Sydney Girls High School

2006
TRIAL HIGHER SCHOOL
CERTIFICATE EXAMINATION

Mathematics

Extension 1

This is a trial paper ONLY. It does not necessarily reflect the format or the contents of the 2006 HSC Examination Paper in this subject.

General Instructions

- Reading Time – 5 mins
- Working Time – 2 hours
- Attempt ALL questions
- ALL questions are of equal value
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Standard integrals are supplied
- Board-approved calculators may be used.
- Diagrams are not to scale
- Each question attempted should be started on a new sheet. Write on one side of the paper only.

Marks

Question 1 **(12 marks)**

a) Differentiate $y = 4\cos^{-1} 3x$ (2)

b) Find the coordinates of the point that divides the interval between (3, 4) and (-5, 1) externally in the ratio 2:3 (2)

c) Find $\int \frac{dx}{4+3x^2}$ (2)

d) Sketch the graph of $y = \sin\left(\sin^{-1} x\right)$ showing clearly the domain and range (3)

e) Find $\int 2xe^{x^2-3}dx$ using the substitution $u = x^2 - 3$ (3)

Marks

Question 2 **(12 marks)**

a) Find the obtuse angle between the lines
$y = 2x$ and $5x + 2y - 3 = 0$ to the nearest minute. (2)

b) Sketch $y = \frac{|x|}{x^2}$ (2)

c) Find the exact value of $\int_0^{\frac{\pi}{6}} \cot x dx$ (3)

d) Find the general solution of $\tan 2\theta = \sqrt{3}$ (2)

e) Find the equation of the normal to the curve
$y = \sin^{-1} 3x$ at the point where $x = 0$ (3)

Marks

Question 3 **(12 marks)**

a) The rate of change in velocity (v) of a falling object is given by
$\frac{dv}{dt} = -k(v - C)$ where k and C are constants.

(i) Show that $v = C + Ae^{-kt}$ is a solution of this equation (1)

(ii) When $C = 500$, the initial velocity is 0 and the velocity (v) after 5 seconds is 21 ms^{-1}.
Find A and k. (2)

(iii) Find the velocity after 20 seconds (1)

(iv) Find the maximum possible velocity (1)

b) A cable car (C) is travelling at a constant distance of 100m above the ground. An observer on the ground at Point A sees the cable car on a bearing of 345° from A with an angle of elevation of 65°. After one minute the cable car has a bearing of 025° from point A and a new angle of elevation of 69°.

Find (i) the distance the cable car has travelled in that minute. (2)

(ii) its speed in metres per second. (1)

c) The acceleration of a particle is given by

$\ddot{x} = -4x \ ms^{-2}$

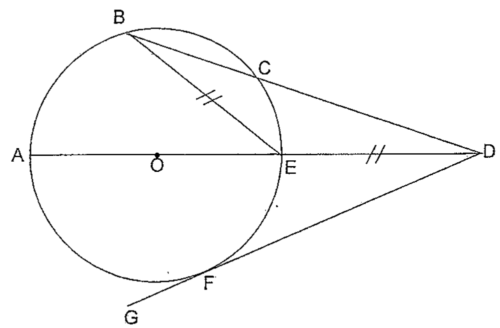
(i) Show that $x = \sin 2t$ is a solution of this differential equation (1)
(ii) Find the times when the displacement will be zero. (1)
(iii) Find the velocity of the particle at these times. (1)
(iv) Hence or otherwise find the equation of its velocity in terms of its displacement. (1)

Marks

Question 4 **(12 marks)**

a) Taking a first approximation of x= 0.6 radians, use 1 application of Newton's method to solve tan x = x correct to 2 decimal places (2)

b)



O is the centre of a circle through points A, B, C, E and F.

Diameter AOE is produced to point D such that BE = ED as shown.

(i) Copy this diagram onto your answer page (1)

(ii) Show that $\angle BEO = 2\angle CDE$ (2)

(iii) Show that $\angle BAO = 90° - \angle BEO$ (2)

(iv) DFG is a tangent to the circle touching at F. Given that BE =5cm and radius of the circle is 3.5cm, find the length of DF in exact form. (2)

c) Solve $\frac{3}{x-1} < 5$ (3)

Marks

Question 5 **(12 marks)**

a) Show that $\tan^{-1}\frac{4}{5} + \tan^{-1}\frac{5}{4} = \frac{\pi}{2}$ (2)

b) Simplify $\frac{|x+1|}{x^2-1}$ for $x \neq \pm 1$ (2)

c) The remainder when polynomial

$P(x) = ax^4 + bx^3 + 15x^2 + 9x + 2$ is divided by $(x-2)$ is 216 and $(x+1)$ is a factor of P (x). Find "a" and "b". (3)

d) Points P $(2ap, ap^2)$ and Q $(2aq, aq^2)$ lie on the parabola $x^2 = 4ay$.

(i) Find the equation of the chord PQ. (1)

(ii) If chord PQ subtends a right angle at the origin show that $pq = -4$. (2)

(iii) Find the equation of the locus of the midpoint of chord PQ. (2)

Marks

Question 6 **(12 marks)**

a) Use the principle of mathematical induction to prove that

$2^n - 1 > 5n + 2$ for $n > 4$ where n = integer (3)

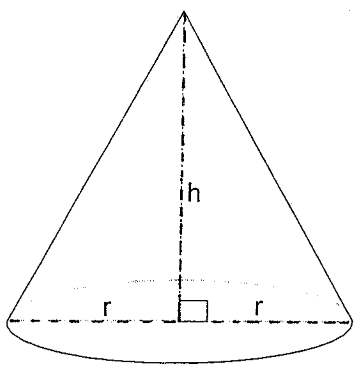
b) (i) Find the point of intersection of the curves $y = x^2$ and $y = (x-2)^2$. (1)

(ii) The area enclosed by the two curves and the x axis from $x = 0$ to $x = 2$ is rotated around the y axis. Find the volume of revolution in exact form. (3)

c) Use the substitution $u^2 = 1 + x^3$ to evaluate correct to 3 decimal places

$$\int_0^1 \frac{3x^2 dx}{2\sqrt{1+x^3}}$$ (2)

d) A funnel is dropping sand at a constant rate of 8m^3 per minute. The sand pile is in the shape of a cone such that the height (h) metres is always twice the radius (r) metres.



Find the rate at which the height is changing when the sand pile is at a height of 2m.. Answer correct to 2 decimal places. (3)

Marks

Question 7 **(12 marks)**

a) (i) Find the inverse function $f^{-1}(x)$ in terms of x for $y = f(x) = x^2 - 4x$ over the restricted domain $x \geq 2$. (2)
Write domain and range of this inverse function

(ii) Hence find the point common to both $f(x)$ and $f^{-1}(x)$ in this domain. (1)

b) Water is flowing at a horizontal velocity of $2ms^{-1}$ over a 5m vertical cliff.

(i) How far from the base of the cliff will it fall? (2)

(ii) At what velocity will it strike the river below? (correct to 1 dec place) (1)

(iii) At what acute angle will it make with the river below? (Use $g = 10\text{ms}^{-2}$) (to the nearest degree) (1)

c) A particle moves in a straight line, its position x metres at time t seconds being given by $x = \sqrt{3}\ \sin t - \cos t$

(i) Express $\sqrt{3}\ \sin t - \cos t$ in the form $A\sin(t-\alpha)$ where α is in radians and $A > 0$ (2)

(ii) Show that the particle is undergoing Simple Harmonic Motion about $x = 0$ (1)

(iii) Find the amplitude and period of the motion. (1)

(iv) At what time does the particle first reach its maximum speed after time $t = 0$ secs? (1)

S.G.H.S. Trial H.S.C Maths 2006 Ext 1

Sydney Girls High School Trial HSC Ext 1 Maths

Q1 a $y = 4\cos^{-1} 3x$

$\frac{dy}{dx} = \frac{-12}{\sqrt{1-9x^2}}$ (2)

b $x_1\ y_1 = 3,\ 4$

$x_2\ y_2 = -5,\ 1$

(externally) $k_1 : k_2 = -2 : 3$

$x = \frac{k_1 x_2 + k_2 x_1}{k_1 + k_2}$

$= \frac{10 + 9}{1} = 19$

$y = \frac{k_1 y_2 + k_2 y_1}{k_1 + k_2}$

$= \frac{-2 + 12}{1} = 10$

$\therefore (x, y) = (19, 10)$ (2)

c $\int \frac{dx}{4 + 3x^2}$

$= \int \frac{dx}{3(\frac{4}{3} + x^2)}$ (2)

$= \frac{1}{3} \int \frac{dx}{(\frac{2}{\sqrt{3}})^2 + x^2}$

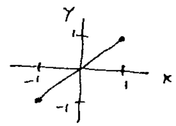
$= \frac{1}{3} \times \frac{\sqrt{3}}{2} \tan^{-1}(\frac{\sqrt{3}x}{2}) + c$

$= \frac{\sqrt{3}}{6} \tan^{-1}(\frac{\sqrt{3}x}{2}) + c$

d $y = \sin(\sin^{-1} x)$

$\therefore y = x \qquad -1 \le x \le 1$

$-1 \le y \le 1$



(3)

e $\int 2x e^{x^2-3} dx$

$u = x^2 - 3$

$\frac{du}{dx} = 2x$

$\therefore du = 2x\, dx$

$\int 2x e^{x^2-3} dx$

$= \int e^u du$

$= e^u + c$ (3)

$= e^{x^2-3} + c$

Question 2

a $y = 2x \qquad m_1 = 2$

$5x + 2y - 3 = 0 \qquad m_2 = -\frac{5}{2}$

$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}$

$= \frac{(2 + 5/2)}{(1 + 2 \times -\frac{5}{2})}$

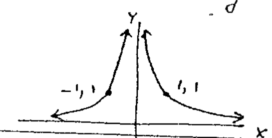
$= \frac{4\frac{1}{2}}{-4}$

obtuse $\theta = 131^\circ 38'$ (2)

b $y = \frac{|x|}{x^2}$

when $x < 0 \qquad y = \frac{-x}{x^2} = -\frac{1}{x}$

when $x > 0 \qquad y = \frac{x}{x^2} = \frac{1}{x}$



(2)

c $\int_0^{\frac{\pi}{6}} \cot x\, dx$

$= \int_0^{\pi/6} \frac{\cos x}{\sin x} dx$ (3)

$= [\log_e \sin x]_0^{\pi/6}$

$= \log_e \sin\frac{\pi}{6} - \log_e \sin 0$

$= \log_e \frac{1}{2} - \log_e 0$ (undefined number)

$= \log_e 1 - \log_e 2$ − undefined number

undefined number

(3)

a $y = \sin^{-1} 3x$

$\frac{dy}{dx} = \frac{3}{\sqrt{1-9x^2}}$

$x = 0, \quad \frac{dy}{dx} = 3$

$\therefore$ Grad normal $= -\frac{1}{3}$

$x = 0, \quad y = \sin^{-1} 0 = 0$

$y - 0 = -\frac{1}{3}(x - 0)$

$\therefore 3y = -x$

$\therefore x + 3y = 0$ is normal

d $\tan 2\theta = \sqrt{3}$

$2\theta = n\pi + \tan^{-1}\sqrt{3}$

$= n\pi + \frac{\pi}{3}$

$\therefore \theta = \frac{n\pi}{2} + \frac{\pi}{6}$

where n = any integer

$(= 0, \pm 1, \pm 2$, etc

(2)

Question 3

a i $\frac{dv}{dt} = -k(v-C)$

$v = C + Ae^{-kt}$

$LHS = \frac{dv}{dt} = 0 - kAe^{-kt}$

$RHS = -k(v-c)$

$= -k(C + Ae^{-kt} - c)$

$= -kAe^{-kt}$

$LHS = RHS$

$\therefore V = C + Ae^{-kt}$ is a soln. (1)

ii $t=0,\ V=0,\ C=500$

$\therefore 0 = 500 + Ae^{-0}$

$\therefore \underline{A = -500}$

$t = 5,\ v = 21$

$\therefore 21 = 500 - 500e^{-5k}$

$500e^{-5k} = 479$ (2)

$e^{-5k} = \frac{479}{500}$

$k = \log_e\left(\frac{479}{500}\right) \div (-5)$

$\therefore \underline{k = 0.0085815}$

iii $t = 20$

$V = 500 - 500e^{-20 \times .0085815}$

$\doteqdot 78.85\ ms^{-1}$ (1)

iv $V = 500 - \frac{500}{e^{kt}}$

Let $t \to \infty$, $V = 500 - 0$

$= 500\ ms^{-1}$

$=$ maximum Vel

(1)

Q3 c $\ddot{x} = -4x$

i $x = \sin 2t$

$\dot{x} = 2\cos 2t$

$\ddot{x} = -4\sin 2t$ (1)

$= -4x$

$\therefore x = \sin 2t$ is a soln

ii Let $x = 0$

$\sin 2t = 0$ (1)

$2t = 0, \pi, 2\pi, 3\pi$ etc

$\therefore t = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$ etc.

iii $\dot{x} = 2\cos 2t$

$t = 0,\ \dot{x} = 2\ ms^{-1}$

$t = \frac{\pi}{2}\ \dot{x} = -2\ ms^{-1}$

$t = \pi\ \dot{x} = 2\ ms^{-1}$ (1)

$t = \frac{3\pi}{2}\ \dot{x} = -2\ ms^{-1}$

iv $\ddot{x} = -4x$

$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = -4x$

$\frac{1}{2}v^2 = \int -4x\, dx$

$= -\frac{4x^2}{2} + c$

$\frac{1}{2}v^2 = -2x^2 + c$

Now $v = 2$ when $x = 0$

$\frac{1}{2} \times 4 = 0 + c$

$c = 2$

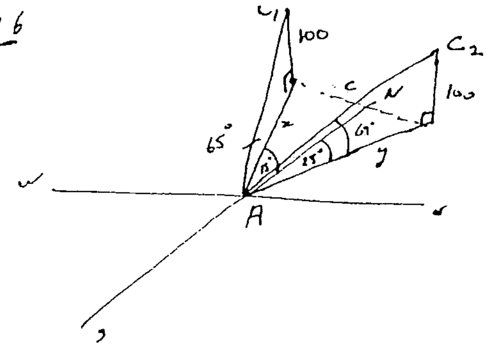
$\frac{1}{2}v^2 = -2x^2 + 2$

$v^2 = 4 - 4x^2$ (1)

$v = \pm\sqrt{4 - 4x^2}$

$\therefore v = \pm 2\sqrt{1-x^2}\ ms^{-1}$

Q3 b



i $\tan 65° = \frac{100}{x}$

$x = \frac{100}{\tan 65} = 46.63$

$\tan 69° = \frac{100}{y}$

$y = \frac{100}{\tan 69} = 38.39$

$c^2 = x^2 + y^2 - 2xy\cos 40°$

$= 46.63^2 + 38.39^2 - 2 \times 46.63 \times 38.39 \times \cos 40°$

$\therefore \underline{c = 30.1\text{ m} = \text{Dist travelled}}$ (2)

ii Speed $= \frac{30.1}{60} = 0.5$ m/s (1)

Q4

a $\tan x = x$

$\therefore \tan x - x = 0$

Let $f(x) = \tan x - x$

$f'(x) = \sec^2 x - 1$

$a_1 = a - \frac{f(a)}{f'(a)}$ (2)

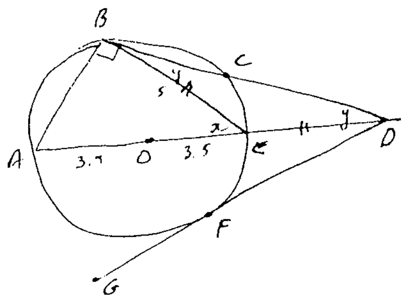
Put $a = 0.6$

$a_1 = 0.6 - \frac{(\tan 0.6 - 0.6)}{(\sec^2 0.6 - 1)}$

$= 0.42$

$\therefore$ Soln is $x \doteqdot 0.62$ (2 d.p)

b i



(1)

ii Aim: Show $\angle BEO = 2 \times \angle CDE$

Proof: $\angle DBC = \angle CDE = y$ (Base ∠s isos Δ)

$\angle BEO = x = 2y$ (Ext ∠ of Δ)

$\therefore \angle BEO = 2 \times \angle CDE$ (2)

iii Aim: Show $\angle BAO = 90^\circ - \angle BEO$

Proof: Draw BA

$\angle ABE = 90^\circ$ (∠ in semi circle)

$\therefore \angle BAO + \angle BEO = 90^\circ$ (∠ sum of Δ)

$\therefore \angle BAO = 90^\circ - \angle BEO$ (2)

iv Aim: Find DF length

Solution: $BE = ED = 5$ cm

$\therefore AD = 12$ cm

$AD \times DE = FD^2$ (Intersect chord + tangent Thm)

$12 \times 5 = FD^2$

$\therefore FD = \sqrt{60} = 2\sqrt{15}$ cm (2)

c $\frac{3x}{x-1} < 5$

(1st c.v) $x = 1$

Let $\frac{3x}{x-1} = 5$

$3x = 5x - 5$

$5 = 2x$

$x = 2\frac12$ (2nd c.v)

✓ × ✓ 1 $2\frac12$ Test in $\frac{3x}{x-1} < 5$

Test $x = 0$: $\frac{0}{-1} < 5$ ✓

Test $x = 3$: $\frac{9}{2} < 5$ ✓

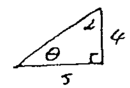
Test $x = 2$: $\frac{6}{1} \not< 5$

Ans: $x < 1$ and $x > 2\frac12$ (3)

Q5

a Let $\tan^{-1}\frac45 = \theta$

$\therefore \tan\theta = \frac45$



Now $\tan\alpha = \frac54$

$\therefore \alpha = \tan^{-1}\frac54$

$\theta + \alpha = \frac{\pi}{2}$ (2)

$\therefore \tan^{-1}\frac45 + \tan^{-1}\frac54 = \frac{\pi}{2}$

b $\frac{|x+1|}{x^2-1} = \frac{x+1}{(x-1)(x+1)}$ if $x > -1$

$= \frac{1}{x-1}$ if $x > -1$ (and $x \neq 1$)

$\frac{|x+1|}{x^2-1} = \frac{-(x+1)}{(x-1)(x+1)}$ if $x < -1$

$= \frac{-1}{x-1}$ if $x < -1$

(or $\frac{1}{1-x}$) if $x < -1$ (2)

c $P(x) = ax^4 + bx^3 + 15x^2 + 9x + 2$

$P(2) = 16a + 8b + 60 + 18 + 2 = 216$

$\therefore 16a + 8b = 136$

$P(-1) = a - b + 15 - 9 + 2 = 0$

$a - b = -8$

$a = b - 8$

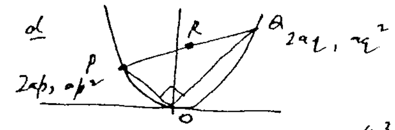
$16(b-8) + 8b = 136$

$24b = 264$

$\therefore b = 11$

$a = 3$ (3)

d



i Chord PQ grad $= \frac{a(p^2-q^2)}{2a(p-q)}$

$= \frac{p+q}{2}$

$y - ap^2 = \frac{p+q}{2}(x - 2ap)$

$y - ap^2 = \left(\frac{p+q}{2}\right)x - ap^2 - apq$

$\therefore y = \left(\frac{p+q}{2}\right)x - apq$ (1)

ii Grad OQ $= \frac{aq^2}{2aq} = \frac{q}{2} = m_1$

Grad OP $= \frac{ap^2}{2ap} = p/2 = m_2$

Since $\angle POQ = 90^\circ$

$m_1 m_2 = -1$

$\therefore \frac{q}{2} \cdot \frac{p}{2} = -1$

$\therefore pq = -4$ (2)

iii Midpt R $= a(p+q), a\frac{(p^2+q^2)}{2}$

$x = a(p+q)$

$\therefore p+q = x/a$

$\frac{a(p^2+q^2)}{2} = y$

$p^2+q^2 = 2y/a$

$(p+q)^2 - 2pq = 2y/a$

$\frac{x^2}{a^2} + 8 = 2y/a$

$x^2 + 8a^2 = 2ya$

$x^2 = 2ya - 8a^2$

$\therefore x^2 = 2a(y-4a)$ is locus of midpt of chord PQ. (2)

Q6 c Prove by M. I.

$2^n - 1 > 5n+2$ for $n > 4$

Step 1 Prove true for $n = 5$

LHS $= 2^5 - 1 = 31$, $5 \times 5 + 2 = 27$ = R.H.S. ∴ LHS > RHS

∴ True for $n = 5$

Step 2 Assume true for $n = k$

$2^k - 1 > 5k + 2$ ∴ $2^k - 1 - 5k - 2 > 0$ ∴ $2^k > 5k + 3$

Step 3 Prove true for $n = k+1$

Aim: Prove $2^{k+1} - 1 > 5(k+1) + 2$ ∴ $2^{k+1} - 1 - 5k - 5 - 2 > 0$

∴ aim: Prove $2^{k+1} - 5k - 8 > 0$

$$2^{k+1} - 5k - 8 = 2 . 2^k - 5k - 8$$

$> 2(5k+3) - 5k - 8$ (From Step 2)

$= 10k + 6 - 5k - 8$

$= 5k - 2$

> 0 since $k > 4$

∴ True for $n = k+1$

Step 4 If true for $n = k$, then true for $n = k+1$

Since true for $n = 5$, then true for $n = 6$ then $n = 7$ etc

∴ By Mathematical Induction $2^n - 1 > 5n + 2$ for all integer $n > 4$

(3)

Q7 d i

$y = x^2$ $\quad y = (x-2)^2$

$x^2 = (x-2)^2$

$x^2 = x^2 - 4x + 4$

$4x = 4$

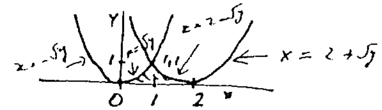
$x = 1$

$y = 1$

∴ Pt of intersection $= (1,1)$

(1)

ii



$(x-2)^2 = y$

$x - 2 = \pm\sqrt{y}$

$x = 2 \pm \sqrt{y}$

∴ use $x = 2 - \sqrt{y}$

now $x^2 = (2 - \sqrt{y})^2$

$= 4 - 4y^{\frac{1}{2}} + y$

$$\text{Vol} = \pi \int_0^1 4 - 4y^{\frac{1}{2}} + y \, dy - \pi \int_0^1 y \, dy$$

$$= \pi \left[4y - \frac{4y^{3/2}}{\frac{3}{2}} + \frac{y^2}{2} \right]_0^1 - \pi \left[y^2/2 \right]_0^1$$

$$= \pi \left[4 - \frac{8}{3} + \frac{1}{2} - 0 \right] - \pi \left[\frac{1}{2} - 0 \right]$$

$$= \frac{4\pi}{3} \text{ units}^3$$

(3)

e

$$\int_0^1 \frac{3x^2 \, dx}{2\sqrt{1+x^3}}$$

Use $u^2 = 1 + x^3$

Change limits $x = 0$, $u^2 = 1$ ∴ $u = 1$

$x = 1$, $u^2 = 2$, $u = \sqrt{2}$

$u^2 = 1 + x^3$

$u = (1+x^3)^{1/2}$

$$\frac{du}{dx} = \frac{1}{2} 3x^2 (1+x^3)^{-\frac{1}{2}} = \frac{3x^2}{2\sqrt{1+x^3}}$$

$$∴ du = \frac{3x^2 \, dx}{2\sqrt{1+x^3}}$$

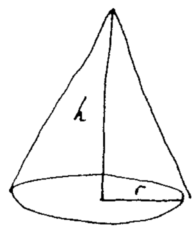
$$∴ \int_0^1 \frac{3x^2 \, dx}{2\sqrt{1+x^3}} = \int_1^{\sqrt{2}} 1 \, du = [u]_1^{\sqrt{2}}$$

$= \sqrt{2} - 1$

$= 0.414$

(2)

Q6 d



$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi \frac{h^2}{4} \cdot h$

$= \pi \frac{h^3}{12}$

$\therefore V = \frac{\pi}{12} h^3$

$h = 2r$

$\therefore r = \frac{h}{2}$

$r^2 = \frac{h^2}{4}$

$\frac{dV}{dt} = 8 \quad m^3/min$

$\frac{dV}{dh} = \frac{3\pi}{12} h^2 = \frac{\pi}{4} h^2$

$\frac{dV}{dt} = \frac{dV}{dh} \cdot \frac{dh}{dt}$

$8 = \frac{\pi}{4} h^2 \cdot \frac{dh}{dt}$

$\therefore \frac{dh}{dt} = \frac{32}{\pi} \frac{1}{h^2}$ (3)

Let $h = 2m$ $\quad \therefore \frac{dh}{dt} = \frac{32}{\pi} \cdot \frac{1}{4} = \frac{8}{\pi}$

$= 2.55$

$\therefore$ Rate of change of height $= 2.55$ m/min

Q7 a

i $f(x): \; y = x^2 - 4x$ $\quad$ for $x \geq 2$, $y \geq -4$

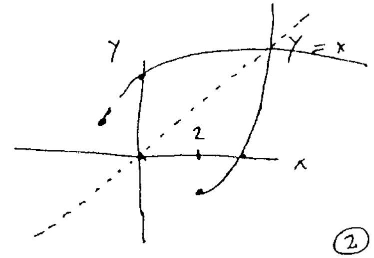
$f^{-1}(x): \; x = y^2 - 4y$

$y^2 - 4y + 4 = x + 4$

$(y-2)^2 = x + 4$

$y - 2 = \pm\sqrt{x+4}$

$y = 2 \pm \sqrt{x+4}$



(2)

$\therefore f^{-1}(x)$ is $y = 2 + \sqrt{x+4}$

$x \geq -4$ is Dom of $f^{-1}(x)$
$y \geq 2$ is Range of $f^{-1}(x)$

ii $f(x)$ and $f^{-1}(x)$ intersect on $y = x$

$\therefore y = x^2 - 4x = x$

$x^2 - 5x = 0$

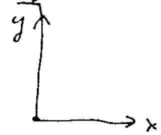
$x(x-5) = 0$

$\therefore x = 5, \; y = 5.$

$\therefore$ Common Pt $= (5, 5)$ (1)

---

b $\dot{x} = 2$



Data: $t = 0, \; x = 0, \; \dot{x} = 2, \; \ddot{x} = 0$

$t = 0, \; y = 5, \; \dot{y} = 0, \; \ddot{y} = -g = -10$

Horiz motion

$\dot{x} = 2$

$x = 2t + C$

$0 = 0 + C$

$\therefore x = 2t$

Vertical motion

$\ddot{y} = -10$

$\dot{y} = -10t + C$

$0 = 0 + C$

$\therefore \dot{y} = -10t$

$y = -\frac{10t^2}{2} + C$

$5 = 0 + C$

$\therefore y = -5t^2 + 5$

i Let $y = 0$

$0 = -5t^2 + 5$

$5t^2 = 5$

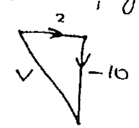
$t^2 = 1$

$\therefore t = 1$

$x = 2 \times 1 = 2$ m (2)

$\therefore$ Water falls 2m from base of cliff

ii $\dot{x} = 2, \; \dot{y} = -10$

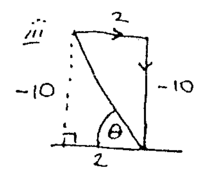


$V^2 = 2^2 + (-10)^2$

$= 104$

$V = \sqrt{104} \doteqdot 10.2$ m/s (1)

iii



$\tan\theta = \frac{10}{2}$

$= 5$

$\therefore \theta = 79°$

(1)

Q7 c $x = \sqrt{3}\sin t - 1\cos t$

i $A\sin(t-\alpha)$

$= A\sin t\cos\alpha - A\cos t\sin\alpha$

$= (A\cos\alpha)\sin t - (A\sin\alpha)\cos t$

$= \sqrt{3}\sin t - 1\cos t$

$A\cos\alpha = \sqrt{3}$ and $A\sin\alpha = 1$

$\frac{\sin\alpha}{\cos\alpha} = \tan\alpha = \frac{1}{\sqrt{3}}$

$\therefore \alpha = \pi/6$

$A^2\cos^2\alpha + A^2\sin^2\alpha = 3+1 = 4$

$\therefore A = 2$

$\therefore \sqrt{3}\sin t - \cos t = 2\sin(t - \pi/6)$ (2)

ii S H M about O is satisfied if $\ddot{x} = -n^2x$ (Defn)

$x = 2\sin(t - \pi/6)$

$\dot{x} = 2\cos(t - \pi/6)$

$\ddot{x} = -2\sin(t - \pi/6)$

$= -1^2x$ (1)

$\therefore$ S. H. M. about $x=0$

iii $-1 \leq \sin(t - \pi/6) \leq 1$

$\therefore -2 \leq 2\sin(t-\pi/6) \leq 2$

$\therefore$ amplitude $= 2$ metres

$n^2 = 1 \quad \therefore n = 1$ (1)

Period $= \frac{2\pi}{n} = 2\pi$ secs.

iv when $t=0$ $x = 2\sin(0 - \pi/6)$

$= -2 \times 1/2 = -1$

Maximum speed occurs at $x=0$

$0 = 2\sin(t - \pi/6)$

$t - \pi/6 = 0$ or π or 2π etc.

$\therefore$ First reaches Max speed when $t = \frac{\pi}{6}$ secs (1)