STUDENT NUMBER: ________________

THE HILLS GRAMMAR SCHOOL

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

2008

MATHEMATICS

EXTENSION 2

Teacher Responsible: Mr R Judge

General Instructions:

- Reading time – 5 minutes
- Working time – 3 hours
- Write using blue or black pen.
- Marks may be deducted for careless, untidy or badly arranged work.
- Board approved calculators may be used.
- A table of standard integrals is supplied at the back of this paper.
- ALL necessary working should be shown in every question.
- **Start each question in a new booklet.**

Total marks – 120

- Attempt Questions 1-8
- All questions are of equal value.

Students are advised that this is a Trial Examination only and cannot in any way guarantee the content or the format of the Higher School Certificate Examination.

Question 1. (15 marks)

a) Find $\int \frac{1}{6x - x^2} dx$ 2

b) Find the exact value of k if $\int_1^{k^2} \frac{e^{\sqrt{x}}}{\sqrt{x}} dx = 6e$ 4

c) If $I_n = \int_1^e x(\log x)^n dx$ where n is a positive integer

i) Show that $I_n = \frac{e^2}{2} - \frac{n}{2} I_{n-1}$ 3

ii) Hence evaluate $\int_1^e x(\log x)^3 dx$ 2

d) Find $\int_0^{\frac{\pi}{3}} \frac{dx}{2 + \cos 2x}$ using the substitution $u = tan\, x$ 4

Question 2 (15 marks)

a) Given $z = 1 - i\sqrt{3}$

i) Write z in modulus - argument form. 1

ii) Hence find z^8 in the form $x + iy$ where x and y are real. 2

iii) Find the least positive value of n such that z^n is real. 2

b) Sketch each of the following regions on a separate Argand diagram

i) $|z - 2 - i| \le 2$ 2

ii) $0 < \arg\left[(1+\mathrm{i})z\right] \le \frac{\pi}{2}$ 2

c) OABC is a square. O represents the complex number 0
A represents the complex number $3 + i$, B represents a complex number z and C represents the complex number w.
D is the point of intersection of the diagonals.

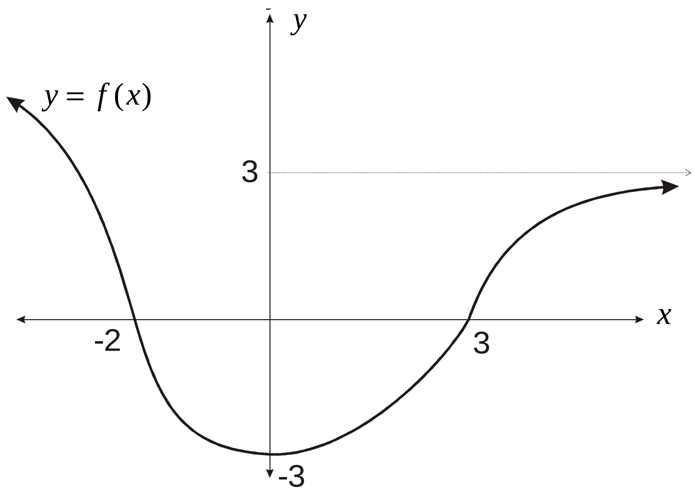
i) Find the complex numbers corresponding to points C and D in the form $x + iy$. 2

ii) Find $\arg\left(\frac{w}{z}\right)$ 2

iii) If E is the fourth vertex of the parallelogram OAEB find the complex number corresponding to E. 2

Question 3 (15 marks)

a) The diagram shows the graph of $y = f(x)$



Sketch graphs of

i) $y = |f(x)|$ 1

ii) $y = \dfrac{1}{f(x)}$ 2

iii) $y^2 = f(x)$ 2

iv) the inverse function $y = f^{-1}(x)$ where the domain of $f(x)$ is restricted to $x \geq 0$ 2

b) For the curve defined by $3x^2 + y^2 - 2xy - 8x + 2 = 0$

i) Show that $\dfrac{dy}{dx} = \dfrac{3x - y - 4}{x - y}$ 2

ii) Find the coordinates of the points on the curve where the tangent to the curve is parallel to the line $y = 2x$ 3

c) Akram estimates that the probability of his winning any one game of tennis against a particular opponent is $\frac{1}{3}$. How many games should they play so that the probability that Akram wins at least one game is greater than 0.9? 3

Question 4 (15 marks)

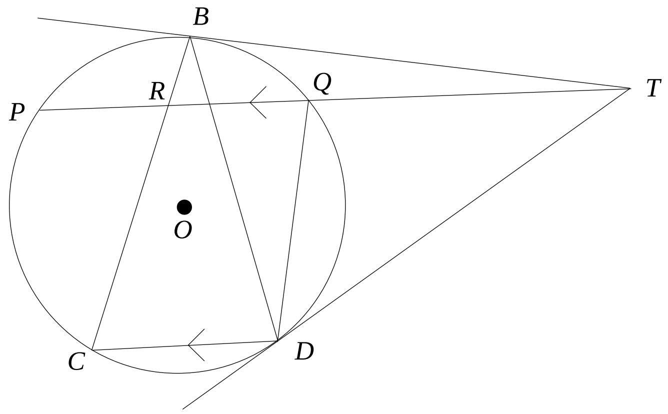
a) i) Find the modulus and argument of the complex number $1+i$ 1

ii) Use the binomial expansion for $(1+i)^n$, where n is a positive integer, to show that 5

α) $1-{}^nC_2+{}^nC_4-\ldots=2^{n/2}\cos\dfrac{n\pi}{4}$

β) ${}^nC_1-{}^nC_3+{}^nC_5-\ldots=2^{n/2}\sin\dfrac{n\pi}{4}$

b)



PQ, CD are parallel chords of a circle, centre O. The tangent at D meets PQ extended at T. B is the point of contact of the other tangent from T. BC meets PQ at R

i) Copy the diagram.

ii) Prove that $\angle BDT=\angle BRT$ and hence state why B, T, D and R are concyclic points 3

iii) Prove $\angle BRT=\angle DRT$. 3

iv) Show that ΔRCD is isosceles. 3

Question 5. (15 marks)

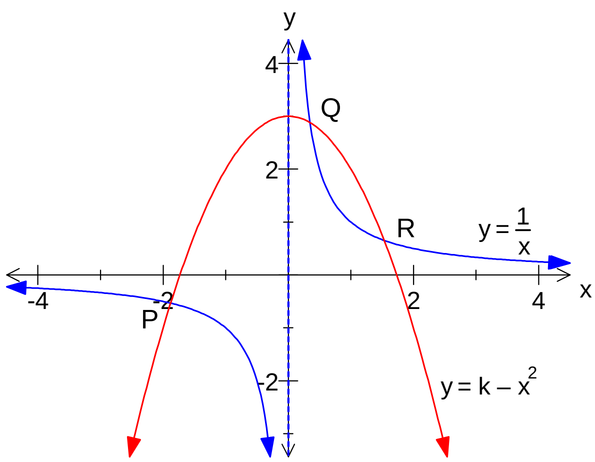
a) i) By considering factors, or otherwise show, that the roots of $z^6 - z^3 + 1 = 0$ are among the roots of $z^9 + 1 = 0$. 1

ii) By selecting the appropriate roots of $z^9 + 1 = 0$, or otherwise, show that 4

$$z^6 - z^3 + 1 = \left(z^2 - 2z\cos\frac{\pi}{9} + 1\right)\left(z^2 - 2z\cos\frac{5\pi}{9} + 1\right)\left(z^2 - 2z\cos\frac{7\pi}{9} + 1\right)$$

iii) Show that $\cos\frac{\pi}{9}\cos\frac{5\pi}{9} + \cos\frac{5\pi}{9}\cos\frac{7\pi}{9} + \cos\frac{7\pi}{9}\cos\frac{\pi}{9} = -\frac{3}{4}$ 3

b)



The curves $y = k - x^2$ for some real k, and $y = \frac{1}{x}$ intersect at the points P, Q and R where $x = \alpha, x = \beta$ and $x = \gamma$ respectively.

i) Show that the monic equations with coefficients in terms of k whose roots are $\alpha^2, \beta^2, \gamma^2$ is given by $x^3 - 2kx^2 + k^2x - 1 = 0$. 3

ii) Show the monic equation, with coefficients in terms of k, whose roots are $\frac{1}{\alpha^2}, \frac{1}{\beta^2}$ and $\frac{1}{\gamma^2}$ is $x^3 - k^2x^2 + 2kx - 1 = 0$ 2

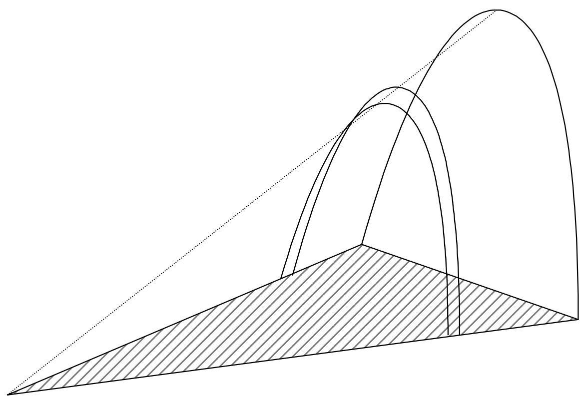
iii) Hence show that $OP^2 + OQ^2 + OR^2 = k^2 + 2k$, where O is the origin. 2

Question 6. (15 marks)

a) The region R in the first quadrant such that $y \le 4x^2 - x^4$ is rotated about the y axis to form a solid of revolution. Use the method of decomposition into cylindrical shells to show that the volume of the solid is $\dfrac{32\pi}{3}$ cubic units 6

b i) Show that the area bounded by the parabola $x^2 = 4ay$ and the latus rectum $y = a$ is equal to $\dfrac{8a^2}{3}$ 3

ii) A particular solid has a triangular base with all sides 6 metres. 6
Cross sections taken parallel to one side of the base are parabolas. Each parabolic cross-section is such that its latus rectum lies in the base of the solid.
Find the volume of the solid.



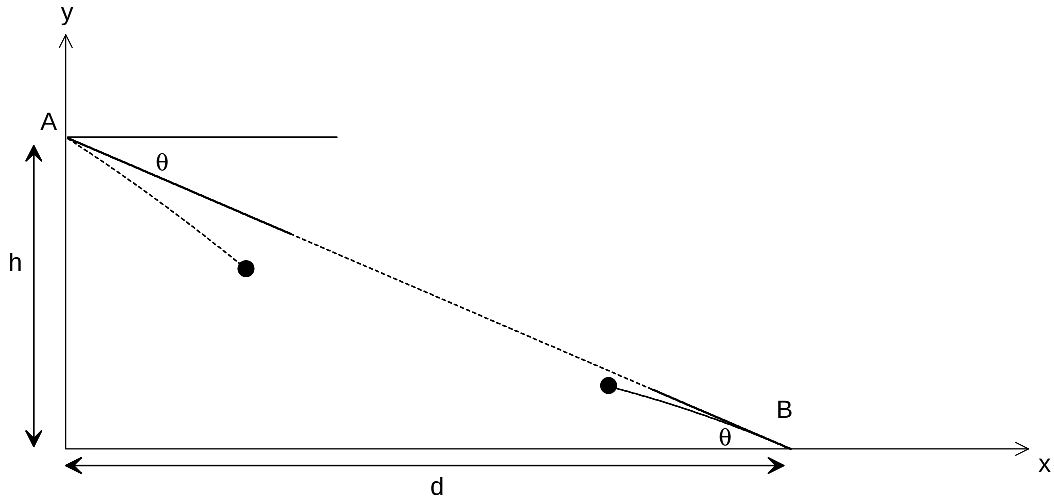
Question 7. (15 marks)

a) A particle of mass 1 kg is projected vertically upwards under gravity with a speed of $2c$ in a medium in which the resistance to motion is $\frac{g}{c^2}$ times the square of the speed, where c is a positive constant.

i) Show that the maximum height (H) reached is $H = \frac{c^2}{2g}\ln 5$. 3

ii) Show that the speed with which the particle returns to its starting point is given by $v = \frac{2c}{\sqrt{5}}$ 4

b) The diagram shows the point A at a height h vertically above the point O. It also shows the point B which is positioned at a horizontal distance d from O.
A projectile is fired from A directly at point B with a Velocity V. At the same instant a projectile is fired from B directly at the point A with the same velocity V.



Let θ be the angle between the horizontal and the angle of projection.

Show that

i) The equations of motion of the two projectiles are 3

$$x_A = Vt\cos\theta \qquad x_B = d - Vt\cos\theta$$

$$y_A = h - Vt\sin\theta - \frac{gt^2}{2} \qquad y_B = Vt\sin\theta - \frac{gt^2}{2}$$

ii) Show the particles will always meet 3

ii) Show that the height H at which they meet is given by. 2

$$H = \frac{h}{2} - \frac{g\left(h^2 + d^2\right)}{8V^2}$$

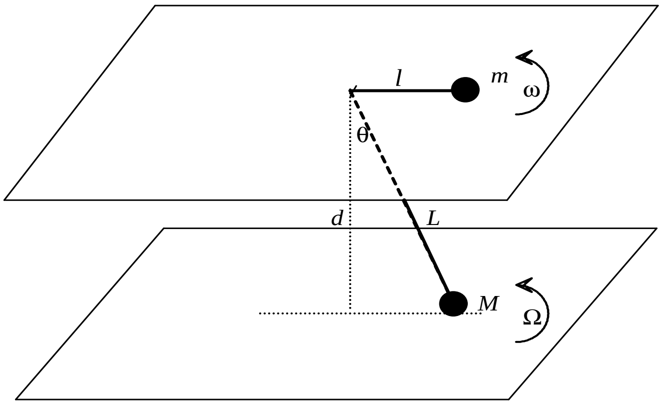
Question 8. (15 marks)

a) Given that $\sin^{-1} x$, $\cos^{-1} x$ and $\sin^{-1}(1-x)$ are acute

i) show that $\sin\left(\sin^{-1} x - \cos^{-1} x\right) = 2x^2 - 1$ 2

ii) solve the equation $\sin^{-1} x - \cos^{-1} x = \sin^{-1}(1-x)$ 3

b)



Two particles are connected by a light inextensible string which passes through a small hole with smooth edges in a smooth horizontal table. One particle of mass m travels in a circle on the table with constant angular motion ω. The second particle of mass M travels in a circle with constant angular velocity Ω on a smooth horizontal floor distance d below the table. The lengths of string on the table and below the table are l and L respectively and L makes an angle θ with the vertical. 10

i) draw diagrams showing the forces acting on each particle.

ii) If the floor exerts a force N on the lower particle, show $N = M\left(g - d\Omega^2\right)$.
State the maximum possible value of Ω for the motion to continue as described. What happens if Ω exceeds this value?

iii) By considering the tension force in the string, show $\dfrac{L}{l} = \dfrac{m}{M}\left(\dfrac{\omega}{\Omega}\right)^2$

iv) If the lower particle exerts zero force on the floor, show that the tension T in the string is given by $T = \dfrac{MgL}{d}$.

v) Given the table is 80cm high and the string is 1.5m long, while the masses on the table and on the floor are 0.4kg and 0.2kg respectively. The particles are observed to have the same angular velocity. If the lower particle exerts zero force on the floor, find, in terms of g the tension in the string.

END OF PAPER

The Hills Grammar Extension 2 Trial 2008 Solutions

Question 1

a) $\int \frac{1}{6x-x^2}dx = \int \frac{dx}{9-(x-3)^2}$

$= \frac{1}{6}\log\frac{x}{6-x}$

b) $\int_1^{k^2} \frac{e^{\sqrt{x}}}{\sqrt{x}}dx = \left[2e^{\sqrt{x}}\right]_1^{k^2}$

$= 2e^{|k|} - 2e = 6e$

$e^{|k|} = 4e$

$k = \pm \ln 4e$

c) i) $I_n = \int_1^e x(\ln x)^n dx = \int_1^e (\ln x)^n \frac{d}{dx}\left(\frac{x^2}{2}\right)dx$

$= \left[\frac{x^2}{2}\ln x\right]_1^e - \int_1^e \frac{x^2}{2}n(\ln x)^{n-1} \times \frac{1}{x}dx$

$= \frac{e^2}{2} - \frac{n}{2}\int_1^e x(\ln x)^{n-1}dx$

$= \frac{e^2}{2} - \frac{n}{2}I_{n-1}$

ii) $I_0 = \int_1^e x dx = \frac{e^2}{2} - \frac{1}{2}$

$I_1 = \frac{e^2}{2} - \frac{1}{2}I_0 = \frac{e^2}{4} + \frac{1}{4}$

$I_2 = \frac{e^2}{2} - I_1 = \frac{e^2}{4} - \frac{1}{4}$

$I_3 = \frac{e^2}{2} - \frac{3}{2}\left(\frac{e^2}{4} - \frac{1}{4}\right)$

$= \frac{e^2}{8} + \frac{3}{8}$

d) $\int_0^{\frac{\pi}{3}} \frac{1}{2+\cos 2x}dx$

let $u = \tan x$ $\qquad \cos 2x = \frac{1-u^2}{1+u^2}$

$du = \sec^2 x\, dx$

$dx = \frac{du}{1+u^2}$

$x = 0 \Rightarrow u = 0$

$x = \frac{\pi}{3} \Rightarrow u = \sqrt{3}$

$= \int_0^{\sqrt{3}} \frac{du}{2+2u^2+1-u^2} = \int_0^{\sqrt{3}} \frac{du}{3+u^2}$

$= \left[\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{u}{\sqrt{3}}\right)\right]_0^{\sqrt{3}} = \frac{1}{\sqrt{3}} \times \frac{\pi}{4} - 0 = \frac{\pi\sqrt{3}}{12}$

Question 2.

a) i) $z = 1 - i\sqrt{3}$

$z = 2cis\left(-\frac{\pi}{3}\right)$ /

ii) $z^8 = 2^8 cis\left(\frac{-8\pi}{3}\right)$ /

$= 2^8 cis\left(\frac{-2\pi}{3}\right)$

$= 2^8\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)$ /

$= -2^7 - 2^7\sqrt{3}i = -128 - 128\sqrt{3}\,i$

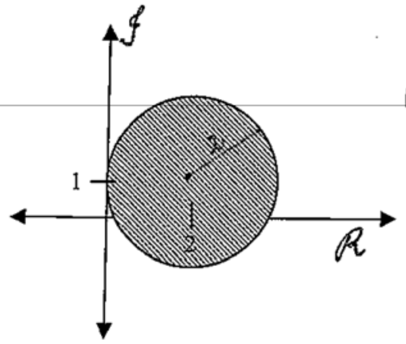
iii) $z^n = 2^n cis\left(-\frac{n\pi}{3}\right)$

real when $\frac{n\pi}{3} = k\pi$ /

for $k = 1$ $n = 3$ /

$\Rightarrow z^3 = -8$

b) i)



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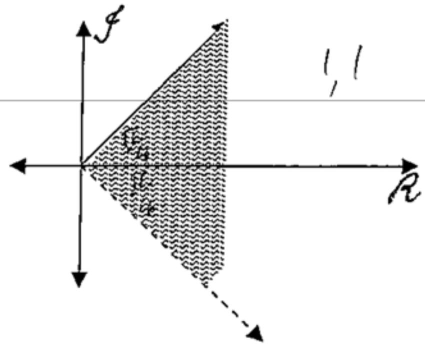
ii) $0 < arg(1+i) + arg\, z \le \frac{\pi}{2}$

$-\frac{\pi}{4} < arg\, z \le \frac{\pi}{4}$



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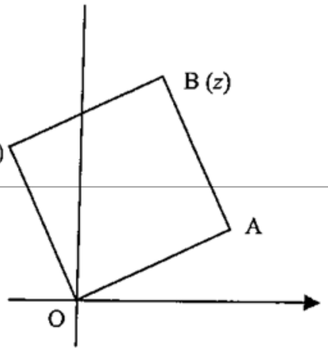
c)



i) $\omega = i\overrightarrow{0A}$

$= -1+3i$

D is mid pt CA

$= (1, 2i)$

ii) $\arg\left(\frac{\omega}{z}\right) = \arg w - \arg z$

$= \mathrm{B\hat{O}C} = \frac{\pi}{4}$

iii) $z = 2\times\overrightarrow{\mathrm{OD}}$

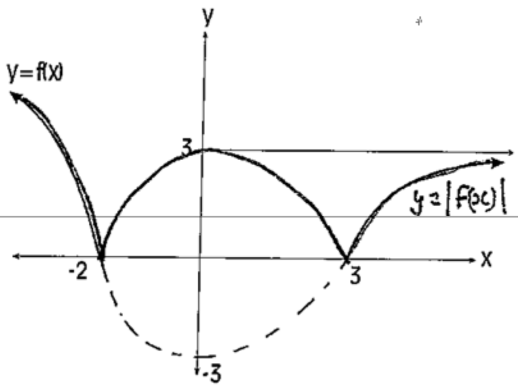
$\overrightarrow{\mathrm{OE}} = \overrightarrow{\mathrm{OA}} + \overrightarrow{\mathrm{DB}}$

$= \binom{3}{1} + 2\binom{1}{2}$

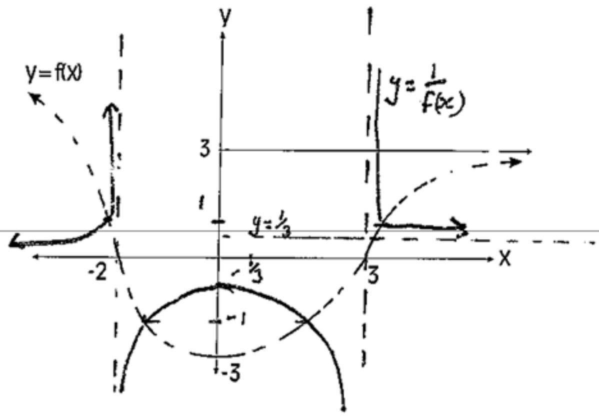
$= \binom{5}{5} \therefore$ E is $5+5i$

Question 3

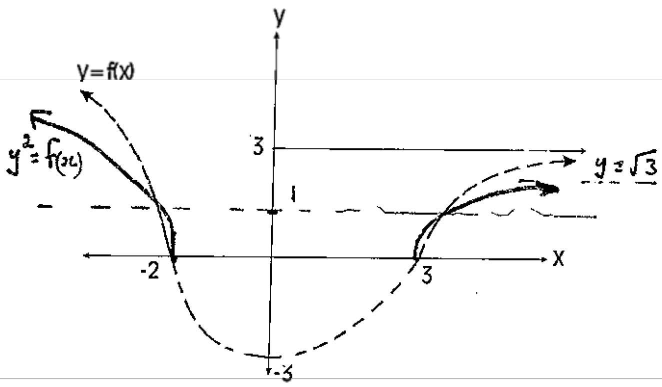
a)i)



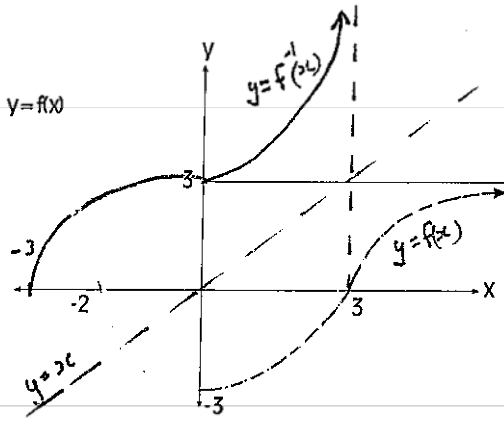
ii)



iii)



iv)



3b i)

$3x^2 + y^2 - 2xy - 8x + 2 = 0$

$\therefore 6x + 2y\dfrac{dy}{dx} - 2x\dfrac{dy}{dx} - 2y - 8 = 0$

$\therefore \dfrac{dy}{dx}(2y - 2x) = 8 - 6x + 2y$

$\therefore \dfrac{dy}{dx} = \dfrac{3x - y - 4}{x - y}$

ii)

if $\dfrac{3x - y - 4}{x - y} = 2$

then $3x - y - 4 = 2x - 2y$

$y = 4 - x$

sub in original

$3x^2 + (16 - 8x + x^2) - 8x + 2x^2 - 8x + 2 = 0$

$6x^2 - 24x + 18 = 0$

$x^2 - 4x + 3 = 0$

$x = 1, 3$

$\therefore y = 3, 1$

Points are $(1,3), (3,1)$

3c) i) Let P (Akran wins) $= p$, $q = 1 - p$

Let number of games $= n$ Consider $(p+q)^n = \Sigma\, {}^nC_r p^r q^{n-r}$

$$P(\text{wins} \geq 1) = 1 - P(\text{wins no games})$$
$$= 1 - q^n$$

let $1 - q^n > 0.9$ $\therefore 0.1 > \left(\frac{2}{3}\right)^n$

$n > \frac{\ln 0.1}{\ln 2/3} \approx 5.68$ $\therefore$ needs to play 6 games

4a) i)

$$1 + i = \sqrt{2}\left(cis\frac{\pi}{4}\right)$$

ii)

$$(1+i)^n = \left(\sqrt{2}cis\frac{\pi}{4}\right)^n$$

$$1 + i\,{}^nC_1 + i^2\,{}^nC_2 + i^3\,{}^nC_3 + \cdots + i^n = \left(\sqrt{2}\right)^n cis\frac{n\pi}{4}$$

Equate real and imaginary parts

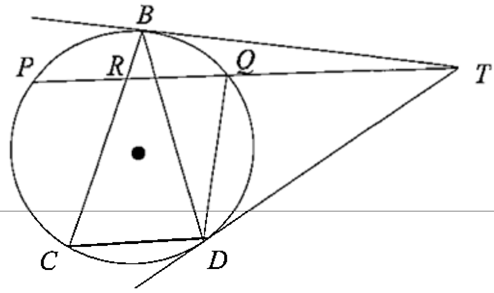
$$\therefore \quad 1 + i^2\,{}^nC_2 + i^4\,{}^nC_4 - \ldots = \left(2^{\frac{1}{2}}\right)^n \cos\frac{n\pi}{4}$$

$$1 - {}^nC_2 + {}^nC_4 - \ldots = 2^{n/2} \cos\frac{n\pi}{4}$$

and $\quad {}^nC_1 + i^3\,{}^nC_3 + i^5\,{}^nC_5 + \ldots = 2^{n/2} \sin\frac{n\pi}{4}$

thus ${}^nC_1 - {}^nC_3 + {}^nC_5 - \ldots = 2^{n/2} \sin\frac{n\pi}{4}$

b



ii) Let $\angle BDT = x$

$\therefore \angle DCB = x$ angle in alternate segment

$\therefore \angle BRT = x$ corresponding angles PT||CD

$\therefore \angle BDT = \angle BRT$

Now as $\angle$BDT and $\angle$BRT are equal angles subtended by the chord BT

BTDR are concyclic

iii) In the cyclic Quad $BTDR$

$BT = DT$ tangents of equal length from external point

$\therefore \angle BRT = \angle DRT$ equal chords subtent equal angles at circumference

iv) Since $\angle DRT = \angle BRT = x$

$\angle RDC = \angle DRT = x$ alternate angles PT||CD

$\therefore \Delta RCD$ is isosceles (base $\angle$'s are equal)

5a) i) $z^9 + 1 = (z^3 + 1)(z^6 - z^3 + 1)$

$= 0$ when $z^3 + 1 = 0$

or $z^6 - z^3 + 1 = 0$

$\therefore$ roots of $z^6 - z^3 + 1 = 0$ are the roots of $z^9 + 1 = 0$ that are not the roots of $z^3 = -1$

5a ii) The 9 roots of $z^9+1=0$ include -1 and are equally spaced around the unit circle in an Argard diagram by an angle of $\dfrac{2\pi}{9}$.

The roots -1 and those spaced at an angle of $\pm\dfrac{2\pi}{3}$ from -1 are the cube roots of -1.

The remaining roots are therefore the roots of $z^6-z^3+1=0$.

Arrange these roots in conjugate pairs $\dfrac{\pi}{9},\dfrac{-\pi}{9},\dfrac{5\pi}{9},\dfrac{-5\pi}{9},\dfrac{7\pi}{9},\dfrac{-7\pi}{9}$

Consider $(z-\alpha)(z-\overline{\alpha})=z^2-2z\Re(\alpha)+|\alpha|$

$$\therefore z^6-z^3+1=\left(z^2-2z\cos\frac{\pi}{9}+1\right)\left(z^2-2z\cos\frac{5\pi}{9}+1\right)\left(z^2-2z\cos\frac{7\pi}{9}+1\right)$$

5a iii) Equate coeff of z^2 $LHS=0$

$$RHS=4\left(\cos\frac{\pi}{9}\cos\frac{5\pi}{9}+\cos\frac{\pi}{9}\cos\frac{7\pi}{9}+\cos\frac{5\pi}{9}\cos\frac{7\pi}{9}\right)+3$$

$$\therefore \cos\frac{\pi}{9}\cos\frac{5\pi}{9}+\cos\frac{\pi}{9}\cos\frac{7\pi}{9}+\cos\frac{5\pi}{9}\cos\frac{7\pi}{9}=-\frac{3}{4}$$

5b) i) $y=k-x^2\ -(1)$

$y=\dfrac{1}{x}\qquad -(2)$

Solve (1) and (2) $\therefore 1=kx-x^3$

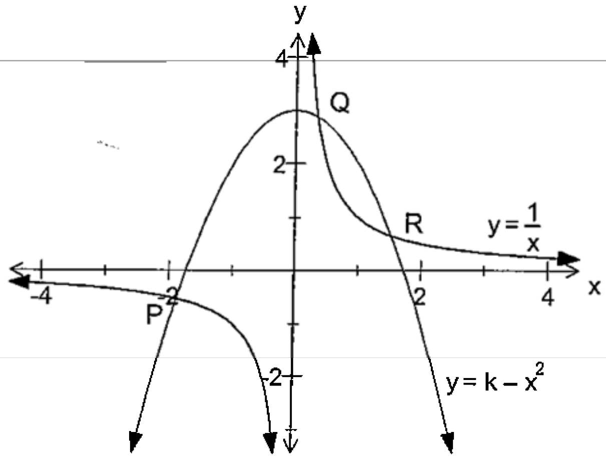
$x^3-kx+1=0$ has roots α,β,γ

let $x=\sqrt{y}$

$\therefore y^{\frac{3}{2}}-ky^{\frac{1}{2}}=$

square b.s. $\quad y^3-2ky^2+k^2y=1$

$\therefore x^3-2kx^2+k^2x-1=0$ has roots $\alpha^2,\beta^2,\gamma^2\ -(\text{A})$



5b ii) Let $x=\dfrac{1}{y}$

$\therefore \dfrac{1}{y^3}-2k\dfrac{1}{y^2}+\dfrac{k^2}{y}-1=0$

$\therefore 1-2ky+k^2y^2-y^3=0$

$\therefore x^3-k^2x^2+2kx-1=0$

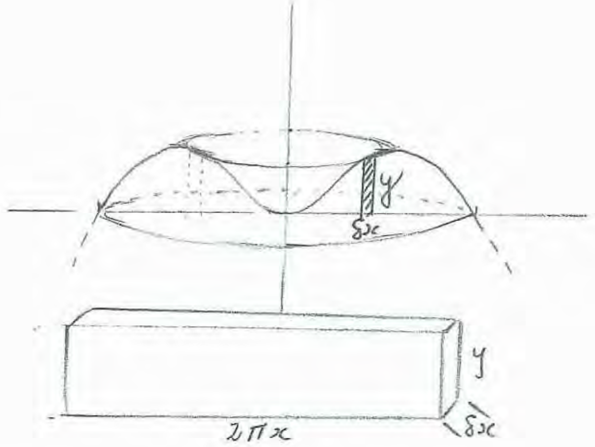
has roots $\dfrac{1}{\alpha^2},\dfrac{1}{\beta^2},\dfrac{1}{\gamma^2}\qquad -(\text{B})$

5b iii) $OP^2=\alpha^2+\dfrac{1}{\alpha^2}$ similarly $OQ^2=\beta^2+\dfrac{1}{\beta^2}$, $\quad OR^2=\gamma^2+\dfrac{1}{\gamma^2}$

$\therefore OP^2+OQ^2+OR^2=\alpha^2+\beta^2+\gamma^2+\dfrac{1}{\alpha^2}+\dfrac{1}{\beta^2}+\dfrac{1}{\gamma^2}$

$=2k+k^2$ by sum of roots from (A) and (B).

6a



$$\delta V \approx 2\pi xy\delta x \quad \therefore V = \lim_{\delta x \to 0}\sum_{0}^{2} 2\pi x\left(4x^2 - x^4\right)\delta x$$

$$\therefore V = 2\pi\int_0^2 \left(4x^3 - x^5\right)dx = 2\pi\left[x^4 - \frac{x^6}{6}\right]_0^2$$

$$= 2\pi\left(16 - \frac{64}{6}\right) = \frac{32\pi}{3}$$

6b) $y = \frac{1}{4a}x^2$

Area under curve $= A = \frac{2}{4a}\int_0^{2a} x^2 dx$

$= \frac{1}{2a}\left[\frac{x^3}{3}\right]_0^2 = \frac{4a^2}{3}$

Shaded area $= 4a \times a - \frac{4a^2}{3}$

$= \frac{8a^2}{3}$

6b) ii) Consider slice at dist x from A

BC = AB tan 30 $= \frac{x}{\sqrt{3}}$

Now BC is semi latus rectum

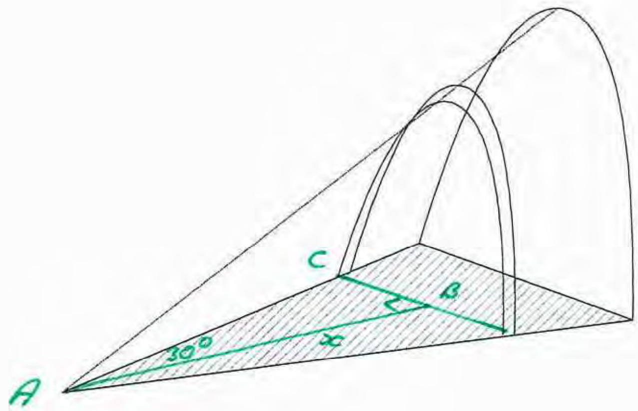
$\therefore a = \frac{x}{2\sqrt{3}}$

A $= \frac{8}{3} \times \frac{x^2}{4\times 3} = \frac{2x^2}{9}$

$\delta V = \frac{2}{9}x^2\delta x$

$$V = \lim_{\delta x \to 0}\sum_{x=0}^{3\sqrt{3}} \delta v = \frac{2}{9}\int_0^{3\sqrt{3}} x^2 dx$$

$$= \frac{2}{9}\left[\frac{x^3}{3}\right]_0^{3\sqrt{3}} = \frac{2}{9} \times \frac{81\sqrt{3}}{3} = 6\sqrt{3}\,u^2$$



7a) i) Take O at point of projection with up as +ve

ResultantForce $= -mg - \frac{gv^2}{c^2}$ $\quad \downarrow mg \ \downarrow \frac{mgv^2}{c^2}$

given $m = 1$

$$\therefore \ddot{x} = v\frac{dv}{dx} = -g\left(1 + \frac{v^2}{c^2}\right)$$

$$\frac{vdv}{\left(1 + \frac{v^2}{c^2}\right)} = -gdx$$

$$\therefore \frac{c^2}{2}\int_{2c}^{0} \frac{\frac{2v}{c^2}dv}{\left(1 + \frac{v^2}{c^2}\right)} = \int_0^H -gdx$$

$$\frac{c^2}{2}\left[\ln\left(1 + \frac{v^2}{c^2}\right)\right]_{2c}^{0} = \left[-gx\right]_0^H$$

$$0 - \frac{c^2}{2}\ln 5 = -gH \Rightarrow H = \frac{c^2}{2g}\ln 5$$

7a) ii) Take O at highest point, down as +ve

$\downarrow g \quad \uparrow \frac{gv^2}{c^2}$

$$\ddot{x} = v\frac{dv}{dx} = g\left(1 - \frac{v^2}{c^2}\right) \quad \therefore \frac{vdv}{\left(1 - \frac{v^2}{c^2}\right)} = gdx \Rightarrow -\frac{c^2}{2}\int_0^V \frac{-\frac{2v}{c^2}dv}{\left(1 - \frac{v^2}{c^2}\right)} = \int_0^H g\,dx$$

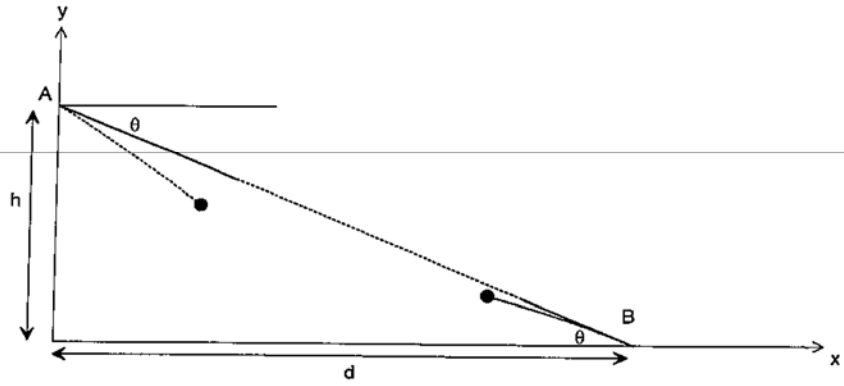
$$-\frac{c^2}{2}\left[\ln\left(1 - \frac{v^2}{c^2}\right)\right]_0^V = \left[gx\right]_0^H$$

$$-\frac{c^2}{2}\ln\left(1 - \frac{V^2}{c^2}\right) = gH \quad \text{sub for } H \text{ from i)}$$

$$\frac{c^2}{2g}\ln\left(\frac{c^2}{c^2 - V^2}\right) = \frac{c^2}{2g}\ln 5 \quad \therefore 5 = \frac{c^2}{c^2 - V^2} \Rightarrow 5c^2 - 5V^2 = c^2$$

$$\therefore 5V^2 = 4c^2 \Rightarrow V = \frac{2c}{\sqrt{5}}$$

7b)



i) $\ddot{x}_A = 0$

$\therefore \dot{x}_A = C \quad t=0, \quad \dot{x} = V\cos\theta$

$\therefore \dot{x}_A = V\cos\theta$

$x_A = V\cos\theta.t + c' \quad t=0 \quad x_A = 0 \therefore c' = 0$

$x_A = V\cos\theta.t$

$\ddot{y}_A = -g$

$\dot{y}_A = -gt + k \quad t=0 \quad \dot{y} = -V\sin\theta$

$\therefore \dot{y}_A = -gt - V\sin\theta$

$y_A = -\frac{1}{2}gt^2 - V\sin\theta.t + k' \quad t=0 \quad y_A = h \therefore k' = h$

$y_A = -\frac{1}{2}gt^2 - V\sin\theta.t + h$

$\ddot{x}_B = 0$

$\dot{x}_B = c'' \quad t=0 \quad \dot{x} = -V\cos\theta$

$\therefore \dot{x}_B = -V\cos\theta$

$x_B = -V\cos\theta.t + c''' \quad t=0 \quad x_B = d \therefore c''' = d$

$x_B = d - V\cos\theta.t$

$\ddot{y}_B = -g$

$\dot{y}_B = gt + k'' \quad t=0 \quad \dot{y}_B = V\sin\theta$

$\dot{y}_B = -gt + V\sin\theta$

$y_B = -\frac{1}{2}gt^2 + V\sin\theta.t + k''' \quad t=0 \quad y_B = 0 \therefore k''' = 0$

$y_B = -\frac{1}{2}gt^2 + V\sin\theta.t$

ii) when $x_A = x_B$

$t.V\cos\theta = d - t.V\cos\theta$

$t_x = \frac{d}{2V\cos\theta} \qquad -(1)$

when $y_A = y_B$

$-\frac{1}{2}\cancel{g}t^2 - tV\sin\theta + h = -\frac{1}{2}\cancel{g}t^2 + V\sin\theta t$

$t_y = \frac{h}{2V\sin\theta} \qquad -(2)$

now $\tan\theta = \frac{h}{d} \therefore d\sin\theta = h\cos\theta$

sub in (1) $t_x = \dfrac{d}{2V\dfrac{d\sin\theta}{h}} = \dfrac{h}{2V\sin\theta} = t_y$

$\therefore$ The two particles have the same x, y coordinates at the same time.

Hence they meet.

iii) $H = -\frac{1}{2}gt^2 + V\sin\theta.t \qquad tan\theta = \frac{h}{d}$

$= -\frac{1}{2}g\frac{h^2}{4V^2\sin^2\theta} + \cancel{V}\cancel{\sin}\theta \times \frac{h}{2\cancel{V}\cancel{\sin}\theta}$

$= -\frac{1}{2}g\dfrac{h^2}{4V^2 h^2/(h^2+d^2)} + \frac{h}{2} = \frac{h}{2} - \frac{g(h^2+d^2)}{8V^2}$

8a) i) $L.H.S.\ \sin(\sin^{-1}x-\cos^{-1}x)$

$$=x\times x-\sqrt{1-x^2}\times\overline{1-x^2}$$

$$=2x^2-1=R.H.S.$$

ii) $\sin^{-1}x-\cos^{-1}x=\sin^{-1}(1-x)$

$$\therefore 2x^2-1=1-x$$

$$2x^2+x-2=0$$

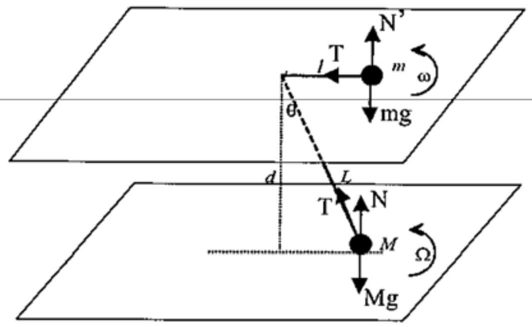
$$x=\frac{1-\pm\sqrt{1+16}}{4}\approx 0.7808,-1.2808$$

but $\sin^{-1}(1-x)$ must be acute

Now $1-(-1.2808)=2.2808^c$ which is not acute

$\therefore$ reject $x=-1.2808$ Solution $x=\frac{1+\sqrt{17}}{4}$

8bi)



ii) Consider M: Hor: $T\cancel{\sin}\theta=MxL\cancel{\sin}\theta.\Omega^2$ $-(1)$

Vert: $N+T\cos\theta=Mg$

$$N=(Mg-T\cos\theta)$$

$$=Mg-ML\cos\theta.\Omega^2$$

$$=M\left(g-L\cos\theta.\Omega^2\right)$$

$$=M\left(g-d\Omega^2\right)$$

$N\geq 0$ $\therefore$ $\Omega^2\leq\frac{g}{d}$ (4)

If $\Omega>\frac{g}{d}$ the mass M will lift off the surface.

iii) Consider m: Vert: $N'=mg$

Hor: $T=ml\omega^2$ $-(2)$

Equate (1) & (2) $ML\Omega^2=ml\omega^2$

$$\frac{L}{l}=\frac{m}{M}\left(\frac{\omega}{\Omega}\right)^2 \quad -(3)$$

iv) If $N=0$ $\Omega^2=\frac{g}{d}$ $T=ML\Omega^2$ $-(4)$

$$=\frac{MLg}{d}$$

v) $d=0.8$ $L+l=1.5$ $m=0.4$ $M=0.2$ $\omega=\Omega$

from (3) $\frac{L}{l}=2$ $\therefore L=2l$

but $L+l=1.5$ (2)

$l=0.5,\ L=1$

$$\therefore\ T=\frac{0.2\times 1\times g}{0.8}=\frac{g}{4}$$