x = 2$.
(Answer correct to 2 decimal places) [2]
- d) Differentiate $\log_e (\sec x + \tan x)$ and hence find $\int_0^{\frac{\pi}{4}} \sec x \, dx$, in simplest exact form. [4]

QUESTION SEVEN

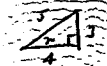
a) A Particle moving on a horizontal line has a velocity of v m / s given by $v^2 = 64 - 4x^2 + 24x$ [5]

- i) Prove that the motion is simple harmonic
- ii) Find the centre of the motion
- iii) Write down the period and amplitude of the motion
- iv) Initially the particle is at the centre of the motion and moving to the left. Write down an expression for the displacement as a function of time.

b) i) Write the expression for $\sqrt{2} \cos \theta + \sin \theta$ in terms of t . [4]
(where $t = \tan \frac{\theta}{2}$)

ii) Hence or otherwise solve $\sqrt{2} \cos \theta + \sin \theta = 1$ for $0^\circ < \theta < 360^\circ$

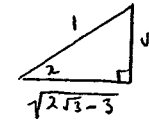
c) Find $\int \frac{x dx}{(25 + x^2)^{\frac{3}{2}}}$ using the substitution $x = 5 \tan \theta$ [3]

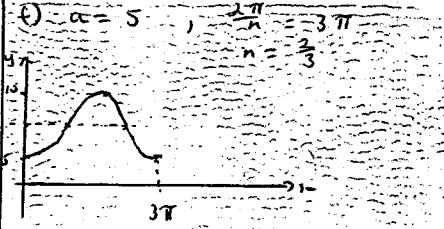
1) a) $3 \cot x = 4$
 $\cot x = \frac{4}{3}$

 $\frac{6 \sin x - 4 \cos x}{\operatorname{cosec} x + \sec x}$
 $= \left[6\left(\frac{3}{5}\right) - 4\left(\frac{4}{5}\right) \right] \div \left[\frac{5}{3} + \frac{5}{4} \right]$
 $= \frac{24}{175}$

b) $\frac{d}{dx} e^{x^2} = 2x e^{x^2}$
 $\therefore \int 2x e^{x^2} dx = e^{x^2} + C$
 $\therefore \int_0^2 x e^{x^2} dx = \frac{1}{2} [e^{x^2}]_0^2$
 $= \frac{1}{2} (e^4 - 1)$

c) $y = x^3 \sin^{-1} 4x$
 put $u = x^3$, $\frac{du}{dx} = 3x^2$
 $v = \sin^{-1} 4x$, $\frac{dv}{dx} = \frac{4}{\sqrt{1-16x^2}}$
 $\frac{dy}{dx} = 3x^2 \sin^{-1} 4x + \frac{4x^3}{\sqrt{1-16x^2}}$

or $\frac{x^3}{\sqrt{1-16x^2}} + 3x^2 \sin^{-1} 4x$
 d) $\log_a \left(\frac{b}{c}\right) + \log_a ac$
 $= (\log_a b - \log_a c) + (\log_a a + \log_a c)$
 $= (0.3 - 0.4) + (1 + 0.4)$
 $= 1.3$

e) $\sin x = \sqrt{3} - 1$

 $\cos 2x = \cos^2 x - \sin^2 x$
 $= 2\sqrt{3} - 3 - (\sqrt{3} - 1)^2$
 $= 2\sqrt{3} - 3 - (4 - 2\sqrt{3})$
 $= 2\sqrt{3} - 3 - 4 + 2\sqrt{3}$
 $= 4\sqrt{3} - 7$



$y = 10 = 5 \cos \frac{2x}{3}$
 or $y = 10 + 5 \cos \left(\frac{2x}{3} - \pi\right)$
 Q2) $y = 5 \sin \left(\frac{2x}{3} - \frac{\pi}{2}\right) + 10$
 all real except $x^2 + 5x + 6 = 0$
 all real except $x = -2, x = -6$

b) $P(x) = x^3 + 5x^2 + 8x + 2$
 $\alpha + \beta + \gamma = -\frac{b}{a} = -5$
 $\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a} = 8$
 $\alpha\beta\gamma = -\frac{d}{a} = -2$
 1) $(\alpha+1) + (\beta+1) + (\gamma+1)$
 $= \alpha + \beta + \gamma + 3$
 $= -5 + 3$
 $= -2$

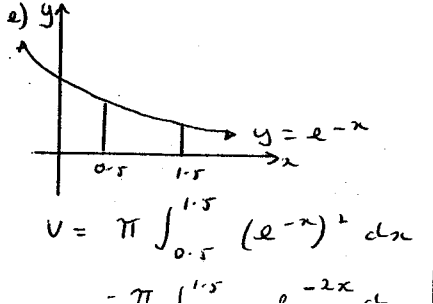
ii) $(\alpha+1)(\beta+1)(\gamma+1)$
 $= (\alpha+1)(\beta\gamma + \beta + \gamma + 1)$
 $= (\alpha\beta\gamma) + (\alpha\beta + \alpha\gamma + \alpha\gamma) + (\alpha + \beta + \gamma) + 1$
 $= -2 + 8 - 5 + 1$
 $= 2$

c) $M = M_0 e^{-kt}$
 when $t = 24$, $m = \frac{M_0}{2}$
 $\therefore \frac{M_0}{2} = M_0 e^{-24k}$
 $0.5 = e^{-24k}$
 $-24k = \log_2 0.5$
 $k = \frac{\log_2 0.5}{-24} [\approx 0.0297]$

If 15% remains
 $m = 0.15 M_0$
 $\therefore 0.15 = e^{-kt}$
 $\log_2 (0.15) = \log_2 e^{-kt}$
 $-kt = \log_2 (0.15)$
 $t = \frac{\log_2 (0.15)}{-k}$
 $= 65 \text{ hrs } 41' 14''$

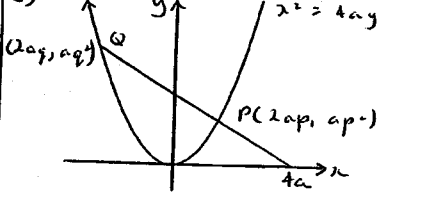
d) $y = e^{\tan^{-1} x}$
 on the y axis $x = 0$
 $y = e^{\tan^{-1}(0)} = e^0 = 1$ i.e. (0,1)
 $\frac{dy}{dx} = \frac{1}{1+x^2} \times e^{\tan^{-1} x}$
 $= \frac{e^{\tan^{-1} x}}{1+x^2}$
 when $x = 0$
 $m = \frac{e^{\tan^{-1}(0)}}{1+0} = 1$

$y - y_1 = m(x - x_1)$
 $y - 1 = 1(x - 0)$
 $y = x + 1$



$\pi \left[-\frac{1}{2} e^{-2x} \right]_{0.5}^{1.5}$
 $= -\frac{\pi}{2} [e^{-3} - e^{-1}]$
 $= \frac{\pi}{2} \left[\frac{1}{e} - \frac{1}{e^3} \right] \text{ units}^3$
 $= 0.50 \text{ (2 d.p.)}$
 a) i) $x = 3 + Ae^{kt}$
 $\frac{dx}{dt} = 5Ae^{kt}$ ②
 $= 5(x-3) [Ae^{kt} = x-3]$

ii) $x = 20$ when $t = 0$
 $20 = 3 + A$
 $A = 17$ ①



i) Gradient PQ = $\frac{ap^2 - aq^2}{2ap - 2aq}$
 $= \frac{p+q}{2}$

Eqn PQ
 $y - y_1 = m(x - x_1)$
 $y - ap^2 = \frac{p+q}{2} (x - 2ap)$
 $2y - 2ap^2 = (p+q)(x - 2ap)$
 subst $x = 4a$, $y = 0$
 $-2ap^2 = (p+q)(4a - 2ap)$
 $-2ap^2 = 4ap - 2ap^2 + 4aq - 2apq$
 $2apq = 4ap + 4aq$
 $pq = 2p + 2q$ ②
 $= 2(p+q)$

ii) Co-ords of M

$$x = \frac{2ap + 2aq}{2}$$

$$y = \frac{ap^2 + aq^2}{2}$$

$$x = a(p+q)$$

$$y = \frac{a}{2}(p^2 + q^2) \quad \text{--- (B)}$$

$$\therefore p+q = \frac{x}{a} \quad \text{--- (A)}$$

$$\text{Now } y = \frac{a}{2} [p^2 + q^2]$$

$$= \frac{a}{2} [(p+q)^2 - 2pq]$$

$$= \frac{a}{2} \left[\left(\frac{x}{a}\right)^2 - 2pq \right] \quad \text{from A}$$

$$= \frac{a}{2} \left[\left(\frac{x}{a}\right)^2 - 2(x)(2)(p+q) \right] \quad \text{from part i)}$$

$$= \frac{a}{2} \left[\left(\frac{x}{a}\right)^2 - 4(p+q) \right]$$

$$= \frac{a}{2} \left[\frac{x^2}{a^2} - 4 \frac{x}{a} \right] \quad \text{from A}$$

$$\text{or } 2ay = x^2 - 4ax$$

$$\text{or } (x-2a)^2 = 2a(y+2a) \quad \text{--- (3)}$$

c) $y = -x, y = \frac{2}{\sqrt{3}}x$
 $\therefore m_1 = -1, m_2 = \frac{2}{\sqrt{3}}$

$$\tan \theta = \frac{m_1 - m_2}{1 + m_1 m_2}$$

$$= \frac{-1 - \frac{2}{\sqrt{3}}}{1 - \frac{2}{\sqrt{3}}}$$

$$\theta = 85^\circ 54'$$

d) $y = \log_e \left(\frac{3+x}{3-x} \right)$
 $= \log_e(3+x) - \log_e(3-x)$

$$\frac{dy}{dx} = \frac{1}{3+x} - \frac{-1}{3-x}$$

$$= \frac{1}{3+x} + \frac{1}{3-x}$$

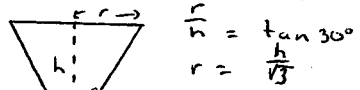
$$= \frac{6}{(3-x)(3+x)}$$

Q4 Let depth be h
 then $\frac{dh}{dt} = 4 \text{ cm s}^{-1}$

Find $\frac{dV}{dt}$ when $h = 9$

$$V = \frac{1}{3} \pi r^2 h$$

$$\frac{dV}{dt} = \frac{1}{3} \pi r^2$$



$$V = \frac{1}{3} \pi r^2 h$$

$$= \frac{1}{3} \pi \left(\frac{h}{\sqrt{3}}\right)^2 h$$

$$= \frac{\pi h^3}{9}$$

$$\frac{dV}{dh} = \frac{\pi h^2}{3}$$

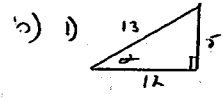
$$\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$$

$$= \frac{\pi h^2}{3} \times 4$$

when $h = 9$

$$\frac{dV}{dt} = \frac{\pi (81)(4)}{3}$$

$$= 108 \pi \text{ cm}^3 \text{ s}^{-1}$$

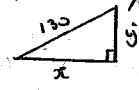


$$\tan \theta = \frac{5}{12}$$

$$\sin \theta = \frac{5}{13}$$

$$\cos \theta = \frac{12}{13}$$

Initially



$$x = 0, y = 0$$

$$\dot{x} = 130 \cos \theta$$

$$= 130 \times \frac{12}{13}$$

$$= 120$$

$$y = 130 \sin \theta$$

$$= 130 \times \frac{5}{13}$$

$$= 50$$

Horizontal Motion

$$\ddot{x} = 0$$

$$\dot{x} = c_1$$

when $t = 0, \dot{x} = 120$

$$\dot{x} = 120$$

$$x = 120t + c_2$$

when $t = 0, x = 0$

$$x = 120t$$

Vertical Motion

$$\ddot{y} = -10$$

$$\dot{y} = -10t + c_1$$

when $t = 0, \dot{y} = 50$

$$\dot{y} = -10t + 50$$

$$y = -5t^2 + 50t + c_2$$

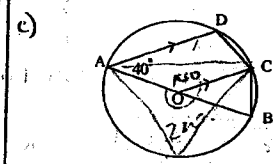
when $t = 0, y = 0$

$$y = -5t^2 + 50t$$

ii) at max range $y = 0$
 $0 = -5t^2 + 50t = 0$
 $10 = 5t \implies t = 10$

$$x_{\text{max}} = 120(10)$$

$$= 1200 \text{ metres}$$



$\angle AOC + 40^\circ = 180^\circ$ (Co-int \angle s)
 $\angle AOC = 140^\circ$
 Major $\angle AOC = 360^\circ - 140^\circ$
 $= 220^\circ$ (\angle at cen
 twice \angle
 circ)
 $\angle ADC = 220^\circ \div 2$
 $= 110^\circ$
 $\angle LOC = 180^\circ - 110^\circ$ (Co-int.)
 $= 70^\circ$ $AD \parallel OC$

Q5

a) i) $R = P(3)$
 $= 3^3 - (k+1)9 + 3k + 1$
 $= 30 - 6k$

ii) If divisible $P(3) = 0$
 $0 = 30 - 6k$
 $k = 5$

iii) $P(x) = x^3 - 6x^2 + 5x + 12$

$$x-3 \overline{) x^3 - 6x^2 + 5x + 12}$$

$$\underline{x^3 - 3x^2 - 4}$$

$$\underline{-3x^2 + 5x}$$

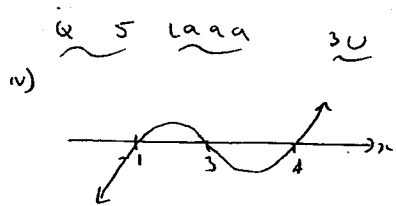
$$\underline{-3x^2 + 9x}$$

$$\underline{-4x + 12}$$

$$\underline{-4x + 12}$$

$$\underline{0}$$

$\therefore P(x) = (x-3)(x^2 - 3x - 4)$
 $= (x-3)(x-4)(x+1)$
 2 zeros at $x = 3, x = 4$



$P(x) > 0$ for $-1 < x < 3$ and $x > 4$

b) $y = \ln x + \sin x$

$a_2 = a_1 - \frac{f(a_1)}{f'(a_1)}$

$j = \frac{1}{x} + \cos x$

$a_2 = 0.5 - \frac{(\ln 0.5 + \sin 0.5)}{(\frac{1}{0.5} + \cos 0.5)}$
 $= 0.574$ (to 3 dp)

Question 6.

i) Step 1. Verify for $n=1$
 i.e. $7^1 + 2 = 9$ which is divisible by 3

Step 2. a) Assume true for $n=k$
 i.e. $7^k + 2 = 3P$ (P integer)

b) Prove true for $n=k+1$

$7^{k+1} + 2 = 7^k \cdot 7 + 2$
 $= 7(3P-2) + 2$ (from assp)
 $= 21P - 14 + 2$
 $= 3(7P-4)$

Since P is an integer, $(7P-4)$ is an integer and

$7^{k+1} + 2$ is divisible by 3 if the assumption is true.

i.e. true for $n=k+1$ if true for $n=k$.

Missing Solns
 Step 3 Since statement is true for $n=1$, it is true for $n=2$. Since true for $n=2$, then true for $n=3$, and so on for all positive integers. (5)

b) $\cos 2x = \sin x$

$1 - 2\sin^2 x = \sin x$

$\therefore 2\sin^2 x + \sin x - 1 = 0$

$(2\sin x - 1)(\sin x + 1) = 0$

$\sin x = \frac{1}{2}$ or $\sin x = -1$

$\therefore x = n\pi + (-1)^n \sin^{-1} \frac{1}{2}$ or

$n\pi + (-1)^n \sin^{-1}(-1)$

i.e. $x = n\pi + (-1)^n (\frac{\pi}{6})$ or

$n\pi + (-1)^n (-\frac{\pi}{2})$ (3)

c) $y > 0$ for all x

(i.e. does not cut x axis)

$\therefore A = \int_{-2}^2 \frac{dx}{\sqrt{25-x^2}}$

$= 2 \int_0^2 \frac{dx}{\sqrt{25-x^2}}$ since fn. is even

$= 2 \left[\sin^{-1} \frac{x}{5} \right]_0^2 = 2 \left(\sin^{-1} \frac{2}{5} \right)$

$\text{Area} \approx 0.82 \text{ u}^2$ (2)

d) $y = \log(\sec x + \tan x)$

let $u = \sec x + \tan x$

$= (\cos x)^{-1} + \tan x$

$\frac{du}{dx} = -(\cos x)^{-2} \cdot -\sin x + \sec^2 x$

$= \frac{\sin x}{\cos^2 x} + \sec^2 x$

$= \tan x \cdot \sec x + \sec^2 x$

(see bottom of next page)

Q6 i) $v^2 = 64 - 4x^2 + 24x$

Q7) For SHM $\ddot{x} = -n^2 x$ or $\dot{x} = -n^2 x$

Now $\frac{d}{dx} \left(\frac{1}{2} v^2 \right) = a = \ddot{x}$

$\therefore \frac{1}{2} v^2 = 32 - 2x^2 + 12x$

$\frac{d}{dx} \left(\frac{1}{2} v^2 \right) = -4x + 12$

i.e. $\ddot{x} = -4(x-3)$ is of form $\ddot{x} = -n^2 x$

\therefore motion is SHM.

(ii) Centre of motion $x=0$ i.e. $x-3=0$ or when $v=0$
 $x=3$

(iii) Period $T = \frac{2\pi}{n}$ where $n=2$

$\therefore T = \frac{2\pi}{2} = \pi$ Period = π secs

Amplitude: is from centre to end

i.e. from 3 to 8

\therefore amplitude = 5 m.

or complete $v^2 = n^2(a^2 - x^2)$

(iv)

$x = -5 \sin 2t + 3$

$x = 5 \sin(-2t) + 3$

$x = 5 \cos(2t - \frac{3\pi}{2}) + 3$

(6)

$v^2 = n^2(a^2 - x^2)$

$v^2 = 4(16 + 6x - x^2)$

$v = 4(25 - (9 - 6x - x^2))$

$v = 4(25 - (x-3)^2)$

$\therefore a = 5$

Q6 (cont'd)

$\frac{dy}{dx} = \frac{\sec x (\tan x + \sec x)}{\sec x + \tan x}$

$= \sec x$

$\therefore \int_0^{\pi/4} \sec x dx = \left[\ln(\sec x + \tan x) \right]_0^{\pi/4}$

$= \ln(\sec \frac{\pi}{4} + \tan \frac{\pi}{4}) - \ln(\sec 0 + \tan 0)$

$= \ln(\sqrt{2} + 1) - \ln(1 + 0)$

$= \ln(\sqrt{2} + 1)$

(4)

$$1) \sqrt{2} \cos \theta + \sin \theta = 1$$

$$\therefore \Rightarrow \sqrt{2} \left(\frac{1-t^2}{1+t^2} \right) + \frac{2t}{1+t^2}$$

$$\Rightarrow \frac{\sqrt{2}(1-t^2) + 2t}{1+t^2}$$

$$(ii) \text{ Now } \sqrt{2} \cos \theta + \sin \theta = 1.$$

$$\therefore \frac{\sqrt{2}(1-t^2) + 2t}{1+t^2} = 1$$

$$\sqrt{2}(1-t^2) + 2t = 1+t^2$$

$$\sqrt{2} - \sqrt{2}t^2 + 2t = 1+t^2$$

$$t^2(1+\sqrt{2}) - 2t + (1-\sqrt{2}) = 0.$$

$$\therefore t = \frac{2 \pm \sqrt{4 - 4(1+\sqrt{2})(1-\sqrt{2})}}{2(1+\sqrt{2})}$$

$$t = \frac{2 \pm \sqrt{4 - 4(1-2)}}{2(1+\sqrt{2})}$$

$$\therefore t = \frac{2(1+\sqrt{2})}{2(1+\sqrt{2})} \text{ or } t = \frac{1-\sqrt{2}}{1+\sqrt{2}} \times \frac{1-\sqrt{2}}{1-\sqrt{2}}$$

$$t = 1 \quad t = \frac{2\sqrt{2}-3}{1}$$

$$(c) \int \frac{x \cdot dx}{(25+x^2)^{\frac{3}{2}}}$$

$$I = \int \frac{25 \tan \theta \sec^2 \theta \cdot d\theta}{125 \sec^3 \theta}$$

$$I = \frac{1}{5} \int \frac{\tan \theta}{\sec \theta} \cdot d\theta$$

$$I = \frac{1}{5} \int \frac{\sin \theta}{\cos \theta} \cdot d\theta$$

$$I = \frac{1}{5} \int \sin \theta \cdot d\theta$$

$$I = -\frac{1}{5} \cos \theta + C$$

$$\therefore I = \frac{-1}{\sqrt{25+x^2}} + C$$

$$(7) \text{ 30 Soln's } \cos \theta = \frac{1-t^2}{1+t^2} \text{ where } t = \tan \frac{\theta}{2}$$

$$\sin \theta = \frac{2t}{1+t^2}$$

$$0^\circ < \theta < 360^\circ$$

$$\text{when } t=1 \quad \text{when } t = \frac{2\sqrt{2}-3}{1}$$

$$\text{then } \tan \frac{\theta}{2} = 1 \quad \tan \frac{\theta}{2} = -0.1715$$

$$\therefore \frac{\theta}{2} = \frac{\pi}{4} \quad \theta = -19^\circ 28'$$

$$* \theta = \frac{\pi}{2} \quad \text{But } 0 < \theta < 360^\circ$$

$$\therefore \theta = 360^\circ - 19^\circ 28'$$

$$\left\{ \begin{array}{l} \theta = 340^\circ 32' \\ \theta = \frac{\pi}{2} \end{array} \right.$$

$$x = 5 \tan \theta, \quad dx = 5 \sec^2 \theta \cdot d\theta$$

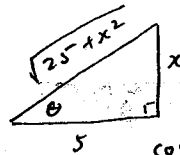
$$\therefore x \cdot dx = 25 \tan \theta \sec^2 \theta \cdot d\theta$$

$$25+x^2 = 25+25 \tan^2 \theta$$

$$= 25 \sec^2 \theta$$

$$(25+x^2)^{\frac{3}{2}} = 125 \sec^3 \theta$$

But



$$\cos \theta = \frac{5}{\sqrt{25+x^2}}$$