Blacktown Boys' High School

2019

HSC Trial Examination

Mathematics Extension 2

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided for this paper
- All diagrams are not drawn to scale
- In Questions 11 – 16, show relevant mathematical reasoning and/or calculations

Total marks: 100

Section I – 10 marks (pages 3 – 7)
- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 8 – 15)
- Attempt Questions 11 – 16
- Allow about 2 hours and 45 minutes for this section

Assessor: X. Chirgwin

Student Name: ______________________________________

Teacher Name: ______________________________________

Students are advised that this is a trial examination only and cannot in any way guarantee the content or format of the 2019 Higher School Certificate Examination.

Office Use Only

Question	**Mark**
Q1	**/1**
Q2	**/1**
Q3	**/1**
Q4	**/1**
Q5	**/1**
Q6	**/1**
Q7	**/1**
Q8	**/1**
Q9	**/1**
Q10	**/1**
Q11 a)	**/1**
Q11 b)	**/2**
Q11 c)	**/4**
Q11 d)	**/3**
Q11 e)	**/5**
Q12 a)	**/4**
Q12 b)	**/3**
Q12 c)	**/8**
Q13 a)	**/5**
Q13 b)	**/2**
Q13 c)	**/8**
Q14 a)	**/3**
Q14 b)	**/3**
Q14 c)	**/4**
Q14 d)	**/5**
Q15 a)	**/5**
Q15 b)	**/6**
Q15 c)	**/4**
Q16 a)	**/5**
Q16 b)	**/5**
Q16 c)	**/5**
Total	**/100**

Section I

10 marks
Attempt Questions 1–10

Use the multiple choice answer sheet provided on page 17 for Questions 1–10.

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1 Let $z = 7 + 10i$. What is the value of $\overline{iz}$?

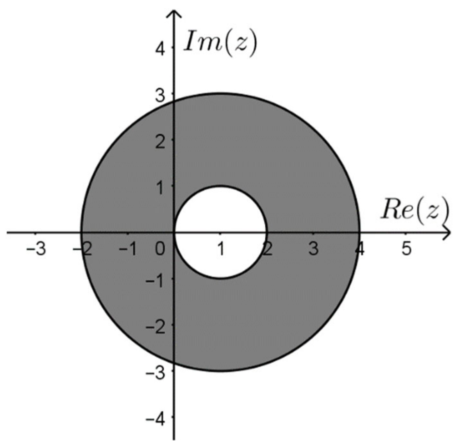
A. $-10 + 7i$

B. $-10 - 7i$

C. $10 + 7i$

D. $10 - 7i$

2 Consider the Argand diagram below.



Which inequality could define the shaded area?

A. $1 \leq |z + 1| \leq 3$

B. $2 \leq |z + 1| \leq 4$

C. $1 \leq |z - 1| \leq 3$

D. $2 \leq |z - 1| \leq 4$

3 What are the equations of the asymptotes of the hyperbola $9y^2 - 25x^2 = 225$?

A. $y = \pm\frac{25}{9}x$

B. $y = \pm\frac{9}{25}x$

C. $y = \pm\frac{5}{3}x$

D. $y = \pm\frac{3}{5}x$

4 The sum of the eccentricities of two different conics is $\frac{3}{2}$.

Which pair of the conics could this be?

A. Circle and ellipse

B. Circle and parabola

C. Parabola and hyperbola

D. Hyperbola and circle

5 Which of the following parametric equations represent $3(x-5)^2 + 4y^2 = 12$

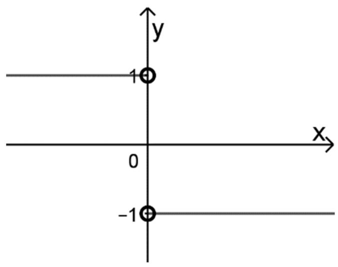
A. $x = 2\cos\theta + 5, y = \sqrt{3}\sin\theta$

B. $x = 2\cos\theta - 5, y = \sqrt{3}\sin\theta$

C. $x = \sqrt{3}\cos\theta + 5, y = 2\sin\theta$

D. $x = 3\cos\theta + 5, y = 4\sin\theta$

6 Given that $f(x) = |x|$ and $g(x) = x$.



Which of the following statement is true for the graph shown above?

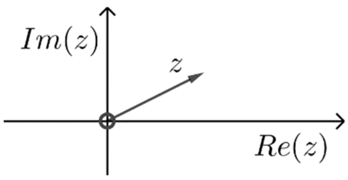
A. $y = \dfrac{f(x)}{g(x)}$

B. $y = -\dfrac{f(x)}{g(x)}$

C. $y = f(x) \times g(x)$

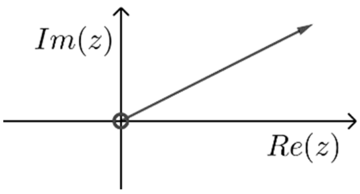
D. $y = -f(x) \times g(x)$

7 The Argand diagram below shows the complex number z.

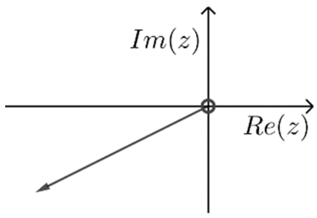


Which Argand diagram best represents $\dfrac{2z}{i}$?

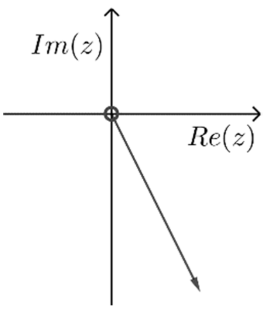
A.



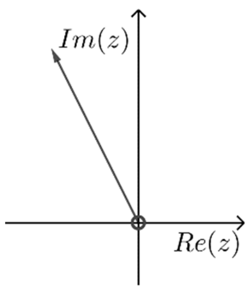
B.



C.



D.



8 There are 9 identical boxes on a table. One person comes to the table and takes three boxes. 2 more people do the same as the first person.
How many ways can this happen?

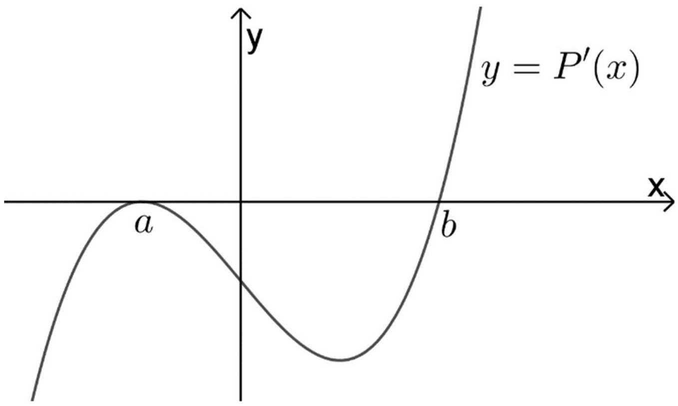
A. 362880

B. 60480

C. 3360

D. 1680

9 The following diagram shows the graph $y = P'(x)$, the derivative of a polynomial $P(x)$. $P'(x)$ has roots at $x = a$ and $x = b$ where $a < 0$ and $b > 0$.



If c is a root of $P(x)$, where $c > b$. Which of the following could be $P(x)$?

A. $P(x) = (x-a)^3(x-c)$

B. $P(x) = (x-a)^3(x+c)$

C. $P(x) = (x+a)^3(x-c)$

D. $P(x) = (x+a)^3(x+c)$

10 Given that a is a constant, which of the following is equivalent to $\int_0^a x(a-x)^{99}dx$?

A. $\dfrac{a}{100}$

B. $\dfrac{a}{101}$

C. $\dfrac{a^{100}}{10000}$

D. $\dfrac{a^{101}}{10100}$

End of Section I

Section II

90 Marks
Attempt Questions 11–16
Answer each question on a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11–16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet.

a) Simplify i^{2019}. **1**

b) Sketch $\frac{x^2}{9} + \frac{y^2}{25} = 1$, indicating the coordinates of its foci and equations of the directrices. **2**

c) i) Express $\sqrt{2} - \sqrt{6}i$ in modulus-argument form. **2**

ii) Hence evaluate $\left(\sqrt{2} - \sqrt{6}i\right)^8$ in the form $x + iy$. **2**

d) Evaluate $\int_2^4 \sqrt{16 - x^2}\,dx$ **3**

e) i) Find the domain and range of $f(x) = x\sin^{-1} x$. **2**

ii) Determine whether $f(x)$ is even, odd, or neither. **1**

iii) Sketch $y = f(x)$, showing clearly the end values. **2**

End of Questions 11

Question 12 (15 marks) Use a SEPARATE writing booklet.

a) Given that $P(x) = x^5 + 6x^4 + 16x^3 + 32x^2 + 48x + 32$.

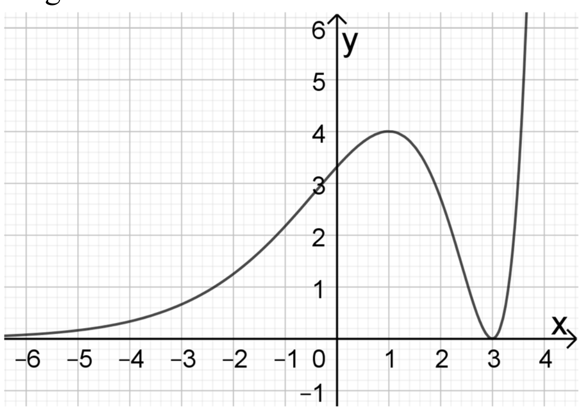
i) Show that $x = -2$ is a root of $P(x)$ of multiplicity three. **2**

ii) Hence, or otherwise, factorise $P(x)$ completely into linear factors. **2**

b) Use the substitution $t = \tan\frac{x}{2}$, or otherwise, show that **3**

$$\int_0^{\frac{\pi}{2}} \frac{dx}{2 + \sin x} = \frac{\sqrt{3}\pi}{9}$$

c) The function $f(x) = (x-3)^2 e^{x-1}$ has stationary points at $x = 1$ and $x = 3$ as shown in the diagram below.



Draw separate half-page sketches of the following graphs. In each case, label any asymptotes and the coordinates of any turning points.

i) $y = f(|x|)$ **2**

ii) $y = \dfrac{1}{f(x)}$ **2**

iii) $y = [f(x)]^2$ **2**

iv) $y^2 = f(x)$ **2**

End of Questions 12

Question 13 (15 marks) Use a SEPARATE writing booklet.

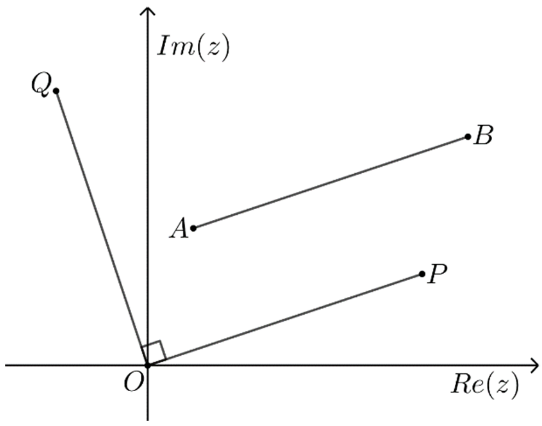
a) The equation $x^3 - 5x + 25 = 0$ has roots α, β and γ. Find:

i) the polynomial with roots $-\alpha, -\beta, -\gamma$. **1**

ii) the polynomial with roots $\alpha^2, \beta^2, \gamma^2$. **2**

iii) the value of $\alpha^3 + \beta^3 + \gamma^3$. **2**

b) In the Argand diagram below, intervals AB, OP and OQ are equal in length, OP is parallel to AB and $\angle POQ = \dfrac{\pi}{2}$. If A and B represent the complex number $1 + 3i$ and $7 + 5i$ respectively, find the complex number which is represented by Q. **2**

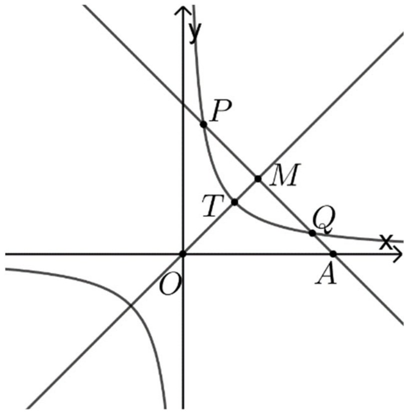


Question 13 continues on page 11

Question 13 (continued)

c) In the diagram $P(cp, cp^{-1})$ and $Q(cq, cq^{-1})$ where $0 < p < q$, are points on the hyperbola $xy = c^2$.

M is the midpoint of PQ and the line PQ cuts the x-axis at A. OM cuts the hyperbola at T.



i) Show that gradient of OM is $-1 \times$ gradient of MA. **2**

ii) Hence, or otherwise, show that $OM = MA$. **1**

iii) Show that the tangent to the hyperbola at T is parallel to the chord PQ. **2**

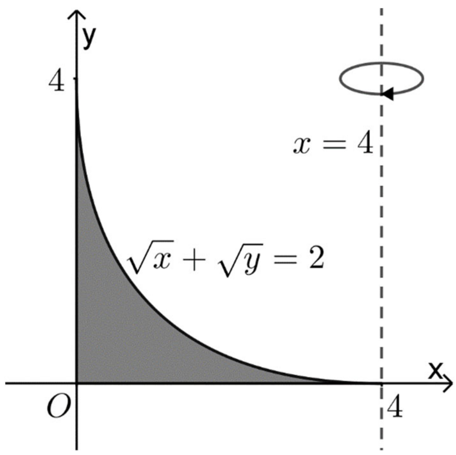
iv) Find the coordinates of R and S where the tangent to the hyperbola at T cuts the x and y axes respectively. **2**

v) Prove that the area of the triangle ORS is a constant. **1**

End of Questions 13

Question 14 (15 marks) Use a SEPARATE writing booklet.

a) The region bounded by the curve $\sqrt{x} + \sqrt{y} = 2$ and the x-axis between $x = 0$ and $x = 4$ is rotated about the line $x = 4$.



i) Use the method of cylindrical shells to show that the volume V of the solid formed is given by: **2**

$$V = 2\pi \int_0^4 \left(16 - 16\sqrt{x} + 4x\sqrt{x} - x^2\right) dx$$

ii) Hence find the exact volume V. **1**

b) A solid has ellipse $16x^2 + 49y^2 = 784$ as its base. If each section perpendicular to the major axis is an equilateral triangle, find the volume of this solid. **3**

c) A eleven-member Wellbeing Committee consists of five students, four teachers, and two parents. The committee meets around a circular table.

i) How many different arrangements of the eleven members around the table are possible if the students sit together as a group and so do the teachers, but no teacher sits next to a student? **2**

ii) One student and one parent are related. Given that all arrangements in part i) are equally likely, what is the probability that these two members sit next to each other? **2**

Question 14 continues on page 13

Question 14 (continued)

d) i) By considering $\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2$, where $x > 0$, prove that the sum of a positive real number and its reciprocal is never less than 2, and is only equal to 2 when $x = 1$. **2**

ii) Hence, or otherwise, find the smallest value of $(a+b)\left(\frac{1}{a}+\frac{1}{b}\right)$ where a and b are positive real numbers. For what values of a and b does this minimum value occur? **3**

End of Questions 14

Question 15 (15 marks) Use a SEPARATE writing booklet.

a) A particle of mass 3 kg moves under the action of a force F so that the velocity is given by $v = 10\sqrt{4 - x^2}\ ms^{-1}$.

i) Find the force F in terms of the displacement x. **2**

ii) If the particle is initially 2 metres to the right of the origin. Find the displacement of the particle at any time t. **3**

b) A particle of mass m falls from rest under gravitational acceleration $g\ ms^{-2}$ and air resistance proportional to the speed $v\ ms^{-1}$. If the constant of proportionality is k.

i) Explain why the acceleration is $g - kv$ for this particle. **1**

ii) Find the velocity of the particle at time t. **2**

iii) Find the terminal velocity. **1**

iv) Express the distance the particle has fallen at time t in terms of t. **2**

c) Given that $I_{2n+1} = \int_0^1 x^{2n+1} e^{x^2}\, dx$, where n is a positive integer.

i) Show that $I_{2n+1} = \frac{1}{2}e - nI_{2n-1}$. **3**

ii) Hence, or otherwise, evaluate $\int_0^1 x^3 e^{x^2}\, dx$. **1**

End of Questions 15

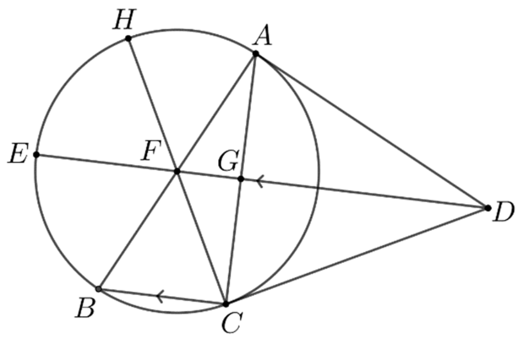
Question 16 (15 marks) Use a SEPARATE writing booklet.

a) A sequence $u_1, u_2, u_3, u_4, \ldots$ satisfies the relationship $u_n = u_{n-1} + u_{n-2}$ for $n \geq 3$.

i) Show that $u_1u_2 + u_2u_3 = {u_3}^2 - {u_1}^2$. **1**

ii) For $n \geq 1$, use mathematical induction to show that **4**
$u_1u_2 + u_2u_3 + u_3u_4 + \cdots + u_{2n-1}u_{2n} + u_{2n}u_{2n+1} = {u_{2n+1}}^2 - {u_1}^2$

b) The triangle ABC is inscribed in a circle. From an external point D, tangents are drawn to the circle, touching it at A and C. The chord ED is drawn parallel to BC, meeting AB at F and AC at G. The line CF is produced to meet the circle at H.



i) Prove that $AFCD$ is a cyclic quadrilateral. **2**

ii) Prove that $HF = AF$. **3**

c) i) If $8x + 4y = \pi$, show that **2**

$$\tan y = \frac{1 - 2\tan x - \tan^2 x}{1 + 2\tan x - \tan^2 x}$$

ii) Hence deduce that $\tan\frac{\pi}{8}$ is a root of the equation **3**

$t^2 + 2t - 1 = 0$, and find the exact value of $\tan\frac{\pi}{8}$.

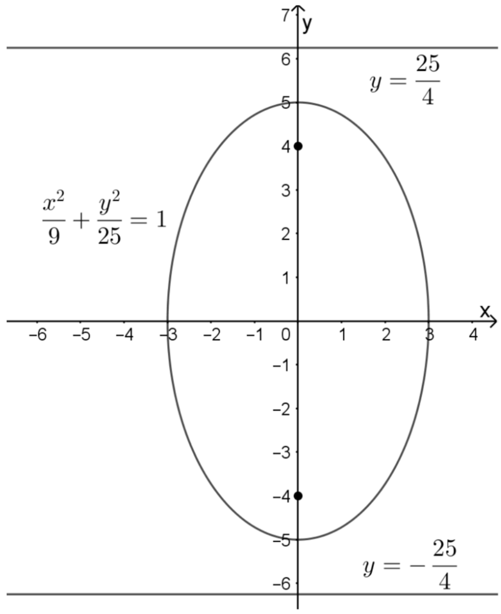
End of Paper

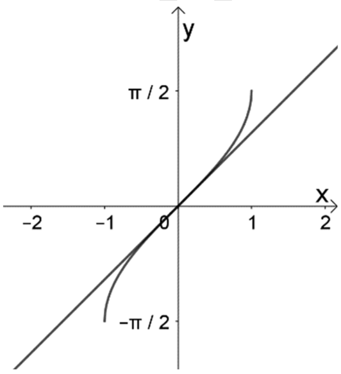
2019 Mathematics Extension 2 AT4 Trial Solutions

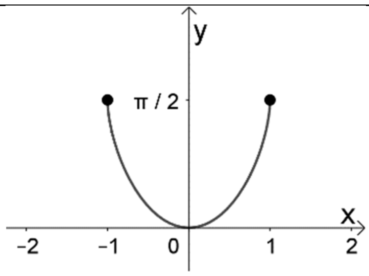
Section 1

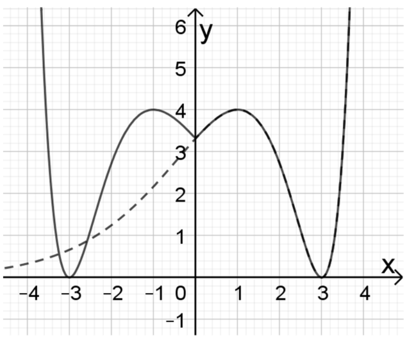
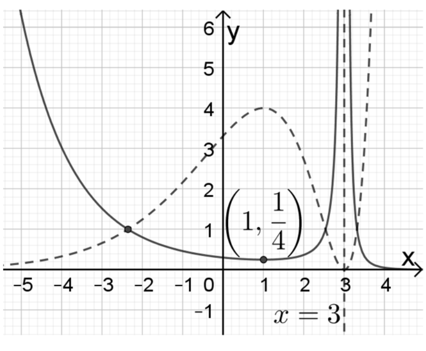
1	**B** $z = 7 + 10i$ $iz = -10 + 7i$ $\overline{iz} = -10 - 7i$	1 Mark
2	**C** Circle centre at $(1, 0)$, inner circle radius 1 and outer circle radius 3 $1 \leq \lvert z - 1 \rvert \leq 3$	1 Mark
3	**C** $9y^2 - 25x^2 = 225$ $\frac{y^2}{25} - \frac{x^2}{9} = 1$ Asymptotes: $y = \pm\frac{5}{3}x$	1 Mark
4	**D** Circle has $e = 0$ Ellipse has $e < 1$ Parabola has $e = 1$ Hyperbola has $e > 1$ $e = \frac{3}{2}$ Circle and ellipse $e < 1$ Circle and parabola $e = 1$ Parabola and hyperbola $e > 2$ Hyperbola and circle $e > 1$ ∴ Hyperbola and circle could be the pair	1 Mark
5	**A** $x = 2\cos\theta + 5, y = \sqrt{3}\sin\theta$ Sub into $3(x-5)^2 + 4y^2 = 12$ $LHS = 3(2\cos\theta + 5 - 5)^2 + 4\left(\sqrt{3}\sin\theta\right)^2$ $LHS = 12\cos^2\theta + 12\sin^2\theta$ $LHS = 12$ $LHS = RHS$	1 Mark
6	**B** $y = -\frac{\lvert x \rvert}{x}$ $y = -\frac{f(x)}{g(x)}$	1 Mark
7	**C** $2z$ is doubling the length of z and then dividing by i indicates rotating it clockwise by $\frac{\pi}{2}$, so the resulting vector should be in the 4th quadrant.	1 Mark

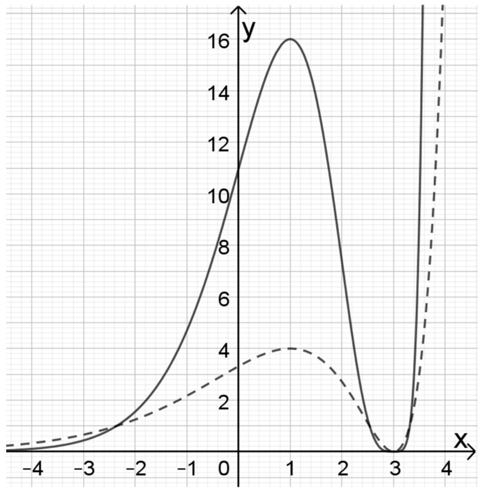
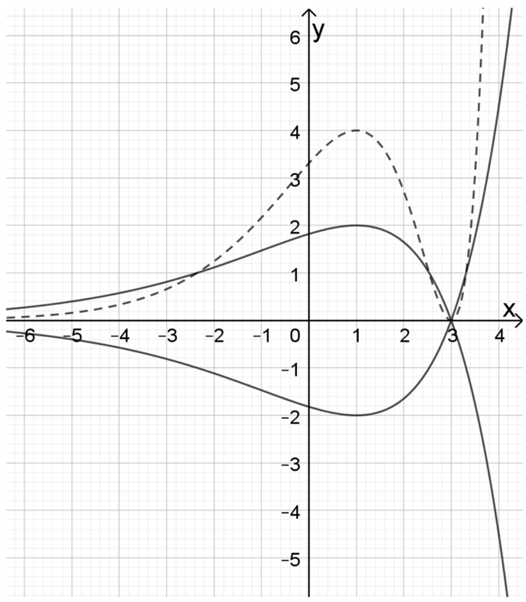
8	**D** First person can select 3 out of 9 boxes = ${}^{9}C_3$ ways Second person can select 3 out of 6 remaining boxes = ${}^{6}C_3$ ways Last person can select the remaining 3 presents in 1 way = ${}^{3}C_3$ way Number of ways = ${}^{9}C_3 \times {}^{6}C_3 \times 1 = 1680$	1 Mark
9	**A** $P'(x)$ has a double root at $x = a$, so $P(x)$ could have a triple root at $x = a$ and a turning point at $x = b$, and given that $x = c$ is a root of $P(x)$ to the right of b. $\therefore P(x) = (x-a)^3(x-c)$ is the possible solution	1 Mark
10	**D** $\int_0^a x(a-x)^{99}dx$ $= \int_0^a (a-x)x^{99}dx$ $= \int_0^a (ax^{99} - x^{100})dx$ $= \left[\frac{ax^{100}}{100} - \frac{x^{101}}{101}\right]_0^a$ $= \left(\frac{a \times a^{100}}{100} - \frac{a^{101}}{101}\right) - 0$ $= \left(\frac{a^{101}}{100} - \frac{a^{101}}{101}\right)$ $= \left(\frac{101a^{101}}{10100} - \frac{100a^{101}}{10100}\right)$ $= \frac{a^{101}}{10100}$	1 Mark

Section 2		
Q11 a)	i^{2019} $= i^{2016} \times i^3$ $= (i^4)^{504} \times i^2 \times i$ $= 1 \times (-1) \times i$ $= -i$	1 Mark Correct solution
Q11 b)	$\frac{x^2}{9} + \frac{y^2}{25} = 1$ $a = 3, b = 5$ $a^2 = b^2(1 - e^2)$ $9 = 25(1 - e^2)$ $\frac{9}{25} = 1 - e^2$ $e^2 = 1 - \frac{9}{25}$ $e^2 = \frac{16}{25}$ $e = \frac{4}{5}$ Foci: $(0, \pm 4)$ Directrices: $y = \pm\frac{25}{4}$  	2 Marks Correct graph with all key features shown clearly 1 Mark Obtains the correct value of eccentricity
Q11 c) i)	$\lvert\sqrt{2} - \sqrt{6}i\rvert = \sqrt{(\sqrt{2})^2 + (\sqrt{6})^2}$ $\lvert\sqrt{2} - \sqrt{6}i\rvert = 2\sqrt{2}$ $\arg(\sqrt{2} - \sqrt{6}i) = \tan^{-1}\left(\frac{-\sqrt{6}}{\sqrt{2}}\right)$ $\arg(\sqrt{2} - \sqrt{6}i) = -\frac{\pi}{3}$ $\sqrt{2} - \sqrt{6}i = 2\sqrt{2}\left(\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right)$ $\sqrt{2} - \sqrt{6}i = 2\sqrt{2}\left(\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}\right)$	2 Marks Correct solution 1 Mark Correct modulus or argument
Q11 c) ii)	$(\sqrt{2} - \sqrt{6}i)^8 = \left[2\sqrt{2}\left(\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}\right)\right]^8$ Using De Moivre's theorem $(\sqrt{2} - \sqrt{6}i)^8 = (2\sqrt{2})^8\left(\cos\frac{8\pi}{3} - i\sin\frac{8\pi}{3}\right)$ $(\sqrt{2} - \sqrt{6}i)^8 = (2\sqrt{2})^8\left(-\frac{1}{2} - i \times \frac{\sqrt{3}}{2}\right)$ $(\sqrt{2} - \sqrt{6}i)^8 = 2^{12}\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)$ $(\sqrt{2} - \sqrt{6}i)^8 = -2^{11}(1 + \sqrt{3}i)\ or\ -2048(1 + \sqrt{3}i)$	2 Marks Correct solution 1 Mark Applies De Moivre's theorem correctly

Q11 d)	$I = \int_2^4 \sqrt{16 - x^2}dx$ Let $x = 4\cos\theta$ $dx = -4\sin\theta\, d\theta$ $x = 4, \quad \theta = 0$ $x = 2, \quad \theta = \frac{\pi}{3}$ $I = \int_{\frac{\pi}{3}}^{0} \sqrt{16 - 16\cos^2\theta} \times -4\sin\theta\, d\theta$ $I = 16\int_0^{\frac{\pi}{3}} \sqrt{1 - \cos^2\theta} \times \sin\theta\, d\theta$ $I = 16\int_0^{\frac{\pi}{3}} \sin^2\theta\, d\theta$ $I = \frac{16}{2}\int_0^{\frac{\pi}{3}} (1 - \cos 2\theta)\, d\theta$ $I = 8\left[\theta - \frac{1}{2}\sin 2\theta\right]_0^{\frac{\pi}{3}}$ $I = 8\left[\frac{\pi}{3} - \frac{1}{2}\sin\left(2 \times \frac{\pi}{3}\right) - 0\right]$ $I = 8\left[\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right]$ $I = \frac{8\pi}{3} - 2\sqrt{3}$	3 Marks Correct solution 2 Marks Finds a correct primitive 1 Mark Uses correct trigonometric substitution
Q11 e) i)	$f(x) = x\sin^{-1}x$ Domain: $-1 \leq x \leq 1$  For $-1 \leq x < 0$, $y = x$ is negative, $y = \sin^{-1}x$ is also negative, so $x\sin^{-1}x$ must be positive. For $0 < x \leq 1$, $y = x$ is positive, $y = \sin^{-1}x$ is also positive, so $x\sin^{-1}x$ must be positive. For $x = 0$, $y = x$ is zero, $y = \sin^{-1}x$ is also zero, so $x\sin^{-1}x$ must be zero. Range: $0 \leq y \leq \frac{\pi}{2}$	2 Marks Correct solution 1 Mark Correct domain or range
Q11 e) ii)	$f(x) = x\sin^{-1}x$ $f(-x) = (-x) \times \sin^{-1}(-x)$ $f(-x) = (-x) \times -\sin^{-1}x$ $f(-x) = x\sin^{-1}x$ $f(x) = f(-x)$ $\therefore f(x)$ is an even function	1 Mark Correct solution

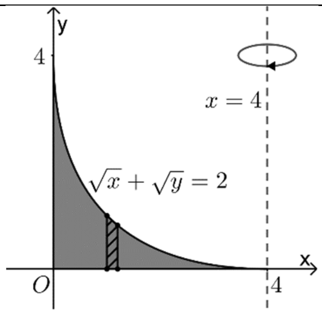
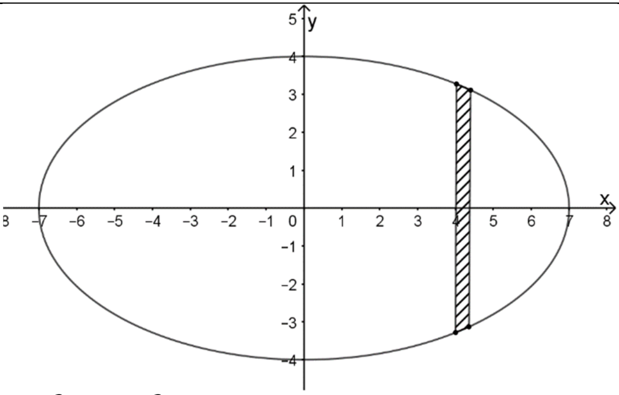
Q11 e) iii)		2 Marks Correct sketch 1 Mark Sketch shows $f(x)$ is even with correct domain and range
Q12 a) i)	$P(x) = x^5 + 6x^4 + 16x^3 + 32x^2 + 48x + 32$ $P'(x) = 5x^4 + 24x^3 + 48x^2 + 64x + 48$ $P''(x) = 20x^3 + 72x^2 + 96x + 64$ $P''(-2) = 20(-2)^3 + 72(-2)^2 + 96(-2) + 64$ $P''(-2) = 0$ $P'(-2) = 5(-2)^4 + 24(-2)^3 + 48(-2)^2 + 64(-2) + 48$ $P'(-2) = 0$ $P(-2) = (-2)^5 + 6(-2)^4 + 16(-2)^3 + 32(-2)^2 + 48(-2) + 32$ $P(-2) = 0$ $\therefore x = -2$ is a root of multiplicity 3.	2 Marks Correct solution 1 Mark Show that $P''(-2) = 0$
Q12 a) ii)	Roots of $P(x)$ are $\alpha, \beta, -2,\ -2,\ -2$ Sum of roots: $\alpha + \beta + (-2) + (-2) + (-2) = -6$ $\alpha + \beta = 0$ Product of roots: $\alpha\beta \times (-2)^3 = -32$ $\alpha\beta = 4$ Let α and β are the solutions to $x^2 - (\alpha + \beta)x + \alpha\beta = 0$ $x^2 + 4 = 0$ $x = \pm 2i$ $\therefore P(x) = (x + 2)^3(x - 2i)(x + 2i)$	2 Marks Correct solution 1 Mark Finds product and sum of remaining two roots
Q12 b)	$t = \tan\frac{x}{2}$ $\frac{dt}{dx} = \frac{1}{2}\sec^2\frac{x}{2}$ $\frac{dt}{dx} = \frac{1}{2}\left(1 + \tan^2\frac{x}{2}\right)$ $\frac{dt}{dx} = \frac{1}{2}(1 + t^2)$ $dx = \frac{2dt}{1 + t^2}$ $x = \frac{\pi}{2},\ \ t = 1$ $x = 0,\ \ t = 0$ $\sin x = \frac{2t}{1 + t^2}$	3 Marks Correct solution 2 Marks Finds a correct primitive 1 Mark Attempts to obtain an integral in terms of t

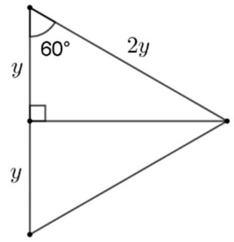
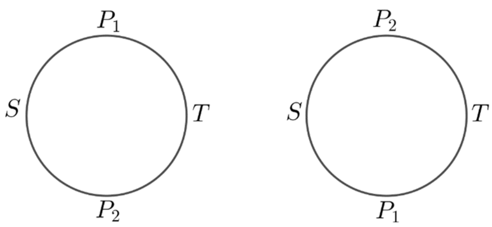
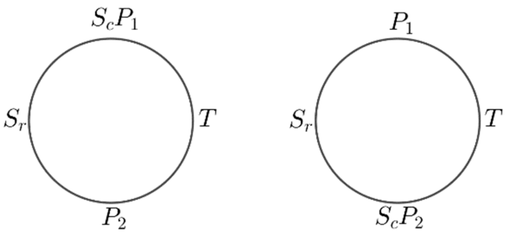
	$\int_0^{\frac{\pi}{2}} \frac{dx}{2+\sin x}$ $= \int_0^1 \frac{1}{2+\frac{2t}{1+t^2}} \times \frac{2dt}{1+t^2}$ $= \int_0^1 \frac{1+t^2}{2+2t^2+2t} \times \frac{2dt}{1+t^2}$ $= \int_0^1 \frac{dt}{t^2+t+1}$ $= \int_0^1 \frac{dt}{t^2+t+\frac{1}{4}+\frac{3}{4}}$ $= \int_0^1 \frac{dt}{\left(t+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$ $= \left[\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{t+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right)\right]_0^1$ $= \left[\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2t+1}{\sqrt{3}}\right)\right]_0^1$ $= \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2\times 1+1}{\sqrt{3}}\right) - \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2\times 0+1}{\sqrt{3}}\right)$ $= \frac{2}{\sqrt{3}}\tan^{-1}\sqrt{3} - \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ $= \frac{2}{\sqrt{3}}\times\frac{\pi}{3} - \frac{2}{\sqrt{3}}\times\frac{\pi}{6}$ $= \frac{\pi}{3\sqrt{3}}$ $= \frac{\sqrt{3}\pi}{9}$	
Q12 c) i)	$y = f(\lvert x \rvert)$  	2 Marks Correct sketch 1 Mark Recognises reflection along the y-axis
Q12 c) ii)	$y = \frac{1}{f(x)}$  	2 Marks Correct sketch 1 Mark Recognises and labels the asymptote

Q12 c) iii)	$y = [f(x)]^2$ 	2 Marks Correct sketch 1 Mark Correct key points shown
Q12 c) iv)	$y^2 = f(x)$ 	2 Marks Correct sketch 1 Mark Correct key points shown
Q13 a) i)	$x^3 - 5x + 25 = 0$ Let $x = -\alpha$ $\alpha = -x$ $(-x)^3 - 5(-x) + 25 = 0$ $-x^3 + 5x + 25 = 0$ $x^3 - 5x - 25 = 0$	1 Mark Correct solution
Q13 a) ii)	$x^3 - 5x + 25 = 0$ Let $x = \alpha^2$ $\alpha = \pm\sqrt{x}$ $\left(\pm\sqrt{x}\right)^3 - 5\left(\pm\sqrt{x}\right) + 25 = 0$ $\pm\sqrt{x}(x-5) = -25$ $x(x-5)^2 = 625$ $x(x^2 - 10x + 25) = 625$ $x^3 - 10x^2 + 25x - 625 = 0$	2 Marks Correct solution 1 Mark Recognises the relationships of the roots and makes the correct substitution

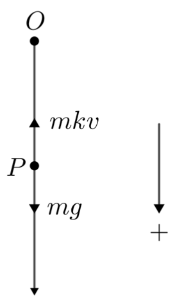
Q13 a) iii)	$x^3 - 5x + 25 = 0$ $\alpha^3 - 5\alpha + 25 = 0 \quad (1)$ $\beta^3 - 5\beta + 25 = 0 \quad (2)$ $\gamma^3 - 5\gamma + 25 = 0 \quad (3)$ $(1) + (2) + (3)$ $\alpha^3 + \beta^3 + \gamma^3 - 5(\alpha + \beta + \gamma) + 25 \times 3 = 0$ $\alpha^3 + \beta^3 + \gamma^3 = 5(\alpha + \beta + \gamma) - 75$ $\alpha^3 + \beta^3 + \gamma^3 = -75$	2 Marks Correct solution 1 Mark Recognises the relationships of the roots and makes the correct substitution
Q13 b)	$\overrightarrow{AB} = (7 + 5i) - (1 + 3i)$ $\overrightarrow{AB} = 6 + 2i$ $\overrightarrow{OP} = 6 + 2i$ $\overrightarrow{OQ} = i\overrightarrow{OP}$ $\overrightarrow{OQ} = i(6 + 2i)$ $\overrightarrow{OQ} = -2 + 6i$	2 Marks Correct solution 1 Mark Finds the complex number that represent P
Q13 c) i)	$M\left(\frac{cp + cq}{2}, \frac{\frac{c}{p} + \frac{c}{q}}{2}\right)$ $M\left(\frac{c}{2}(p + q), \frac{c}{2}\left(\frac{p + q}{pq}\right)\right)$ $m_{OM} = \frac{\frac{c}{2}\left(\frac{p+q}{pq}\right) - 0}{\frac{c}{2}(p + q) - 0}$ $m_{OM} = \frac{1}{pq}$ $m_{MA} = m_{PQ}$ $m_{MA} = \frac{\frac{c}{p} - \frac{c}{q}}{cp - cq}$ $m_{MA} = \frac{\frac{q - p}{pq}}{p - q}$ $m_{MA} = -\frac{1}{pq}$ $-1 \times m_{MA} = \frac{1}{pq}$ $\therefore m_{OM} = -1 \times m_{OM}$	2 Marks Correct solution 1 Mark Finds the gradient of OM or MA
Q13 c) ii)	$\tan \angle MOA = m_{OM}$ $\tan \angle MOA = \frac{1}{pq}$ $\tan \angle MAx = m_{MA}$ $\tan \angle MAx = -\frac{1}{pq}$ $\angle MAx = \pi - \angle MOA$ $\therefore \angle MAO = \angle MOA$ $\therefore OM = MA$ (equal sides opposite equal angles in a triangle)	1 Mark Correct solution

Q13 c) iii)	Equation of OM $y = \frac{1}{pq}x$ T is the intersection point of OM and hyperbola $xy = c^2$ $x \times \frac{1}{pq}x = c^2$ $x^2 = c^2pq$ $x = c\sqrt{pq}$ $y = c^2x^{-1}$ $\frac{dy}{dx} = -c^2x^{-2}$ At $x = c\sqrt{pq}$ Gradient of tangent at T $m_T = \frac{-c^2}{(c\sqrt{pq})^2}$ $m_T = -\frac{1}{pq}$ $m_T = m_{PQ}$ ∴ tangent at T is parallel to chord PQ.	2 Marks Correct solution 1 Mark Find the x value of T
Q13 c) iv)	At T $x = c\sqrt{pq}$ $y = \frac{c\sqrt{pq}}{pq}$ Equation of tangent at T $y - \frac{c\sqrt{pq}}{pq} = -\frac{1}{pq}(x - c\sqrt{pq})$ $pqy - c\sqrt{pq} = -x + c\sqrt{pq}$ $x + pqy - 2c\sqrt{pq} = 0$ At R, $y = 0$ $x = 2c\sqrt{pq}$ $R(2c\sqrt{pq}, 0)$ At S, $x = 0$ $pqy - 2c\sqrt{pq} = 0$ $y = \frac{2c\sqrt{pq}}{pq}$ $S\left(0, \frac{2c\sqrt{pq}}{pq}\right)$	2 Marks Correct solution 1 Mark Finds the equation of tangent at T
Q13 c) v)	$A = \frac{1}{2} \times OR \times OS$ $A = \frac{1}{2} \times 2c\sqrt{pq} \times \frac{2c\sqrt{pq}}{pq}$ $A = 2c^2$ ∴ Area of triangle ORS is a constant since c is a constant.	1 Mark Correct solution

Q14 a) i)	 $A(x) = 2\pi(4 - x)y$ $\sqrt{y} = 2 - \sqrt{x}$ $y = (2 - \sqrt{x})^2$ $\Delta V = 2\pi(4 - x)y\Delta x$ $\Delta V = 2\pi(4 - x)(2 - \sqrt{x})^2 \Delta x$ $V = 2\pi \int_0^4 (4 - x)(2 - \sqrt{x})^2 dx$ $V = 2\pi \int_0^4 (4 - x)(4 - 4\sqrt{x} + x) dx$ $V = 2\pi \int_0^4 (16 - 16\sqrt{x} + 4x - 4x + 4x\sqrt{x} - x^2) dx$ $V = 2\pi \int_0^4 (16 - 16\sqrt{x} + 4x\sqrt{x} - x^2) dx$	2 Marks Correct solution 1 Mark Finds the correct area in terms of x
Q14 a) ii)	$V = 2\pi \int_0^4 (16 - 16\sqrt{x} + 4x\sqrt{x} - x^2) dx$ $V = 2\pi \left[16x - \frac{16x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{4x^{\frac{5}{2}}}{\frac{5}{2}} - \frac{x^3}{3} \right]_0^4$ $V = 2\pi \left[\left(16 \times 4 - \frac{16 \times 4^{\frac{3}{2}}}{\frac{3}{2}} + \frac{4 \times 4^{\frac{5}{2}}}{\frac{5}{2}} - \frac{4^3}{3} \right) - 0 \right]$ $V = 2\pi \left(\frac{128}{15} \right)$ $V = \frac{256\pi}{15}\ units^3$	1 Mark Correct solution
Q14 b)	 $16x^2 + 49y^2 = 784$ $\frac{x^2}{49} + \frac{y^2}{16} = 1$	3 Marks Correct solution 2 Marks Makes significant progress in finding the volume 1 Mark Finds the correct cross sectional area

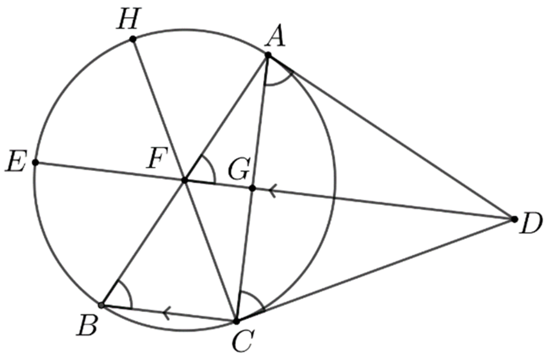
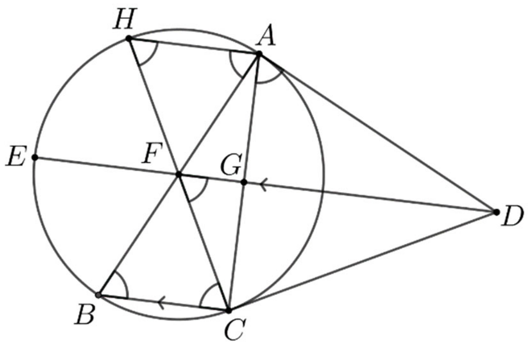
	 Cross-sectional area $A = \frac{1}{2} \times 2y \times 2y \times \sin 60°$ $A = \sqrt{3}y^2$ $\Delta V = \sqrt{3}y^2 \Delta x$ $\Delta V = \sqrt{3} \times 16\left(1 - \frac{x^2}{49}\right)\Delta x$ $V = 16\sqrt{3}\int_{-7}^{7}\left(1 - \frac{x^2}{49}\right)dx$ $V = 16\sqrt{3}\left[x - \frac{x^3}{147}\right]_{-7}^{7}$ $V = 16\sqrt{3}\left[\left(7 - \frac{7^3}{147}\right) - \left((-7) - \frac{(-7)^3}{147}\right)\right]$ $V = 16\sqrt{3} \times \frac{28}{3}$ $V = \frac{448\sqrt{3}}{3}\ units^3$	
Q14 c) i)	If S and T denote the sets 'students and 'teachers' respectively, and P_1, P_2 are the parents, then all the possible arrangements are shown below.  There are 5! ways of arranging the students and 4! ways of arranging the teachers. So the total number of arrangements of the 11 members is $2 \times 5! \times 4! = 5760$.	2 Marks Correct solution 1 Mark Recognises the arrangement of students and teachers
Q14 c) ii)	Let S_C be the related student. They could be related to either P_1 or P_2. Let S_r denote the remaining set of students and T be the set of teachers. Then the possible arrangements are shown below.  There are 4! Ways of arranging the students S_r and 4! Ways of arranging the teachers T. Thus the number of arrangements of the 11 members is $2 \times 4! \times 4! = 1152$ Hence the probability of this pair of people sit together is: $\frac{1152}{5760} = \frac{1}{5}$	2 Marks Correct solution 1 Mark Finds the correct number of arrangements

Q14 d) i)	$\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2 = x-2+\frac{1}{x}$ $\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2 \geq 0$ $x-2+\frac{1}{x}\geq 0$ $x+\frac{1}{x}\geq 2$ Hence the sum of a positive number and its reciprocal is never less than 2. If $x=1$, then $x+\frac{1}{x}=2$	2 Marks Correct solution 1 Mark Deduce $x+\frac{1}{x}\geq 2$
Q14 d) ii)	$(a+b)\left(\frac{1}{a}+\frac{1}{b}\right)=\frac{a}{b}+\frac{b}{a}+2$ $Let\ A=\frac{a}{b}$ $\frac{1}{A}=\frac{b}{a}$ Using Part i) $A+\frac{1}{A}\geq 2$ $\frac{a}{b}+\frac{b}{a}\geq 2$ $\frac{a}{b}+\frac{b}{a}+2\geq 4$ $\therefore (a+b)\left(\frac{1}{a}+\frac{1}{b}\right)\geq 4$ Hence the minimum value is 4 and this minimum value occurs when $a=b$.	3 Marks Correct solution 2 Marks Show that the minimum value is 4 1 Mark Shows $\frac{a}{b}+\frac{b}{a}\geq 2$
Q15 a) i)	$v=10\sqrt{4-x^2}$ $v=10(4-x^2)^{\frac{1}{2}}$ $\ddot{x}=v\frac{dv}{dx}=10(4-x^2)^{\frac{1}{2}}\times 10\times\frac{1}{2}\times -2x\times(4-x^2)^{-\frac{1}{2}}$ $\ddot{x}=-100x$ Or $\frac{1}{2}v^2=\frac{1}{2}\times\left(10(4-x^2)^{\frac{1}{2}}\right)^2$ $\frac{1}{2}v^2=50(4-x^2)$ $\frac{d}{dx}\left(\frac{1}{2}v^2\right)=50\times -2x$ $\frac{d}{dx}\left(\frac{1}{2}v^2\right)=-100x$ $F=m\ddot{x}$ $F=-300x$	2 Marks Correct solution 1 Mark Expresses acceleration in terms of displacement

Q15 a) ii)	$v = \frac{dx}{dt} = 10(4 - x^2)^{\frac{1}{2}}$ $\frac{dt}{dx} = \frac{1}{10(4 - x^2)^{\frac{1}{2}}}$ $t = \int \frac{1}{10(4 - x^2)^{\frac{1}{2}}} dx$ $t = \frac{1}{10} \int \frac{1}{\sqrt{4 - x^2}} dx$ $t = \frac{1}{10} \sin^{-1} \frac{x}{2} + C$ When $x = 2, t = 0$ $0 = \frac{1}{10} \sin^{-1} \frac{2}{2} + C$ $C = -\frac{\pi}{20}$ $t = \frac{1}{10} \sin^{-1} \frac{x}{2} - \frac{\pi}{20}$ $t + \frac{\pi}{20} = \frac{1}{10} \sin^{-1} \frac{x}{2}$ $10t + \frac{\pi}{2} = \sin^{-1} \frac{x}{2}$ $\frac{x}{2} = \sin\left(10t + \frac{\pi}{2}\right)$ $\therefore x = 2 \sin\left(10t + \frac{\pi}{2}\right)$	3 Marks Correct solution 2 Marks Finds the correct value of C 1 Mark Expresses t as a primitive function in terms of x
Q15 b) i)	Take O as the point of release of the particle P. Take motion downwards as positive. There are two forces acting on P: its weight force acting downwards and its resistance of mkv acting upwards.  $F = mg - mkv$ $m\ddot{x} = mg - mkv$ $\ddot{x} = g - kv$	1 Mark Correct solution
Q15 b) ii)	$\frac{dv}{dt} = g - kv$ $\frac{dt}{dv} = \frac{1}{g - kv}$ $t = \int \frac{1}{g - kv} dv$ $t = -\frac{1}{k} \ln(g - kv) + C$	2 Marks Correct solution 1 Mark Find the primitive function of t in terms of v with correct constant value

	$t=0, v=0$ $0=-\frac{1}{k}\ln g+C$ $C=\frac{1}{k}\ln g$ $t=-\frac{1}{k}\ln(g-kv)+\frac{1}{k}\ln g$ $t=\frac{1}{k}\ln\left(\frac{g}{g-kv}\right)$ $kt=\ln\left(\frac{g}{g-kv}\right)$ $e^{kt}=\frac{g}{g-kv}$ $\frac{g-kv}{g}=\frac{1}{e^{kt}}$ $g-kv=ge^{-kt}$ $-kv=ge^{-kt}-g$ $kv=g\left(1-e^{-kt}\right)$ $v=\frac{g}{k}\left(1-e^{-kt}\right)$	
Q15 b) iii)	$t\to\infty, e^{-kt}\to 0$ $v\to\frac{g}{k}(1-0)$ $v\to\frac{g}{k}$	1 Mark Correct solution
Q15 b) iv)	$\frac{dx}{dt}=\frac{g}{k}\left(1-e^{-kt}\right)$ $x=\frac{g}{k}\int\left(1-e^{-kt}\right)dt$ $x=\frac{g}{k}\left[t+\frac{1}{k}e^{-kt}\right]+C$ $t=0, x=0$ $0=\frac{g}{k}\left[0+\frac{1}{k}e^{0}\right]+C$ $C=-\frac{g}{k^2}$ $x=\frac{g}{k}\left[t+\frac{1}{k}e^{-kt}\right]-\frac{g}{k^2}$ $x=\frac{g}{k}\left(t+\frac{1}{k}e^{-kt}-\frac{1}{k}\right)$	2 Marks Correct solution 1 Mark Find the primitive function
Q15 c) i)	$I_{2n+1}=\int_0^1 x^{2n+1}e^{x^2}dx$ $u=x^{2n}$ $\quad v'=xe^{x^2}$ $u'=2nx^{2n-1}$ $\quad v=\frac{1}{2}e^{x^2}$	3 Marks Correct solution 2 Marks Applies integration by parts correctly

	$I_{2n+1}=\int_0^1 x^{2n}\times xe^{x^2}dx$ $I_{2n+1}=\left[x^{2n}\times\frac{1}{2}e^{x^2}\right]_0^1-\int_0^1 2nx^{2n-1}\times\frac{1}{2}e^{x^2}dx$ $I_{2n+1}=\left[\left(1^{2n}\times\frac{1}{2}e^1\right)-0\right]-n\int_0^1 x^{2n-1}\,e^{x^2}dx$ $I_{2n+1}=\frac{1}{2}e-nI_{2n-1}$	1 Mark Correctly identifies u and v' and finds corresponding u' and v
Q15 c) ii)	$I_3=\int_0^1 x^3e^{x^2}dx$ $I_3=\frac{1}{2}e-1\times I_1$ $I_1=\int_0^1 xe^{x^2}dx$ $I_1=\left[\frac{1}{2}e^{x^2}\right]_0^1$ $I_1=\left[\frac{1}{2}e^1-\frac{1}{2}e^0\right]_0^1$ $I_1=\frac{1}{2}(e-1)$ $I_3=\frac{1}{2}e-1\times\frac{1}{2}(e-1)$ $I_3=\frac{1}{2}$	1 Marks Correct solution
Q16 a) i)	$u_n=u_{n-1}+u_{n-2}$ $u_3=u_2+u_1$ $u_2=u_3-u_1$ $u_1u_2+u_2u_3$ $=u_2(u_1+u_3)$ $=(u_3-u_1)(u_3+u_1)$ $={u_3}^2-{u_1}^2$	1 Mark Correct solution
Q16 a) ii)	$u_1u_2+u_2u_3+u_3u_4+\cdots+u_{2n-1}u_{2n}+u_{2n}u_{2n+1}={u_{2n+1}}^2-{u_1}^2$ 1. Prove statement is true for $n=1$ $LHS=u_1u_2+u_2u_3$ $RHS={u_{2\times1+1}}^2-{u_1}^2$ $RHS={u_3}^2-{u_1}^2$ $u_1u_2+u_2u_3={u_3}^2-{u_1}^2$ (proven in part i)) $LHS=RHS$ ∴ statement is true for $n=1$ 2. Assume statement is true for $n=k$ $u_1u_2+u_2u_3+u_3u_4+\cdots+u_{2k-1}u_{2k}+u_{2k}u_{2k+1}={u_{2k+1}}^2-{u_1}^2$ 3. Prove statement is true for $n=k+1$ i.e. $(u_1u_2+u_2u_3+u_3u_4+\cdots+u_{2k-1}u_{2k}+u_{2k}u_{2k+1})+$ $u_{2k+1}u_{2k+2}+u_{2k+2}u_{2k+3}={u_{2k+3}}^2-{u_1}^2$ $LHS=(u_1u_2+u_2u_3+u_3u_4+\cdots+u_{2k-1}u_{2k}+u_{2k}u_{2k+1})$ $+u_{2k+1}u_{2k+2}+u_{2k+2}u_{2k+3}$ $LHS={u_{2k+1}}^2-{u_1}^2+u_{2k+2}(u_{2k+1}+u_{2k+3})$ (from step 2)	4 Marks Correct solution 3 Marks Makes significant progress in the body of proof 2 Marks Correct $k+1$ statement 1 Mark Prove initial statement is true

	Since $u_n = u_{n-1} + u_{n-2}$ $u_{2k+3} = u_{2k+2} + u_{2k+1}$ $u_{2k+2} = u_{2k+3} - u_{2k+1}$ $LHS = {u_{2k+1}}^2 - {u_1}^2 + (u_{2k+3} - u_{2k+1})(u_{2k+3} + u_{2k+1})$ $LHS = {u_{2k+1}}^2 - {u_1}^2 + {u_{2k+3}}^2 - {u_{2k+1}}^2$ $LHS = {u_{2k+3}}^2 - {u_1}^2$ $LHS = RHS$ ∴ statement is true by mathematical induction for all $n \geq 1$	
Q16 b) i)	 $\angle CAD = \angle ABC$ (Alternate segment theorem) $\angle ABC = \angle AFD$ (corresponding angles are equal, $ED \parallel BC$) $AD = CD$ (tangents from an external point are equal in length) $\angle CAD = \angle ACD$ (angles opposite equal sides of ΔACD are equal) $\therefore \angle AFD = \angle ACD$ Since angles in the same segment are equal $\therefore AFCD$ is a cyclic quadrilateral	2 Marks Correct solution 1 Mark Makes significant progress
Q16 b) ii)	Join HA  Since $AFCD$ is a cyclic quadrilateral $\angle DAC = \angle DFC$ (angles in the same segment are equal) $\angle FCB = \angle DFC$ (alternate angles are equal, $ED \parallel BC$) Since $\angle FBC = \angle FCB$ (proven above that $\angle CAD = \angle ABC$) $\therefore \Delta FBC$ is an isosceles triangle $\angle HAB = \angle HCB$ (angles in the same segment) $\angle AHC = \angle ABC$ (angles in the same segment) $\therefore \Delta HFA \vert\vert\vert \Delta CFB$ (equiangular) $\therefore \Delta HFA$ is also an isosceles triangle $\therefore HF = AF$ (equal sides of isosceles triangle)	3 Marks Correct solution 2 Marks Shows $\Delta HFA \vert\vert\vert \Delta CFB$ 1 Marks Shows $\angle FBC = \angle FCB$

Q16 c) i)	$8x + 4y = \pi$ $y = \frac{\pi}{4} - 2x$ $\tan y = \tan\left(\frac{\pi}{4} - 2x\right)$ $\tan y = \frac{\tan\frac{\pi}{4} - \tan 2x}{1 + \tan\frac{\pi}{4}\tan 2x}$ $\tan y = \frac{1 - \tan 2x}{1 + \tan 2x}$ $\tan y = \frac{1 - \frac{2\tan x}{1 - \tan^2 x}}{1 + \frac{2\tan x}{1 - \tan^2 x}}$ $\tan y = \frac{\frac{1 - \tan^2 x - 2\tan x}{1 - \tan^2 x}}{\frac{1 - \tan^2 x + 2\tan x}{1 - \tan^2 x}}$ $\tan y = \frac{1 - \tan^2 x - 2\tan x}{1 - \tan^2 x + 2\tan x}$	2 Marks Correct solution 1 Mark Finds $\tan y = \frac{1 - \tan 2x}{1 + \tan 2x}$
Q16 c) ii)	If $y = 0$ $0 = \frac{\pi}{4} - 2x$ $x = \frac{\pi}{8}$ $\tan 0 = \frac{1 - \tan^2\frac{\pi}{8} - 2\tan\frac{\pi}{8}}{1 - \tan^2\frac{\pi}{8} + 2\tan\frac{\pi}{8}}$ $1 - \tan^2\frac{\pi}{8} - 2\tan\frac{\pi}{8} = 0$ Let $t = \tan\frac{\pi}{8}$ $1 - t^2 - 2t = 0$ $t^2 + 2t - 1 = 0$ $\therefore \tan\frac{\pi}{8}$ must a root of of $t^2 + 2t - 1 = 0$ $t = \frac{-2 \pm \sqrt{2^2 - 4 \times 1 \times (-1)}}{2 \times 1}$ $t = \frac{-2 \pm \sqrt{8}}{2}$ $t = -1 \pm \sqrt{2}$ $\tan\frac{\pi}{8} > 0$ $\therefore \tan\frac{\pi}{8} = -1 + \sqrt{2}$	3 Marks Correct solution 2 Marks Deduce $\tan\frac{\pi}{8}$ is a root and finds the values of t 1 Mark Deduce $\tan\frac{\pi}{8}$ is a root of the equation