Total marks – 120
Attempt Questions 1 – 8
All questions are of equal value

Answer each question on a SEPARATE piece of paper clearly marked Question 1, Question 2, etc. Each piece of paper must show your name.

Question 1 *(15 marks)* Use a *separate* piece of paper — *Marks*

a) Find $\int \frac{\sin\theta d\theta}{\cos^5\theta}$ — 2

b) By completing the square, evaluate $\int_{-1}^{0} \frac{dx}{\sqrt{1-2x-x^2}}$ — 3

c) (i) Find A, B and C if $\frac{16}{(x^2+4)(2-x)} = \frac{Ax+B}{(x^2+4)} + \frac{C}{(2-x)}$ — 2

(ii) Hence find $\int \frac{16}{(x^2+4)(2-x)} dx$ — 2

d) Use the substitution $x = 3\sin\theta$ to find $\int \frac{x^2}{\sqrt{9-x^2}} dx$ — 3

e) Using $t = \tan\frac{x}{2}$ find $\int \frac{dx}{1+\sin x}$ — 3

Question 2 *(15 marks)* Use a *separate* piece of paper

a) Let $z = 2 - i$ and $\omega = 1 + 3i$, find;

(i) $z\overline{\omega}$ — 1

(ii) $\frac{2}{z}$ — 1

b) Let $\alpha = \frac{4i}{-1+i\sqrt{3}}$

(i) Express α in modulus argument form — 2

(ii) Express α^5 in modulus argument form — 2

(iii) Hence express α^5 in the form $x + iy$ — 1

c) Sketch the region on the Argand diagram where the inequalities — 3

$$\frac{\pi}{4} \le \arg(z-i) \le \frac{3\pi}{4} \quad \text{and} \quad |z-i| \le 2$$

hold simultaneously

Question 2 *(continued)* — *Marks*

d) The point A represents the real number $z_1 = 1$. The point B represents the complex number $z_2 = (1+\sqrt{3}) + i$. If $ABCD$ is a square in anti-clockwise rotational order, find;

(i) the complex number z_3 represented by the point D. — 2

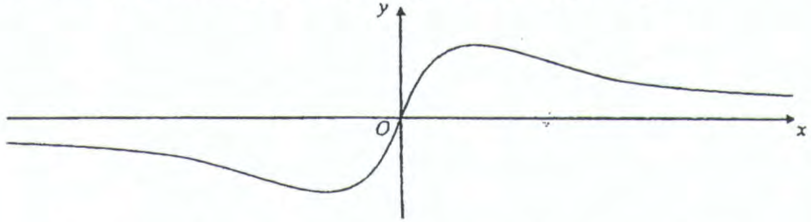
(ii) the complex number z_4 represented by the point C. — 2

(iii) the complex number z_5 represented by the point of intersection of the diagonals AC and BD. — 1

Question 3 *(15 marks)* Use a *separate* piece of paper

a) Sketch $y = \frac{3x^2}{4-x^2}$ showing all asymptotes. — 3

b) The diagram shows the graph of $y = f(x)$ which has range $-1 \le y \le 1$



Draw separate one-third page sketches of the graphs of the following;

(i) $y = |f(x)|$ — 1

(ii) $y = f(x+1)$ — 1

(iii) $y = [f(x)]^2$ — 2

(iv) $y = \frac{1}{f(x)}$ — 2

(v) $y = e^{f(x)}$ — 2

c) It is know that $P(x) = x^4 - 4x^3 + 5x^2 + ax + b$ is divisible by $(x-2)^2$

(i) Find the values of a and b. — 3

(ii) Factorise $P(x)$ into linear factors — 1

Question 4 *(15 marks)* Use a *separate* piece of paper — *Marks*

a) A curve has the equation $x^3 + 2xy - 4y^2 = 10$. — 2

Find an expression for $\dfrac{dy}{dx}$ as a function of x and y.

b) A cylindrical hole of radius 1 cm is bored through the centre of a sphere of radius 3 cm. Using the method of cylindrical shells, calculate the exact volume of the sphere that remains. — 4

c) For the conic $\dfrac{x^2}{25} + \dfrac{y^2}{16-\lambda} = 1$ find ;

(i) the values of λ that makes the conic an ellipse with the foci on the x axis. — 2

(ii) the values of λ that makes the conic a rectangular hyperbola. — 1

(iii) the coordinates of the foci and the equations of the directrices when $\lambda = 0$ — 2

d) How many different ways are there of seating four married couples at a circular table with men and women in alternate positions and no wife next to her husband? — 4

Question 5 *(15 marks)* Use a *separate* piece of paper

a) Suppose α, β and γ are the three roots of the polynomial equation $x^3 + x + 12 = 0$

(i) Find $\alpha^2 + \beta^2 + \gamma^2$ — 2

(ii) Hence explain why only one of the roots is real — 1

(iii) The real root is denoted by α. Prove that $-3 < \alpha < -2$ — 1

(iv) Hence prove that the modulus of each of the other roots lies between 2 and $\sqrt{6}$ — 3

b) A woman of mass M kg jumps vertically (feet first) from a rock ledge into a river below.

When she is falling at v m/s, she encounters air resistance equal to $\dfrac{Mv}{10}$ Newtons.

She hits the water at a speed of V m/s.

Let x be the displacement below the rock ledge at time t seconds after jumping.

(i) Show that $\ddot{x} = g - \dfrac{v}{10}$, where g is the acceleration due to gravity — 1

(ii) If it takes one second for her feet to hit the water, using $g = 10\,\text{m/s}^2$ show that; — 3

$$V = 100\left(1 - e^{-\frac{1}{10}}\right)$$

(iii) Find the height of the rock ledge above the water, to the nearest $0\cdot 1$ metre — 4

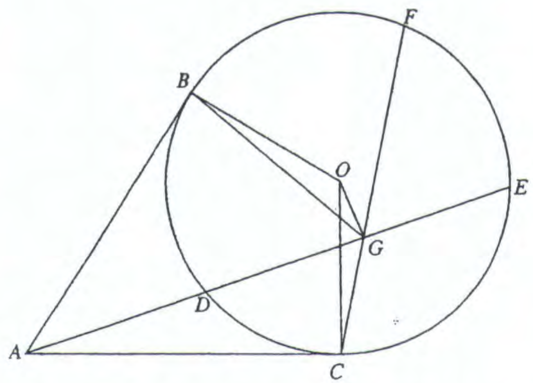
Question 6 *(15 marks)* Use a *separate* piece of paper — *Marks*

a) By considering the series $1 + t + t^2 + t^3 + \cdots + t^n$, or otherwise;

(i) sum the series $1 + 2t + 3t^2 + 4t^3 + \cdots + nt^{n-1}$ — 2

(ii) hence evaluate $1 + 2\times 2 + 3\times 2^2 + 4\times 2^3 + \cdots + 20\times 2^{19}$ — 1

b)



In the diagram, AB and CA are the tangents from A to the circle with centre O, meeting the circle at B and C.

ADE is a secant of the circle. G is the midpoint of DE. CG produced meets the circle at F.

(Note: on page 10 there is a copy of the above diagram that you can use, please detach and include with your solutions)

(i) Show that $ABOC$ is a cyclic quadrilateral — 2

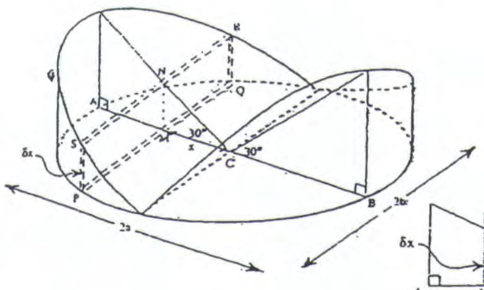
(ii) Show that $AOGC$ is a cyclic quadrilateral — 2

(iii) Hence prove that $BF \parallel AE$ — 3

Question 6 *(continued)* *Marks*

c) The roof of a sports stadium has an elliptical base with a major axis of length $2a$ and minor axis of length $2b$. The two identical sloping tops are inclined at 30° to the base.

$PQRS$ represents a rectangular cross-section of thickness δx taken x units from the centre O of the ellipse.



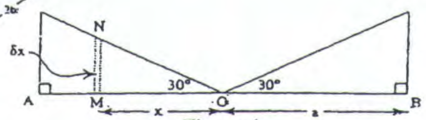


Figure A

Figure A shows a side view of the stadium if sliced in half along AB

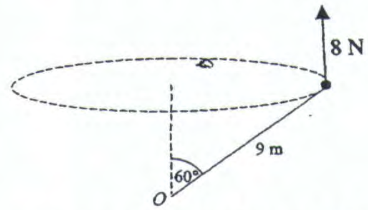
(i) Find the height MN of the rectangular cross-section $PQRS$ — 1

(ii) Show that the area A of the rectangle $PQRS$ is given by $A = \frac{2b}{a\sqrt{3}}x\sqrt{a^2 - x^2}$ — 2

(iii) Calculate the volume of the stadium roof. — 2

Question 7 *(15 marks)* Use a *separate* piece of paper *Marks*

a)



A toy aircraft of mass $0 \cdot 5$ kg is attached to one end of a string of length 9 m. The other end of the string is attached to a fixed point O and moves with constant angular velocity.

The string is taut, and makes an angle of 60° with the upward vertical at O.

In a simplified model of the motion, the aircraft is treated as a particle and the force of the air on the aircraft is taken to act vertically upwards with magnitude 8 Newtons. (Use $g = 10\ \text{m/s}^2$)

(i) Resolve the forces on the aircraft in the horizontal and vertical directions — 2

(ii) Find the tension in the string — 1

(ii) Find the speed of the aircraft in m/s. — 2

b) $P\left(cp, \frac{c}{p}\right)$ is a point on the hyperbola $xy = c^2$

(i) Show that the equation of the tangent at P has the equation $x + p^2 y = 2cp$ — 2

(ii) The tangent at P cuts the x axis and y axis at A and B respectively. Find the coordinates of A and B. — 1

(iii) Q is the fourth vertex of the rectangle $OAQB$. Show that the locus of Q is another rectangular hyperbola. — 2

c) (i) Let $I_n = \int \tan^n \theta \, d\theta$, show that $I_n = \frac{1}{n-1}\tan^{n-1}\theta - I_{n-2}$ — 3

(ii) Hence find the exact value of $\int_0^{\frac{\pi}{4}} \tan^4 \theta \, d\theta$ — 2

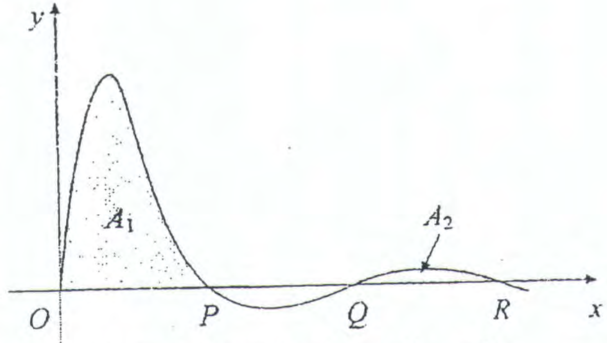
Question 8 *(15 marks)* Use a *separate* piece of paper

Marks

a) (i) Given that $y = x\sin x$, find $\dfrac{d^2y}{dx^2}$ 1

(ii) Prove by induction that $\dfrac{d^{2n}y}{dx^{2n}} = (-1)^n(x\sin x - 2n\cos x)$ 3

b)



The diagram shows a sketch of part of the curve C with equation

$$y = e^{-x}\sin x, \quad x \geq 0$$

(i) Find the coordinates of the points P, Q and R where C cuts the x axis. 1

(ii) Use integration by parts to show that; 3

$$\int e^{-x}\sin x\,dx = -\frac{1}{2}e^{-x}(\sin x + \cos x) + c$$

(iii) The terms $A_1, A_2, \ldots, A_n$ represent areas between C and the x axis for successive portions of C where y is positive. The areas represented by A_1 and A_2 are shown in the diagram. 2

Show that $A_n = \frac{1}{2}\left(e^{(1-2n)\pi} + e^{(2-2n)\pi}\right)$

(iv) Show that $A_1 + A_2 + A_3 + \cdots$ is a geometric series and that $S_\infty = \dfrac{e^\pi}{2(e^\pi - 1)}$ 3

(v) Given that $\int_0^\infty e^{-x}\sin x\,dx = \frac{1}{2}$, find the exact value of 2

$$\int_0^\infty \left|e^{-x}\sin x\right| dx$$

Question 1 (15)

a) $\int \frac{\sin\theta\,d\theta}{\cos^5\theta}$ $\quad u = \cos\theta$, $du = -\sin\theta\,d\theta$

$= -\int \frac{du}{u^5}$

$= \frac{1}{4u^4} + c$

$= \frac{1}{4\cos^4\theta} + c$ (2)

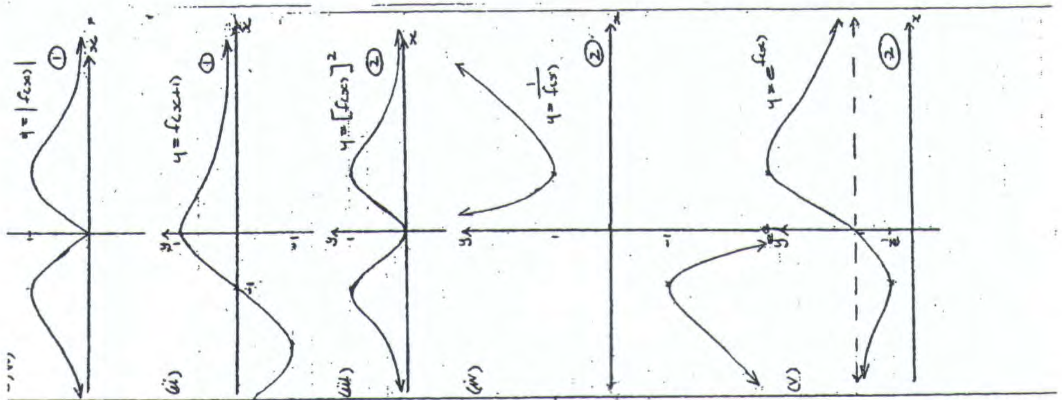
b) $\int_{-1}^{0} \frac{dx}{\sqrt{1-2x-x^2}}$

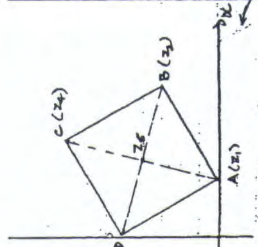
$= \int_{-1}^{0} \frac{dx}{\sqrt{2-(x+1)^2}}$

$= \left[\sin^{-1}\left(\frac{x+1}{\sqrt{2}}\right)\right]_{-1}^{0}$

$= \sin^{-1}\frac{1}{\sqrt{2}} - \sin^{-1}0$

$= \frac{\pi}{4}$ (3)





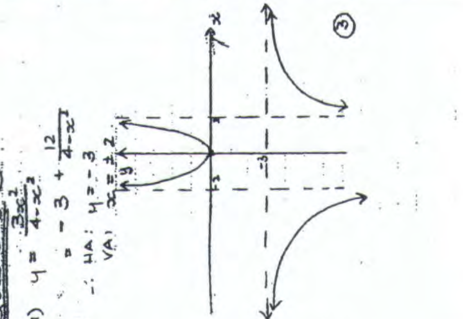
Question 3 (15)

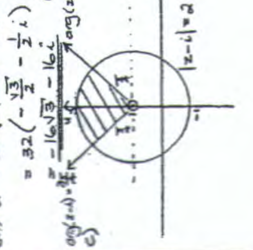
a) $y = \frac{3x^2}{4-x^2}$

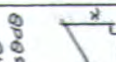
$= -3 + \frac{12}{4-x^2}$

HA: $y = -3$

VA: $x = \pm 2$







c)(i) $P(x)=x^4-4x^3+5x^2+ax+b$

$P'(x)=4x^3-12x^2+10x+a$

$P(2)=0$

$16-32+20+2a+b=0$

$2a+b=-4$

$P'(2)=0$

$32-48+20+a=0$

$a=-4,\ \therefore b=4$ ③

(ii) $P(x)=x^4-4x^3+5x^2-4x+4$

$=(x^2-4x+4)(x^2+1)$

$=(x-2)^2(x+i)(x-i)$ ①

Question 4 ⑮

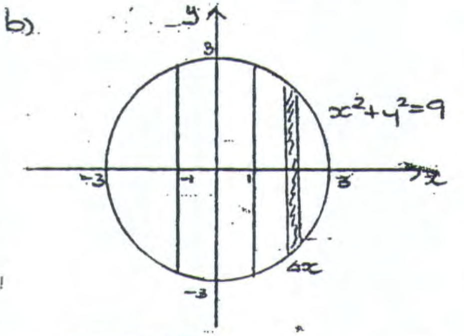
a) $x^3+2xy-4y^2=10$

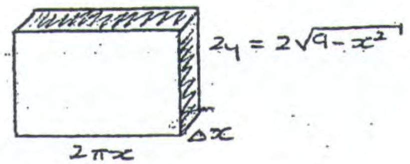
$3x^2+(2x)\frac{dy}{dx}+(y)(2)-8y\frac{dy}{dx}=0$

$(2x-8y)\frac{dy}{dx}=-3x^2-2y$

$\frac{dy}{dx}=\frac{3x^2+2y}{8y-2x}$ ②

b)





$A(x)=4\pi x\sqrt{9-x^2}$

$\Delta V=4\pi x\sqrt{9-x^2}\,\Delta x$

$V=\lim_{\Delta x\to 0}\sum_{x=1}^{3}4\pi x\sqrt{9-x^2}\,\Delta x$

$=2\pi\int_1^3 2x\sqrt{9-x^2}\,dx$

$=-2\pi\left[\frac{2}{3}(9-x^2)^{\frac{3}{2}}\right]_1^3$

$=-\frac{4\pi}{3}(0-8\sqrt{8})$ ④

c) ...

$-16<-\lambda<9$

$-9<\lambda<16$ ②

(ii) $16-\lambda=-25$

$\lambda=41$ ①

(iii) $\frac{x^2}{25}+\frac{y^2}{16}=1$

$a^2=25$ $\quad b^2=a^2(1-e^2)$

$a=5$ $\quad 16=25(1-e^2)$

$1-e^2=\frac{16}{25}$

$e^2=\frac{9}{25}$

$e=\frac{3}{5}$ ②

$\therefore$ foci $(\pm 3,0)$ directrices $x=\pm\frac{25}{3}$

d) Place men first = 3!

A, B, D, C, (W_A)

Wife A has 2 spots this leaves 1 spot for each of the other wives

Ways = $3!\times 2\times 1\times 1\times 1$

$=12$ ④

Question 5 ⑮

a)(i) $x^3+x+12=0$

$\sum\alpha^2=(\sum\alpha)^2-2\sum\alpha\beta$

$=0^2-2(1)$

$=-2$ ②

(ii) as $\sum\alpha^2<0$ there must be an imaginary root. However when coefficients are real, roots appear in conjugate pairs ie there are two imaginary roots ∴ there is one real root ①

(iii) $P(-3)=-27-3+12=-18<0$ $\quad P(-2)=-8-2+12=2>0$

as polynomial is continuous $-3<\alpha<-2$ ①

(iv) $\alpha\beta\gamma=-12$

$4<\beta\gamma<6$

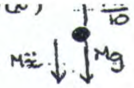
$\beta\gamma$ are conjugates

$z\bar{z}=|z|^2$

$\therefore 4<|z|^2<6$

$2<|z|<\sqrt{6}$ ③

b)(i)



$M\ddot{x}=Mg-\frac{Mv}{10}$

$\ddot{x}=g-\frac{v}{10}$ ①

(ii) $\frac{dv}{dt}=\frac{10g-v}{10}$

$t=-10\int_0^V\frac{-dv}{10g-v}$

when $t=1$, $v=V$

$1=-10\int_0^V\frac{-dt}{10g-v}$

$-\frac{1}{10}=\left[\log(10g-v)\right]_0^V$

$=\log\left(\frac{10g-V}{10g}\right)$

$e^{-\frac{1}{10}}=\frac{10g-V}{10g}$

$10g\,e^{-\frac{1}{10}}=10g-V$

$V=10g(1-e^{-\frac{1}{10}})$ ③

$V=100(1-e^{-\frac{1}{10}})$ ($\because g=10$)

(iii) $v\frac{dv}{dx}=\frac{10g-v}{10}$

$x=10\int_0^V\frac{v\,dv}{10g-v}$

$x=-10\int_0^V\left(1-\frac{10g}{10g-v}\right)dv$

$=-10\left[v+10g\log(10g-v)\right]_0^V$

$=-10\left(V+10g\log\left(\frac{10g-V}{10g}\right)\right)$

$=-10\left(100(1-e^{-\frac{1}{10}})+100(-\frac{1}{10})\right)$

$=4.8\text{m}$ ④

Question 6 ⑮

a)(i) $1+t+t^2+t^3+\dots+t^n=\frac{t^n-1}{t-1}$

$t+2t+3t^2+\dots+nt^{n-1}=\frac{(t-1)(nt^{n-1})-(t^n-1)(1)}{(t-1)^2}$

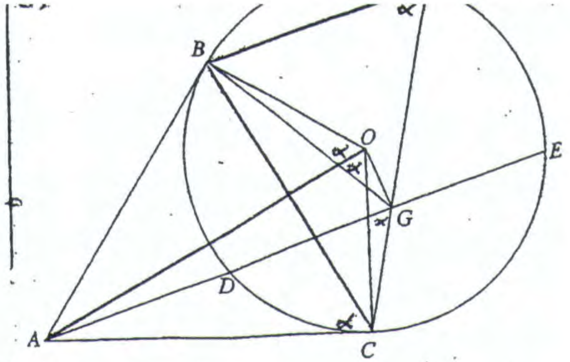
$=\frac{(n-1)t^n-nt^{n-1}+1}{(t-1)^2}$ ②

(ii) $t=2,\ n=20$

$1+2\times2+3\times2^2+\dots+20\times2^{19}$

$=19\times2^{20}-20\times2^{19}+1$

$=9\,437\,185$ ①



(i) $\angle OBA=90°$ (radius ⊥ tangent)

$\angle OCA=90°$ (")

∴ ABOC is cyclic quadrilateral (opposite ∠'s supplementary) ②

(ii) $\angle OGA=90°$ (⊥ centre bisects chord)

$\therefore \angle OGA=\angle OCA$

AOGC is cyclic quadrilateral (∠'s in same segment =) ②

(iii) Let $\angle BFC=\alpha$

$\angle BOC=2\alpha$ (∠ at centre twice ∠ at circumference on same arc)

$\angle BCA=\alpha$ (∠ in alternate segment)

$\therefore\angle BOA=\alpha$ (∠'s in same segment = in ABOC)

$\angle BOC=\angle AOB+\angle AOC$ (common ∠)

$2\alpha=\alpha+\angle AOC$

$\angle AOC=\alpha$

$\angle AOC=\angle AGC$ (∠'s in same segment = in AOGC)

$\therefore\angle AGC=\alpha$

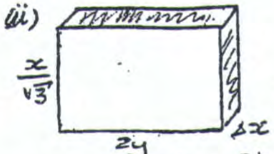
$\angle AGC=\angle BFC=\alpha$

∴ BF ∥ AE (corresponding ∠'s =) ③

c) $\frac{MN}{x}=\tan 30°$

$MN=\frac{x}{\sqrt{3}}$ ①

(ii)



$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

$a^2y^2=a^2b^2-b^2x^2$

$y^2=\frac{b^2(a^2-x^2)}{a^2}$

$A(x)=\frac{x}{\sqrt{3}}\times\frac{2b\sqrt{a^2-x^2}}{a}$

$=\frac{2b}{a\sqrt{3}}x\sqrt{a^2-x^2}$ ②

(iii) $\Delta V = \frac{2b}{a\sqrt{3}} x\sqrt{a^2-x^2}\, \Delta x$

$$V = \lim_{\Delta x \to 0} \sum_{x=-a}^{a} \frac{2b}{a\sqrt{3}} x\sqrt{a^2-x^2}\,\Delta x$$

$$= \frac{2b}{a\sqrt{3}} \int_0^a 2x\sqrt{a^2-x^2}\,dx$$

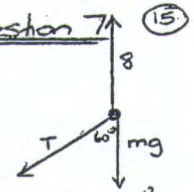
$$= \frac{-2b}{a\sqrt{3}} \left[\frac{2}{3}(a^2-x^2)^{\frac{3}{2}} \right]_0^a$$

$$= \frac{4b}{3a\sqrt{3}}\left(a^3\right)$$

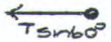
$$= \frac{4a^2b}{3\sqrt{3}} \text{ units}^3 \quad (2)$$

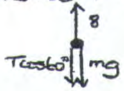
Question 7 (15)

a)



horizontal $F = \frac{mv^2}{r}$ vertical $F = 0$





$T\sin 60° = \frac{mv^2}{r}$

$T\cos 60° + mg - 8 = 0$ (2)

(ii) $\frac{T}{2} = 8 - (0.5)(10)$

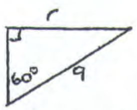
$T = 6\ N$ (1)

(iii) $\frac{\sqrt{3}T}{2} = \frac{mv^2}{r}$

$\frac{\sqrt{3}(6)}{2} = \frac{(0.5)v^2}{\frac{9\sqrt{3}}{2}}$

$v^2 = 3\sqrt{3} \times \frac{9\sqrt{3}}{2} \times 2$

$= 81$

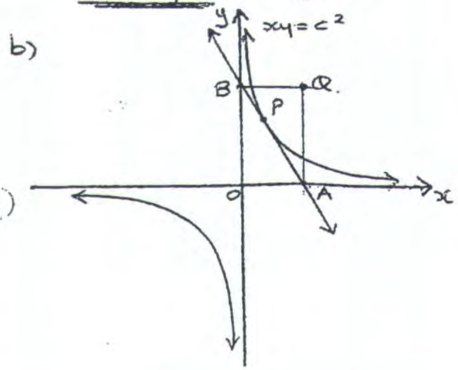


$\frac{r}{9} = \sin 60°$

$r = \frac{9\sqrt{3}}{2}$

$v = 9\ m/s$ (2)

b)



$y = \frac{c^2}{x}$

$\frac{dy}{dx} = \frac{-c^2}{x^2}$

when $x = cp$, $\frac{dy}{dx} = -\frac{1}{p^2}$

$y - \frac{c}{p} = -\frac{1}{p^2}(x - cp)$

$p^2y - pc = -x + cp$

$x + p^2y = 2cp$ (2)

(ii) A: $y = 0$ B: $x = 0$

$\therefore x = 2cp$ $p^2y = 2cp$

$y = \frac{2c}{p}$

$A(2cp, 0)$ $B: (0, \frac{2c}{p})$ (1)

(iii) $Q(2cp, \frac{2c}{p})$

$xy = 2cp \times \frac{2c}{p}$

$xy = 4c^2$

$\therefore$ locus of Q is $xy = 4c^2$ (2)

c)(i) $I_n = \int \tan^n\theta\, d\theta$

$= \int \tan^{n-2}\theta(\sec^2\theta - 1)\,d\theta$

$= \int \tan^{n-2}\theta \sec^2\theta\, d\theta - \int \tan^{n-2}\theta\, d\theta$ $u = \tan\theta$

$= \int u^{n-2}\,du - I_{n-2}$ $du = \sec^2\theta\, d\theta$

$= \frac{u^{n-1}}{n-1} - I_{n-2}$

$= \frac{1}{n-1}\tan^{n-1}\theta - I_{n-2}$ (3)

(ii) $\int_0^{\frac{\pi}{4}} \tan^4\theta = I_4$

$= \left[\frac{1}{3}\tan^3\theta\right]_0^{\frac{\pi}{4}} - I_2$

$= \left[\frac{1}{3}\tan^3\theta - \tan\theta\right]_0^{\frac{\pi}{4}} + I_0$

$= \left[\frac{1}{3}\tan^3\theta - \tan\theta\right]_0^{\frac{\pi}{4}} + \int_0^{\frac{\pi}{4}} d\theta$

$= \left[\frac{1}{3}\tan^3\theta - \tan\theta + \theta\right]_0^{\frac{\pi}{4}}$

$= \frac{1}{3} - 1 + \frac{\pi}{4}$

$= \frac{\pi}{4} - \frac{2}{3}$

$= \frac{3\pi - 8}{12}$ (2)

Question 8

a)(i) $y = x\sin x$

$\frac{dy}{dx} = (x)(\cos x) + (\sin x)(1)$

$= x\cos x + \sin x$

$\frac{d^2y}{dx^2} = (x)(-\sin x) + (\cos x)(1) + \cos x$

$= 2\cos x - x\sin x$ (1)

(ii) Prove true for $n=1$

LHS $= \frac{d^2y}{dx^2}$ RHS $= (-1)(x\sin x - 2\cos x)$

$= 2\cos x - x\sin x$ $= 2\cos x - x\sin x$

$\therefore$ LHS = RHS

Hence the result is true for $n=1$

Assume the result is true for $n=k$

ie $\frac{d^{2k}y}{dx^{2k}} = (-1)^k(x\sin x - 2k\cos x)$

Prove the result is true for $n=k+1$

ie $\frac{d^{2k+2}y}{dx^{2k+2}} = (-1)^{k+1}(x\sin x - 2(k+1)\cos x)$

Proof:

$\frac{d^{2k+1}y}{dx^{2k+1}} = (-1)^k\left[(x)(\cos x) + \sin x + 2(\ldots)\right]$

$= (-1)^k(x\cos x + (2k+1)\sin x)$

$\frac{d^{2k+2}y}{dx^{2k+2}} = (-1)^k\left[(x)(-\sin x) + (\cos x)(1) + (2k+1)\cos x\right]$

$= (-1)^k(-x\sin x + (2k+2)\cos x)$

$= (-1)^{k+1}(x\sin x - 2(k+1)\cos x)$

Hence the result is true for $n=k+1$ if it is also true for $n=k$.

Hence the result is true for all positive integral values of n by induction. (3)

b)(i) $e^{-x}\sin x = 0$

$\sin x = 0$

$x = 0, \pi, 2\pi, \ldots$ (1)

$\therefore P(\pi, 0), Q(2\pi, 0), R(3\pi, 0)$

(ii) $\int e^{-x}\sin x\, dx$

$u = e^{-x}$ $v = -\cos x$

$du = -e^{-x}dx$ $dv = \sin x\, dx$

$= -e^{-x}\cos x - \int e^{-x}\cos x\, dx$

$u = e^{-x}$ $v = \sin x$

$du = -e^{-x}dx$ $dv = \cos x\, dx$

$= -e^{-x}\cos x - e^{-x}\sin x - \int e^{-x}\sin x\, dx$

$\therefore 2\int e^{-x}\sin x\, dx = -e^{-x}(\sin x + \cos x) + c$

$\int e^{-x}\sin x\, dx = -\frac{1}{2}e^{-x}(\sin x + \cos x) + c$ (3)

(iii) $A_1 = \int_0^{\pi} e^{-x}\sin x\, dx$

$A_2 = \int_{2\pi}^{3\pi} e^{-x}\sin x\, dx$

$\therefore A_n = \int_{(2n-2)\pi}^{(2n-1)\pi} e^{-x}\sin x\, dx$

$= -\frac{1}{2}\left[e^{-x}(\sin x + \cos x)\right]_{(2n-2)\pi}^{(2n-1)\pi}$

$= -\frac{1}{2}\left(-e^{-(2n-1)\pi} - e^{-(2n-2)\pi}\right)$

$= \frac{1}{2}\left(e^{(2-2n)\pi} + e^{(1-2n)\pi}\right)$ (2)

(iv) $A_1 = \frac{1}{2}(e^{-\pi} + 1)$

$A_2 = \frac{1}{2}(e^{-3\pi} + e^{-2\pi})$

$A_3 = \frac{1}{2}(e^{-5\pi} + e^{-4\pi})$

$\therefore$ geometric series $r = e^{-2\pi}$

$S_\infty = \frac{\frac{1}{2}(e^{-\pi} + 1)}{1 - e^{-2\pi}}$

$= \frac{e^{\pi} + e^{2\pi}}{2(e^{2\pi} - 1)}$

$= \frac{e^{\pi}(1 + e^{\pi})}{2(e^{\pi}+1)(e^{\pi}-1)}$

$= \frac{e^{\pi}}{2(e^{\pi} - 1)}$ (3)

A_n = areas above x axis

(v) Let $A_1 + A_2 + A_3 + \ldots = X$

a_n = areas under x axis

$a_1 + a_2 + a_3 + \ldots = Y$

$\therefore \int_0^{\infty} e^{-x}\sin x\, dx = X - Y$

$\frac{1}{2} = \frac{e^{\pi}}{2(e^{\pi}-1)} - Y$

$Y = \frac{e^{\pi} - (e^{\pi} - 1)}{2(e^{\pi} - 1)}$

$= \frac{1}{2(e^{\pi} - 1)}$

$\int_0^{\infty} \left|e^{-x}\sin x\right| dx = X + Y$

$= \frac{e^{\pi}}{2(e^{\pi}-1)} + \frac{1}{2(e^{\pi}-1)}$

$= \frac{e^{\pi} + 1}{2(e^{\pi} - 1)}$ (2)