QUESTION ONE (Start a new answer booklet)

Marks

[2] (a) Evaluate $\displaystyle\int_0^3 \frac{dx}{9+x^2}$, giving your answer in exact form.

[2] (b) Find the acute angle, in radians correct to two decimal places, between the lines $y = \frac{1}{3}x + 3$ and $y = -\frac{2}{3}x + 3$.

[2] (c) Differentiate $y = \ln(\sin^{-1} x)$ with respect to x.

[3] (d) Solve the inequality $x \geq \dfrac{1}{x}$.

[3] (e) The polynomial equation $3x^3 - 2x^2 + 3x - 4 = 0$ has roots α, β an[illegible]. Find the exact value of $\dfrac{1}{\alpha\beta} + \dfrac{1}{\alpha\gamma} + \dfrac{1}{\beta\gamma}$.

QUESTION TWO (Start a new answer booklet)

Marks

[3] (a) Evaluate $\displaystyle\int_{-2}^{0} \frac{x^2}{\sqrt{1-x^3}}\,dx$ using the substitution $u = 1 - x^3$.

[3] (b) Prove the identity $\dfrac{\cos x - \cos 2x}{\sin 2x + \sin x} = \operatorname{cosec} x - \cot x$.

[3] (c) Two parallel lines have equations $x - my + 1 = 0$ and $x - my - 1 = 0$.

(i) Show that the point $(0, \frac{1}{m})$ lies on $x - my + 1 = 0$.

(ii) Find m given that the perpendicular distance between the lines is 1 unit.

[3] (d) The func[illegible] $= f(x)$ is defined by:

$$f(x) = \begin{cases} \sin^{-1} x, & \text{for } -1 \leq x < 0, \\ \frac{\pi}{8}, & \text{for } x = 0, \\ \cos^{-1} x, & \text{for } 0 < x \leq 1. \end{cases}$$

(i) Dra[illegible]at sketch of $y = f(x)$, for $-1 \leq x \leq 1$.

(ii) Evalu[illegible]$\frac{1}{2}) - f(-\frac{1}{2})$.

QUESTION THREE (Start a new answer booklet)

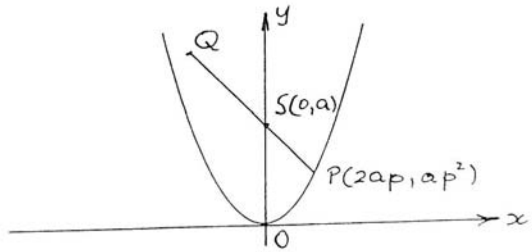
[4] (a) (i) Given ${}^nC_r = \dfrac{n!}{r!(n-r)!}$ show that $\dfrac{r \times {}^nC_r}{{}^nC_{r-1}} = n - r + 1$.

(ii) Hence prove that:

$$\frac{{}^nC_1}{{}^nC_0} + \frac{2 \times {}^nC_2}{{}^nC_1} + \frac{3 \times {}^nC_3}{{}^nC_2} + \cdots + \frac{n \times {}^nC_n}{{}^nC_{n-1}} = \frac{n}{2}(n+1).$$

[4] (b) The coefficients of the terms in x^3 and x^{-3} in the expansion of $\left(mx + \dfrac{n}{x^2}\right)^6$ are equal, where m and n are non-zero real numbers. Prove that $\dfrac{m^2}{n^2} = \dfrac{10}{3}$.

[4] (c)



In the diagram above, $P(2ap, ap^2)$ is a point on the parabola $x^2 = 4ay$ with focus $S(0, a)$. The point Q lies on PS produced, and Q divides PS externally so that $PQ : QS = -3 : 2$.

(i) Prove that Q has coordinates $\left(-4ap, a(3 - 2p^2)\right)$.

(ii) Show that as P varies the locus of Q is another parabola. Find its equation, and write down the coordinates of its vertex and its focus in terms of a.

[illegible]nues overleaf

QUESTION FOUR (Start a new answer booklet)

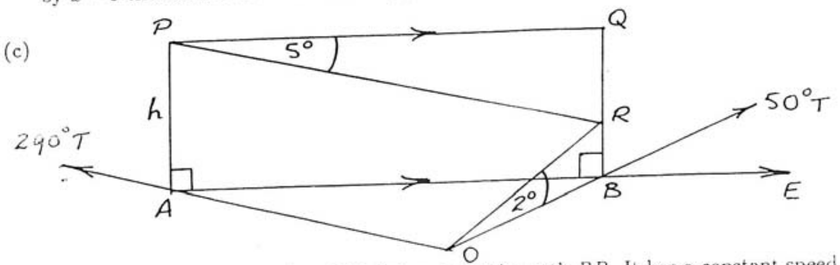
Marks

4 (a) (i) Prove that $\cos 2\theta = \dfrac{1-t^2}{1+t^2}$, where $t = \tan\theta$.

(ii) Hence show that $\tan\frac{\pi}{8} = \sqrt{2} - 1$.

4 (b) $P(x)$ is an odd polynomial of degree 3. It has $x+4$ as a factor, and when it is divided by $x-3$ the remainder is 21. Find $P(x)$.

4 (c)



In the diagram above an aircraft is flying along the path PR. It has a constant speed of 300 km/hr and is descending at a steady angle of 5°. It flies directly over beacons at A and B, where B is due East of A. An observer at O first sights the aircraft over A at a bearing of 290°T. The observer sights the aircraft again 10 minutes later over B at a bearing of 50°T and with an angle of elevation of 2°.

(i) Show that the aircraft has travelled 50 km in the 10 minutes between observations.

(ii) Show that $\angle AOB = 120°$.

(iii) Prove that the observer at O is 19 670 metres, to the nearest 10 metres, from the beacon at B.

(iv) Find the altitude h of the aircraft, to the nearest 10 metres, when it was originally sighted over A.

continues next page

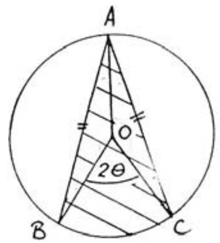
QUESTION FIVE (Start a new answer booklet)

rks

3 (a) The radius of a spherical bubble is expanding at a constant rate of 0·02 mm/sec. Find the rate of increase of the volume when the radius is 20 mm.

3 (b) The displacement x metres of a particle at time t seconds moving in simple harmonic motion is $x = 4\cos(\frac{\pi}{2}t+\epsilon)$, where $0 \le \epsilon \le 2\pi$. When $t = \frac{2}{3}$, $x = 2$ and $\dot{x} = \pi\sqrt{3}$ m/sec. Find the value of ϵ.

6 (c)



In the diagram above, AB and AC are equal chords of a circle with centre O and radius r. Let $\angle BOC = 2\theta$.

(i) The triangles ABO and ACO are congruent. Show that each has area $\frac{1}{2}r^2\sin\theta$.

(ii) Write down an expression in terms of r and θ for the area of the minor sector OBC.

(iii) The shaded region, bounded by the two chords AB and AC and the minor arc BC, has area half that of the circle. Prove that $\theta + \sin\theta = \frac{\pi}{2}$.

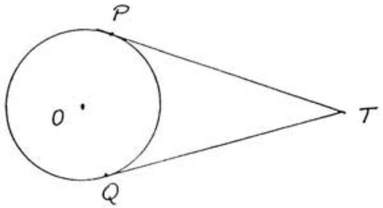
(iv) Taking $\theta = 0{\cdot}8$ as a first approximation to the solution of $\theta + \sin\theta = \frac{\pi}{2}$, use one application of Newton's method to find a second approximation correct to two decimal places.

Exam continues overleaf

QUESTION SIX (Start a new answer booklet)

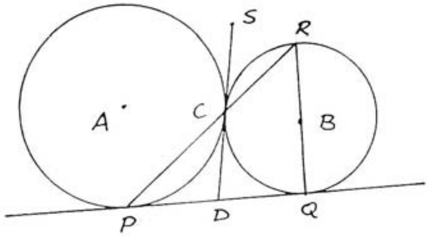
Marks

6 (a) (i)



In the diagram above, TP and TQ are tangents drawn from an external point T to a circle centre O. Prove that $TP = TQ$.

(ii)



In the figure above, two circles with centres A and B touch externally at point C. The common tangent at C intersects the direct common tangent PQ at D. The line QB produced meets the circumference of the smaller circle in R.

Copy the diagram into your answer booklet and prove that P, C and R [illegible] collinear.

6 (b) The rate of change of a quantity [illegible] with respect to t is given by $\frac{dI}{dt} = \frac{V}{L} - \frac{R}{L}I$, [illegible] R, L and V are constants.

(i) Show that $I = \frac{V}{R} + Ae^{[illegible]}$ [illegible]ties this equation, where A is a constant.

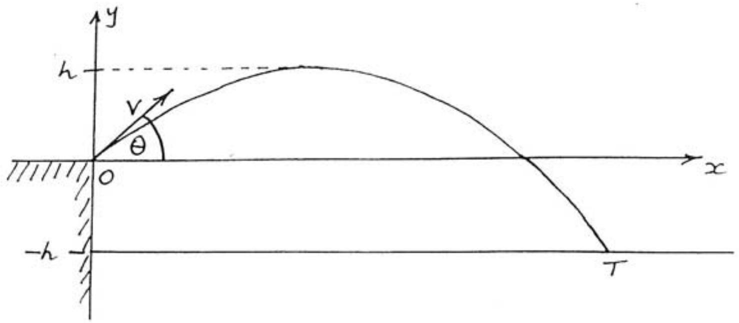
(ii) Deduce a relationship betw[illegible] V and R as t increases without bound.

(iii) Initially $I = 0$, and it is k[illegible] that $V = 5$, $R = 2{\cdot}2 \times 10^3$, and $L = 6{\cdot}5$ [illegible]
Find the value of I, in scien[illegible]notation to 2 significant figures, when $t = 2$ [illegible]

QUESTION SEVEN (Start a new answer booklet)

[illegible]arks

6 (a)



The diagram above shows the path of a projectile fired from the top O of a cliff. Its initial velocity is V m/s, its initial angle of elevation is θ and it rises to a maximum height h metres above O. It strikes a target T situated on a horizontal plane h metres below O.

The horizontal and vertical components of displacement in metres at time t seconds are given by $x = Vt\cos\theta$ and $y = Vt\sin\theta - \frac{1}{2}gt^2$ respectively.

(i) Prove that $h = \dfrac{V^2\sin^2\theta}{2g}$.

(ii) Prove that the time taken for the projectile to reach its target is:

$$\frac{V\sin\theta(1+\sqrt{2})}{g} \text{ seconds.}$$

(iii) Hence show that the distance from the target to the base of the cliff is:

$$\frac{V^2(1+\sqrt{2})\sin 2\theta}{2g} \text{ metres.}$$

(b) (i) Show that $\sin x + \cos x = \sqrt{2}\sin($[illegible]

(ii) Prove that the derivative of $y = e^x$ [illegible] given by $\frac{dy}{dx} = \sqrt{2}\,e^x\sin(x + \frac{\pi}{4})$.

(iii) Given the function $y = e^x \sin x$ p[illegible] mathematical induction that the nth derivative for positive integral n is:

$$\frac{d^n y}{dx^n} = (\sqrt{[illegible]} \ldots \sin(x + \tfrac{n\pi}{4}).$$

QUESTION ONE

Marks

[2] (a) $\displaystyle\int_0^3 \frac{dx}{9+x^2} = \left[\frac{1}{3}\tan^{-1}\frac{x}{3}\right]_0^3$ ✓

$= \frac{\pi}{12}.$ ✓

[2] (b) $\tan\theta = \left|\dfrac{\frac{1}{3}+\frac{2}{3}}{1-\frac{2}{9}}\right|$

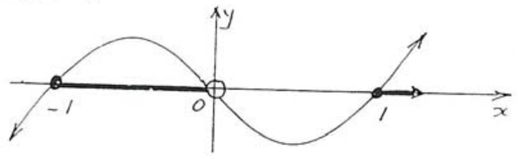
$= \frac{9}{7}.$ ✓

So $\theta \doteqdot 0{\cdot}91$ radians, to two decimal places. ✓

[2] (c) $y = \ln(\sin^{-1} x)$

$\dfrac{dy}{dx} = \dfrac{1}{(\sin^{-1} x)\sqrt{1-x^2}}.$ ✓✓

[3] (d)



$x \ge \dfrac{1}{x}$

[$\times x^2$] $x^3 \ge x$ and $x \ne 0$

$x(x-1)(x+1) \ge 0,$ ✓

so $-1 \le x < 0$ or $x \ge 1.$ ✓✓

[3] (e) $3x^3 - 2x^2 +$ [illegible] $= 0.$

$\alpha + $ [illegible] $= \frac{2}{3},$

[illegible] $= \frac{1}{3},$ ✓ for both

$\dfrac{1}{\alpha\beta} + \dfrac{1}{\alpha\gamma}$ [illegible] $= \dfrac{\alpha+\beta+\gamma}{\alpha\beta\gamma}$ ✓

$= \frac{1}{2}.$ ✓

QUESTION TWO

Marks

[3] (a) $\displaystyle\int_{-2}^0 \frac{x^2\,dx}{\sqrt{1-x^2}} = -\frac{1}{3}\int_9^1 \frac{du}{u^{\frac{1}{2}}}$ ✓

$= -\frac{1}{3}\left[2u^{\frac{1}{2}}\right]_9^1$ ✓

$= -\frac{2}{3} + \frac{2}{3}\times 3$

$= \frac{4}{3}.$ ✓

[3] (b) LHS $= \dfrac{\cos x - \cos 2x}{\sin 2x - \cos x}$

$= \dfrac{\cos x - 2\cos^2 x + 1}{2\sin x\cos x + \sin x}$ ✓

$= \dfrac{-(2\cos^2 x - \cos x - 1)}{\sin x(2\cos x + 1)}$

$= \dfrac{-(2\cos x + 1)(\cos x - 1)}{\sin x(2\cos x + 1)}$

$= \dfrac{1-\cos x}{\sin x}$ ✓

$= \operatorname{cosec} x - \cot x$ ✓

$=$ RHS.

[3] (c) (i) LHS $= x - my + 1$

$= 0 - \dfrac{1}{m}\times m + 1$

$= 0$

$=$ RHS. ✓

So the point lies on the line.

(ii) $1 = \left|\dfrac{0 - m \times \frac{1}{[illegible]} - 1}{[illegible]}\right|$ ✓

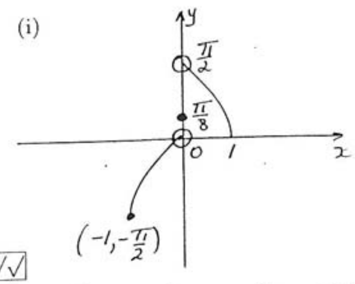
$1 = \dfrac{2}{\sqrt{1 [illegible]}}$

$\sqrt{1+m^2} = 2$

$m^2 = 3$

so $m = \sqrt{3}$ or [illegible] $-\sqrt{3}.$ ✓

[3] (d) (i)



✓✓

(ii) $f(\frac{1}{2}) - f(-\frac{1}{2}) = \cos^{-1}\frac{1}{2} - \sin^{-1}(-\frac{1}{2})$

$= \frac{\pi}{3} - (-\frac{\pi}{6})$

$= \frac{\pi}{2}.$ ✓

QUESTION THREE

Marks

[4] (a) (i) $\dfrac{r \times {}^nC_r}{{}^nC_{r-1}} = \dfrac{r \times n!}{r!\,(n-r)!} \times \dfrac{(r-1)!\,(n-r+1)!}{n!}$

$= \dfrac{r(r-1)!\,(n-r+1)(n-r)!}{r(r-1)!\,(n-r)!}$ ✓

$= n - r + 1.$ ✓

(ii) $\text{LHS} = \dfrac{{}^nC_1}{{}^nC_0} + \dfrac{2 \times {}^nC_2}{{}^nC_3} + \dfrac{3 \times {}^nC_3}{{}^nC_2} + \ldots + \dfrac{n \times {}^nC_n}{{}^nC_{n-1}}$

$= (n-1+1) + (n-2+1) + (n-3+1) + \ldots + (n-n+1)$

$= n + (n-1) + (n-2) + \ldots + 1$ ✓

$= \dfrac{n}{2}\Big(2n + (n-1)(-1)\Big)$

$= \dfrac{n}{2}(n+1)$ ✓

$= \text{RHS}.$

[4] (b) $u_{r+1} = {}^6C_r(mx)^{6-r}(nx^{-2})^r$

$= {}^6C_r m^{6-r} n^r x^{6-3r}.$ ✓

For term in x^3: $6 - 3r = 3$

$r = 1$

so $u_2 = {}^6C_1 m^5 n x^3.$ ✓

For term in x^{-3}: $6 - 3r = -3$

$r = 3$

so $u_4 = {}^6C_3 m^3 n^3 x^{-3}.$ ✓

Now ${}^6C_1 m^5 n = {}^6C_3 m^3 n^3$

$6m^5 n = 20 m^3 n^3$

$\dfrac{m^2}{n^2} = \dfrac{10}{3}.$ ✓

[4] (c) (i) $x = \dfrac{0 - 4ap}{3-2}$

$= -4ap$

$y = \dfrac{3a - 2ap^2}{3-2}$

$= a(3 - 2p^2).$

So Q is $(-4ap, a(3-2p^2)).$ ✓

(ii) $x = -4ap$

so $p^2 = \dfrac{x^2}{16a}$

$y = a\left(3 - \dfrac{2x^2}{16a^2}\right)$

$= 3a - \dfrac{x^2}{8a}$

so $x^2 = -8a(y - 3a).$

This is another parabol

Vertex: $V(0, 3a)$.

Focal Length: $2a$.

Focus: $S(0, a)$. ✓✓

QUESTION FOUR

[4] (a) (i) $\text{LHS} = \cos 2\theta$

$= \cos^2\theta - \sin^2\theta$

$= \dfrac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin^2\theta}$ ✓

$= \dfrac{1 - \tan^2\theta}{1 + \tan^2\theta}$

$= \dfrac{1 - t^2}{1 + t^2}$ ✓

$= \text{RHS}.$

(ii) Let $\theta = \frac{\pi}{8}.$

$\cos\frac{\pi}{4} = \dfrac{1-t^2}{1+t^2}$ ✓

$\dfrac{1}{\sqrt{2}} = \dfrac{1-t^2}{1+t^2}$

$t^2(1+\sqrt{2}) = \sqrt{2} - 1$

$t^2 = \dfrac{\sqrt{2}-1}{\sqrt{2}+1}$

$= \dfrac{\sqrt{2}-1}{\sqrt{2}+1} \times \dfrac{\sqrt{2}-1}{\sqrt{2}-1}$

$= (\sqrt{2}-1)^2$

so $t = \sqrt{2} - 1, \ t > 0.$ ✓

[4] (b) $P(x)$ is odd, of degree 3, so x and $x - 4$ are also factors.

Now $P(x) = ax(x+4)(x-4)$ ✓

$P(3) = 21$ ✓

$21 = 3a \times 7 \times (-1).$

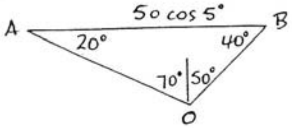
$a = -1.$ ✓

Thus $P(x) = -x(x+4)(x-4)$

$= -x^3 + 16x.$ ✓

(c) (i) 300 km/hr = 50 km in 10 minutes. ☑

(ii)

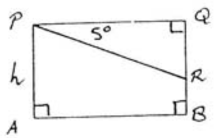


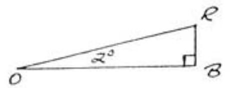
$\angle AOB = 70° + 50°$
$= 120°$. ☑

(iii)
$$AB = PQ$$
$$\cos 50° = \frac{PQ}{50}$$
$$PQ = 50\cos 5°$$
so $AB = 50\cos 5°$.

Now $\dfrac{AB}{\sin 120°} = \dfrac{OB}{\sin 20°}$
$$OB = \frac{AB\sin 20°}{\sin 120°}$$
$$= \frac{50\cos 5° \sin 20°}{\sin 120°}$$
$\doteqdot 19\,670$ metres, to nearest 10 metres. ☑

(iv)





$$\sin 5° = \frac{QR}{50\,000}$$
so $QR = 50\,000\sin 5°$.
$$\tan 2° = \frac{RB}{19\,670}$$
so $RB = 19\,670\tan 2°$.

Now $h = QR + RB$
$\doteqdot 5\,040$ metres, to nearest 10 metres. ☑

QUESTION FIVE

Marks

6 (a) (i) $\dfrac{dV}{dt} = \dfrac{dV}{dr} \times \dfrac{dr}{dt}$ ☑
$= 4\pi r^2 \times 0{\cdot}02.$
$= 4\pi \times 400 \times 0{\cdot}02$ ☑
$= 32\pi \text{ mm}^3/\text{s}.$ ☑

(ii) $x = 4\cos(\frac{\pi}{2}t + \varepsilon).$

When $t = \frac{2}{3},\ x = 2$
$2 = 4\cos(\frac{\pi}{3} + \varepsilon)$
$\cos(\frac{\pi}{3} + \varepsilon) = \frac{1}{2}.$ ☑

Now $\dot{x} = -2\pi\sin(\frac{\pi}{2}t + \varepsilon).$

When $t = \frac{2\pi}{3},\ \dot{x} = \pi\sqrt{3}$
$\pi\sqrt{3} = -2\pi\sin(\frac{\pi}{3} + \varepsilon)$
$\sin(\frac{\pi}{3} + \varepsilon) = -\frac{\sqrt{3}}{2}.$ ☑

Since $\cos(\frac{\pi}{3} + \varepsilon) > 0$ and $\sin(\frac{\pi}{3} + \varepsilon) < 0$
$\frac{\pi}{3} + \varepsilon = \frac{5\pi}{3}$
$\varepsilon = \frac{4\pi}{3}.$ ☑

6 (b) (i) reflex $\angle BOC = 2\pi - 2\theta$
$\angle AOB = \angle AOC$
$= \frac{1}{2}(2\pi - 2\theta)$
$= \pi - \theta.$ ☑

Area $= \frac{1}{2}r^2\sin(\pi - \theta)$
$= \frac{1}{2}r^2\sin\theta.$ ☑

(ii) Area $= \frac{1}{2}r^2 \times 2\theta$
$= r^2\theta.$ ☑

(iii) $2 \times \frac{1}{2}r^2\sin\theta + r^2\theta = \frac{1}{2} \times \pi r^2$
$r^2\sin\theta + r^2\theta = \frac{\pi r^2}{2}$
$\theta + \sin\theta = \frac{\pi}{2}.$ ☑

(iv) $\theta + \sin\theta = \frac{\pi}{2}$

$\theta + \sin\theta - \frac{\pi}{2} = 0$

Let $f(\theta) = \theta + \sin\theta - \frac{\pi}{2}$

$f'(\theta) = 1 + \cos\theta$

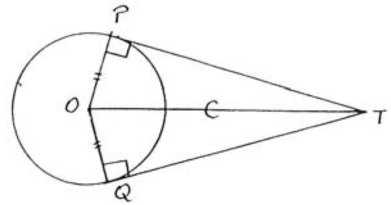
$$\theta_2 = \theta_1 - \frac{f(\theta_1)}{f'(\theta_1)}$$

$$= 0{\cdot}8 - \frac{0{\cdot}8 + \sin 0{\cdot}8 - \frac{\pi}{2}}{1 + \cos 0{\cdot}8} \quad \boxed{\surd}$$

$\doteqdot 0{\cdot}83$ correct to 2 decimal places. $\boxed{\surd}$

QUESTION SIX

6 (a) (i)



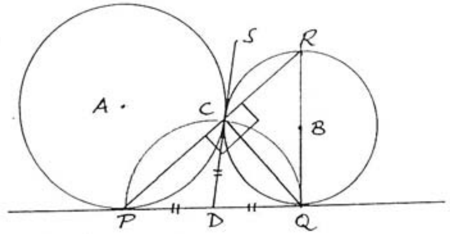
Join PO, OQ and OT.
In $\triangle OPT$ and $\triangle OQT$:

1. $PO = OQ$ (equal radii),
2. $\angle OPT = \angle OQT$ (radius perpendicular to tangent at point of contact),
3. $OT = OT$ (common side),

so $\triangle OPT \equiv \triangle OQT$ (RHS),

so $PT = QT$ (matching sides of congruent triangles). $\boxed{\surd\surd}$

(ii)



Join CQ.
$PD = DC = DQ$ (tangents from an external point),
so circle with PQ as diameter passes through C. $\boxed{\surd}$

So $\angle PCQ = 90°$ (angle in a semicircle, diameter PQ). $\boxed{\surd}$
also $\angle RCQ = 90°$ (angle in a semicircle, diameter QR). $\boxed{\surd}$
so $\angle PCR = 180°$,
So P [illegible] R are collinear. $\boxed{\surd}$

6 (b) (i) $I =$ [illegible] $Ae^{-\frac{R}{L}t}$

$\frac{dI}{dt} =$ [illegible] $e^{-\frac{R}{L}t}$ $\boxed{\surd}$

$=$ [illegible] $I - \frac{V}{R})$

$=$ [illegible] $+ \frac{V}{L}$

$=$ [illegible] $\frac{R}{L} I$. QED $\boxed{\surd}$

(ii) $\lim_{t\to\infty} I = \lim_{t\to\infty}\left(\frac{V}{R} + Ae^{-\frac{R}{L}t}\right)$

so $I = \frac{V}{R}$. ☑

(iii) When $t = 0,\ I = 0,$

so $0 = \frac{V}{R} + Ae^{0}$

so $A = -\frac{V}{R}$. ☑

Now $I = \frac{V}{R} - \frac{V}{R}e^{-\frac{R}{L}t}$

$= \frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right)$

$= \frac{5}{2\cdot2\times10^{3}}\left(1 - e^{\frac{-2\cdot2\times10^{3}}{6\cdot5\times10^{-3}}\times2\times10^{-6}}\right)$ ☑

$\doteqdot 1{\cdot}1\times10^{-3}$ to two significant figures. ☑

QUESTION SEVEN

Marks

6 (a) (i) $y = Vt\sin\theta - \frac{1}{2}gt^2$

so $\dot{y} = V\sin\theta - gt.$

Let $\dot{y} = 0,$

so $t = \dfrac{V\sin\theta}{g}$. ☑

Now $y = V\sin\theta\left(\dfrac{V\sin\theta}{g}\right) - \frac{1}{2}g\left(\dfrac{V\sin\theta}{g}\right)^2$

$= \dfrac{V^2\sin^2\theta}{g} - \dfrac{V^2\sin^2\theta}{2g}$

$= \dfrac{V^2\sin^2\theta}{2g}$ ☑

(ii) Projectile reaches target where $y = -\dfrac{V^2\sin^2\theta}{2g}$.

so $-\dfrac{V^2\sin^2\theta}{2g} = Vt\sin\theta - \frac{1}{2}gt^2$ ☑

so $g^2t^2 - 2gVt\sin\theta - V^2\sin^2\theta = 0$

so $t = \dfrac{2gV\sin\theta \pm \sqrt{4g^2V^2\sin^2\theta + 4g^2V^2\sin^2\theta}}{2g^2}$

$= \dfrac{2gV\sin\theta \pm 2\sqrt{2}gV\sin\theta}{g}$

$= \dfrac{V\sin\theta(1+\sqrt{2})}{g}$ since $t > 0$. ☑

(iii) $x = Vt\cos\theta$

$= V\cos\theta \times \dfrac{V\sin\theta(1+\sqrt{2})}{g}$ ☑

$= \dfrac{V^2\sin2\theta(1+\sqrt{2})}{2g}$. ☑

(b) (i) LHS $= \sqrt{2}\sin\left(x + \frac{\pi}{4}\right)$

$= \sqrt{2}\left(\sin x\cos\frac{\pi}{4} + \cos x\sin\frac{\pi}{4}\right)$

$= \sqrt{2}\left(\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x\right)$

$= \sin x + \cos x$

= RHS. ☑

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(ii) $y = e^x \sin x$

$$\frac{dy}{dx} = e^x \cos x + e^x \sin x$$
$$= e^x(\sin x + \cos x)$$
$$= \sqrt{2}\, e^x \sin\left(x + \tfrac{\pi}{4}\right).\ \boxed{\checkmark}$$

(iii) Prove $\dfrac{d^n y}{dx^n} = \left(\sqrt{2}\right)^n e^x \sin\left(x + \tfrac{n\pi}{4}\right)$, for positive integral n.

Step 1: Consider $n = 1$, LHS $= \dfrac{dy}{dx}$

$$= \sqrt{2}\, e^x \sin\left(x + \tfrac{\pi}{4}\right)$$
$$= \text{RHS.}\ \boxed{\checkmark}$$

So the result is true for $n = 1$.

Step 2. Assume the result is true for positive integral k.

So $\dfrac{d^k y}{dx^k} = \left(\sqrt{2}\right)^k e^x \sin\left(x + \tfrac{k\pi}{4}\right).\ \boxed{\checkmark}$

We shall prove the result is true for $k + 1$,

that is we prove, $\dfrac{d^{k+1} y}{dx^{k+1}} = \left(\sqrt{2}\right)^{k+1} e^x \sin\left(x + \tfrac{(k+1)\pi}{4}\right).$

Now LHS $= \dfrac{d^{k+1} y}{dx^{k+1}}$

$$= \frac{d}{dx}\left(\frac{d^k y}{dx^k}\right)$$
$$= \frac{d}{dx}\left(\left(\sqrt{2}\right)^k e^x \sin\left(x + \tfrac{k\pi}{4}\right)\right) \text{ by induction hypothesis } \boxed{\checkmark}$$
$$= \left(\sqrt{2}\right)^k e^x \sin\left(x + \tfrac{k\pi}{4}\right) + \left(\sqrt{2}\right)^k e^x \cos\left(x + \tfrac{k\pi}{4}\right)$$
$$= \left(\sqrt{2}\right)^k e^x \left(\sin\left(x + \tfrac{k\pi}{4}\right) + \cos\left(x + \tfrac{k\pi}{4}\right)\right)$$
$$= \left(\sqrt{2}\right)^k \times \sqrt{2}\, e^x \sin\left(x + \tfrac{k\pi}{4} + \tfrac{\pi}{4}\right).$$
$$= \left(\sqrt{2}\right)^{k+1} e^x \sin\left(x + \tfrac{(k+1)\pi}{4}\right).\ \boxed{\checkmark \text{ with conclusion}}$$
$$= \text{RHS.}$$

Conclusion: It follows from steps 1 and 2 above by mathematical induction [illegible] the result is true for positive integral n.