Sydney Girls High School

2004

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

Extension 1

This is a trial paper ONLY. It does not necessarily reflect the format or the contents of the 2004 HSC Examination Paper in this subject.

Candidate Number

General Instructions

- Reading Time - 5 mins
- Working time - 2 hours
- Attempt **ALL** questions
- **ALL** questions are of equal value
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Standard integrals are supplied
- Board-approved calculators may be used.
- Diagrams are not to scale
- Each question attempted should be started on a new sheet. Write on one side of the paper only.

QUESTION 1 (12 marks)

Marks

(a) Find the coordinates of the point that divides the interval joining $(-2, 5)$ and $(4, -3)$ externally in the ratio $3:2$ **3**

(b) Solve $\dfrac{2x+1}{x-2} > 1$ **3**

(c) Evaluate $\lim_{x \to 0}\left(\dfrac{\tan 4x}{\frac{x}{2}}\right)$ **2**

(d) Use the substitution $u = 1 + x$ to evaluate $\int_0^1 \dfrac{x}{\sqrt{(1+x)^3}}\,dx$ **4**

Question 2. (12 marks) Marks

(a) Sketch the graph of $y=2\sin^{-1}(\frac{x}{2})$ 3

On your graph indicate the domain and range.

(b) Differentiate and express in simplest form: $y=\sin^{-1}\left(\frac{x}{2}+1\right)$ 3

(c) Evaluate $\int_{-\frac{1}{3}}^{\frac{1}{3}} \frac{dx}{\sqrt{4-9x^2}}$ 3

(d) (i) Express $2\sin\theta - \cos\theta$ in the form $A\sin(\theta-\alpha)$ 1

where α is in radians, correct to 4 decimal places.

(ii) Hence, or otherwise, solve for $0 \le \theta \le 2\pi$ 2

$$2\sin\theta - \cos\theta = \frac{\sqrt{5}}{2}$$

Give answer in radians correct to 2 decimal places.

Question 3. (12 marks) Marks

(a) A particle is moving in a straight line with its position (x) metres at time t seconds given by $x=2\cos(t+\frac{\pi}{4})$.

(i) Show that the particle is moving in simple harmonic motion. 1

(ii) Write the period of its motion. 1

(iii) Find its maximum displacement. 1

(iv) Find its maximum velocity. 1

(v) Find the first time the particle is at the origin. 1

(b) Prove by mathematical induction that $3^{2n}-1$ is divisible by 8 for all integers $n \ge 1$ 3

(c) The area of a equilateral triangle of side length x cm is increasing at the rate of 2 $cm^2.sec^{-1}$

(i) Show that the area of the triangle is given by $A=\frac{\sqrt{3}}{4}x^2 cm^2$ 2

(ii) Find the exact rate of increase of the side (x) of the triangle when $x=2\sqrt{3}$cm 2

Question 4. (12 marks) — Marks

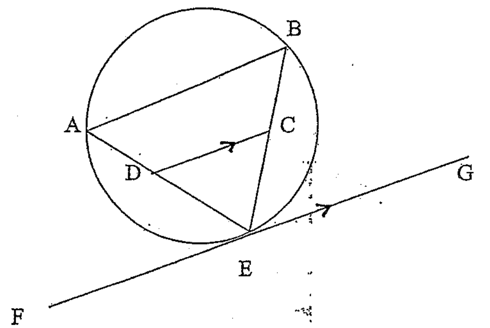
(a) Taking $x = 0.6$ as a first approximation, use one application of Newton's method to find a second approximation to the root of $\tan x = x$ correct to 2 decimal places. — 3

(b) (i) Find the zeros of the polynomial function $P(x) = x^4 + 3x^3 + 2x^2$ — 2

(ii) Hence, without using calculus, sketch the polynomial function showing these zeros on the graph.

(c) $DC \parallel FG$, FEG is a tangent at point E. — 3

Copy this diagram and prove that A, B, C, D are concyclic points.



(d) A particle moves with velocity $v = x - 5$ metres.sec^{-1}

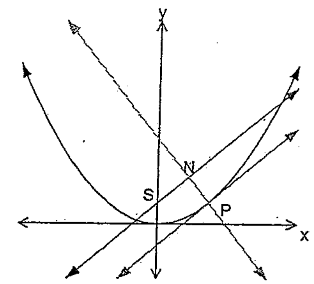
If $x = 6$ metres initially

(i) Show that the acceleration is the same as the velocity for all positions x — 2

(ii) Find x when $t = 4$ seconds — 1

Question 5 — Marks

(a)



The diagram shows the parabola $x^2 = 4ay$. P is the point $(2ap, ap^2)$ and $S(0,a)$ is the focus

(i) Derive the equation of the normal to the parabola $x^2 = 4ay$ at the point $P\ (2ap,\ ap^2)$ — 1

(ii) A line SN is drawn parallel to the tangent at P and intersects the normal PN at point N. Find the coordinates of point N. — 2

(iii) Show that the locus of N as P varies is a parabola and determine the vertex and focus of that parabola — 3

(b) Two of the roots of $x^3 - px^2 - qx - 20 = 0$ are 3 and 5

(i) Find the other root — 1

(ii) Find p and q — 2

(c) (i) Find a general solution to $1 - 2\cos 2x = 0$ — 2

(ii) Sketch the curve $y = 1 - 2\cos 2x$ for $0 \le x \le 2\pi$ — 1

Question 6 (12 marks) | Marks

(a) Find $\int (\cos x + \sin x)^2 dx$ 3

(b) (i) Find the domain and range of function $f : y = \dfrac{2}{x-1}$ 2

(ii) Find the inverse function f^{-1} in terms of x. 1

(iii) Sketch both functions on the same set of axes and state the co-ordinates of any common points. 3

(c) From the top of a lighthouse 50 metres tall on a headland which is 750 metres above sea level, a tanker is seen on a bearing 320^0T at an angle of depression of 12^0. A tugboat is also sighted at a bearing of 032^0T at an angle of depression of 20^0. Calculate the distance between the vessels 3

Question 7 | Marks

(a) A skyrocket is fired from a height of 30 metres at an angle of 60^0 to the horizontal with a velocity of $20\,ms^{-1}$. Use $g = 10ms^{-2}$ to find in simplest exact form

(i) The equations of motion for the horizontal and vertical components of displacement. 1

(ii) The maximum height above ground that the rocket will reach. 2

(iii) The total time the skyrocket is in flight. 2

(iv) How far from the launching position will it land? 1

(v) The speed at which the rocket hits the ground 2

(b) The rate of change of temperature (T) for a substance cooling to room temperature (A) is given by

$$\frac{dT}{dt} = -k(T-A) \quad \text{where k and A are constants}$$

(i) Show that $T = A + Ce^{-kt}$ is a solution of this equation 1

(ii) Initially the temperature is 130^0. The temperature after one minute is 100^0 and after 2 minutes is 80^0. Determine the room temperature

End of Exam

MATHS   Ext 1 Trial 2004

Question 1

(3) a Externally 3 : 2

$= -3 : 2$

$= k_1 : k_2$

$x = \dfrac{k_1 x_2 + k_2 x_1}{k_1 + k_2}$

$= \dfrac{-3 \times 4 + 2 \times -2}{-3 + 2}$

$= \dfrac{-16}{-1}$

$= 16$

$y = \dfrac{k_1 y_2 + k_2 y_1}{k_1 + k_2}$

$= \dfrac{-3 \times -3 + 2 \times 5}{-3 + 2}$

$= \dfrac{9 + 10}{-1}$

$= -19$

$\therefore$ Point $= (16, -19)$

(3) b $\dfrac{2x+1}{x-2} > 1$

1st c.v. $x = 2$

Let $\dfrac{2x+1}{x-2} = 1$

$2x + 1 = x - 2$

$x = -3$ 2nd c.v.

$-3 \qquad 2$

Test $x = -4$ in ineq.

$\dfrac{-8+1}{-4-2} = \dfrac{-7}{-6} = 1\tfrac{1}{6} > 1$

Test $x = 0$ in ineq.

$\dfrac{1}{-2} \not> 1$

Test $x = 3$ in ineq.

$\dfrac{6+1}{3-2} = \dfrac{7}{1} > 1$

Ans: $x < -3$, $x > 2$

(2) c $\displaystyle\lim_{x \to 0} \frac{4x}{\tan \frac{x}{2}}$

$= 8 \displaystyle\lim_{x \to 0} \frac{x/2}{\tan x/2}$

$= 8 \times 1$

$= 8$

(4) d Let $u = 1 + x$

$du = dx$

$x = 0, \quad u = 1$

$x = 1, \quad u = 2$

$\displaystyle\int_0^1 \frac{x\,dx}{\sqrt{(1+x)^3}} = \int_1^2 \frac{u-1}{u^{1\frac{1}{2}}}\,du$

$= \displaystyle\int_1^2 u^{-1/2} - u^{-3/2}\,du$

$= \left[2u^{1/2} + 2u^{-1/2}\right]_1^2$

$= 2\left[\sqrt{u} + \dfrac{1}{\sqrt{u}}\right]_1^2 = 2\left[\sqrt{2} + \dfrac{1}{\sqrt{2}} - 2\right]$

(d) $= 2\sqrt{2} + \sqrt{2} - 4$

$= 3\sqrt{2} - 4$

Question 2

(3) a $y = 2\sin^{-1}\left(\dfrac{x}{2}\right)$

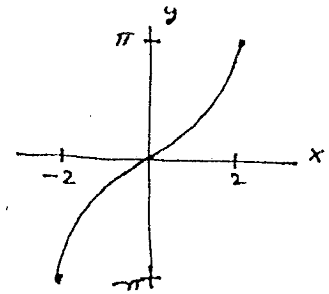
$-1 \le \dfrac{x}{2} \le 1$

$\therefore -2 \le x \le 2$ Dom

$-\dfrac{\pi}{2} \le \sin^{-1}\left(\dfrac{x}{2}\right) \le \dfrac{\pi}{2}$

$-\pi \le 2\sin^{-1}\dfrac{x}{2} \le \pi$

$\therefore -\pi \le y \le \pi$ Range



(3) b $y = \sin^{-1}\left(\dfrac{x}{2} + 1\right)$ Let $u = \dfrac{x}{2} + 1$

$y = \sin^{-1}(u)$ $\dfrac{du}{dx} = \dfrac{1}{2}$

$\dfrac{dy}{du} = \dfrac{1}{\sqrt{1-u^2}}$

$\dfrac{dy}{dx} = \dfrac{1}{\sqrt{1-\left(\frac{x}{2}+1\right)^2}} \cdot \dfrac{1}{2}$

$= \dfrac{1}{2\sqrt{1 - \frac{x^2}{4} - x - 1}}$

$= \dfrac{1}{2\sqrt{\frac{-x^2-4x}{4}}}$

$= \dfrac{1}{\sqrt{-x^2-4x}}$

(3) c $\displaystyle\int_{-\frac{1}{3}}^{\frac{1}{3}} \frac{dx}{\sqrt{4-9x^2}}$

$= \displaystyle\int_{-\frac{1}{3}}^{\frac{1}{3}} \frac{dx}{3\sqrt{\frac{4}{9} - x^2}}$

$= \dfrac{1}{3}\displaystyle\int_{-\frac{1}{3}}^{1/3} \frac{dx}{\sqrt{\left(\frac{2}{3}\right)^2 - x^2}}$

$= \dfrac{1}{3}\left[\sin^{-1}\left(\dfrac{3x}{2}\right)\right]_{-1/3}^{1/3}$

$= \dfrac{1}{3}\left[\left(\sin^{-1}\dfrac{1}{2}\right) - \left(\sin^{-1}\left(-\dfrac{1}{2}\right)\right)\right]$

$= \dfrac{1}{3}\left[\dfrac{\pi}{6} + \dfrac{\pi}{6}\right]$

$= \dfrac{\pi}{9}$

Question 2 (ctd)

1 d i $2\sin\theta - 1\cos\theta$

$$A\sin(\theta-\alpha) = A\sin\theta\cos\alpha - A\cos\theta\sin\alpha$$
$$= (A\cos\alpha)\sin\theta - (A\sin\alpha)\cos\theta$$

$A\cos\alpha = 2$ $\quad$ $A\sin\alpha = 1$

$$A^2\cos^2\alpha + A^2\sin^2\alpha = 5$$
$$A^2(\cos^2\alpha + \sin^2\alpha) = 5$$
$$A = \sqrt{5}$$

$$\frac{A\sin\alpha}{A\cos\alpha} = \frac{1}{2}$$
$$\tan\alpha = 1/2$$
$$\therefore \alpha = \tan^{-1}\tfrac{1}{2} \doteqdot 0.4636 \quad (4\text{ d.p})$$

Hence $2\sin\theta - \cos\theta = \sqrt{5}\sin(\theta - 0.4636)$

2 ii Solve for $0 \le \theta \le 2\pi$

$$2\sin\theta - \cos\theta = \frac{\sqrt{5}}{2}$$
$$\sqrt{5}\sin(\theta - 0.4636) = \frac{\sqrt{5}}{2}$$
$$\sin(\theta - 0.4636) = \frac{1}{2}$$
$$\theta - 0.4636 = 0.5236 \text{ or } 2.6180$$

ans $\theta = 3.08$ or 0.99

Question 3

1 a i
$$x = 2\cos(t + \tfrac{\pi}{4})$$
$$\dot{x} = -2\sin(t + \tfrac{\pi}{4})$$
$$\ddot{x} = -2\cos(t + \tfrac{\pi}{4})$$

$\therefore \ddot{x} = -1x$ which is in form $\ddot{x} = -n^2x$

$\therefore$ S.H.M.

1 ii $n^2 = 1$

$\therefore n = 1$ $\quad$ $T = \frac{2\pi}{n} = \frac{2\pi}{1} = 2\pi$ secs.

1 iii $x = a\cos(nt + \alpha)$

$\therefore a = 2$ m = Maximum displacement.

1 iv Maximum velocity occurs at $x = 0$
$$v^2 = n^2(a^2 - x^2)$$
$$= 1(4 - 0)$$
$$\therefore v = 2 \text{ ms}^{-1} = \text{Max. velocity.}$$

1 v Let $x = 0$
$$0 = 2\cos(t + \tfrac{\pi}{4})$$
$$\cos(t + \tfrac{\pi}{4}) = 0$$
$$\therefore t + \tfrac{\pi}{4} = \pi/2$$
$$\therefore t = \tfrac{\pi}{4}$$
= 1st time the particle is at origin

Question 3

3 (b) Prove true for $n=1$

Let $n=1$, $3^{2n}-1 = 8$ which is divisible by 8

$\therefore$ True for $n=1$

Assume true for $n=k$

$\therefore 3^{2k}-1 = 8m$ which is divisible by 8

Prove true for $n=k+1$

$$3^{2(k+1)}-1 = 3^{2k+2}-1 = 3^{2k}.3^2-1$$
$$= (3^{2k}-1)9 + 8$$
$$= (8m)9 + 8$$
$$= 8(9m+1) \text{ which is divisible by 8}$$

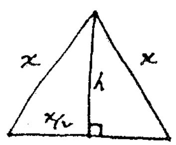
If true for $n=k$, then true for $n=k+1$

Since true for $n=1$, then true for $n=2$, $n=3$ etc

Hence by math. induction, true for all $n \geq 1$

c i

2



Data $\frac{dA}{dt} = 2$ cm/s

$$h^2 = x^2 - \frac{x^2}{4} = \frac{3x^2}{4}$$
$$h = \frac{\sqrt{3}x}{2}$$
$$\therefore A = \frac{1}{2} \times x \times \frac{\sqrt{3}x}{2}$$
$$= \frac{\sqrt{3}x^2}{4}$$

2 ii

$$\frac{dA}{dx} = \frac{2\sqrt{3}x}{4}$$
$$= \frac{\sqrt{3}x}{2}$$

$$\frac{dA}{dt} = \frac{dA}{dx}.\frac{dx}{dt}$$
$$2 = \frac{\sqrt{3}}{2}x.\frac{dx}{dt}$$

$$\therefore \frac{dx}{dt} = \frac{4}{\sqrt{3}}.\frac{1}{x} \quad \text{put } x = 2\sqrt{3}$$
$$= \frac{4}{\sqrt{3}}.\frac{1}{2\sqrt{3}}$$
$$= \frac{2}{3} \text{ cm/s} = \text{Rate of increase of side.}$$

Question 4

3 a

$$\tan x = x$$
$$\tan x - x = 0$$
$$\therefore f(x) = \tan x - x$$
$$f'(x) = \sec^2 x - 1$$

Let $a = 0.6$

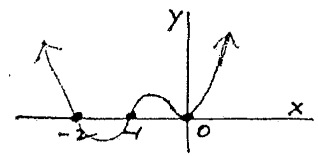
$$a_1 = a - \frac{f(a)}{f'(a)}$$
$$= 0.6 - \left(\frac{\tan(0.6) - 0.6}{\sec^2(0.6) - 1}\right)$$
$$= 0.42 = 2^{nd} \text{ approx to } x$$

1 b i

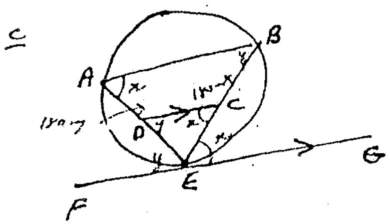
$$P(x) = x^4 + 3x^3 + 2x^2$$
$$= x^2(x^2 + 3x + 2)$$
$$= x^2(x+2)(x+1)$$

$\therefore$ Zeros are $x = 0$ (double root), -2, -1

1 ii Test $x = 1$, $P(1) = 1 + 3 + 2 = 6$



3 c



Aim: Prove A, B, C, D concyclic

Proof: $\angle BEG = \angle BAE$ ($\angle$ in alt seg.) $= x$

$\angle BEG = \angle DCE$ (alt $\angle$s, DC // FG)

$= x$

$\angle DCB = 180 - \angle DCE = 180 - x$ ($\angle$s st line)

$\therefore \angle BAD + \angle DCB = 180°$

$\angle AEF = \angle ABE = y$ ($\angle$ in alt seg)

$\angle DEF = \angle CDE = y$ (alt $\angle$s, DC // FG)

$\angle ADC = 180 - \angle CDE = 180 - y$ ($\angle$s st line)

$\therefore \angle ABC + \angle ADC = 180°$

Hence A, B, C, D are concyclic points (opp $\angle$s are supp.)

Question 6

(2) **d** i
$$V = x - 5$$
$$V^2 = (x-5)^2$$
$$= x^2 - 10x + 25$$
$$\tfrac{1}{2}V^2 = \tfrac{1}{2}x^2 - 5x + \tfrac{25}{2}$$
$$\text{Acc} = \ddot{x} = \frac{d}{dx}\left(\tfrac{1}{2}V^2\right)$$
$$= x - 5$$

Data. $t = 0,\ x = 6$

Since $\ddot{x} = V$ $\therefore$ acc is the same as vel for all x

(2) ii
$$V = \frac{dx}{dt} = x - 5$$
$$\frac{dx}{x-5} = dt$$
$$\int dt = \int \frac{dx}{x-5}$$
$$t = \log_e(x-5) + c$$

$t = 0,\ x = 6$ $\therefore$ $0 = \log_e(6-5) + c$
$$0 = \log_e 1 + c$$
$$\therefore c = 0$$
$$t = \log_e(x-5)$$

Put $t = 4$ $\quad 4 = \log_e(x-5)$
$$\therefore x - 5 = e^4$$
$$x = e^4 + 5 \text{ metres}$$
$$= 59.6 \text{ m}$$

Question 5

a i $x = 2ap$ $\quad \frac{dx}{dp} = 2a$

$y = ap^2$ $\quad \frac{dy}{dp} = 2ap$

$\frac{dy}{dx} = p$ = Tan grad

$\therefore$ Grad of normal $= -\frac{1}{p}$

Eqn of normal is
$$y - ap^2 = -\frac{1}{p}(x - 2ap)$$
$$py - ap^3 = -x + 2ap$$
$$\therefore x + py = ap^3 + 2ap$$ (1)

ii Grad of SN $= p$
$$y - a = p(x - 0)$$
$$\therefore y = px + a$$

Solve Sim $y = px + a$ and $x + py = ap^3 + 2ap$
$$x + p^2x + ap = ap^3 + 2ap$$
$$x(1 + p^2) = ap^3 + ap$$
$$x = ap(1+p^2)/(1+p^2)$$
$$x = ap$$
$$y = ap^2 + a$$

$\therefore$ Point $N = (ap,\ ap^2 + a)$ (2)

iii $x = ap$ $\quad \therefore p = x/a$
$$y = a(p^2 + 1)$$
$$= a\left(\frac{x^2}{a^2} + 1\right)$$
$$y = \frac{x^2}{a} + a$$
$$ay = x^2 + a^2$$
$\therefore x^2 = a(y - a)$ is locus of N which is a parabola

Vertex $= (0, a)$ $\quad$ Focus $= \left(0, \frac{5a}{4}\right)$ (3)

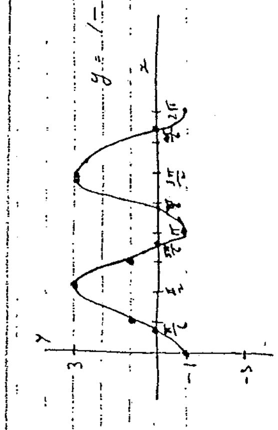
Q6 **b** $\quad x^3 - px^2 - qx - 20 = 0$

i Let roots be $\alpha, 3, 5$
$$\alpha + 3 + 5 = p$$
$$3\alpha + 5\alpha + 15 = q$$
$$15\alpha = 20$$
$$\therefore \alpha = 20/15 = 1\tfrac{1}{3}$$ (1)

ii
$$p = 1\tfrac{1}{3} + 3 + 5 = 9\tfrac{1}{3}$$
$$q = 3\left(\tfrac{4}{3}\right) + 5\left(\tfrac{4}{3}\right) + 15$$
$$= 25\tfrac{2}{3}$$ (2)

c i $1 - 2\cos 2x = 0$
$$2\cos 2x = 1$$
$$\cos 2x = \tfrac{1}{2}$$
$$2x = \cos^{-1}\tfrac{1}{2} = \tfrac{\pi}{3}$$
General Soln: $\quad 2x = 2n\pi \pm \frac{\pi}{3}$
$$\therefore x = n\pi \pm \pi/6 \quad (n = \text{any integer})$$ (2)

ii



(1)

Question 6

(a) $\int (\cos x + \sin x)^2 \, dx$

$(\cos x + \sin x)^2$
$= \cos^2 x + 2\cos x \sin x + \sin^2 x$
$= 1 + \sin 2x$

$= \int 1 + \sin 2x \, dx$

$= x - \frac{1}{2}\cos 2x + c$ (3)

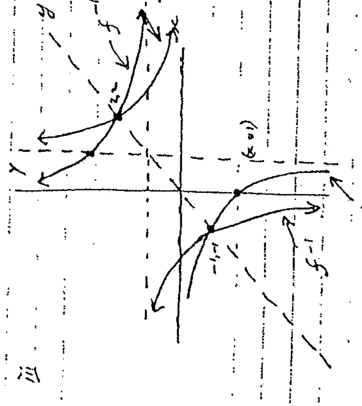
6 b i $f: y = \frac{2}{x-1}$ Dom = all real x but $x \neq 1$
Range = all real y but $y \neq 0$ (2)

ii $f^{-1}: x = \frac{2}{y-1}$

$y - 1 = \frac{2}{x}$

$\therefore y = \frac{2}{x} + 1$ (1) Dom: all real x but $x \neq 0$
Range: all real y but $y \neq 1$

iii



(1)

For common points Solve sim $\begin{cases} y = \frac{2}{x-1} \\ y = x \end{cases}$

$\frac{2}{x-1} = x$

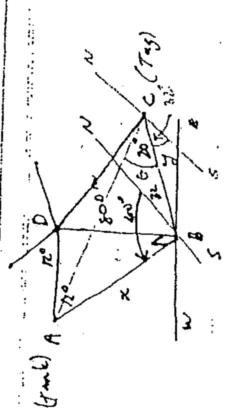
$2 = x^2 - x$

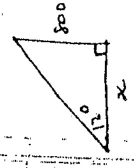
$0 = x^2 - x - 2$

$0 = (x-2)(x+1)$ (1)

$\therefore (x = 2, y = 2)$ and $(x = -1, y = -1)$ (1)

(c)

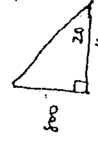




$\tan 12^\circ = \frac{800}{x}$

$x = \frac{800}{\tan 12}$

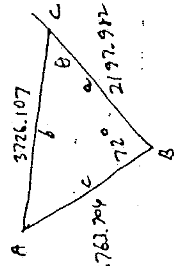
$= 3763.704$



$\tan 20 = \frac{800}{y}$

$y = \frac{800}{\tan 20}$

$= 2197.98$



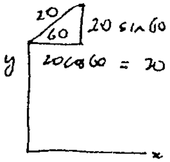
Cosine Rule

$b^2 = a^2 + c^2 - 2ac \cos B$

$b = 3726.107$

Distance between vessels = 3726 metres (3)

Q7



$20 \sin 60 = \frac{20\sqrt{3}}{2} = 10\sqrt{3}$

$20 \cos 60 = 20 \times \frac{1}{2} = 10$

$g = 10$

i

Vert.

$t = 0, \quad \dot{y} = 10\sqrt{3}, \quad y = 30$

$\ddot{y} = -g$

$\dot{y} = -gt + c$

$10\sqrt{3} = c$

$\dot{y} = -gt + 10\sqrt{3}$

$y = -g\frac{t^2}{2} + 10\sqrt{3}\,t + c$

$30 = c$

$y = -10\frac{t^2}{2} + 10\sqrt{3}\,t + 30$

$y = -5t^2 + 10\sqrt{3}\,t + 30$

Horiz

$t = 0, \; x = 0, \; \dot{x} = 10$

$\ddot{x} = 0$

$\dot{x} = 0 + c$

$10 = c$

$\dot{x} = 10$

$x = 10t + c \quad \therefore c = 0$

$x = 10t$

(1)

ii At max height $\dot{y} = 0$

$0 = -10t + 10\sqrt{3}$

$t = \sqrt{3}$

$\therefore$ Max height $= y$

$= -5 \times 3 + 10\sqrt{3} \times \sqrt{3} + 30$

$= -15 + 30 + 30$

$= 45$ metres (2)

iii Let $y = 0$

$5t^2 - 10\sqrt{3}\,t - 30 = 0$

$t^2 - 2\sqrt{3}\,t - 6 = 0$

$t = \frac{2\sqrt{3} \pm \sqrt{12 + 24}}{2}$

$= \frac{2\sqrt{3} + 6}{2}$

$= \sqrt{3} + 3$ secs

= total flight time. (2)

iv $x = 10 \times t$

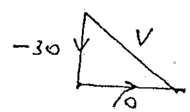
$= 10(\sqrt{3} + 3)$ metres

= range. (1)

v Let $t = \sqrt{3} + 3$

$\dot{y} = -10(\sqrt{3} + 3) + 10\sqrt{3} = -30$

$\dot{x} = 10$



$V^2 = (-30)^2 + 10^2$

$= 900 + 100$

$= 1000$

$V = \sqrt{1000}$

$= 10\sqrt{10}$ m/s.

= speed it hits the ground (2)

Q 7 b i $T = A + Ce^{-kt}$

$\frac{dT}{dt} = -kCe^{-kt}$

$-k(T-A) = -k(Ce^{-kt})$ $\quad \therefore \frac{dT}{dt} = -k(T-A)$

Hence $T = A + Ce^{-kt}$ is a solution (1)

ii $t=0,\ T=130$ $\quad 130 = A + C$

$t=1,\ T=100$ $\quad 100 = A + Ce^{-k}$

$t=2,\ T=80$ $\quad 80 = A + Ce^{-2k}$

$A = 130 - C$

$100 = 130 - C + Ce^{-k}$

$\therefore C(1-e^{-k}) = 30$ (1)

$80 = 130 - C + Ce^{-2k}$

$\therefore C(1-e^{-2k}) = 50$ (2)

(1) ÷ (2)

$\frac{1-e^{-k}}{1-e^{-2k}} = \frac{3}{5}$ (3)

$\frac{1-m}{1-m^2} = \frac{3}{5}$ $\quad$ Put $e^{-k} = m \quad \therefore e^{-2k} = m^2$

$5 - 5m = 3 - 3m^2$

$3m^2 - 5m + 2 = 0$

$(3m-2)(m-1) = 0$

$\therefore m = 2/3$ or $m = 1$

Now $e^{-k} = 2/3$ or $e^{-k} = 1$

Taking $e^{-k} = 1$ means (3) becomes $\frac{1-1}{1-1} = \frac{0}{0} \neq \frac{3}{5}$

$\therefore e^{-k} = 2/3$ only

Sub into (1)

$C = 30/(1-2/3) = 90$

$\therefore A = 130 - 90 = 40$

Thus Room Temp $= 40°$ (3)