CRANBROOK SCHOOL

ESSE QUAM VIDERI

HSC Trial Examination

Mathematics Extension 2

Friday July 23, 2010

General Instructions

- Reading time – 5 minutes
- Writing time – 3 hours
- All 8 questions should be attempted
- Total marks available - 120
- All questions are worth 15 marks
- Begin a new 8 page booklet for each question
- An approved calculator may be used
- A table of standard integrals can be found at the back of the paper
- All relevant working should be shown for each question

Question 1 (15 marks) Use a new 8 page booklet **Marks**

(a) Find : $\int x\sqrt{3x-1}\,dx$ **3**

(b) By using the substitution $t = \tan\frac{\theta}{2}$, evaluate

$$\int_0^{\frac{\pi}{2}} \frac{d\theta}{2+\sin\theta}$$ **3**

(c) (i) Split into partial fractions: $\frac{8}{(x+2)(x^2+4)}$ **2**

(ii) Hence evaluate: $\int_0^2 \frac{8\ dx}{(x+2)(x^2+4)}$ **3**

(d) If $I_n = \int_0^{\frac{\pi}{2}} \cos^n x\ dx, \quad (n \geq 2)$

(i) Show that $I_n = (n-1)\,I_{n-2} - (n-1)\,I_n$ **2**

(ii) Hence evaluate $\int_0^{\frac{\pi}{2}} \cos^6 x\ dx$ **2**

Question 2 (15 marks) Use a new 8 page booklet **Marks**

(a) If $z = 3 + 2i$, plot on an Argand diagram

(i) z and $\overline{z}$ **1**

(ii) iz **1**

(iii) $z(1 + i)$ **1**

(b) (i) Find all pairs of integers a and b such that $(a + ib)^2 = 8 + 6i$ **1**

(ii) Hence solve: $z^2 + 2z(1+2i) - (11+2i) = 0$ **2**

(c) (i) If $z = \cos\frac{\pi}{3} + i\sin\frac{\pi}{3}$, *find* z^6 **2**

(ii) Plot on an argand diagram, all complex numbers that are the solutions of $z^6 = 1$ **2**

(d) Sketch the locus of the following. Draw separate diagrams.

(i) $\arg(z - 1 - 2i) = \frac{\pi}{4}$ **1**

(ii) $z\overline{z} - 3(z + \overline{z}) \le 0$ **2**

(iii) $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{3}$ **2**

Question 3 (15 marks) Use a new 8 page booklet **Marks**

(a) For the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$

(i) Find the eccentricity. **1**

(ii) Find the coordinates of the foci S and S'. **1**

(iii) Find the equations of the directricies. **1**

(iv) Sketch the curve $\frac{x^2}{25}+\frac{y^2}{16}=1$ **1**

(v) Show that the coordinates of any point P can be represented by ($5\cos\theta$, $4\sin\theta$) **2**

(vi) Show that $PS+PS'$ is independent of the position of P on the curve. **2**

(vii) Show that the equation of the normal at the point P on the ellipse is $5x\sin\theta-4y\cos\theta-9\sin\theta\cos\theta=0$ **2**

(viii) If the normal meets the major axis at L and the minor axis at M, prove that $\frac{PL}{PM}=\frac{16}{25}$ **2**

(ix) Show that the normal bisects $\angle SPS'$ **3**

Question 4 (15 marks) Use a new 8 page booklet **Marks**

(a) (i) Find $\int \frac{\sin 2x}{2+\sin^2 x}\, dx$ **2**

(ii) Evaluate $\int_0^{\frac{1}{2}} \sqrt{\frac{1+x}{1-x}}\, dx$ **4**

(b) If a > 0, b > 0 and c > 0, show that

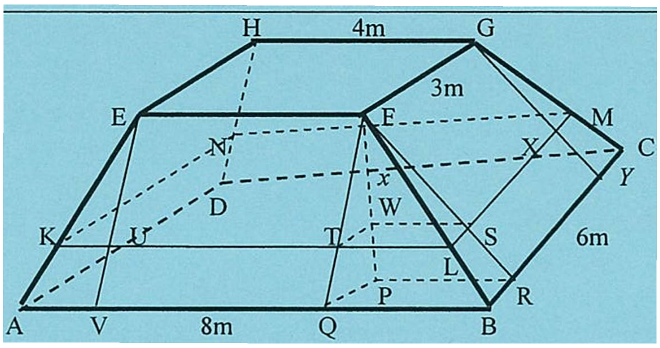
(i) $a^2 + b^2 + c^2 - ab - bc - ca \geq 0$ **2**

(ii) $\frac{a+b+c}{3} \geq \sqrt[3]{abc}$ **4**

(iii) $(a+b+d)(b+c+d)(c+a+d)(a+b+c) \geq 81abcd$ **3**

Question 5 (15 marks) Use a new 8 page booklet **Marks**

(a) A concrete beam of height $15m$ has plane sides. Cross-sections parallel to the ends are rectangular. The beam measures $4m$ by $3m$ at one end and $8m$ by $6m$ at the other end as shown.



In the figure KLMN is the cross-section and its distance from the top is x metres. FW $= x$ metres

(i) Show that an expression for the area of a cross-section at a distance x metres from the smaller end is given by $A(x) = 12 + \dfrac{24x}{15} + \dfrac{4x^2}{75}$. **3**

(ii) Find the volume of the beam. **2**

(b) Find the exact volume of the solid generated by rotating the area bounded by the curve $y = \log_e x$, the x - axis and the line $x = 4$ about the y-axis. Use the method of cylindrical shells and include sketches with your answer. **5**

(c) By slicing perpendicular to the y-axis, determine the volume formed when the region bounded by the curve $y = -3x^4 + 12x^2$ and the x-axis between $x = 0$ and $x = 2$ is rotated about the y-axis. Include sketches with your answer. **5**

Question 6 (15 marks) Use a new 8 page booklet **Marks**

(a) A wasp after leaving its hive O, flies 2km North East, 4km 30^0 West of North and then 6km 210^0 True North.

(i) Convert each of the wasp's flights into the form $z = r\,cis\theta$. **1**

(ii) Draw a vector diagram showing the wasp's flights relative to its hive O. **1**

(iii) Determine the resultant vector of the wasp's flights. **1**

(iv) Hence determine how far to the nearest 0.1km and in what direction in radians to 1 decimal place is the wasp from its hive. **2**

(b) (i) Given that $P(x)$ has a rational zero, find this zero and hence factorise $P(x)$ over the complex field of numbers if $P(x) = 2x^3 - 3x^2 + 2x - 3$. **2**

(ii) If α, β and γ are the roots of the equation $x^3 + qx^2 + r = 0$, where $r \neq 0$ determine a cubic equation whose roots are $\frac{1}{\alpha}, \frac{1}{\beta}$ and $\frac{1}{\gamma}$. **2**

(iii) The equation $2x^3 - 13x^2 - 26x + 16 = 0$ has roots in geometric progression. Find these roots. **2**

(c) The tangent at $P(a\sec\theta, b\tan\theta)$ on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ meets a directrix at Q. S is the corresponding focus.

(i) Find the equation of the tangent at P. **1**

(ii) Find the coordinates of Q. **1**

(iii) Show that PQ subtends a right angle at S. **2**

Question 7 (15 marks) Use a new 8 page booklet **Marks**

(a) Given $y = \dfrac{x^3}{x^2 - 4}$

(i) Find the coordinates of all stationary points. **2**

(ii) Find the points of intersection with the coordinate axes and the position of all asymptotes. **1**

(iii) Hence sketch the curve $y = \dfrac{x^3}{x^2 - 4}$ **1**

(b) Use the graph $y = \dfrac{x^3}{x^2 - 4}$ to find the number of roots of the equation $x^3 - k(x^2 - 4) = 0$ for varying value of *k*. **2**

(c) (i) Show that $\int_{-a}^{a} f(x)\,dx = \int_0^a [f(x) + f(-x)]\,dx$ **2**

(ii) Hence show that $\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} \dfrac{1}{1 + \sin x}\,dx = \int_0^{\frac{\pi}{4}} \dfrac{2}{\cos^2 x}\,dx$ **2**

(iii) Hence evaluate $\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} \dfrac{1}{1 + \sin x}\,dx$ **1**

(d) A sociologist believes that the fraction $y(t)$ of a population who have heard a rumour after t days can be modelled by a continuous function given by $y(t) = \dfrac{y_0 e^{kt}}{(1 - y_0) + y_0 e^{kt}}$, $t \geq 0$, where y_0 is the fraction, $0 \leq y_0 < 1$ for all $t \geq 0$, who have heard the rumour at time $t = 0$ and k is a positive constant.

(i) Show that $y_0 \leq y(t) < 1$ for all $t \geq 0$. **1**

(ii) Find the rate of change of y with respect to t. **1**

(iii) If $k = 0{\cdot}2$ and $y_o = 0{\cdot}1$, show that $y(5) = \dfrac{e}{e+9}$ **1**

(iv) Give an interpretation of the above results (i), (ii) and (iii) in terms of the sociological model. **1**

Question 8 (15 marks) Use a new 8 page booklet **Marks**

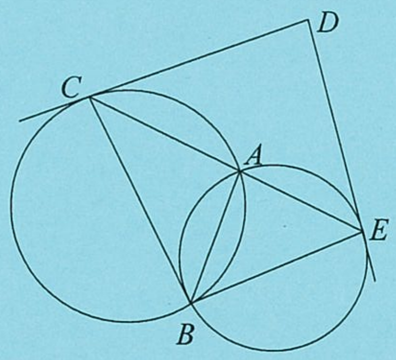
(a) (i) Find a polynomial $p(x)$ with real coefficients having $3i$ and $1+2i$ as zeros. **2**

(ii) Find all zeros of the equation
$6x^4 - 7x^3 - 28x^2 + 35x - 10 = 0$. **3**

(b) (i) If k is a positive integer such that $k \geq 4$, show that
$2k^3 > 3k^2 + 3k + 1$. **2**

(ii) Hence show by mathematical induction for positive integers n, $n \neq 3$, that $3^n > n^3$. **4**

(c)



Two circles intersect at A and B. CAE is a straight line where C is a point on the first circle and E is a point on the second circle. The tangent at C to the first circle and the tangent at E to the second circle meet at D.

(i) Copy the diagram.

(ii) Prove that $BCDE$ is a cyclic quadrilateral. **4**

EXT 2 PAPER — KAMBALA — MR NAGY'S SOLUTIONS

Q.1. (16)

(a) $\sqrt{5^2+2^2} = \sqrt{29}$ (1)

(b) $\frac{1}{(-3-4i)} \times \frac{-3+4i}{(-3+4i)} = \frac{-3+4i}{9+16} = -\frac{3}{25} + \frac{4}{25}i$ (2)

(c) $\frac{1+i^5}{1-i} = \frac{1+i}{1-i} \times \frac{1+i}{1+i} = \frac{(1+i)^2}{1+1} = \frac{1+2i+i^2}{2} = \frac{2i}{2} = i$ (2)

(d) (i) $|-1+i| = \sqrt{2}$ & $|\sqrt{3}+i| = \sqrt{4} = 2$ Hence $|z| = \frac{\sqrt{2}}{2}$

$\arg(-1+i) = \frac{3\pi}{4}$ & $\arg(\sqrt{3}+i) = \frac{\pi}{6}$ Hence $\arg(z) = \frac{7\pi}{12}$

ie. $z = \frac{\sqrt{2}}{2} \text{cis}\left(\frac{7\pi}{12}\right)$ (2)

(ii) $Re(z) = \frac{\sqrt{2}}{2}\cos\frac{7\pi}{12} = Re\left(\frac{(-1+i)}{(\sqrt{3}+i)} \times \frac{(\sqrt{3}-i)}{(\sqrt{3}-i)}\right)$

$= Re\left(\frac{-\sqrt{3}+1+i(\sqrt{3}+1)}{3+1}\right)$

$\therefore \frac{\sqrt{2}}{2}\cos\frac{7\pi}{12} = \frac{1-\sqrt{3}}{4}$

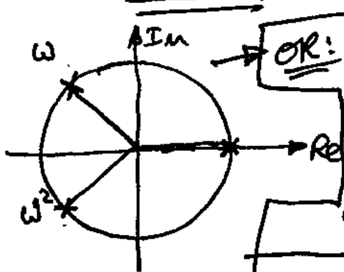
Thus $\cos\frac{7\pi}{12} = \frac{1-\sqrt{3}}{2\sqrt{2}} = \frac{\sqrt{2}-\sqrt{6}}{4}$ (2)

(e) (i) $\omega = \text{cis}\frac{2\pi}{3}$ or $-\text{cis}\frac{2\pi}{3}$

then $\omega^2 = -\text{cis}\frac{2\pi}{3}$ or $\text{cis}\frac{2\pi}{3}$

$\omega + \omega^2 = 2\cos\frac{2\pi}{3} = -1$

Hence $\omega^2+\omega+1=0$ as required.



OR: ω satisfies $x^3-1=0$

$(x-1)(x^2+x+1)=0$

since $x=1$ is a real solution, ω satisfies $\omega^2+\omega+1=0$ as required

(2)

(ii) $RHS = (b+c)(b+c\omega)(b+c\omega^2) = (b+c)(b^2+bc\omega^2+bc\omega+c^2\omega^3)$

$= b^3+b^2c\omega^2+b^2c\omega+bc^2\omega^3+b^2c+bc^2\omega^2+bc^2\omega+c^3\omega^3$

$= b^3+b^2c(\omega^2+\omega+1)+bc^2(\omega^3+\omega^2+\omega)+c^3\omega^3$

$= b^3+b^2c \times 0 \quad + bc^2(1+\omega^2+\omega) + c^3$ since $\omega^3=1$

$= b^3+c^3 = LHS$ as required. (2)

(f) $P(x) = (x-2i)(x+2i)(x-1+3i)(x-1-3i) = (x^2+4)((x-1)^2+9) = (x^2+4)(x^2-2x+10)$

$P(x) = x^4-2x^3+10x^2+4x^2-8x+40 = x^4-2x^3+14x^2-8x+40$ (3)

2. (16)

(a) $\displaystyle\int \frac{x}{\sqrt{x+1}}\,dx$ Let $u^2 = x+1$ $\quad x = u^2 - 1$

$2u\,du = dx$

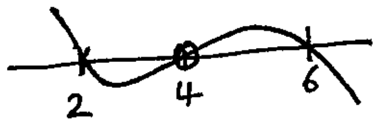
$$= \int \frac{u^2-1}{u} \cdot 2u\,du = \frac{2u^3}{3} - 2u + c = \frac{2}{3}u(u^2-3) + c$$

$$= \frac{2}{3}\sqrt{x+1}\,(x+1-3) + c = \underline{\underline{\frac{2\sqrt{x+1}\,(x-2)}{3} + c}}$$ (2)

(b) $\dfrac{x^2-5x}{4-x} + 3 \leq 0$

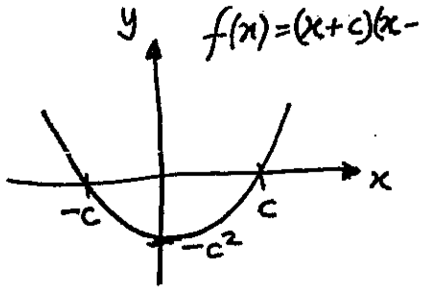
$\dfrac{x^2-5x+3(4-x)}{4-x} \leq 0$ $\quad \therefore \dfrac{x^2-8x+12}{4-x} \cdot (4-x)^2 \leq 0$

$(x-6)(x-2)(4-x) \leq 0 \;, x \neq 4$



Thus $\underline{\underline{2 \leq x < 4 \text{ or } x \geq 6}}$ (3)

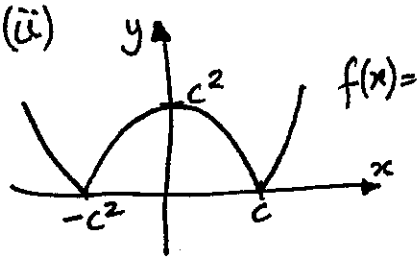
(c) (i)



(1)

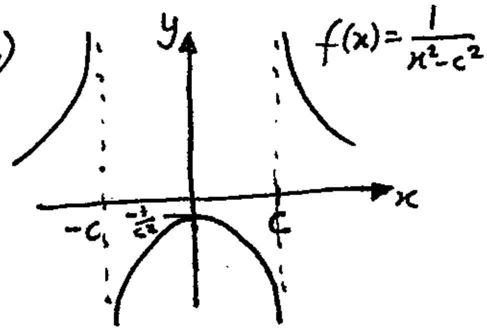
(ii)

(A)



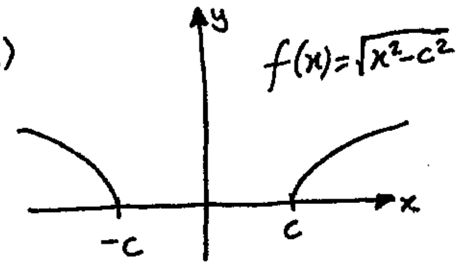
(2)

(B)



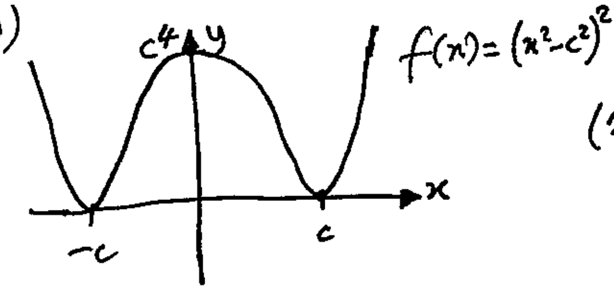
(2)

(C)



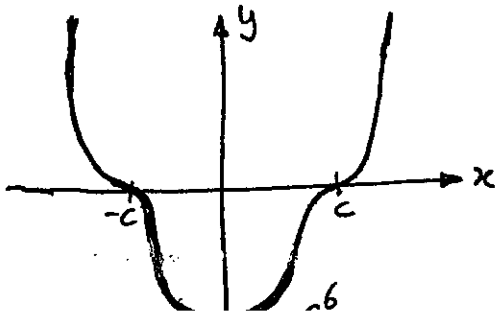
(2)

(D)



(2)

(E)



(2)

3. (16)

(a) $f(x) = x - \ln(1+x^2)$

$f'(x) = 1 - \frac{2x}{1+x^2}$

$f'(x) = \frac{1+x^2-2x}{1+x^2} = \frac{(x-1)^2}{1+x^2}$

let $y = \ln u$ where $u = 1+x^2$

$\frac{dy}{du} = \frac{1}{u}$ & $\frac{du}{dx} = 2x$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \frac{2x}{u} = \frac{2x}{1+x^2}$

(3)

Since square numbers ≥ 0, $(x-1)^2 \geq 0$ and $1+x^2 \geq 0$ thus $f'(x) \geq 0$ for all x, as required

(b) (i) In ΔABX and ΔADC,

$\angle ABX = \angle ADC$ (Angles subtended by chord AC at the circumference in the same segment are equal)

$\angle CAD = \angle XAB$ (AD bisects angle $\angle BAC$ - given).

Therefore $\Delta ABX \; ||| \; \Delta ADC$ (Two angles the same). (2)

(ii) Since $\Delta ABX \; ||| \; \Delta ADC$, $\frac{AB}{AX} = \frac{AD}{AC}$ $\therefore$ $AB.AC = AD.AX$ as required. (1)

(iii) $\Delta CXD \; ||| \; \Delta AXB$ since $\angle ABX = \angle ADC$ as above and $\angle CXD = \angle BXA$ (vertically opposite angles equal).

Thus $\frac{XD}{XB} = \frac{XC}{AX}$ $\therefore$ $AX.XD = XC.XB$ So $XD = \frac{XC.XB}{AX}$ --①

from (ii), $AB.AC = AD.AX = (AX + XD)AX = AX^2 + XD.AX$

and from ①, $AB.AC = AX^2 + \left(\frac{XC.XB}{AX}\right).AX$

$AB.AC = AX^2 + BX.XC$ as required

(2)

(c)(i) $(l+m+n)^2 = l^2+m^2+n^2+2(lm+ln+mn) = 29 + 2(lm+ln+mn)$

Thus $lm+ln+mn = \frac{(-3)^2 - 29}{2} = -10$

Let the cubic equation be monic,

ie. $x^3 + bx^2 + cx + d = 0$ where $-b = l+m+n = -3$ $\therefore b = 3$

$c = lm+mn+ln = -10$ $\therefore c = -10$

$-d = lmn = -6$ $\therefore d = 6$ (2)

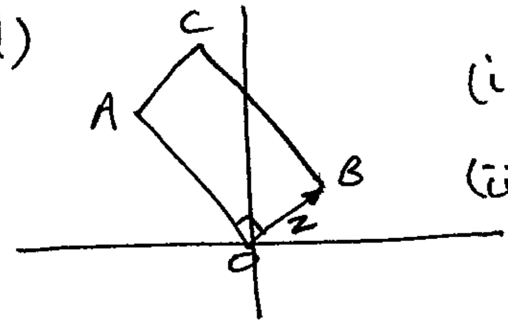
Hence monic cubic is $x^3+3x^2-10x+6=0$ as required.

(ii) root at $x=1$ by inspection, ie. $(x-1)(x^2+4x-6)=0$

Hence other roots of $x = \frac{-4 \pm \sqrt{16+24}}{2} = -2 \pm \sqrt{10}$

Thus l, m and n are $1, -2+\sqrt{10}, -2-\sqrt{10}$ in any order (3)

3. (d)



(i) $\underline{\underline{A = 2iz}}$ by inspection (1)

(ii) $\vec{OD} = \frac{1}{2}\vec{OC} = \frac{1}{2}(\vec{OB} + \vec{OA})$

$= \frac{1}{2}(z + 2iz)$

$= \underline{\underline{\frac{z}{2} + iz \text{ or } z\left(\frac{1}{2} + i\right)}}$ (2)

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4. (a) Let $\cos\theta = x$ then $\cos 3\theta = 4x^3 - 3x$

$8x^3 - 6x + 1 = 0$

$2(4x^3 - 3x) + 1 = 0$

$2\cos 3\theta + 1 = 0$

$\cos 3\theta = \frac{-1}{2}$

$3\theta = \pm\frac{2\pi}{3} + 2n\pi \quad , n \in \mathbb{Z}$

$\therefore \theta = \pm\frac{2\pi}{9} + \frac{2n\pi}{3} \quad , n \in \mathbb{Z}$ (3)

Thus $x = \underline{\underline{\cos\frac{2\pi}{9}, \cos\frac{8\pi}{9}, \cos\frac{14\pi}{9}}} \approx 0.766, -0.9397, 0.1736$

(or equivalent!)

(b)(i) $(x-y)^2 \geq 0$ since a square of a real number is positive. (2)

$x^2 + y^2 - 2xy \geq 0 \quad \therefore \underline{\underline{x^2 + y^2 \geq 2xy \text{ as required}}}$

BAD QUESTION!

(ii) $a^2 + b^2 \geq 2ab$

$c^2 + d^2 \geq 2cd \quad \therefore a^2 + b^2 + c^2 + d^2 \geq 2ab + 2cd$

The question is incorrect (take $a = 2, b = 1, c = 1, d = 1$)

$2^2 + 1^2 + 1^2 + 1^2 = 7 \not\geq 4 \times 2 \times 1 \times 1 \times 1$

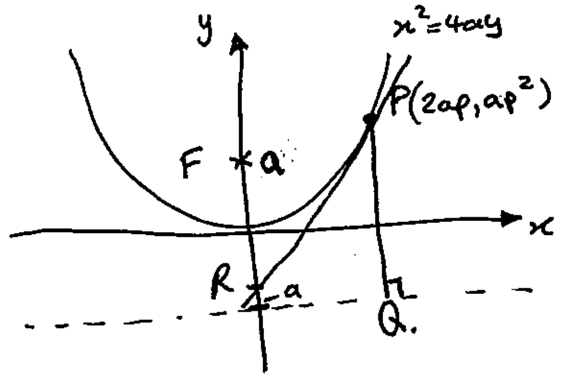
perhaps the question meant: (2?)

$(a^2 + b^2)(c^2 + d^2) \geq 4abcd$?

True since $a^2 + b^2 \geq 0$ and $c^2 + d^2 \geq 0$

So $a^2 + b^2 + c^2 + d^2 \geq 0$

4. (c)



(i) $\frac{dy}{dx} = \frac{dy}{dp} \times \frac{dp}{dx} = 2ap \times \frac{1}{2a} = p$ ∴ Eqn of tangent at P is:

$y - ap^2 = p(x - 2ap)$ (2)

$y = px - 2ap^2 + ap^2$

∴ $px - y - ap^2 = 0$ as required.

(ii) Midpoint of PR is $x = ap$ and $y = p(ap) - ap^2$ (from above) (3)

i.e. $(ap, 0)$

Midpoint of FQ is $(ap, \frac{a - -a}{2})$ or $(ap, 0)$

Hence since midpoints coincide, they bisect each other. as required

(iii) Gradient of PR $= M_{PR} = p$ from (i)

Gradient of FQ $= M_{FQ} = \frac{-2a}{2ap} = \frac{-1}{p}$

$M_{PR} \times M_{FQ} = -1$ (2)

∴ perpendicular as required

(iv) PQRF is a rhombus since the diagonals bisect at right angles. (2)

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5. (a) (i) $y = uv$ and $y' = uv' + u'v$ from the product rule. (2)

$y'' = uv'' + u'v' + u'v' + u''v = uv'' + 2u'v' + u''v$ as required.

(ii) $y''' = u'v'' + uv''' + 2u''v' + 2u'v'' + u'''v + u''v'$

$y''' = uv''' + 3u''v' + 3u'v'' + u'''v$ (2)

BORING QUESTION!

Similarly $y^{iv} = uv^{iv} + 4u'v''' + 6u''v'' + 4u'''v' + u^{iv}v$

and $y^{v} = uv^{v} + 5u'v^{iv} + 10u''v''' + 10u'''v'' + 5u^{iv}v' + u^{v}v$

(iii) $\frac{d^5}{dx^5}\left((1-x^2)e^{-x}\right) = (1-x^2)(-e^{-x}) + 5(2x)e^{-x} + 10(2)(-e^{-x})$ (2)

$= e^{-x}(x^2 - 1 + 10x - 20) = (x^2 + 10x - 21)e^{-x}$

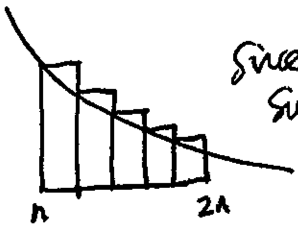
Q.5. (b) (i) $t_n = \frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{2n-1} + \frac{1}{2n}$

$\therefore t_n + \frac{1}{2n} = \frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{2n-1} + \frac{2}{2n}$ and $\frac{2}{2n} = \frac{1}{n}$

$t_n + \frac{1}{2n} = \frac{1}{n} + \frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{2n-1}$ as required. (1)

(ii) Area under graph from $x=n$ to $2n$ $= \int_n^{2n} \frac{1}{x}\,dx = \ln\left(\frac{2n}{n}\right) = \ln 2$

and area of the rectangles (width 1) from n to $2n-1$ is $\frac{1}{n} + \frac{1}{n+1} + \cdots + \frac{1}{2n-1} = t_n + \frac{1}{2n}$



Since the area formed from summing the rectangles > area under graph, (3)

then $t_n + \frac{1}{2n} > \ln 2$ as required.

(iii) for $n=1$, $S_n = 1 - \frac{1}{2} = \frac{1}{2}$ and $t_n = \frac{1}{2}$ $\therefore$ true for $n=1$

Assume true for $n=k$, ie. $S_k = t_k$

then for $n=k+1$, $S_{k+1} = S_k + \frac{1}{2k+1} - \frac{1}{2k+2}$ ------ ①

$t_{k+1} = \frac{1}{k+2} + \frac{1}{k+3} + \cdots + \frac{1}{2k+1} + \frac{1}{2k+2} = t_k + \frac{1}{2k+1} + \frac{1}{2k+2} - \frac{1}{k+1}$

Hence $t_{k+1} = S_k + \frac{1}{2k+1} + \frac{1-2}{(2(k+1))}$ (4)

$= S_k + \frac{1}{2k+1} - \frac{1}{2k+2} = S_{k+1}$ from ①

Hence true for S_{k+1}

Therefore, true for $n=1$ and if true for $n=k$, then also true for $n=k+1$

$\therefore$ true for all $n \in \mathbb{Z}$, $n \geq 1$ by induction.

(iv) $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots + \frac{1}{9999} - \frac{1}{10000} = S_{5000} = t_{5000}$

As $n \to \infty$, $t_n + \frac{1}{2n} \to \ln 2$ or $t_n \to \ln 2 - \frac{1}{2n}$ (2)

Hence $t_{5000} \approx \ln 2 - \frac{1}{10000} \approx \underline{0.693 \ (3.d.p.)}$

$s_1 = 1 - \frac{1}{2} = \frac{1}{2}$ $\qquad$ $t_1 = \frac{1}{2}$

$s_2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} = \frac{7}{12}$ $\qquad$ $t_2 = \frac{1}{3} + \frac{1}{4} = \frac{7}{12}$

$s_3 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} = \frac{37}{60}$ $\qquad$ $t_3 = \frac{1}{4} + \frac{1}{5} + \frac{1}{6} = \frac{37}{60}$

S23

RTP $\sum_{r=1}^{n} \frac{r}{(r+1)!} = \frac{(n+1) - 1!}{(n+1)!}$

Prove true for $n = 1$

LHS $= \frac{1}{(r+1)!}$ RHS $= \frac{1+1 - 1!}{1+1}$

$= \frac{1}{2}$ $= \frac{1}{2}$

~~LHS = $\frac{3}{(3+1)!}$~~

LHS $= \frac{1}{2} + \frac{1}{3} + \frac{1}{8}$ RHS $= \frac{23}{24}$

$= \frac{21}{24}$

$= \frac{7}{8}$

$9^n - 7^n$ is even

prove true for $n = 1$

LHS $= 9 - 7$

$= 2$

$\therefore$ true for $n = 1$

assume true for $n = k$

ie $9^k - 7^k = 2M$ $m \in \mathbb{Z}^+$

prove true for $n = k+1$

$9^{k+1} - 7^{k+1} = 2N$ $N \in \mathbb{Z}^+$

LHS $= 9^{k+1} - 7^{k+1}$

$=$