

Student Name FILE

Teacher's Name: _____

Extension 1 Mathematics

TRIAL HSC

August 2020

- General Instructions**
- Working time - 120 minutes + 10 minutes reading time
 - Write using black pen
 - NESA approved calculators may be used
 - A reference sheet is provided at the back of this paper
 - In questions 11-14, show relevant mathematical reasoning and/or calculations

-
- Total marks:** 70
- Section I – 10 marks**
- Attempt Questions 1-10
 - Allow about 15 minutes for this section

- Section II – 60 marks**
- Attempt questions 11-14
 - Allow about 1 hours and 45 minutes for this section

Section I

10 Marks

Attempt Questions 1-10

Allow about 15 minutes for this section

Use the multiple choice answer sheet for Questions 1-10.

1. A coin is biased such that the probability of a head is 0.8. The probability that exactly four tails will be observed when the coin is flipped ten times is:

(A) $10 \times 0.2^6 \times 0.8^4$

(B) ${}^{10}C_4 \times 0.2^4 \times 0.8^6$

(C) ${}^{10}C_4 \times 0.2^6 \times 0.8^4$

(D) $10 \times 0.2^4 \times 0.8^6$

2. Which one of the following vectors is parallel to the vector $\vec{OP} = 12\vec{i} - 6\vec{j}$?

(A) $\vec{OA} = 12\vec{i} + 6\vec{j}$

(B) $\vec{OB} = -\vec{i} + 2\vec{j}$

(C) $\vec{OC} = 2\vec{i} + \vec{j}$

(D) $\vec{OD} = -2\vec{i} + \vec{j}$

3. Which of the following is the correct expression for $\int \frac{dx}{\sqrt{9-x^2}}$?

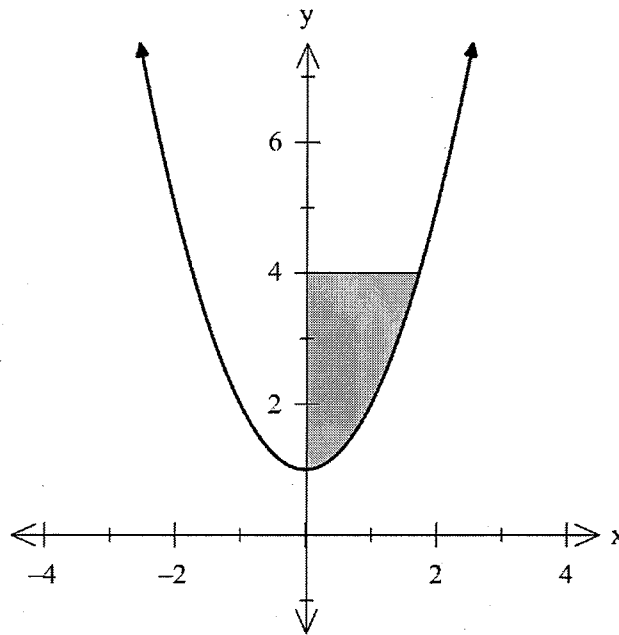
(A) $\sin^{-1} 3x + c$

(B) $\cos^{-1} 3x + c$

(C) $\sin^{-1} \frac{x}{3} + c$

(D) $\cos^{-1} \frac{x}{3} + c$

4. The region made between the curves $y = x^2 + 1$ and $y = 4$, and the y -axis is shown below.

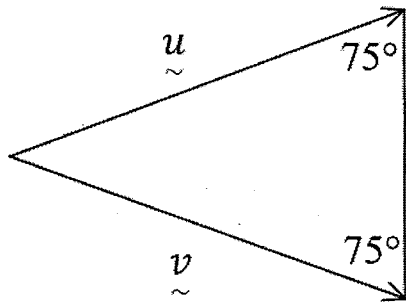


Which of these expressions gives the area of the region shown?

- (A) $\int_0^{\sqrt{3}} x^2 + 1 \, dx$
- (B) $\int_1^4 y - 1 \, dy$
- (C) $\pi \int_1^4 y - 1 \, dy$
- (D) $\int_1^4 \sqrt{y - 1} \, dy$
5. The number of elephants, N , in a population at time t is given by $N = Ae^{kt} + 750$, with constants $A > 0$ and $k > 0$. Which of the following is the correct differential equation?

- (A) $\frac{dN}{dt} = -k(N + 750)$
- (B) $\frac{dN}{dt} = k(N + 750)$
- (C) $\frac{dN}{dt} = -k(N - 750)$
- (D) $\frac{dN}{dt} = k(N - 750)$

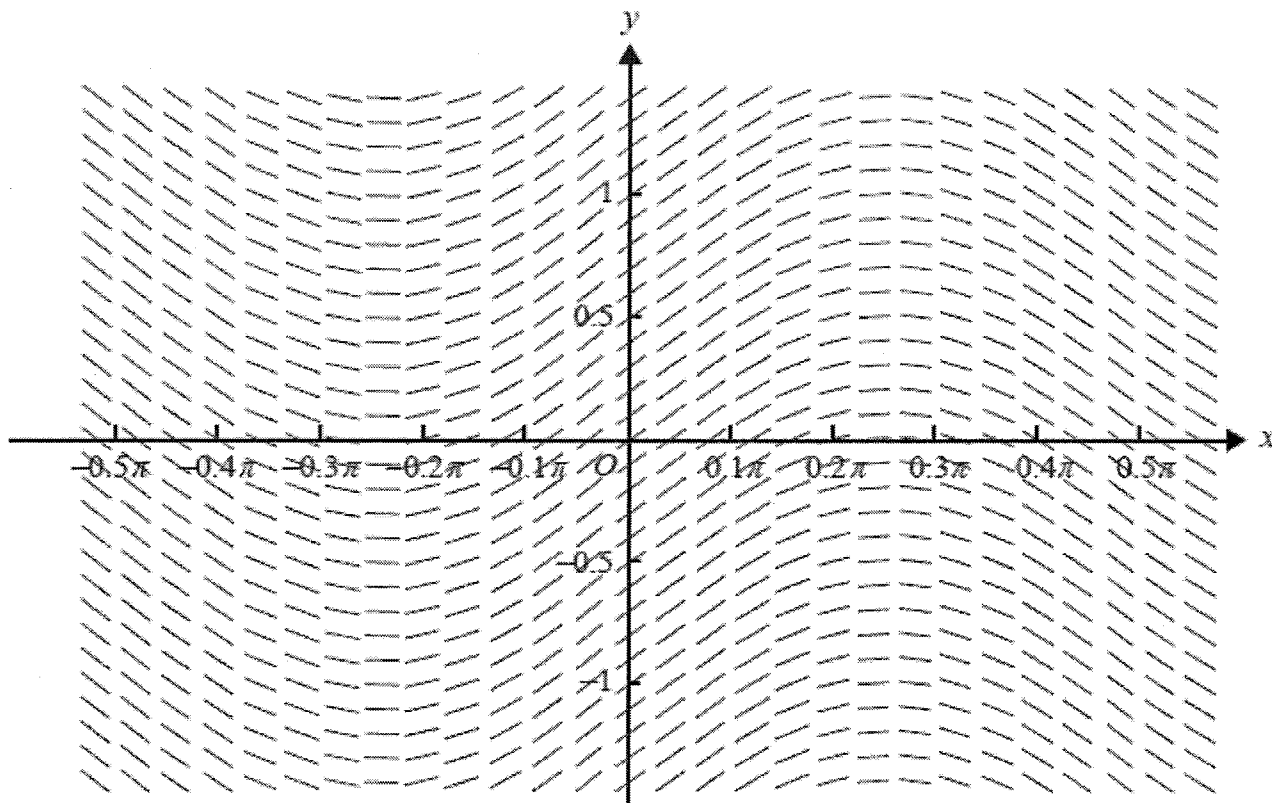
6. In the triangle below, $|\vec{u}| = |\vec{v}| = 4$.



What is the value of $\vec{u} \cdot \vec{v}$?

- (A) 16
(B) $8\sqrt{2}$
(C) 8
(D) $8\sqrt{3}$
7. Which polynomial has a multiple root at $x = 1$
- (A) $x^3 + 3x^2 - 4$
(B) $x^3 - 3x + 2$
(C) $x^3 - 3x - 4$
(D) $x^3 + 3x^2 + 2$
8. When $\cos x + \sin x$ is rewritten in the form $R \cos(x - \alpha)$, then:
- (A) $R = \sqrt{2}$ and $\alpha = \frac{\pi}{4}$
(B) $R = 2$ and $\alpha = \frac{\pi}{4}$
(C) $R = \sqrt{2}$ and $\alpha = \frac{3\pi}{4}$
(D) $R = 2$ and $\alpha = \frac{3\pi}{4}$

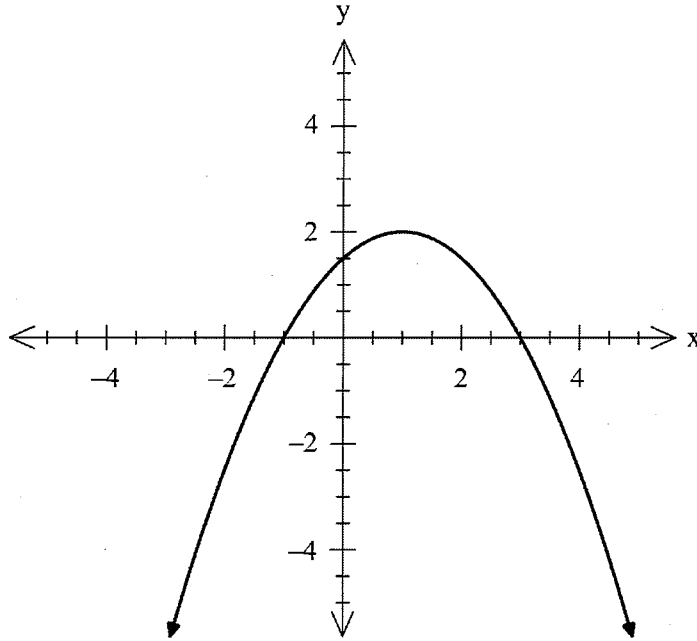
9. The slope field below is for a first-order differential equation.



Which of the following is a possible differential equation?

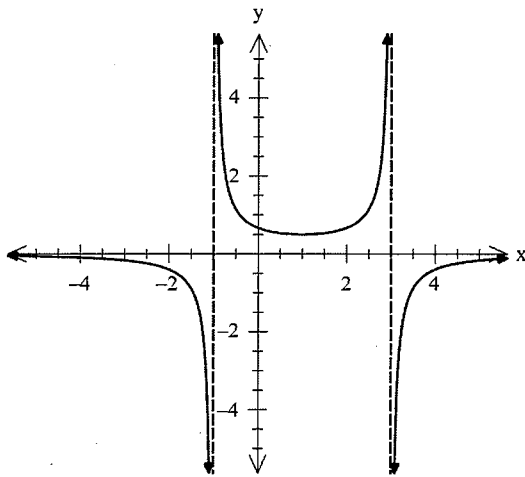
- (A) $\frac{dy}{dx} = \sin 2x$
- (B) $\frac{dy}{dx} = \sin 2y$
- (C) $\frac{dy}{dx} = \cos 2x$
- (D) $\frac{dy}{dx} = \cos 2y$

10. The graph of the function $y = f(x)$ is below.

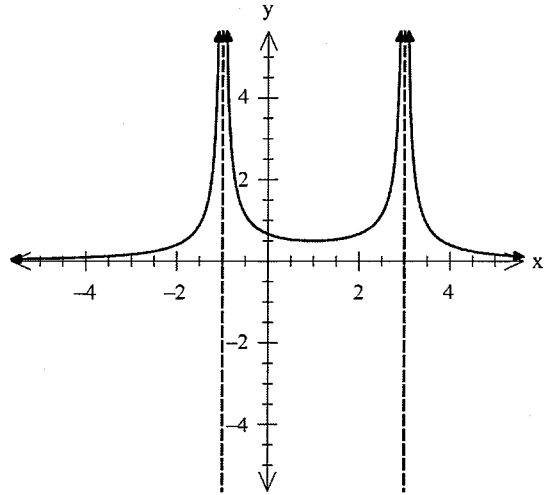


Which of the following is a graph of $y = \frac{1}{|f(x)|}$?

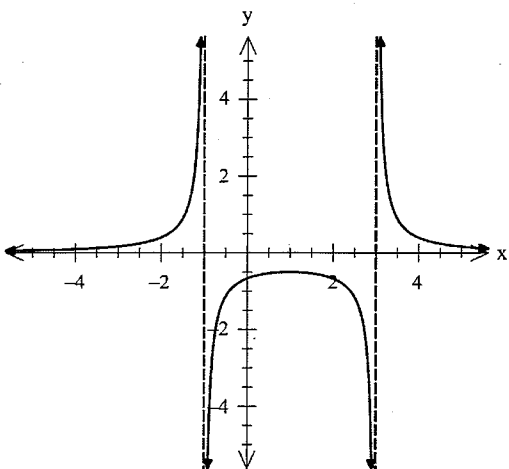
(A)



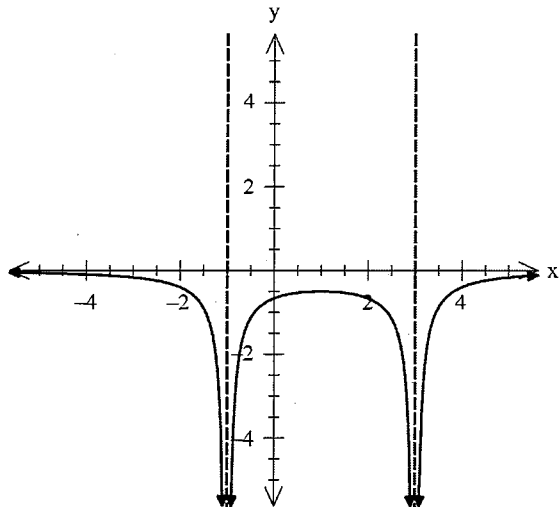
(B)



(C)



(D)



Section II

Total marks – 60

Attempt Question 11-14

Allow about 1 hour and 45 minutes for this section

Begin each question on a NEW page

In Questions 11-14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Begin a NEW page.

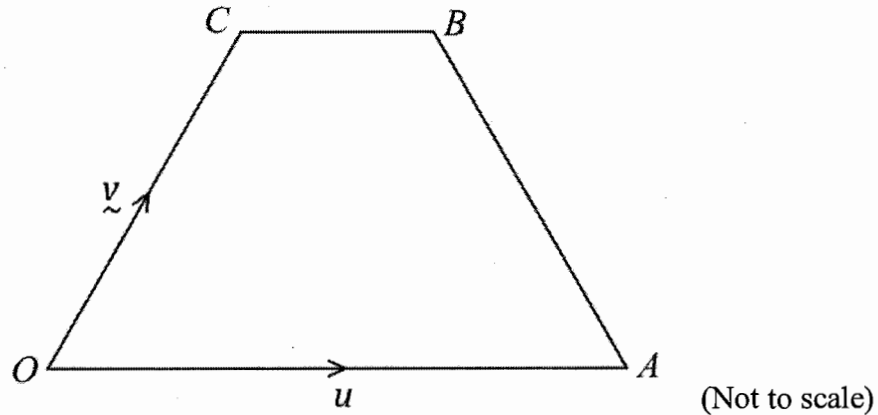
- a) Nine people are arranged around a circular table
- i. How many ways can they be arranged? 1
 - ii. If they are seated randomly, what is the probability that two individuals, Lois and Clark, are sitting next to each other? 1
- b) The heights in a population are normally distributed with a mean of 173 cm and a standard deviation of 7cm. Use the empirical rule to find the approximate probability that a randomly selected person has a height under 159 cm? 2
- c) A standard die is rolled 5 times. What is the probability of rolling a four at least two times? 2
- d) Find $\frac{dy}{dx}$ if $y = e^{2x} \cos^{-1} x$ 2
- e) Find $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$ if $x = 2 \cos t$ and $y = 2 \sin t$ 2
- f) Solve $\frac{2x}{x-3} \geq x + 4$ 3
- g) Use the substitution $u = 2 - x^4$, or otherwise, to find $\int 7x^3(2 - x^4)^5 dx$ 2

End of Question 11

Question 12 (15 marks) Begin a NEW page.

a) Find the particular solution to $\frac{dy}{dx} = \frac{2x}{3y^2}$ where $y(0) = 1$ 3

b) In the trapezium below, $\vec{OA} = \underline{u}$, $\vec{OC} = \underline{v}$, and $|\vec{OA}| = 2|\vec{CB}|$ 2



Express the vector \vec{BA} in terms of \underline{u} and \underline{v} .

- c) An unbiased coin is flipped 6400 times. The random variable X is the number of heads recorded.
- i. Assuming that X can be accurately approximated by a normal distribution, find the values of μ and σ such that $X \sim N(\mu, \sigma^2)$ 1
 - ii. Find the z -scores of flipping 3260 heads and of flipping 3100 heads. 2
 - iii. Hence, use the table below to estimate the probability of flipping between 3 100 and 3 260 heads. 2

First Decimal Place										
z	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	0.5000	0.5398	0.5793	0.6179	0.6554	0.6915	0.7257	0.7580	0.7881	0.8159
1	0.8413	0.8643	0.8849	0.9032	0.9192	0.9332	0.9452	0.9554	0.9641	0.9713
2	0.9772	0.9821	0.9861	0.9893	0.9918	0.9938	0.9953	0.9965	0.9974	0.9981
3	0.9987	0.9990	0.9993	0.9995	0.9997	0.9998	0.9998	0.9999	0.9999	1.0000

Question 12 is continued on the next page

d) If $t = \tan \frac{\theta}{2}$

i. Show that $3 \sin \theta - 4 \cos \theta - 4 = \frac{6t-8}{1+t^2}$ 2

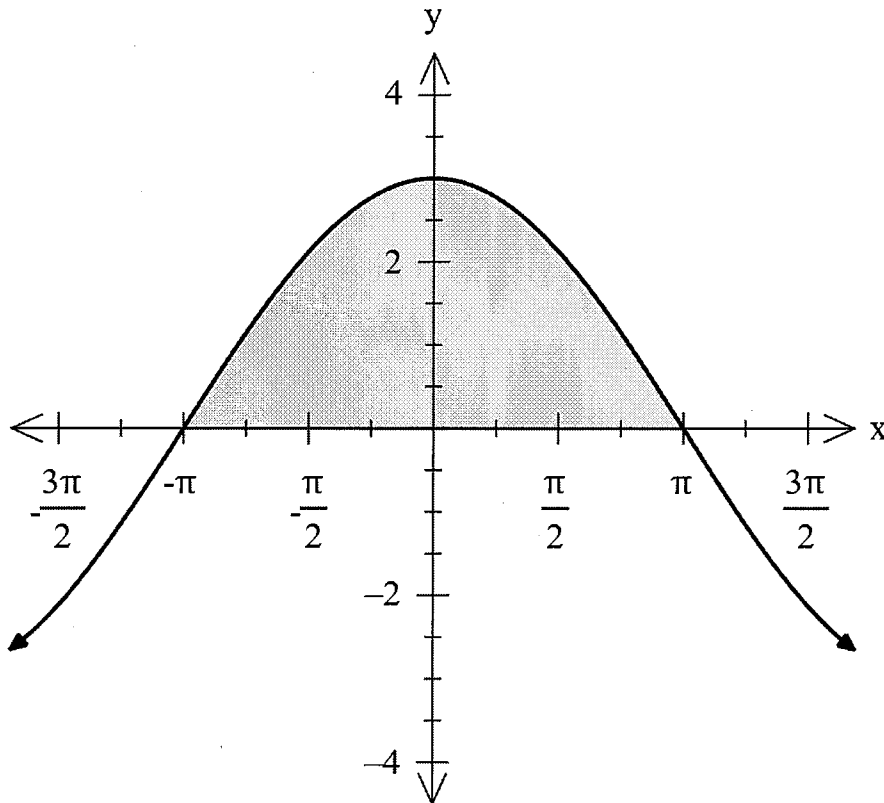
ii. Hence solve $3 \sin \theta - 4 \cos \theta = 4$ for $0 \leq \theta \leq 2\pi$ 3

End of Question 12

Question 13 (15 marks) Begin a NEW page.

- a) The radius of the base of a cylinder is increasing at a rate of 5 cm/min. The height of the cylinder is fixed at 30 cm and the formula for the volume of a cylinder is $V = \pi r^2 h$. Find the exact rate of change of the volume of the cylinder at the instant where the radius is 10 cm. 2

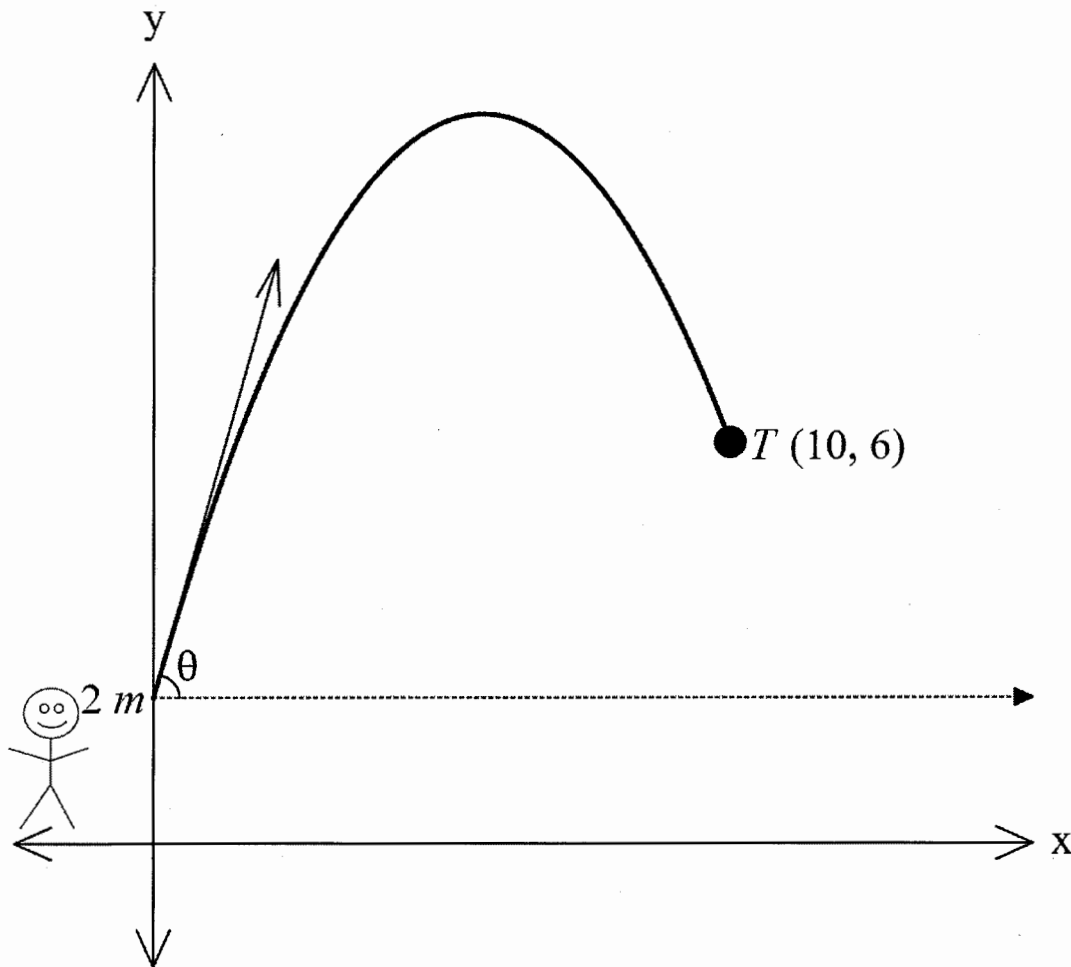
- b) Below, the region between $y = 3 \cos \frac{x}{2}$ and the x -axis is shaded between $x = -\pi$ and $x = \pi$.



- If the region is rotated around the x -axis, find the volume of the solid formed. 3
Leave your answer to 2 decimal places.

- c) Use mathematical induction to prove that $7^{2n} + 7^n + 4$ is divisible by 6 for integers $n \geq 0$. 4

- d) Billy throws a pebble from a height of 2 metres at an angle of θ to the horizontal, with a velocity of 30m/s.



- i. Show that the expressions for the horizontal and vertical displacement at t seconds after projection are $x = 30t\cos \theta$ and $y = -5t^2 + 30t\sin \theta + 2$ respectively. (Take the acceleration due to gravity as -10 m/s^2 and take the origin to be the ground directly below Billy). 2

- ii. Show that the equation of the path of the particle is 2

$$y = \frac{-x^2}{180} (1 + \tan^2 \theta) + x \tan \theta + 2$$

- iii. If Billy manages to hit a target at point T, which is 10 away on the ground and 6 metres high, find two possible angles of projection, to the nearest degree. 2

End of Question 13

Question 14 (15 marks) Begin a NEW page.

a) 25 quokkas are introduced to an island to promote the survival of the species. The growth of their population, Q , over t years, can be modelled by $\frac{dQ}{dt} = 0.0006Q(15\,000 - Q)$.

i. Show that $\frac{1}{0.0006Q(15\,000 - Q)} = \frac{1}{9} \left(\frac{1}{Q} + \frac{1}{15\,000 - Q} \right)$ 1

ii. Hence, solve $\frac{dQ}{dt} = 0.0006Q(15\,000 - Q)$, using integration, to show that 3
 $Q = \frac{15\,000}{1 + Be^{-9t}}$, where B is some constant.

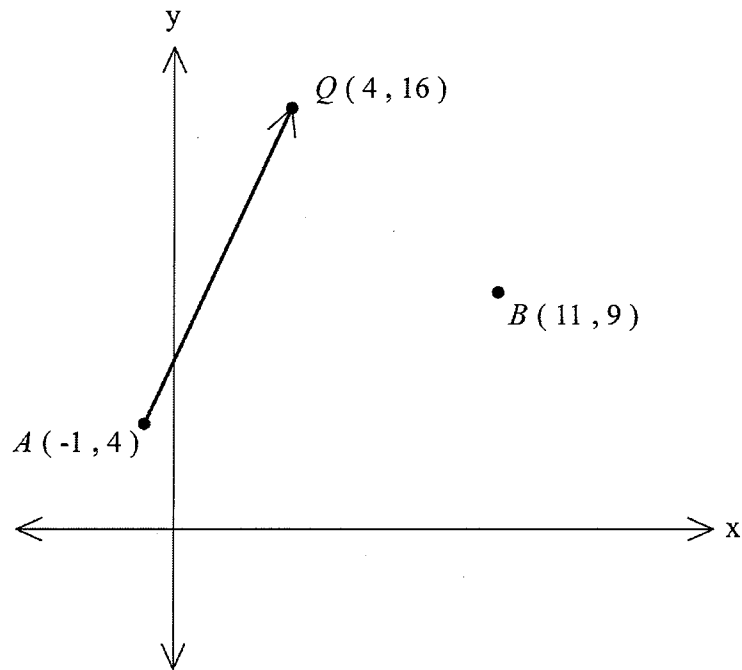
iii. Find the value of B , and hence use the model to estimate the number of quokkas on the island after two months. 2

iv. What is the maximum number of quokkas that the island will support? 1

v. After how many months is the number of quokkas to greater than half of the maximum amount? 2

Question 14 is continued on the next page

- b) Adam walks in a straight line from the point $A(-1, 4)$ to the point $Q(4, 16)$ with constant speed. His position vector can be expressed in the form $\underline{p} = \underline{a} + t\underline{u}$, where t is the time after he starts walking. Adam arrives at the point Q at $t = 3$.



- i. State the vectors \underline{a} and \underline{u} . 2
- ii. Bob is at point $B(11,9)$. During Adam's walk from A to Q , Bob wishes to throw a ball to Adam. Bob decides to throw the ball when Adam is at the closest point to B .
- α) Write a vector \underline{w} that is perpendicular to \underline{u} . 1
- β) Hence or otherwise, find value of t when Bob throws the ball to Adam. 3

End of Exam

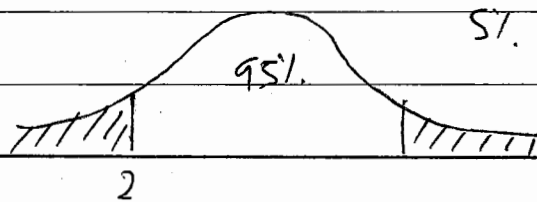
1	MC
2	
3	1. B
4	
5	2. D
6	
7	3. C
8	
9	4. D
10	
11	5. D
12	
13	6. D
14	
15	7. B
16	
17	8. A
18	
19	9. C
20	
21	10. B
22	
23	
24	
25	
26	

11.

1 a) i/ $8! = 40320$

2
3 ii/ $\frac{1 \times 2 \times 7!}{40320} = \frac{1}{4}$

6 b) $159 = 173 - 2 \times 7$



11 $P(X < 159) = \frac{5\%}{2} = 2.5\%$

13 c) $P(\text{At least two}) = 1 - \left({}^5C_0 \left(\frac{5}{6}\right)^5 + {}^5C_1 \left(\frac{1}{6}\right) \times \left(\frac{5}{6}\right)^4 \right)$
 14 $= \frac{763}{3888}$

16 d) $u = e^{2x}$ $v = \cos^{-1}x$
 17 $u' = 2e^{2x}$ $v' = -\frac{1}{\sqrt{1-x^2}}$

19 $\therefore \frac{dy}{dx} = 2e^{2x} \cos^{-1}x - \frac{e^{2x}}{\sqrt{1-x^2}}$

21 e) $\frac{dx}{dt} = -2\sin t$ $\frac{dy}{dt} = 2\cos t$

22
23 $\therefore \frac{dy}{dx} = \frac{-2\sin t}{2\cos t} = -\tan t$

25 At $t = \frac{\pi}{4}$, $\frac{dy}{dx} = -\tan \frac{\pi}{4}$
 26 $= -1$

$$1 \quad f) \quad \frac{2x}{x-3} \times (x-3)^2 \geq (x+4)(x-3)^2 \quad x \neq 3$$

$$3 \quad 2x(x-3) \geq (x+4)(x-3)^2 \geq 0$$

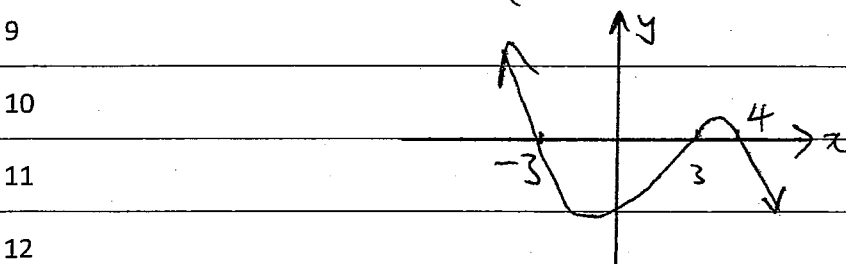
$$4 \quad (x-3)[2x - (x+4)(x-3)] \geq 0$$

$$5 \quad (x-3)(2x - (x^2 + x - 12)) \geq 0$$

$$6 \quad (x-3)(-x^2 + x + 12) \geq 0$$

$$7 \quad -(x-3)(x^2 - x - 12) \geq 0$$

$$8 \quad -(x-3)(x-4)(x+3) \geq 0$$



$$13 \quad \therefore x \leq -3, \quad 3 < x \leq 4$$

$$15 \quad g) \quad u = 2 - x^4$$

$$16 \quad du = -4x^3 dx \quad \text{and} \quad dx = \frac{du}{-4x^3}$$

$$18 \quad \therefore \int 7x^3 (2-x^4)^5 dx$$

$$20 \quad = 7 \int x^3 u^5 \times \frac{du}{-4x^3}$$

$$22 \quad = \frac{-7}{4} \int u^5 du$$

$$24 \quad = \frac{-7}{4} \times \frac{u^6}{6} + C$$

$$25 \quad = \frac{-7(2-x^4)^6}{24} + C$$

$$26 \quad = \frac{-7(2-x^4)^6}{24} + C$$

12.

1 a) $\frac{dy}{dx} = \frac{2x}{3y^2}$

2

3 $\int 3y^2 dy = \int 2x dx$

4

$y^3 = x^2 + c$

5

At $x=0, y=1$

6

$1 = 0 + c$

7

$c = 1$

8

So $y^3 = x^2 + 1$

9

$y = \sqrt[3]{x^2 + 1}$

10

11 b) $\vec{BA} = \frac{1}{2}\hat{i} - \hat{j} + \hat{k}$

12

$= \frac{1}{2}\hat{i} - \hat{j}$

13

14 c) i) $\mu = 6400 \times \frac{1}{2}$
 $= 3200$

15

$\sigma^2 = 6400 \times \frac{1}{2} \times \frac{1}{2}$

$\sigma^2 = 1600$

16

$\sigma = 40$

17

18 ii) For 3260

For 3100

19

$\frac{3260 - 3200}{40} = 1.5$

$\frac{3100 - 3200}{40} = -2.5$

20

$z = \frac{3260 - 3200}{40} = 1.5$

$z = \frac{3100 - 3200}{40} = -2.5$

21

22 iii) $P(3100 \leq X \leq 3260)$

23

$= P(-2.5 \leq Z \leq 1.5)$

24

$= P(Z \leq 1.5) - P(Z \geq 2.5)$

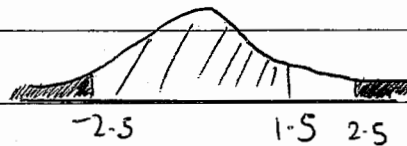
25

$= P(Z \leq 1.5) - (1 - P(Z \leq 2.5))$

26

$= 0.9332 - 1 + 0.9938$

$= 0.927$



1	d) i) LHS = $3\sin\theta - 4\cos\theta - 4$	
2	$= \frac{3 \times 2t}{1+t^2} - \frac{4(1-t^2)}{1+t^2} - \frac{4(1+t^2)}{1+t^2}$	
3	$= \frac{6t - 4 + 4t^2 - 4 - 4t^2}{1+t^2}$	
4	$= \frac{6t - 8}{1+t^2}$	
5	$= \frac{6t - 8}{1+t^2}$	
6	$= \frac{6t - 8}{1+t^2}$	
7	$= \frac{6t - 8}{1+t^2}$	
8		
9	ii) $3\sin\theta - 4\cos\theta = 4$	
10	$3\sin\theta - 4\cos\theta - 4 = 0$	
11	$\frac{6t - 8}{1+t^2} = 0$	
12	Hence $\frac{6t - 8}{1+t^2} = 0$ from i)	
13	$t = \frac{4}{3}$	
14		
15	$\therefore \tan \frac{\theta}{2} = \frac{4}{3}$ $0 \leq \theta \leq 2\pi$	
16	$\frac{\theta}{2} = 0.927\dots$ $0 \leq \frac{\theta}{2} \leq \pi$	
17	$\theta = 1.85$ (2dp)	
18		
19	Check: If $\theta = \pi$ LHS = $3\sin\pi - 4\cos\pi$	
20	$= 4$	
21	$= \text{RHS}$	
22		
23	$\therefore \theta = 1.85$ (2dp), π	
24		
25		
26		

13

1

2

$$a) \frac{dr}{dt} = 5$$

$$V = \pi r^2 \times 30$$

3

$$\frac{dV}{dr} = 60\pi r$$

4

5

$$\text{Now } \frac{dV}{dt} = \frac{dr}{dt} \times \frac{dV}{dr}$$

6

$$= 5 \times 60\pi r = 300\pi r$$

7

$$\text{At } r = 10$$

8

$$\frac{dV}{dt}$$

9

$$= 3000\pi$$

10

11

$$b) V = 2\pi \int_0^{\pi} (3\cos \frac{x}{2})^2 dx$$

$$\cos x = 2\cos^2 \frac{x}{2} - 1$$

$$\cos^2 \frac{x}{2} = \frac{1}{2}(1 + \cos x)$$

12

13

$$= 18\pi \int_0^{\pi} \cos^2 \frac{x}{2} dx$$

14

15

$$= 9\pi \int_0^{\pi} 1 + \cos x dx$$

16

17

$$= 9\pi [x + \sin x]_0^{\pi}$$

18

19

$$= 9\pi (\pi + \sin \pi - (0 + \sin 0))$$

20

21

$$= 9\pi^2 u^3$$

22

23

$$= 88.83 \text{ (2dp) } u^3$$

24

25

26

1	c) Show true for $n=0$	
2	$7^0 + 7^0 + 4 = 6$, which is divisible by 6	
3	\therefore Statement is true for $n=1$.	
4		
5	Assume true for $n=k$, i.e. assume	
6	$7^{2k} + 7^k + 4 = 6M$, where M is an integer	
7	$\therefore 7^k = 6M - 4 - 7^{2k}$	
8		
9	Hence prove true for $n=k+1$, i.e. aim to prove	
10	$7^{2(k+1)} + 7^{k+1} + 4$ is divisible by 6.	
11		
12	$= 7^{2k+2} + 7^{k+1} + 4$	
13	$= 49 \times 7^{2k} + 7 \times 7^k + 4$	
14	$= 49 \times 7^{2k} + 7(6M - 4 - 7^{2k}) + 4$, by assumption.	
15	$= 49 \times 7^{2k} + 42M - 28 - 7 \times 7^{2k} + 4$	
16	$= 42 \times 7^{2k} + 42M - 24$	
17	$= 6(7 \times 7^{2k} + 7M - 4)$	
18	$= 6L$, where L is an integer.	
19		
20	\therefore If the statement is true for $n=k$, it is	
21	also true for $n=k+1$.	
22		
23	As the statement is true for $n=0$, it is also	
24	true for $n=1+1=2, 3, 4, \dots$	
25	Hence, by mathematical induction, it is true for	
26	all integers $n \geq 0$	

1	$d) i) \ddot{x} = 0$		$\ddot{y} = -10$	
2	$\ddot{x} = C_1$		$\dot{y} = 30 \sin \theta$	$\dot{y} = -10t + C_3$
3	At $t=0, \dot{x} = 30 \cos \theta$		At $t=0, \dot{y} = 30 \sin \theta$	
4	$\therefore C_1 = 30 \cos \theta$		$\ddot{x} = 30 \cos \theta$	$\therefore C_3 = 30 \sin \theta$
5	So $\dot{x} = 30 \cos \theta$			So $\dot{y} = -10t + 30 \sin \theta$
6	$x = 30t \cos \theta + C_2$			$y = -5t^2 + 30t \sin \theta + C_4$
7	At $t=0, x=0$		At $t=0, y=2$	
8	$\therefore C_2 = 0$		$\therefore C_4 = 2$	
9	So $x = 30t \cos \theta$		So $y = -5t^2 + 30t \sin \theta + 2$	
10				
11	ii/			
12	So $t = \frac{x}{30 \cos \theta}$ — (1)	and $y = -5t^2 + 30t \sin \theta + 2$ — (2)		
13				
14	Sub (1) into (2)			
15				
16			$y = -5 \left(\frac{x}{30 \cos \theta} \right)^2 + 30 \sin \theta \times \frac{x}{30 \cos \theta} + 2$	
17			$= \frac{-5}{30^2} x^2 \sec^2 \theta + x \tan \theta + 2$	
18			$= \frac{-x^2}{180} (1 + \tan^2 \theta) + x \tan \theta + 2$	
19				
20				
21				
22				
23				
24				
25				
26				

$$1 \quad \text{At } x=6, \quad y=10$$

$$2 \quad 6 = \frac{-10^2}{180} (1 + \tan^2 \theta) + 10 \tan \theta + 2$$

$$3 \quad 6 = \frac{-5}{9} (1 + \tan^2 \theta) + 10 \tan \theta + 2$$

$$4 \quad 54 = -5 - 5 \tan^2 \theta + 90 \tan \theta + 18$$

$$5 \quad 5 \tan^2 \theta - 90 \tan \theta + 41 = 0$$

$$6 \quad \tan^2 \theta = \frac{90 \pm \sqrt{90^2 - 4 \times 5 \times 41}}{2 \times 5}$$

$$7 \quad \tan^2 \theta = \frac{90 \pm \sqrt{7280}}{10}$$

$$8 \quad = \frac{90 \pm \sqrt{7280}}{10}$$

$$9 \quad = 10$$

10

$$11 \quad \therefore \tan \theta = \frac{90 + \sqrt{7280}}{10}, \quad \tan \theta = \frac{90 - \sqrt{7280}}{10}$$

$$12 \quad \therefore \tan \theta = \frac{90 + \sqrt{7280}}{10}, \quad \tan \theta = \frac{90 - \sqrt{7280}}{10}$$

13

14

15

16

17

18

19

20

21

22

23

24

25

26

(Note: θ is acute).

$$\therefore \theta = 87^\circ, 25^\circ$$

$$\begin{aligned}
 1 \quad a) \quad & \text{RHS} = \frac{1}{9} \left(\frac{1}{Q} + \frac{1}{15000-Q} \right) \\
 2 \quad & \\
 3 \quad & = \frac{1}{9} \left(\frac{15000 - Q + Q}{Q(15000-Q)} \right) \\
 4 \quad & \\
 5 \quad & = \frac{15000}{9} \left(\frac{1}{Q(15000-Q)} \right) \\
 6 \quad & \\
 7 \quad & \\
 8 \quad & = \frac{1}{\frac{9}{15000} Q(15000-Q)} \\
 9 \quad & \\
 10 \quad & = 0.0006 Q(15000-Q) \\
 11 \quad & = \text{LHS}
 \end{aligned}$$

$$\begin{aligned}
 12 \quad & \\
 13 \quad \text{ii/ Hence } & \frac{dt}{dQ} = \frac{1}{0.0006 Q(15000-Q)} \\
 14 \quad & \frac{dt}{dQ} = \frac{1}{9} \left(\frac{1}{Q} + \frac{1}{15000-Q} \right) \\
 15 \quad & \\
 16 \quad & t = \frac{1}{9} \left(\int \frac{1}{Q} dQ - \int \frac{-1}{15000-Q} dQ \right) \\
 17 \quad & \\
 18 \quad & t = \frac{1}{9} \ln|Q| - \ln|15000-Q| + C \\
 19 \quad & 9(t-C) = \ln \left| \frac{Q}{15000-Q} \right| \\
 20 \quad & \therefore \left| \frac{Q}{15000-Q} \right| = e^{9t-9C} \\
 21 \quad & \frac{Q}{15000-Q} = A e^{9t}, \quad \text{where } A = \pm e^{-9C} \\
 22 \quad & Q = 15000 A e^{9t} - Q A e^{9t} \\
 23 \quad & Q(1 + A e^{9t}) = 15000 A e^{9t} \\
 24 \quad & Q = \frac{15000 A e^{9t}}{1 + A e^{9t}} \\
 25 \quad & \frac{15000}{\frac{1}{A e^{9t}} + 1} \\
 26 \quad & Q = \frac{15000}{\frac{1}{A e^{9t}} + 1}
 \end{aligned}$$

$$Q = \frac{15000}{1 + B e^{-9t}}, \quad \text{where } B = \frac{1}{A}$$

1 iii/ At $t=0$, $Q=25$

2
$$15000$$

3
$$25 = \frac{15000}{1 + Be^{-9 \times 0}}$$

4
$$25 + 25B = 15000$$

5
$$25B = 14975$$

6
$$B = 599$$

7

8 \therefore At $t = \frac{2}{12}$ (t is in years)

9
$$15000$$

10
$$Q = \frac{15000}{1 + 599e^{-9 \times \frac{2}{12}}}$$

11
$$= 111.395$$

12 \therefore Estimate of 111 quokkas after 2 months.

13

14

15 i/ carrying capacity as $t \rightarrow \infty$

16
$$15000$$

17
$$\therefore Q \rightarrow \frac{15000}{1+0} = 15000$$

18

19 \therefore The maximum number supported

20 is 15000.

21

22

23

24

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26

1

2 v/ Hence, set $Q = 7500$

3

4
$$7500 = \frac{15000}{1 + 599e^{-9t}}$$

5

6
$$1 + 599e^{-9t} = 2$$

7

8
$$e^{-9t} = \frac{1}{599}$$

9

10
$$-9t = \ln\left(\frac{1}{599}\right)$$

11

12
$$t = -\frac{1}{9} \ln\left(\frac{1}{599}\right)$$

13

$$= 0.71 \dots \text{ years}$$

14

$$= 8.527 \dots \text{ months.}$$

15

16 \therefore After 9 months the amount of
17 quokkas exceeds half of the maximum.
18

19

20

21

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26

$$1) \vec{AQ} = \begin{bmatrix} 4 - (-1) \\ 16 - 4 \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$$

and Adam gets to Q from A at $t=3$,

$$\text{Hence } \vec{a} = \begin{bmatrix} -1 \\ 4 \end{bmatrix} \text{ and } \vec{u} = \frac{1}{3} \times \begin{bmatrix} 5 \\ 12 \end{bmatrix} \\ = \begin{bmatrix} \frac{5}{3} \\ 4 \end{bmatrix}$$

ii) α) A vector perpendicular is

$$\vec{w} = \begin{bmatrix} -4 \\ \frac{5}{3} \end{bmatrix} \text{ or any scalar multiple.}$$

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B) Equation of the throw is: $\begin{bmatrix} 11 \\ 9 \end{bmatrix} + \lambda \begin{bmatrix} -4 \\ \frac{5}{3} \end{bmatrix}$,
 as it goes from $(11, 9)$ in the direction \underline{w} .

Hence $\begin{bmatrix} -1 \\ 4 \end{bmatrix} + t \begin{bmatrix} \frac{5}{3} \\ 4 \end{bmatrix} = \begin{bmatrix} 11 \\ 9 \end{bmatrix} + \lambda \begin{bmatrix} -4 \\ \frac{5}{3} \end{bmatrix}$
 $\begin{bmatrix} -1 + \frac{5t}{3} \\ 4 + 4t \end{bmatrix} = \begin{bmatrix} 11 - 4\lambda \\ 9 + \frac{5\lambda}{3} \end{bmatrix}$

$\therefore -1 + \frac{5t}{3} = 11 - 4\lambda$ and $4 + 4t = 9 + \frac{5\lambda}{3}$
 $-3 + 5t = 33 - 12\lambda$
 $12\lambda = 36 - 5t$ $12 + 12t = 27 + 5\lambda$ — (2)
 $\lambda = \frac{36 - 5t}{12}$ — (1)

Sub (1) into (2)

$12 + 12t = 27 + 5 \times \frac{36 - 5t}{12}$
 $75 + 12t = \frac{5(36 - 5t)}{12}$
 $-180 + 144t = 180 - 25t$
 $169t = 360$
 $t = \frac{360}{169}$ (or $t = 2.13$ (2dp))