Section I

#### 10 Marks Attempt Questions 1–10 Allow about 20 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10

- 1 Let  $\alpha$ ,  $\beta$  and  $\gamma$  be roots of the equation  $x^3 + 3x^2 + 4 = 0$ . Which of the following polynomial equations have roots  $\alpha^2$ ,  $\beta^2$  and  $\gamma^2$ ?
- A.  $x^3 9x^2 24x 4 = 0$
- B.  $x^3 9x^2 12x 4 = 0$
- C.  $x^3 9x^2 24x 16 = 0$
- D.  $x^3 9x^2 12x 16 = 0$
- 2. If  $w = \sqrt{3}cis(\pi)$ , then  $w^4$  is equal to
- A. 9*cis*(0)
- B. 9*cis*  $(\pi)$
- C. 81*cis*  $(4\pi)$
- D.  $4\sqrt{3}cis(\pi^4)$
- 3.  $\int \frac{3x^2}{\sqrt{x^3 2}} dx$  is equal to
- A.  $2\sqrt{x^3 2} + c$ B.  $\frac{\sqrt{x^3 - 2}}{2} + c$
- C.  $\frac{2}{3(x^3-2)^{\frac{3}{2}}}+c$
- D.  $\frac{3}{2(x^3-2)^{\frac{3}{2}}}+c$

- 4. If  $y = \tan 2x$ , then  $\frac{d^2y}{dx^2}$  is equal to
- A.  $\frac{8\sin 2x}{\cos^3 2x}$
- B.  $2 \sec^2 2x$
- C.  $\frac{8\sin 2x}{\cos 2x}$
- D.  $\frac{-8\sin 2x}{\cos 2x}$
- 5. A light elastic spring of natural length 2 metres hangs vertically from a ceiling. A mass of 0.5 kg is attached to the free end of the spring and hangs in equilibrium. Given that the stiffness of the spring is 0.25g (where g is gravity in m/s<sup>2</sup>), then the length of the spring in metres would be
  - A.  $\frac{2}{g}$ B. 2 C.  $2 + \frac{2}{g}$ D. 4
  - 6. The displacement x, in metres, of a particle from a fixed point at time t, in seconds,  $t \ge 0$ , is given by  $x = 2\cos(3t)$ .

The number of oscillations made by the particle per second is

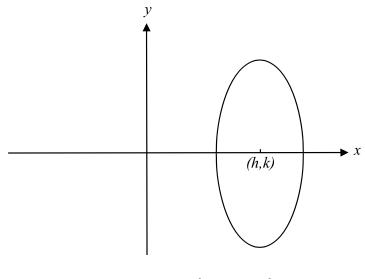
- A.  $\frac{1}{\pi}$
- B.  $\frac{3}{2\pi}$
- C. 2
- D. 3

7. If z = x + yi, then the set of points in the complex plane given by  $(2-i)z + (2+i)\overline{z} = 1$  is

- A. a horizontal line through (0, 2)
- B. a vertical line through (-2, 0)
- C. a straight line with gradient 2 and y-intercept -1
- D. a straight line with gradient -2 and y-intercept  $\frac{1}{2}$

8. A solution to the differential equation  $\frac{dy}{dx} + \frac{1}{\sqrt{1-x^2}} = 1$  is

- A.  $x \sin^{-1}x$
- B.  $x \cos^{-1} x$
- C.  $x 2\sqrt{1 x^2}$
- D.  $\frac{-2x^3}{3}\sqrt{1-x^2}$
- 9. An ellipse is shown on the set of axes below. The scale on the x and y axes is the same. The centre of the ellipse is the point (h, k).



The general form of the equation of an ellipse is  $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ 

For the ellipse shown above

- A. h < k and a < b
- B. h < k and a = b
- C. h = k and a > b
- D. h > k and a < b

10. The eccentricity of a hyperbola with parametric equations  $x = 3 \sec \theta$  and  $y = 4 \tan \theta$  is:

A.  $\frac{5}{3}$ B.  $\frac{3}{5}$ C.  $\frac{5}{4}$ D.  $\frac{4}{3}$ 

# **Section II**

# 90 Marks Attempt Questions 11-16 Allow about 2 hours and 40 minutes for this section

Answer each question in a separate writing booklet. Extra writing booklets are available

All necessary working should be shown in every question.

## Question 11 (15 marks) Use a SEPARATE writing booklet

a) Find 
$$\int \frac{dt}{\sqrt{7+6t-t^2}}$$
 2

b) i) Show that 
$$\frac{1}{(x^2+3)(x^2+1)} = \frac{1}{2} \left[ \frac{1}{(x^2+1)} - \frac{1}{(x^2+3)} \right]$$
 2

ii) Hence evaluate 
$$\int_{0}^{1} \frac{1}{(x^2+3)(x^2+1)} dx$$
 2

- c) The equation  $z^2 (a+ib)z 6i = 0$ , where *a* and *b* are real, has roots  $\alpha$  and  $\beta$  such that  $\alpha^2 + \beta^2 = 5$ .
  - i) Show that  $a^2 b^2 = 5$  and ab = -6 2
  - ii) Hence find the values of *a* and *b*.

3

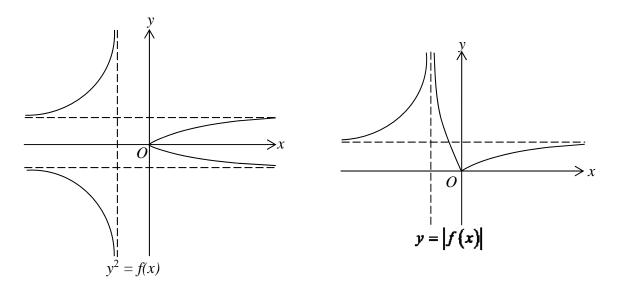
4

d) If 
$$I_m = \int_0^k (k^2 - x^2)^m dx$$
, for  $m \ge 1$ , show that  
 $I_m = \frac{2k^2m}{2m+1} I_{m-1}$ 

$$\left[ \text{Hint } \frac{x^2}{k^2 - x^2} = \frac{k^2}{k^2 - x^2} - 1 \right]$$

# Question 12 (15 marks) Use a SEPARATE writing booklet

a) Given the graphs of  $y^2 = f(x)$  and y = |f(x)| as shown.

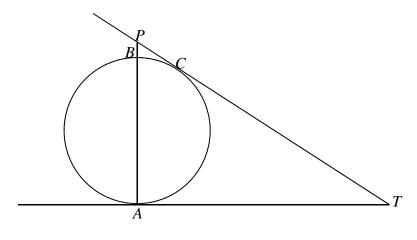


i) Explain the shape and position of y = f(x) in relation to the above graphs.

iii) Hence sketch the graph of y = f|(x)|. 2

2

b) *AB* is a diameter of a circle *ABC*. The tangent at *A* and *C* meet at *T*. The lines *TC* and *AB* are produced to meet at *P*. Join *AC* and *CB*.



i) Prove that  $\angle BCP = 90^{\circ} - \angle CAT$ .

2

2

3

- ii) Explain why ATCB could never be a cyclic quadrilateral.2You must support your answers with geometrical reasons.2
- c) i) Prove that if a polynomial P(x) has a root  $\alpha$  of multiplicity r, then P'(x) has a root of multiplicity (r-1).
  - ii) Given that x = 1 is a double root of the equation  $x^4 5x^3 + 16x^2 21x + 9 = 0$ , and using the result of ci), or otherwise, find the other roots.

# Question 13 (15 marks) Use a SEPARATE writing booklet

| a) | i) | Show that $\sin x + \sin 3x = 2\sin 2x \cos x$ . | 2 |  |
|----|----|--|---|--|
|    |    |  |   |  |

- ii) Hence, or otherwise, solve  $\sin x + \sin 2x + \sin 3x = 0$  for  $0 \le x \le 2\pi$ .
- b) A hyperbola has eccentricity  $\frac{3}{2}$  and directrices x = -4 and x = 4. Find the Cartesian equation of this hyperbola.

3

c) The point  $P(a\cos\theta, b\sin\theta)$  lies on the ellipse,  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ . The points *T* and *T'* are the feet of the perpendiculars from the foci *S* and *S'* respectively to the tangent through *P*.

i) Show that 
$$ST = \frac{|e\cos\theta - 1|}{\sqrt{\frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2}}}$$
 2

- ii) Hence prove  $ST.S'T' = b^2$ .
- d) A corner on a race track is an arc of a circle of radius 100m. The track is banked such that there is no tendency for a vehicle to move sideways when cornering at 100km/h. Find the angle, to the nearest minute, of banking. Take g to be  $10 \text{ m/s}^2$ . 2

#### Question 14 (15 marks) Use a SEPARATE writing booklet

a) If 
$$z = \cos \theta + i \sin \theta$$
,

i) Show that 
$$z^{z} + z^{-z} = 2\cos n \theta$$
 for  $n = 1, 2, 3, ....$  2

ii) Hence show that 
$$4\cos\theta\cos 2\theta\cos 3\theta = 1 + \cos 2\theta + \cos 4\theta + \cos 6\theta$$
 3

- iii) Hence find the general solution of  $\cos^2 \theta + \cos^2 2\theta + \cos^2 3\theta = 1$ . 3
- b) The region bounded by the graphs of  $y = x^2$  and y = x + 2 is revolved around the line x = 3. Express the volume of the resulting solid as a definite integral. Do not calculate the value of this integral.
- c) A solid has, as its base, the circular region in the *xy* plane bounded by the graph of  $x^2 + y^2 = a^2$ , where a > 0. Find the volume of the solid if every cross section by a plane perpendicular to the *x*-axis is an equilateral triangle with one side in the base.

3

4

### Question 15 (15 marks) Use a SEPARATE writing booklet

- a) An ellipse can be described as the locus of a point moving so that the sum of its distance from two fixed points (foci) is a constant.
  - i) If the two fixed points are A(-4,0) and B(4,0) and the sum of the distances of P(x, y)from these points is 10units, show that the equation of the ellipse is  $\frac{x^2}{25} + \frac{y^2}{9} = 1.$  2
  - ii) Show that the ellipse can be represented parametrically by the equations  $x = 5 \cos \theta$  and  $y = 3 \sin \theta$  and find the equation, in general

form of the tangent to the ellipse at  $\theta = \frac{\pi}{6}$ .

- b) The roots of  $x^3 + 3px + q = 0$  are  $\alpha, \beta$  and  $\gamma$  none of which are equal to 0.
  - i) Find the monic equation with roots  $\frac{\beta\gamma}{\alpha}, \frac{\alpha\gamma}{\beta}, \frac{\alpha\beta}{\gamma}$ , giving the coefficients in terms of p and q.
  - ii) Deduce if  $\alpha\beta = \gamma$ , then  $(3p q^2) + q = 0.$  2
- c) Given that  $\sin^{-1} x, \cos^{-1} x$  and  $\sin^{-1} \sqrt{(1-x^2)}$  are acute, show that:

$$\sin\left(\sin^{-1} x - \cos^{-1} x\right) = 2x^2 - 1.$$

4

### Question 16 (15 marks) Use a SEPARATE writing booklet

a) A particle of mass *m* is projected vertically upward under gravity with a speed *v* in a medium where resistance is *mk* times the speed, where *k* is a positive constant. If the particle reaches its greatest height *H*, in time *T*, show that v = gT + kH.

b)

Given that  $\sin(x+y) + \sin(x-y) = 2\sin x \cos y$  and  $\cos(x+y) + \cos(x-y) = 2\cos x \cos y$  and by using the fact that  $A + B + C = \pi$  for triangle ABC.

Show that 
$$\frac{\sin A + \sin B}{\cos A + \cos B} = \cot \frac{C}{2}$$
 4

c) If 
$$\cos \alpha = \tan x \cot y$$
, show that  $\tan^2 \frac{1}{2}\alpha = \sin(y-x)\csc(y+x)$  3

d) Evaluate 
$$\int_{0}^{\frac{\pi}{3}} \frac{1}{9 - 10\sin^2 x} dx$$
 4

#### End of Assessment Task

| CAAADB DADA |
|-------------|
|-------------|

|       | Fights 20004. Soln ExT2  |  |
|-------|--|--|
| ·Q1.  | $\therefore$ Let $y = x^{\frac{1}{2}}$   |  |
|       | $y^{32} + 3y = -t$<br>=> $y^{32} = -3y - 4$                                    |  |
|       | $= 3 y'^{2} = -3y - 4$   |  |
|       | expand + squere  |  |
|       | $= 3 g^{3} - 9 y^{2} - 24 y - 16 = 0$  |  |
| C     |  |  |
|       |  |  |
| -A 2. | · c as Pay has real  |  |
| A     | Coefficients complex roots   |  |
|       | occur in conjugate pours.  |  |
|       | 3 0  |  |
| Q2    | 2 w = BCIST  | •  |
|       | w4 = (13) 4 cr s(41)   |  |
|       | = 90150 1. (A)   |  |
|       |  |  |
| 3     | $\int \frac{3\kappa^2}{1z^3-2} dx = \int \frac{dy}{dx} \cdot \frac{1}{x^2} dx$ |  |
|       | J iz- z Jour   |  |
|       | $ e u=x^2-2 = \int u^{-\frac{1}{2}} du$  | the second of the second second  |
|       | $\frac{1}{dy} = 3x^2, \qquad = 2x^2 + c  A$                                    | and the second |
|       | $\frac{\partial x}{\partial x} = 2 \left[ \frac{x^2}{x^2} + C \right]$         |  |
|       | - 21 x-2+ 0.   |  |
| 14    |  |  |
|       | y = tan 25c  |  |
|       | $\frac{dy}{dx} = 2 \sec^2 2xc$ $\frac{dy}{dx} = 2 (\cos 2x)^{-2}$              |  |
|       | = 4 ( = = + + +  |  |
|       | $\frac{d^2y}{dx^2} = -4 \left(\cos 2x\right)^{-3} - 2\sin 2x$                  |  |
|       | $\frac{dx^2}{\cos^3 2x} = \frac{8 \sin 2x}{6}$                                 |  |
|       |  |  |
| <     |  |  |
| 5,    | 6  |  |
|       |  |  |
|       |  |  |
|       |  |  |
|       |  |  |

| 0     |   |   |
|-------|---|---|
| 5.    |   |   |
|       | T T= Bx   | (10)·A  |
|       | = 0,25gsc   | $\begin{array}{c} \hline b \\ e \\ e \\ \hline a^2 + b^2 \\ \hline a^2 \end{array}$ |
|       |   | 1 az  |
|       | 0,5 g T= 0,5 g · .x = 2<br>Since x is the extension in  |   |
|       | the spring beyond its natural   | =   9+16  |
|       | length, the length of the sting<br>is 2+2 = 4m. D   | 51  |
|       | is 2+2 = 4 m D  | = 5/3.  |
| 6.    | no of oscillations passes is f.   |   |
|       | : $f = \frac{h}{2\pi}$ , $h = 3$ . for $x = 2(c_3(b_1))$  |   |
|       | $f = \frac{3}{2\pi}$ B  |   |
| (5)   | $(2-L) 2 + (2+L) \overline{2} = 1$  |   |
| U.    | 23-22 チンモナンモー1  |   |
|       | If z = x+gi.  |   |
|       | $= \sum_{x \in i} (x + iy) - i (x + iy) + 2(x - iy) + i(x - iy)$  | =1  |
|       | 2x +2ig -ix+ y+2x-2ig+ix+y=1  |   |
|       | 4xc+ 2 y = 1  |   |
|       | 2y = -4x+1  |   |
|       | y= South<br>m your D  |   |
|       | m grus D  |   |
| (\$). | dy = 1  |   |
| 8.    |   |   |
|       | dy = 1 - 1<br>de Ti-er  |   |
|       | $y = \int 1 - \frac{1}{1+x^2} dx$   |   |
|       | = x - Shutx+C (A)   |   |
| (a)   | For centre of ellipse, B=0, h>0   |   |
| Ú.    | So h> &. The lengter of servi-major   | n a nicht a thair   |
| **    | So hyle. The lengter of semi-major<br>axis is b, This runs II to yaxis<br>semi never is a ; II to ocaxis. | (2 · · · · · · · · · · · · · · · · · · ·  |
|       | Servi nenor is a ; Il to ocaris.  |   |

|     | Question 11.  |              |
|-----|---|--------------|
| 101 | l ab  |              |
| (9) | $\int \frac{db}{\sqrt{1+6t-t^2}}$   |              |
|     | r dt  |              |
|     | $= \int \frac{dt}{\left(16 - \left(t - s\right)^2\right)}$  |              |
|     | $= \sin^{-1}(\frac{4-3}{4}) + c$ (2)  | $\checkmark$ |
|     | - sin ( 4) te   |              |
| (2) | il Let I - A + B  |              |
| .7  | $\frac{\partial}{\partial c^2 + 3} \frac{\partial}{\partial c^2 + 1} = \frac{A}{2c^2 + 3} + \frac{B}{2c^2 + 1}$   |              |
|     | $(1 - A(x^2 + 1) + B(x^2 + 3))$   |              |
|     | het sc2 = -1 1=23   |              |
|     | Het xt = -3 1=A-2 2   |              |
|     | $A = -\frac{1}{2}$  | $\checkmark$ |
|     |   |              |
|     | $\frac{1}{(2+3)(2+1)} = \frac{1}{2} \left[ \frac{1}{(2+3)} - \frac{1}{(2+3)} \right]$   |              |
|     |   |              |
|     | $(i)_{2}\int_{0}^{1}\left(\frac{1}{x^{2}+1}-\frac{1}{x^{2}+3}\right)dx$   |              |
|     | = 1 [tautoc - totaut 15]  |              |
|     |   |              |
|     | $= \frac{1}{2} \begin{bmatrix} \frac{\pi}{4} & -\frac{1}{13} & \frac{\pi}{5} \end{bmatrix}$<br>= $\frac{\pi}{4} \begin{bmatrix} \frac{1}{2} & -\frac{\pi}{5} \end{bmatrix}$ (2) | 1            |
| _   |   |              |
|     | OR students can expand  |              |
|     | RHS to become LHS.  |              |
|     |   |              |
|     |   |              |
|     |   |              |
| _   |   |              |

| Question 11.   | 11(c) (U) x+B= a+ib   |                 |
|--|---|-----------------|
| dt   | AB = -ib  |                 |
| $(9) \int \frac{at}{(7+6t-t^2)}$   | $(\alpha + \beta)^{2} = \alpha^{2} + \beta^{2} + 2\alpha\beta$  |                 |
| J 17+61-t-   | (a+ib)2= 5-12i  |                 |
| [ db   | $a^2 - b^2 + 2iab = s - 12i$  | /               |
| $= \int \frac{dL}{(16-(t-5))^2}$   | equate real + imag.   |                 |
|  | $a^{2}-b^{2}=5$ (2)   |                 |
| $= \sin^{-1}(\frac{4-3}{4}) + c$ (2)   | ab = -6.  |                 |
| (b) (c) Let $1 = A + B$<br>$6^{2}+3(6^{2}+1) = 2^{2}+1$  | (1) $a^2 - b^2 = 5$   |                 |
|  | $a^4 - a^2 b^2 = 5a^2$  |                 |
| $(1 - A(x^2 + 1) + B(x^2 - 13))$   | at-sa2-36 =0 1  |                 |
| het sc2 = -1 1=23  | (a2-9)(a2-4)=0  |                 |
| B=1  | (a=3, b==2], (3)  |                 |
| het xt = -3 I = A-2 (2)  | $a_{-3}, b_{-2}$ (3)<br>$a_{a_{-3}}, b_{-2}$ (3)  |                 |
| A = -2   |   |                 |
|  | $I(d) I_{m} = \int_{0}^{k} (x^{2} - x^{2})^{m} I dx$  |                 |
| $\frac{1}{2} \frac{1}{2} \frac{1}$ | 0   |                 |
|  | Let u = (-22-2) <sup>m</sup> v'=1 ]. 5  | 0% of students  |
| $\left(\frac{1}{x^{2}}\right) = \int_{0}^{1} \left(\frac{1}{x^{2}+1} - \frac{1}{x^{2}+3}\right) dx$  |   | could not start |
| 10   |   | out concetly, t |
| = 1 [taut x - tstaut K]]   |   | Couldnot and d  |
|  |   | question        |
| $= \frac{1}{2} \begin{bmatrix} \frac{1}{4} & -\frac{1}{13} & \frac{1}{16} \end{bmatrix}$   | $= \left[i\mathcal{R}\left(\mathcal{R}^{2}-\mathcal{R}^{2}\right)^{m}-o\right]+2m\int_{0}^{\infty}\int_{0}^{\infty}z^{2}\left(\mathcal{R}^{2}-z^{2}\right)^{m-1}dz$ | 1               |
| = = [ = - =] 2   | $= 2m \int_{0}^{\frac{1}{2}} \frac{1}{\kappa^{2} (\kappa^{2} - \kappa^{2})^{m}} d\kappa$  |                 |
|  |   |                 |
| OR students can expand   | $= 2m k^2 \int_0^k (-k^2 - x^2)^{m-1} dx - 2m \int (k^2 - m^2)^m dx.$   |                 |
| RHS to became LHS.   |   |                 |
| - to accore ons,   | - Im = 2 km Jm-n = 2 m Jm   |                 |
|  | $I_{m}(1+2m) = 2k^{2}m I_{m-1}(4)$  |                 |
|  | $\frac{1}{7} \qquad p e^2 \qquad T \qquad (4)$  |                 |
|  | $I_m = \frac{2R^2}{2m+1} I_m - 1$   |                 |
|  |   |                 |
|  |   |                 |

| 0   |   |                       | 0   |       |
|-----|---|-----------------------|---|-------|
| QD  |   |                       | 12(b)(c) < CAT = LABC (Lbetfort chad)                                     |       |
|     | i From the graph y'= flu), value            | graphs answered       | bad and L BCP = LCAB ( " ") Reasa   | ue we |
| 1.4 | of fix between vertical asymptote           | v well.               | but ( BAT = 90° ( diam's Having / poaly                                   |       |
|     | ty-axis is negative. (1)                    |                       | ¿ CBCP= 90°- CCAT.  |       |
|     | ' the part of the graph                     |                       | 0.0   |       |
|     | y=  f(sc)  must be obtained by 2            | 1                     | (i) Since LBCPio ext 2  |       |
|     | reflection about sc-axis. (1)               | V                     | and its = LBAC which is   |       |
|     |   | position wit asymp () | part of int opp L, 1 (2)  |       |
|     | (ii) / ·                                    | shape O               | ATBC cannot be cyclic quad  |       |
|     | 2   |                       | as theorem exts = intopps /   |       |
|     |   |                       | does not hold.  |       |
|     |   |                       |   |       |
|     | q = f(x)                                    | 1                     | $(q)$ i) Let $P(x) = (x-x)^r \alpha(x)$                                   |       |
|     |   |                       | $p(x) = Q(x) [r(r-x)^{r-1}] + [(x-x)^{r-1}(x)]$                           |       |
|     | 12 1  |                       | [ie ploc) = (x -x)^-1 [rQ (x)+(xx) a'00] Some shuden                      | y st  |
|     |   |                       | :. Plac has a root .1 but did not   | : fac |
|     | (11) y=lf(t) 1 is symmetrical               |                       | X=x of multiplicity (r-1) (x-x)r-1 + sho                                  | w-fer |
|     | about y-axis.                               |                       | Since $P(\alpha) = P'(\alpha) = 0$ by subst. $P'(\alpha) = P(\alpha) = 0$ | 0.    |
|     | about y-axis.<br>Since f (Fiel) = f ( loci) |                       |   |       |
|     | far > < > 0                                 |                       | (1) Let $P(x) = x^4 - 5x^3 + 16x^2 - 21x + 9$                             |       |
|     | - graph of y= f (Isci) is                   |                       | $p(cx) = 4x^3 - 15x^2 + 322 - 21$   |       |
|     | 1 TY  |                       | we know $P(i) = P(i) = 0$ (i)   |       |
|     | 2.  | position wit asymp () | P(1) = 4 - 15 + 32 + 21 = 0 = P(1)  |       |
|     |   | shape D               | To find other roots, several approve<br>$HB P(x) = (x-i)^2 Q(s_c)$        |       |
|     | y   | U.                    | $HB^{\prime}P(x) = (x-i)^2 Q(s_c)$  |       |
|     |   |                       | by inspect $Q(x) = x^2 - 3x + q$  |       |
|     |   |                       | OP Divide (x-1)2 into P(x) tuse   |       |
|     |   |                       | long: to find p(2)  |       |
|     |   |                       | OP 1, 1, a, à are roots sina (200)  |       |
|     |   |                       | $b \perp 0$   |       |
|     |   |                       | $5 \ 20$<br>$3t-3x+q=0, x=3\pm 727 = 3\pm 373i$<br>$2 \ 2 \ 3$            |       |
|     |   |                       |   |       |
|     |   |                       |   |       |

|   |                                   | 0  |  |                   |
|---|-----------------------------------|----|--|-------------------|
|   |                                   |    |  |                   |
|   | Act soln                          | 13 | da)  |                   |
| = Sin (2x-xc)+ 814 (2xc+xc)   | Sin Stain T=2sin (SET)            |    | ST, S'T = $\frac{1-e^{-}ce^{-}ce^{-}ce^{-}}{ce^{-}c$ |                   |
| = Fin 2xc cosx - cos 2xc Finxc  | $\cos\left(\frac{z-T}{2}\right)$  |    | $ST, S'T' = \frac{1 - e^2 cos^2 ce}{\frac{cos^2 ce}{a^2} + \frac{s^2 v^2 ce}{b^2}}$  |                   |
| · + sin 2x cosx + cos 2x sinx 1   | LHS= 2 sin ( 2) cos (x-3x         |    |  |                   |
| = 2 8in 200 cosx. (2)   | = 2 Sin 2x cos (-xc)              |    | For ellipse b=a2(1-e2)   |                   |
|   | = 2 sinz z cosx                   |    | >) e <sup>2</sup> = 1 - b <sup>2</sup> /   | · students lost i |
| (i) Sinx + Sin2x + Sin3x = 0  | =RHS QED                          |    | ar ar  |                   |
| 28112xcossc + Sin2x =0  |                                   |    | ST. STI= 1-costa + 52<br>cr costa  | Alg simplificat   |
|   |                                   |    |  |                   |
| $g_{111} 2x (2cos x + 1) = 0$   |                                   |    | $\frac{\cos^2 \alpha}{\sin^2 \alpha} + \frac{\sin^2 \alpha}{\sin^2 \alpha}$  |                   |
| $\sin 2x = 0  \cos x = -\frac{1}{2}$  |                                   |    | 0  |                   |
| 22-0, 17, 27, 37,47 / 2= 3,5  |                                   |    | $= 1 - \cos^2 \alpha \left( 1 - \frac{b^2}{\alpha z} \right)$  |                   |
|   |                                   |    | CO20 + 8420 /  |                   |
|   |                                   |    | C12 62   |                   |
| b) e= 3, dividences k= 14   | · incorrect formula               |    | $= a^2 b^2 (1 - e^2 co^2 a)$   |                   |
| x2 + 42 - 1 eandicet  | a= 6° (e'-1) X                    |    | 62 003 00 + 02 81 4 0  |                   |
| $\frac{x^{2}}{a^{2}} + \frac{y^{2}}{b^{2}} = 1  \text{equdicef}$ $\frac{x^{2}}{b^{2}} + \frac{y^{2}}{b^{2}} = 1  \text{equdicef}$ $\frac{x^{2}}{b^{2}} + \frac{y^{2}}{b^{2}} = 1  \text{equdicef}$ $\frac{x^{2}}{b^{2}} + \frac{y^{2}}{b^{2}} = 1  \text{equdicef}$ |                                   |    | 0 605 0 10 0 10  |                   |
|   |                                   |    | $= a b^{2} (1 - e^{2} cos^{2} a)$  |                   |
| and $b^2 = a^2(e^2 - 1)$  |                                   |    | a2(1-c2)costa+asina  |                   |
| and $b^2 = a^2 (e^2 - 1)$   | , jougway                         |    | a-(1-c) cos ara sua  |                   |
| = 36 (9-1)  | -slongway<br>* bossextasiney-ab-0 |    | 2121   |                   |
| = 41 2  |                                   |    | $= \frac{a^2 b^2 (1 - e^2 \cos^2 \alpha)}{a^2 (\cos^2 \alpha + \sin^2 \alpha) - a^2 e^2 \cos^2 \alpha}$  |                   |
| 00 equ x2 - y2 - 1  | equitant: y=brino=-arive(x-acc    |    | 2(cos20+8120)-ate2costo  |                   |
| 36 45 V   | -asine y+abin 0 = biosor-at       |    |  |                   |
|   | * costo                           |    | = a2 b2 (1-e2-coste)   |                   |
| c(i) The tangent to the elipse w  | * bcosa (ae)+ asva (d-ab)         |    | $a^{2}(1-e^{2}\cos^{2}a)$  | 1                 |
| C(i) The tangent to the elipse with<br>22 yl lass equ x core y since<br>art br = 1 has equ x core y since = 1   | Torcaso tarsin20                  |    | 3  |                   |
|   | Breato ta sin a                   |    | $= b^2$  |                   |
| at > (acoso, bsing)   | = abecoso - abl                   |    |  |                   |
| T   | ab/ 00520 + 51420 ) a,b>0         |    | = RHS QED.   |                   |
| an a  | and at t sing                     |    | B  |                   |
| -h  |                                   | d) |  | , a few couldnot  |
|   | ST= ab ecose -1                   |    | A Umg A-B=0  | vesaeve forces    |
| ST = lecoso-1 (Idiot four line)   | ab ar suite                       |    | Ab Umg it with coso = mg sino  | edp A-B = 0.      |
| 1 Costa sinta   |                                   |    | $\frac{100 \times 1000}{100 \times 1000} = \frac{1}{9} \left( \frac{250}{9} \right)^2$   | p                 |
|   | = 10000-11                        |    | - V = 3600 / - tand = (210)  |                   |
| 10 100 No. 20 10 10 10 10 10 10 10  | V COSTO + SINZO QEO               |    | $= \frac{250}{4} m(s) = \frac{1000}{5} m(s)$   |                   |

|  |                        | ( |
|--|------------------------|---|
| Q14  |                        |   |
| (a) (i) 2+2 - cosh a+ iskina   |                        |   |
| + cos(-nce) + isin(-nce)   |                        |   |
| = cosha tisuna -isuna brate cos<br>cucat<br>sin odd)                 |                        | _ |
| $= 2\cos n \theta$ , 2,  |                        |   |
| $(4)$ $Z+2^{-1}=2050, 2^{2}+2^{-2}=200520$                           |                        |   |
| $z^{3} + z^{-3} = 2\cos 30$  |                        |   |
| Scese cos 20 cos 3 0=(2+2-1) (2+2)(2+2)                              |                        |   |
| $= 2^{6} + 1 + 2^{2} + 2^{4} + 2^{-4} + 2^{-2} + 1 + 2^{-6}$         | , Insufficient warting |   |
| $= 2 + 2^{2} + 2^{-2} + 2^{4} + 2^{-4} + 2^{6} + 2^{-6}$             | 0 0                    |   |
| $= 2 + 2 \cos 200 + 2\cos 400 + 2\cos 600$                           |                        |   |
| $= 1 + \cos 26 + \cos 46 + \cos 60$                                  |                        |   |
| $ \tilde{1}  2\cos^2 \omega + 2\cos^2 2\omega + 2\cos^2 3\omega = 2$ |                        |   |
| 1-cos20 + cos40 + cos60 = 0  | , V poorly answered    |   |
| 4 cos a cos 2 a cos 3 a = 0<br>a = 2 to 1 = 17, V                    | 1                      |   |
| $ar = 2e\pi + \pi/4 \qquad 3$  |                        |   |
| a a - 2 k TI ± TI 6  |                        |   |
| 3  |                        |   |
|  |                        |   |
|  |                        |   |
|  |                        |   |
|  |                        |   |
|  |                        |   |
|  |                        |   |
|  |                        |   |

| 0  |                    |
|--|--------------------|
| $\sim \uparrow^{\gamma}$   |                    |
| year They = et 2.  |                    |
| 1446   |                    |
|  |                    |
| -1 2 3   |                    |
|  | · answed a poorly  |
| pts of intersection x=-1, 2.   | · could not obtain |
| Creat Contraction of the Contrac | volume on terms of |
| 17:52 Rotale ship to fam a shell.  | votation around x3 |
| · Valshell OV = 2 TT ROXY r= 3-5K  |                    |
| : Volshell $\sigma V = 2\pi r R \sigma_{2} y_{1} - y_{2}$<br>$R = y_{1} - y_{2}$   |                    |
| $V = \frac{z-2}{2\pi(3-z)(x+2-z^2)} = x+2-zz$  | 22<br>2            |
| x=-1 / 3   |                    |
| V = lim = 27 (3-3c)(x+2-3c)/05c  |                    |
| Osc->o x=-1  |                    |
| $V = 2\pi \int_{-2\pi}^{2} (3-5c) (5c+2-5c^2) dsc$   |                    |
| V -27] (3-50)(5((2-10))))  |                    |
|  |                    |
| (c). $p(x,g)$  |                    |
| -a Da  |                    |
| Gie  | o answers well.    |
|  | o answered where   |
| $24 \sqrt{24}  \nabla V = \frac{1}{2} 24 \cdot 24 \cdot 54 \cdot 60^{\circ} \cdot \sigma_{32}$ $24 = 44 \sqrt{2} \cdot 3 \cdot \sigma_{32}$ $V = 42 \sqrt{2} \sqrt{3} \cdot \sigma_{32}$ $V = 42 \sqrt{3} \cdot \sigma_{32}$   |                    |
| 29 = 12/3 5 22   |                    |
| V=, 5 y2/305c  |                    |
| V=lim ==== 1/2 13 JSC  |                    |
| 0x30 x=-a  | 1                  |
| $= \lim_{x \to 0} \frac{x_{z=2}}{z} (a^2 - x^2) F_3 J_3 L$   |                    |
| (TV-)C   |                    |
| = 13 (" (a2-si2) de  |                    |
| -a   |                    |
| = 2/3 f (a2-x2) de q   |                    |
| -a 17 km 1   |                    |
| $= 13 \int_{-a}^{a} (a^{2} - sc^{2}) dz$<br>= 213 $\int_{-a}^{a} (a^{2} - sc^{2}) dz$<br>= 213 $\int_{-a}^{a} (a^{2} - sc^{2}) dz$<br>= 213 $\begin{bmatrix} a^{2} - sc^{2} \\ -a \end{bmatrix} dz$  |                    |
|  |                    |
| $=\frac{4}{2}a^{2}$  |                    |
| 5 V  |                    |

| 0     |  |                     |
|-------|--|---------------------|
|       | DY   |                     |
| QP IS | +(o,b)   |                     |
| 网位    | $\frac{1}{A \cdot \psi} = \frac{1}{A \cdot \psi} = $ |                     |
| 6910  | A Ly (P ( g g)   |                     |
|       |  |                     |
|       | OP=3 (by Brten)  |                     |
|       | -'.b=3.  |                     |
|       | when PQ B(a, 0), a=s   |                     |
|       | Sub 22 + 92 = 1  |                     |
|       | e G  |                     |
|       | $eq_{1} = \frac{x^{2}}{2c} + \frac{y^{2}}{4c} = 1.$  | 1                   |
|       |  |                     |
| fi ,  | $\frac{x^2}{2r} + \frac{y^2}{q} = \frac{3cos^2\alpha}{2r} + \frac{3^2siu^2\alpha}{q}$  |                     |
|       | $= \cos^2 \alpha + \sin^2 \alpha$ $= 1 = PHS \cdot QED,$   |                     |
|       | = I = M. QED.  |                     |
|       |  |                     |
|       | x = gcose  |                     |
|       | dr.<br>de = - 5 Sin Ce   |                     |
|       | y = 3 8140   | o Shudents used equ |
|       | dy = 3 coste /   | of tain rather than |
|       |  | finding il          |
|       | oly: dx × de = 3000 · -1<br>obc = Ja × Jy = 3000 · -1<br>Jsina   | v                   |
|       | π  |                     |
|       | when $\sigma = \frac{1}{6}$ -36  |                     |
|       | $\frac{dy}{dy} = \frac{-3 \times \frac{3}{2}}{5 \times \frac{3}{2}} = \frac{-36}{5}$   |                     |
|       | L  | · messy algebra.    |
|       | x = 5×5 = 55,00  |                     |
|       | $y = 3 \times \frac{1}{2} = \frac{3}{2}$ (4)   |                     |
|       | $y - \frac{3}{2} = -\frac{36}{5}(x - \frac{56}{5}),$   |                     |
|       | 0  |                     |
|       | 3/310 + 59 - 30 = 2  |                     |
|       | -  |                     |
|       |  |                     |
|       |  |                     |

| N(6) | WK+B+Y=0. JV   |                      |
|------|--|----------------------|
|      | ab+ a8 + p8= 36  |                      |
|      |  |                      |
| -    | $\frac{\beta \delta}{\alpha} + \frac{\alpha \delta}{\beta} + \frac{\alpha \delta}{\beta} = (\frac{\beta \delta}{\beta})^2 + (\alpha \delta)^2 + (\alpha \delta)^2$   | overy poorly answer  |
| _    | XP8 WO   |                      |
| i se | $= \left(\beta \cdot \frac{1}{5} + \frac{1}{5$   | Bra) . shudents      |
| 1.1  |  | failed to set up     |
|      | $= (\beta + \alpha + \alpha + \beta^2 - 2\alpha \beta + (\alpha + \beta + \beta))$   | step 1.              |
|      | a pg   | 1                    |
|      |  |                      |
|      | $= (3p)^2 - 2q(0)$   |                      |
|      | $= \frac{q}{p^2} \qquad \qquad$   |                      |
|      | - 1b <sup>2</sup>  |                      |
|      | -q.  |                      |
|      | AV + X + X + A + B X = X + B + X   | 2                    |
|      | $\frac{\beta_{X}}{\beta_{X}} + \frac{\alpha_{X}}{\beta_{X}} + \frac{\alpha_{X}}{\beta_{X}} + \frac{\beta_{X}}{\beta_{X}} + \frac{\beta_{X}}{\beta$ |                      |
|      | = 0 - 2.3p = -6p   |                      |
|      |  |                      |
| **   | °°, <u>b</u> 8 , <u>x</u> 8 , <u>x</u> <u>8</u> = / <u>x</u> <u>8</u> 8 = -q.  |                      |
|      | regol equ 3+9622=60x+a=0   | a normalized and the |
|      | regal equ 23+ 9 p2 2-6 px+q=0  |                      |
|      | i) xp =1 is a rood V   | . Hence could not    |
|      |  | anteur. (i)          |
|      | $\frac{1}{q^2} + \frac{qp^2}{q^2} - 6ptq = 0$  | Autor i ilo          |
|      | 94 1912 (10 102 0  |                      |
|      | $(3p-q)^2 + q = 0$ (2)   |                      |
|      |  |                      |
|      |  |                      |
|      |  | 63                   |
|      |  |                      |
|      |  |                      |
|      |  |                      |
|      |  |                      |
|      |  |                      |

Let A=sin'z + B=cos'z Az 1/2 Trav z=snu A x=cosB IS d) TI-12+= cosA and TI-x+ = Sing LHS = sin(sin'x - cos'x)= sin(A-B) =  $\sin A - \cos B - \cos A \sin B$ =  $x \cdot x - \overline{1 - x^2} = 2x^2 - 1$ 

| Qlag in hal position @ origin, + in hal direct the  | 16b) Let A=x+y   |
|---|--|
| V=-g-lev  | B=22-Y   |
| in that conductors: $t=0, x=0, V=u$ .   | -: A+B = 2x  |
| Iehn bet at w   | $X = (\underline{A+B})$  |
| udu = - (g+ku)  | Y = (A - B)  |
|   | 2.   |
| $-dc = \frac{v dv}{g + k v}$  | : 81 A + 81 N B = 281 ( 4+B) (05 ( +B) . She   |
| = & du = & u du<br>g+&u)  |  |
|   | COSA + COSB = 2 COS( 2/COS( 2) / 11  |
| $- \mathcal{R} dx = \left[ 1 - \frac{q}{g + k \omega} \right] du$                             | SinA+SUB - 28in (Att) cos(AB)  |
| - &x2 + c = &v-g lu g+ &u assured N.  | poorly cos Atcos B 2cos(AtB) cos (ATB)   |
| x=0, N= H=> C= ku-g lu g + ku only ( shud   | 1 aut  |
| P. I. M.  |  |
| - X = R + E la gtku V- U  |  |
| If tarticle reaches greatest height   | NOW AFBIC = TT   |
| It at a time T, its speed w = 0.  | · A+B=T-C (4) & could  |
|   | LHS tan TI-C   |
| H= ten en 1 greul   | $= \tan(\frac{\pi}{2} - \frac{c}{2}) \sqrt{\frac{\pi}{2}}$   |
|   | - cot 2  |
| Rehn bet with   | = 2HS. QED   |
|   |  |
| du = - (g + k u)  | a VIII - toud  |
| -dt_ du & t= - Edu  | $(16c) \text{ Let } t = \frac{t - t \alpha}{2}$ $= 2 \cos \alpha = \frac{1 - t^2}{1 + t^2} \qquad (18t) \text{ At at}$ |
| $-dt = \frac{dw}{g+\varepsilon w}, -\varepsilon dt = \frac{-\varepsilon dw}{g+\varepsilon w}$ | $= 2 \cos \alpha = \frac{1}{1+\epsilon_2}$ in Not at<br>i. (+t <sup>2</sup> ) $\cos \alpha = 1-t^2$                    |
| R. Edu P. Carbul  | $(tt)\cos x = t-t^{2}$   |
| - led = ledu :, _ lette = lu[g+leu]   | $t^{2} = \frac{1 - \cos \alpha}{1 + \cos \alpha} = \frac{1 - \tan 2 \cot y}{1 + \tan 2 \cot y}$                        |
|   | towerly  |
| $-t = -t en \left(\frac{g+ber}{g+bu}\right) - 2$  | t <sup>2</sup> = <u>1-tanic colu</u> × fing cosx<br><u>1+tanic coly</u> sinic cos g                                    |
|   | since cos g  |
| T = - te en 19+ ku Kom (2)  | = singlesse - sinx cosy  |
| · ln 9 = - & T  | Styly COSX + Sinx Cosy   |
| gtkul   | = sin (y->c) cosec(y+x)  |
| $\Rightarrow$ H = $\frac{4}{k} - \frac{gkT}{k2}$  |  |
| $v \cdot v = kH + gT$   | tent to a for (y->) cosec (y+x)  |

| Rlag in hal positor @ origin, + in hal direct Atue                   | 16 b) Let A=x+y   |
|--|---|
| v = -g - kv  | B = x - y   |
| initial condutions: $t=0, x=0, V=u$ .                                | -A+B = 2x   |
| Iehn bet at w  | X = (A+B)   |
| udu = - (greu)   | Y = (A-B)   |
|  | $\frac{1}{2}$   |
| $-d_{DC} = \frac{v dv}{g + k v}$                                     | isinAt SinB = 25in ( HB/cos (AB) . Shidente failed to   |
| - Rabe = Rudu /  | at the second to  |
| = ledic = lev du<br>g+ lev   | cosA + cosB = 2 cos (AB) cos (AB) apply sum to prod for   |
| $- \& dx = \left[ 1 - \frac{g}{g + \varepsilon_{\alpha}} \right] du$ | STUATEUB - 2814 (Att) cos(AB)   |
| - kx2 + c = kv - g ku   g + ku   arsuned N. poorly                   | COS A + COS B 2COS (A+B) COS (A+B)  |
| x=0, N= H=> C= ku-g ln g+ku Only (shud-got                           |   |
| P. A. WA   | $= \tan \left( \frac{A \pm B}{2} \right) \sqrt{\frac{A \pm B}{2}}$  |
| ix = i + # ln g+ku v- 0 jue mandes                                   |   |
| If particle reaches greatest height                                  | NOW A+B+C = TT  |
| if at a time T, its speed w = 0.                                     |   |
|  |   |
| It= 4 en great   | LHS tan T-C   |
| Ins ter ter torighter  | $= \tan(\frac{\pi}{2} - \frac{c}{2}) \sqrt{\frac{c}{2}}$  |
| Rean bet with  | I cot 2   |
| du - hou ban   | = EHS. QED  |
| dv = - (g + k v)   | a VI C , tand   |
| -dt du Bil- bdu  | $(16c)$ Let $t = \frac{tand}{2}$  |
| $-dt = \frac{dw}{g+Ew}, - k dt = \frac{-kdw}{g+Ew}$                  | => $\cos x = \frac{1-t^2}{1+t^2}$ ind attempted by many   |
|  | $(t+t)\cos x = (-t)$  |
| - lel = ledu :- lebte = lugteu                                       | $t^{2} = \frac{1 - \cos \alpha}{1 + \cos \alpha} = \frac{1 - 4 \cos \alpha \cos \alpha}{1 + 4 \cos \alpha \cos \alpha}$ |
| $- t = -t ln \left( \frac{g+by}{g+bu} \right) = 2$                   | 1-tenecotu com  |
| - t. to ligten (   | t <sup>2</sup> = 1 - tank colu x Fing cosx<br>1+ tank coly Sinx cos g   |
| T = - te en g+ Bu Jom E  |   |
| 1 le un 1g+ku 1 gom C  | = singles sc - singlesg   |
| or ln I - BT   | stifty cosx + Sinx cosy   |
| 0  | = sin (y-x) cosec(y+x)  |
| $\Rightarrow H = \frac{1}{k} - \frac{gkT}{g}$                        |   |

| 0   |  |   |
|-----|--|---|
| 100 | $\frac{1}{9-108in^{2}c} = \frac{8ec^{2}c}{9-bn^{2}x}$  |   |
|     | Since $\sin^2 x = \frac{\tan^2 x}{1 + \tan^2 x}$ , we get<br>$\frac{1}{q - 10 \sin^2 x} = \left[ \frac{a - 10}{1 + \tan^2 x} \right]^{-1}$<br>$= \left[ \frac{a - \tan^2 x}{1 + \tan^2 x} \right]^{-1}$<br>$= \frac{1 + \tan^2 x}{1 + \tan^2 x}$<br>$= \frac{1 + \tan^2 x}{9 - \tan^2 x}$<br>$= \frac{5c^2 x}{9 - \tan^2 x}$<br>Hence using sub $u = \tan x$<br>$du = \frac{5c^2 x}{5c^2 x}$<br>$du = \frac{5c^2 x}{5c^2 x}$<br>$du = \frac{5c^2 x}{5c^2 x}$<br>$du = \frac{5c^2 x}{5c^2 x}$<br>$du = \frac{5c^2 x}{5c^2 x}$ | , very few god 2/4<br>, many sub sino hært<br>but god læst in<br>Alg.<br>, 2. som good attempts |
|     | $\int_{6}^{\frac{1}{3}} \frac{1}{9^{-105/22c}} d_{2}$ $= \int_{6}^{\frac{1}{3}} \frac{1}{9^{-12}} d_{2}$   |   |
|     | $= \frac{1}{6} \left[ l_{11} \left[ \frac{3t_{4}}{3^{-\alpha}} \right] \right]_{0}^{T_{3}}$ $= \frac{1}{6} l_{11} \frac{3t_{13}}{3^{-12}}$ (   |   |
|     | $= \frac{1}{6} \ln \left( \frac{3+15}{3^2-(3)^2} \right)^2 = \frac{1}{6} \ln \left( \frac{24}{13} \right)$ $= \frac{1}{6} \ln \left( \frac{12+6}{6} \right)$   |   |
|     | - to (n (2+53)<br>or see shud<br>attemp.   | ents  |
|     |  |   |

Shidents affenist. Start here d.  $\int_{0}^{\pi/3} \frac{1}{9 - 10 \sin^2 \kappa} d\kappa$ .  $=\int_{-\infty}^{\pi/3} \frac{1}{q'-10(1-\cos^{2}x)} dx$  $= \int_{0}^{\pi/3} \frac{1}{-1(\frac{10}{4}cos^{2}\kappa)} d\kappa$  $= \int_{0}^{\pi/3} \frac{1}{100} d\kappa d\kappa.$  $|t| = tank \qquad |x=\pi/3, t= fs$ dt = sec2x dx k=0, t=0  $dt = \sec x \cos x$   $dt = (1+t^2)dx$   $\log \cos^2 x = 10$   $H + t^2$  $dx = \frac{dt}{dt}$  $\overline{I} = \int_{0}^{\sqrt{5}} \frac{1}{\frac{10}{1142} - 1} \cdot \frac{dt}{1+t^2}$  $= \int_{0}^{\sqrt{3}} \frac{1}{\frac{10 - (1 + \sqrt{2})}{1 + \sqrt{2}}} \cdot \frac{dt}{1 + t^{2}}$  $= \int_{0}^{t_{3}} \frac{1+t^{2}}{4-t^{2}} \cdot \frac{dt}{1+t^{2}}$  $= \int_{-1}^{1} \frac{1}{3-t} dt = \int_{-1}^{1} \frac{1}{(3-t)(3+t)} dt$  $= \frac{1}{t} \int_{0}^{t_{3}} = \frac{1}{t} \left[ \frac{1}{2} \ln (3-t) + \ln (3+t) \right]_{0}^{t_{3}}$  $= \frac{1}{t} \left[ -\ln (3-t_{3}) + \ln (3+t_{3}) + \ln (3+t_{3}) \right]_{0}^{t_{3}}$  $= m \frac{1}{\sqrt{3}}$ = 1 lu 3+F3

Office Use Only - Do NOT write anything, or make any marks, below this line.

- 1 -