Student Number: ....

Barker College

2012
YEAR 12
TRIAL
EXAMINATION

Mathematics Extension 2

ANSWER SHEET

Section I – Multiple Choice
Choose the best response and fill in the response oval completely.

Start Here →

1.	A ○	B ○	C ○	D ○
2.	A ○	B ○	C ○	D ○
3.	A ○	B ○	C ○	D ○
4.	A ○	B ○	C ○	D ○
5.	A ○	B ○	C ○	D ○
6.	A ○	B ○	C ○	D ○
7.	A ○	B ○	C ○	D ○
8.	A ○	B ○	C ○	D ○
9.	A ○	B ○	C ○	D ○
10.	A ○	B ○	C ○	D ○

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2012
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TRIAL
EXAMINATION

Mathematics
Extension 2

Staff Involved:

- **VAB**
- **BHC**
- **WMD***
- **MRB***

Number of copies: 35

PM TUESDAY 31ST JULY

General Instructions

- Reading time - 5 minutes
- Working time - 3 hours
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- Show all necessary working in Questions 11 - 16

Total marks – 100

Section I **Page 2 - 3**

10 marks

- Attempt Questions 1 - 10
- Allow about 20 minutes for this section

Section II **Pages 4 - 13**

90 marks

- Attempt Questions 11 - 16
- Allow about 2 hours 40 minutes for this section

Section I — Multiple Choice

10 marks

Attempt Questions 1 to 10.

Use the multiple-choice answer sheet for Questions 1-10.

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1. The roots of $z^2 + bz + c = 0$ are $3+\sqrt{2}i$ and $3-\sqrt{2}i$. The value of c is:

(A) -6 (B) 11 (C) 6 (D) 7

2. The points A, B, C and D represent the complex numbers a, b, c and d on an Argand diagram. If $a+c=b+d$ then, being as specific as possible, the quadrilateral $ABCD$ is definitely a:

(A) trapezium (B) rectangle (C) rhombus (D) parallelogram

3. If z is such that $|z+1+i| \le 1$ then the minimum value of $|z|$ is:

(A) 1 (B) $\sqrt{2}$ (C) $\sqrt{2}-1$ (D) $\sqrt{2}+1$

4. If z is a complex number with locus $|z-2|+|z+2|=6$ then the graph of the locus of z is an ellipse with:

(A) major axis 3 units, minor axis $\sqrt{5}$ units

(B) major axis 4 units, minor axis 3 units

(C) major axis 6 units, minor axis 4 units

(D) major axis 6 units, minor axis $2\sqrt{5}$ units

5. The eccentricity of a hyperbola with parametric equations $x = 3\sec\theta$ and $y = 4\tan\theta$ is:

(A) $\frac{5}{3}$ (B) $\frac{3}{5}$ (C) $\frac{5}{4}$ (D) $\frac{4}{3}$

6. The gradient of the tangent to the curve $x^3 - xy^2 + 8 = 0$ at the point $(1,3)$ is:

(A) 1 (B) -1 (C) $\frac{1}{2}$ (D) $\frac{1}{3}$

7. The directrices of the rectangular hyperbola $xy = 4$ are:

(A) $x + y = \pm 2\sqrt{2}$ (B) $x - y = \pm 2\sqrt{2}$ (C) $x + y = \pm\sqrt{2}$ (D) $x - y = \pm\sqrt{2}$

8. $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{x^3}{\cos x} dx$

(A) $= 2\int_{0}^{\frac{\pi}{4}} \frac{x^3}{\cos x} dx$ (B) $= \frac{\pi}{2}$ (C) $= 0$ (D) is undefined

9. For the function $f(x) = \cos^{-1}(e^x)$:

(A) $D_f : -1 \le x \le 1$ and $R_f : 0 \le y \le \pi$

(B) $D_f : x \le 0$ and $R_f : 0 \le y \le \frac{\pi}{2}$

(C) $D_f : x \le 0$ and $R_f : 0 \le y < \frac{\pi}{2}$

(D) $D_f : x \le 0$ and $R_f : 0 \le y < \pi$

10. Mike asked Virginia, Boyd and Graham how they would produce the graph of $y = f(2 - x)$ if they were given the graph of $y = f(x)$.

- Boyd said, "shift $y = f(x)$ two units to the left then reflect that in the $y-$axis."
- Virginia said, "shift $y = f(x)$ two units to the right then reflect that in the line $x = 2$."
- Graham said, "just reflect $y = f(x)$ in the line $x = 1$."

Who was correct?

(A) Nobody

(B) Only Boyd

(C) Boyd and Virginia

(D) Boyd, Virginia and Graham

End of Section I

Section II

Attempt Questions 11 to 16.

All questions are of equal value.

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Question 11 (15 marks) **[Use a SEPARATE writing booklet]** **Marks**

(a) For $f(x) = x(x+2)(2x-3)$ graph each of the following, showing important features.

It is not necessary to find stationary points or points of inflexion.

However, show axis intercepts and asymptotes where appropriate.

(i) $y = f(x)$ **1**

(ii) $y^2 = f(x)$ **2**

(iii) $y = [f(x)]^2$ **2**

(iv) $y = e^{f(x)}$ **2**

(v) $y = \dfrac{1}{f(x)}$ **2**

(vi) $|y| = f(|x|)$ **2**

(b) Given that $\dfrac{x^3+3x^2+6x-10}{x^2+2x-3} = \dfrac{(x-1)(x^2+4x+10)}{(x-1)(x+3)}$, **4**

sketch $y = \dfrac{x^3+3x^2+6x-10}{x^2+2x-3}$, showing asymptotes, the y-intercept and points of discontinuity. It is not necessary to find stationary points.

End of Question 11

Question 12 (15 marks) **[Use a SEPARATE writing booklet]** **Marks**

(a) Find $\int \frac{1-x}{1+\sqrt{x}}dx$. **2**

(b) (i) Show that $\int_{-a}^{a} f(x)dx = \int_0^a f(x)dx + \int_0^a f(-x)dx$. **2**

(ii) Hence, or otherwise, evaluate $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{dx}{1-\sin x}$. **3**

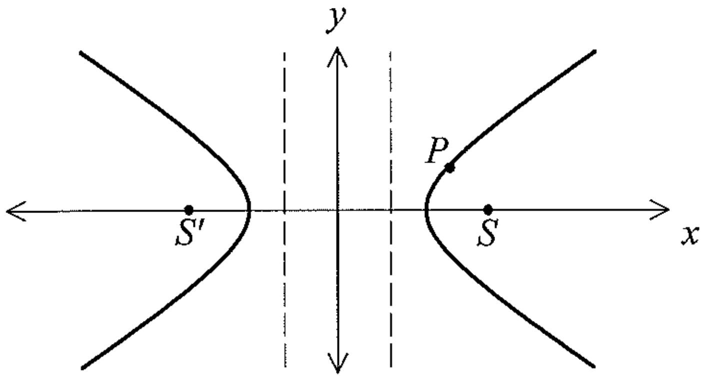
(c) By splitting $\frac{2x+2}{(x-1)(x^2+1)}$ into partial fractions, or otherwise, **4**

find $\int \frac{2x+2}{(x-1)(x^2+1)}\, dx$.

Question 12 continues on the next page...

Question 12 (continued) **Marks**

(d) **4**



$P(a\sec\theta,\ b\tan\theta)$ is a point on the hyperbola $\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1$ shown on the diagram above. The directrices and foci of the hyperbola have been indicated on the diagram.

A tangent, $\dfrac{x\sec\theta}{a}-\dfrac{y\tan\theta}{b}=1$, is drawn to the hyperbola at P.

This tangent to the hyperbola at P meets the nearest directrix at Q.

Prove that PQ subtends a right angle at the nearest focus S.

End of Question 12

Question 13 (15 marks) **[Use a SEPARATE writing booklet]** **Marks**

(a) A polynomial $P(x)$ gives remainders of 3 and 5 when divided by $x-1$ and $x+1$ respectively. What is the remainder when $P(x)$ is divided by x^2-1? **2**

(b) (i) Show that the equation $3x^5+20x^3+45x=c$, where 'c' is a real constant, can have only one real root. **2**

(ii) If the sum of the other (complex roots) is -2, find the value of the constant 'c'. **2**

(c) (i) The equation $x^3+px^2+qx+r=0$ (where p, q and r are non-zero real numbers) has roots α, β and γ such that $\frac{1}{\alpha}, \frac{1}{\beta}$ and $\frac{1}{\gamma}$ are consecutive terms of an arithmetic sequence. Show that $\beta=-\frac{3r}{q}$. **3**

(ii) The equation $x^3-26x^2+216x-576=0$ has roots α, β and γ such that $\frac{1}{\alpha}, \frac{1}{\beta}$ and $\frac{1}{\gamma}$ are consecutive terms of an arithmetic sequence. **3**

Find the values of α, β and γ.

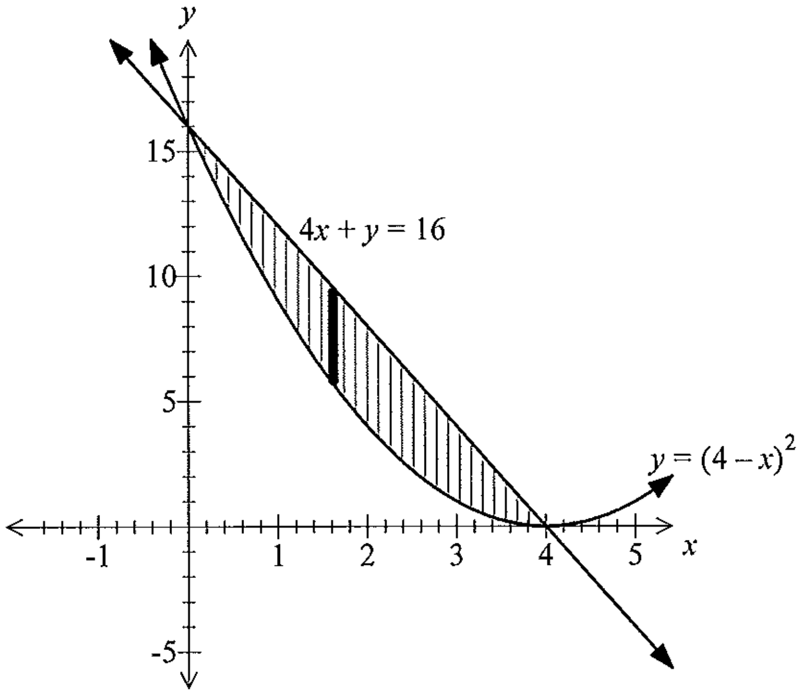
(d) The area between the curve $y=\sin x$ and the line $y=1$ is rotated about $y=1$. **3**

Use a slicing technique to find the volume of the solid of revolution formed by the portion from $x=0$ to $x=\frac{\pi}{2}$.

End of Question 13

Question 14 (15 marks) **[Use a SEPARATE writing booklet]** **Marks**

(a)



The region enclosed by the curve $y=(4-x)^2$ and the line $4x+y=16$ is shaded in the diagram above. A solid is formed with this region as its base.

When the solid is sliced perpendicular to the x-axis, each cross-section is an equilateral triangle with its base in the xy-plane.

(i) Show that the area of the cross-section x units to the right of the y-axis is **2**

$$\frac{\sqrt{3}}{4}x^2(4-x)^2, \text{ where } 0 \le x \le 4.$$

(ii) Hence find the volume of the solid. **2**

(b) Find all the complex numbers $z = a+ib$, where a, b are real, such that **4**

$|z|^2 + i\overline{z} = 11 + 3i$.

Question 14 continues on the next page...

Question 14 (continued) **Marks**

(c) The complex number z is represented by the point P. If $\frac{z-1}{z-2i}$ is purely imaginary, **4**

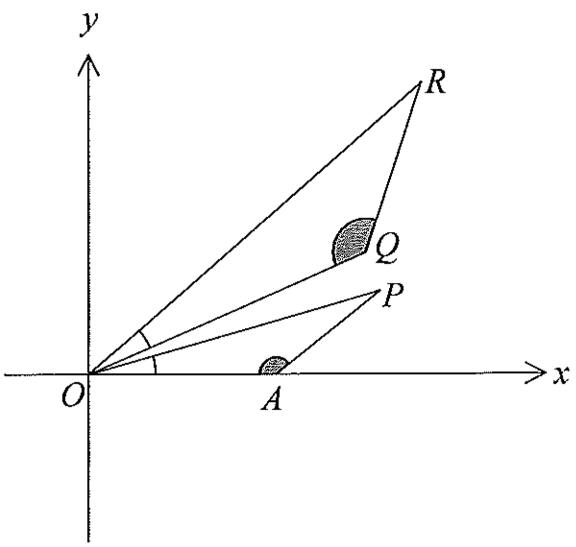
find the locus of P, showing that it is a circle excluding two points.

State the centre of the circle and give the coordinates of the two excluded points.

(d) In the figure below, the points P, Q and A represent the complex numbers **3**

z_1, z_2 and 1 respectively. By construction, $\angle OAP = \angle OQR$ and $\angle AOP = \angle QOR$.

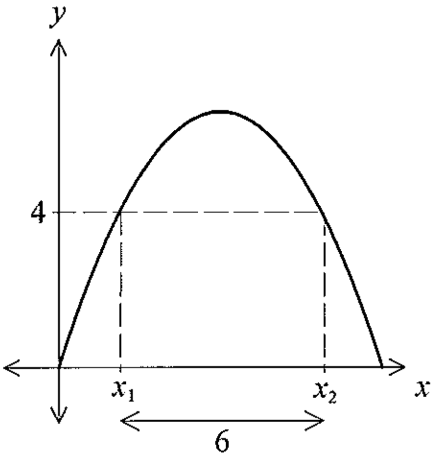
Explain why the point R represents the complex number $z_1 z_2$.



End of Question 14

Question 15 (15 marks) **[Use a SEPARATE writing booklet]** **Marks**

(a)



A particle is projected from the origin with velocity V m/s at an angle of α to the horizontal.

(i) Assuming that the coordinates of the particle at time t are $(Vt\cos\alpha,\ Vt\sin\alpha - \frac{gt^2}{2})$, prove that the horizontal range R of the particle is $\dfrac{V^2\sin 2\alpha}{g}$. **2**

(ii) Hence prove that the path of the particle has equation $y = x\left(1 - \dfrac{x}{R}\right)\tan\alpha$. **2**

(iii) Suppose that $\alpha = 45°$ and that the particle passes through two points 6 metres apart and 4 metres above the point of projection, as shown in the diagram. Let x_1 and x_2 be the x-coordinates of the two points.

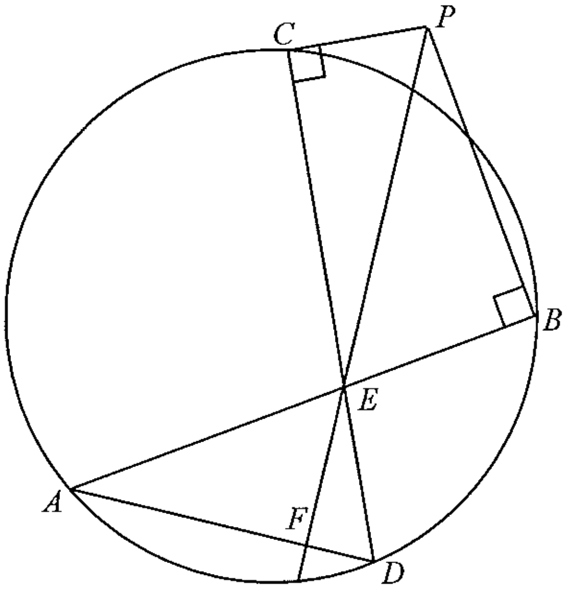
(α) Show that x_1 and x_2 are the roots of the equation $x^2 - Rx + 4R = 0$. **2**

(β) Use the identity $(x_2 - x_1)^2 = (x_2 + x_1)^2 - 4x_2x_1$ to find R. **2**

Question 15 continues on the next page...

Question 15 (continued) **Marks**

(b)



AB and CD are two chords of the circle, intersecting at E.
PC and PB are perpendicular to CD and AB respectively.

(i) Trace or copy the diagram onto your own page and explain why $CEBP$ is a cyclic quadrilateral. **1**

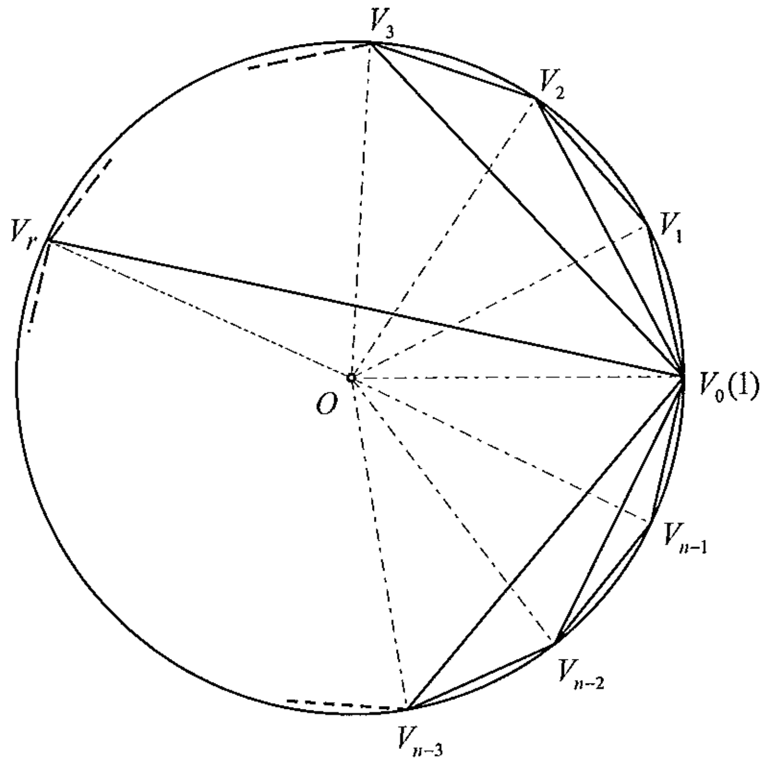
(ii) Prove that the line PE is perpendicular to AD at F. **3**

(c) Find all six x values such that $\sin x = \sin 7x$ and $0 < x < \pi$. **3**

End of Question 15

Question 16 (15 marks) **[Use a SEPARATE writing booklet]** **Marks**

(a)



A regular n-sided polygon, with vertices V_0, V_1, V_2,......, V_{n-1} is inscribed in the unit circle on the complex plane, **part** of which is shown in the diagram above.

V_0 lies on the positive real axis; thus the number one (1) is represented by V_0.

The other vertices of the polygon are in anti-clockwise order going around the circle.

Let $\omega = \cos\left(\frac{2\pi}{n}\right) + i\sin\left(\frac{2\pi}{n}\right)$ and let P be the product of the lengths of the line intervals from V_0 to all the other points V_1, V_2,......, V_{n-1}.

(i) (α) Show that 1 and ω are roots of the equation $z^n - 1 = 0$. **1**

(β) Find the other roots of $z^n - 1 = 0$ in terms of powers of ω. **1**

(ii) By considering the sum of the geometric series $1 + z + z^2 + z^3 + \ldots\ldots + z^{n-1}$, or otherwise, show that: **2**

$$(z-\omega)(z-\omega^2)(z-\omega^3)\ldots\ldots(z-\omega^{n-1}) = 1 + z + z^2 + z^3 + \ldots\ldots + z^{n-1}.$$

Question 16 continues on the next page...

Question 16 (continued) **Marks**

(iii) (α) Explain why the length of the line interval joining V_0 to V_r (r an integer and $1 \le r \le n-1$) is given by $\left|1-\omega^r\right|$. **1**

(β) Hence show that $P = n$. **2**

(iv) By finding a relationship between the length of V_0V_r and the size of $\angle V_0OV_r$ in triangle V_0OV_r (r an integer and $1 \le r \le n-1$), or otherwise, show that: **3**

$$\sin\left(\frac{\pi}{n}\right)\sin\left(\frac{2\pi}{n}\right)\ldots\sin\left(\frac{(n-1)\pi}{n}\right) = \frac{n}{2^{n-1}}.$$

(v) Use the expression in (iv) to show that $\sin\left(\frac{\pi}{7}\right)\sin\left(\frac{2\pi}{7}\right)\sin\left(\frac{3\pi}{7}\right) = \frac{\sqrt{7}}{8}$. **1**

(b) Find $\int \sqrt{x^2+6}\,dx$. **4**

Use a suitable integration technique, and if necessary,

the standard integral $\displaystyle\int \frac{1}{\sqrt{x^2+a^2}}dx = \ln\left(x+\sqrt{x^2+a^2}\right)$.

End of Paper

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STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1 \; ; \; x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$