WESTFIELDS SPORTS
First in Australia

YEAR 12

TRIAL HSC

MATHEMATICS EXTENSION 1

TIME ALLOWED – 2 HOURS
(PLUS 5 MINUTES READING TIME)

GENERAL INSTRUCTIONS

- Reading Time – 5 minutes.
- Working time – 2 hours.
- Check that you have the correct paper.
- Approved calculators may be used.
- ***Do not write on this booklet.***
- All necessary working should be shown.
- Marks may be deducted for careless or badly arranged work.
- Write using black or blue pen only.
- Formula sheet is provided at the back.

Total marks: 84
Attempt Questions 1-7
All questions are equal value.
Start a new page for each question

Question 1 (12 marks)

(a) Evaluate $\lim_{x \to 0} \frac{\tan 2x}{3x}$ (2)

(b) Find the acute angle (to the nearest degree) between the lines $y = 4x$ and $3x + 2y - 4 = 0$. (2)

(c) Solve

$$\frac{x+1}{x-3} \geq 2$$ (3)

(d) Evaluate $\int_0^{\frac{\pi}{3}} \sin^2 3x dx$ (3)

(e) The interval PQ has endpoints $P(2,3)$ and $Q(-3,5)$. Find the coordinates of the point T, which divides the interval PQ externally in the ratio 3:1. (2)

Question 2 (12 marks) **Start a new page**

(a) Differentiate (i) $\sin^{-1}(3-2x)$ (2)

(ii) $\cot^2(5x)$ (2)

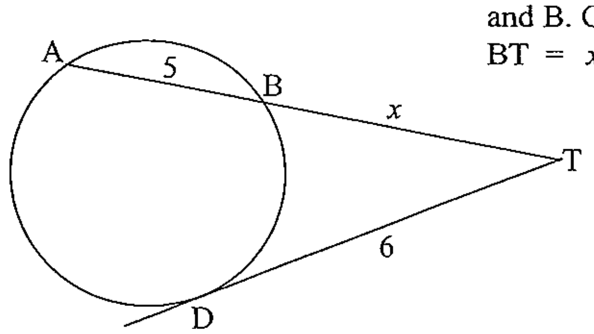
(b) Find the gradient of the curve $xy+y=3x^2$ at the point $(2,4)$. (2)

(c) Six identical yellow discs and four identical blue discs are placed in a straight line.

(i) How many arrangements are possible? (1)

(ii) Find the probability that all blue discs are together. (1)

(d)



The line DT is a tangent to the circle at D and AT is a secant meeting the circle at A and B. Given that DT = 6, AB = 5 and BT = x, find the value of x.

(2)

(e) Find the general solution of $\tan 3\theta = 1$ (2)

Question 3 (12 marks) **Start a new page**

(a) Consider the function $f(x)=\frac{1}{2}\cos^{-1}(1-3x)$

(i) State the domain and range (2)

(ii) Hence or otherwise sketch the graph of $y=f(x)$ (1)

(b) A ball is thrown with initial velocity $20\,m/s$ at an angle of elevation of $\tan^{-1}\frac{4}{3}$. Take $g=10\,m/s^2$

(i) Show that the parabolic path of the ball has parametric equations

$x=12t$ and $y=16t-5t^2$. (2)

(ii) Hence find the horizontal range of the ball, and its greatest height. (2)

(c)

(i) Express $\sqrt{3}\cos x-\sin x$ in the form $R\cos(x+\alpha)$ where $R>0$ and $0<\alpha<\frac{\pi}{2}$ (2)

(ii) Hence, sketch the graph of the equation for $y=\sqrt{3}\cos x-\sin x$ for $-\frac{\pi}{6}<x<2\pi$ (1)

(d) Evaluate exactly $\cos\left[-\tan^{-1}\frac{8}{15}\right]$ (2)

Question 4 (12 marks) **Start a new page**

(a) Prove that $5^n + 11$ is divisible by 4 for all integers $n \geq 0$ by mathematical induction. (3)

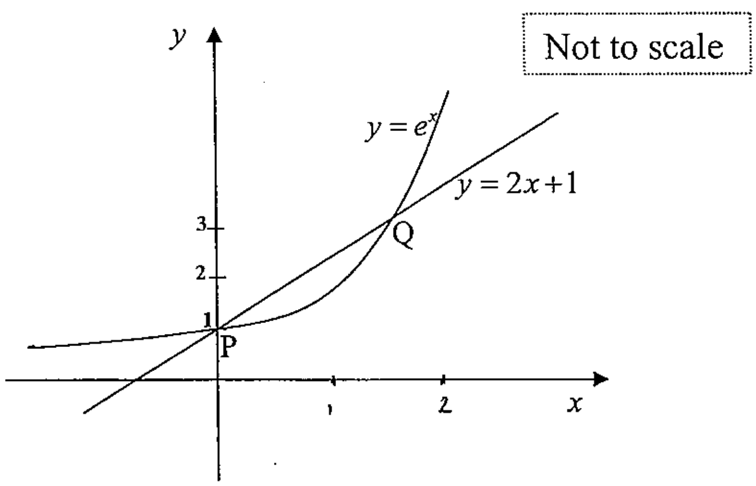
(b) α, β and γ are the roots of the equation $x^3 + 2x^2 - 3x + 5 = 0$.

(i) State the values of $\alpha\beta + \alpha\gamma + \beta\gamma$ and $\alpha\beta\gamma$. (1)

(ii) Find the values of $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$ (2)

(c) $Q(x) = x^3 + ax^2 + 2x + b$. Given that $Q(x)$ has a factor of $(x+3)$ and when $Q(x)$ is divided by $(x-1)$ the remainder is 4. Find the values of a and b. (3)

(d) The diagram below shows the curve $y = e^x$ and the line $y = 2x + 1$ intersecting at the point $P(0,1)$ and point Q.
Use Newton's method once to find a better approximation for the x - ordinate of the point Q. Leave your answer correct to 1 decimal place. (3)



Question 5 (12 marks) **Start a new page**

(a) A fair six faced die with faces numbered 1,2,3,4,5,6 is tossed seven times. What is the probability that a "6" occurs on exactly two of the seven tosses? (2)

(b) Find the constant term in the expansion $\left(2x^3 - \frac{1}{x}\right)^{12}$. (2)

(c) Consider the function $f(x) = \frac{e^x}{e^x + 2}$

(i) Show that $f(x)$ has no stationary point. (2)

(ii) Given that $f''(x) = \frac{2e^x\left(2 - e^x\right)}{\left(e^x + 2\right)^3}$ find the coordinates of the point of inflexion. (2)

(iii) Explain why $0 < f(x) < 1$ for all x. (1)

(iv) What happens to $f(x)$ as $x \to \infty$ and $x \to -\infty$. (1)

(v) Sketch the curve $y = f(x)$. (2)

Question 6 (12 marks) **Start a new page**

(a) A function is defined by $x = \sin y$ for $\frac{\pi}{2} \le y \le \pi$. Find $\frac{dy}{dx}$ in terms of x. (2)

(b) A class of 20 students consists of 12 girls and 8 boys. For a discussion section, 4 students are chosen at random to form a committee.

(i) How many committees can be formed? (1)

(ii) If the committee is to include 4 females members, how many committees can be formed? (1)

(iii) How many of these committees have at least 1 male member? (1)

(c) A curve has gradient function $\frac{e^{2x}}{1+e^{4x}}$ and passes through the point $\left(0, \frac{\pi}{8}\right)$.

Use the substitution $u = e^{2x}$ to find its equation. (4)

(d) Find the equation of the tangent to the parabola represented by the equation $x = 4t$, $y = 2t^2$ at the point $t = 1$. (3)

Question 7 (12 marks) **Start a new page**

(a) A spherical bath capsule dissolves in the bath so that its decrease in volume is proportional to its surface area. If its shape remains spherical as it dissolves, show that the radius of the capsule will decrease at a constant rate. (2)

(b) The temperature T degrees inside a heated room at time t hours obeys Newtons Law of Cooling, which states that the rate of change of temperature is proportional to $(T-A)$, where A is the air temperature outside the room.

(i) Show the $T = A + Ce^{kt}$ (where C and k are constants) satisfies Newton's Law of Cooling. (1)

(ii) The outside air temperature A is 5^0 and a heating system breakdown causes the inside temperature of a room to fall from 20^0 to 17^0 in half an hour. After how many hours has the temperature inside the room dropped to 10^0? (3)

(c) A particle P is projected from a point on horizontal ground with velocity V at an angle of projection α.

You may assume that the equations of the motion are

$$\ddot{y} = -g \qquad \ddot{x} = 0$$

$$\dot{y} = V\sin\alpha - gt \qquad \dot{x} = V\cos\alpha$$

$$y = Vt\sin\alpha - \frac{1}{2}gt^2 \qquad x = Vt\cos\alpha$$

(i) Show that the particle's maximum height is $\dfrac{V^2\sin^2\alpha}{2g}$. (2)

(ii) A second particle Q is projected from the same point on horizontal ground with velocity $\sqrt{\dfrac{5}{2}}\,V$ at an angle $\dfrac{\alpha}{2}$ to the horizontal. Both particles reach the same maximum height. Show that $\alpha = \cos^{-1}\left(\dfrac{1}{4}\right)$. (4)

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$