HORNSBY GIRLS HIGH SCHOOL

Mathematics Extension 2

Year 12 Higher School Certificate
Trial Examination Term 3 2018

STUDENT NUMBER: ______________________________

General Instructions

- Reading Time – 5 minutes
- Working Time – 3 hours
- Write using black or blue pen
  Black pen is preferred
- NESA-approved calculators and drawing templates may be used
- A reference sheet is provided separately
- In Questions 11 – 16, show relevant mathematical reasoning and/or calculations
- Marks may be deducted for untidy and poorly arranged work
- Do not use correction fluid or tape
- Do not remove this paper from the examination room

Total marks – 100

Section I Pages 3 – 6

10 marks

Attempt Questions 1 – 10

Answer on the Objective Response Answer Sheet provided

Section II Pages 7 – 18

90 marks

Attempt Questions 11 – 16

Start each question in a new writing booklet

Write your student number on every writing booklet

Question Total	***1-10***	***11***	***12***	***13***	***14***	***15***	***16***	***Total***
	/10	*/15*	*/15*	*/15*	*/15*	*/15*	*/15*	*/100*

This assessment task constitutes 45% of the Higher School Certificate Course School Assessment

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the Objective Response answer sheet for Questions 1 – 10

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1 The ellipse with equation $\frac{x^2}{9}+\frac{y^2}{4}=1$ has foci:

(A) $\left(0,\pm\sqrt{5}\right)$

(B) $\left(\pm\sqrt{5},0\right)$

(C) $\left(0,\frac{\pm\sqrt{5}}{3}\right)$

(D) $\left(\pm\frac{5}{\sqrt{3}},0\right)$

2 The equation $x^3+2x^2+3=0$ has roots α, β and γ.
Which equation has roots 2α, 2β and 2γ?

(A) $\left(x-2\right)^3+2\left(x-2\right)^2+3=0$

(B) $2x^3+4x^2+6=0$

(C) $8x^3+8x^2+3=0$

(D) $x^3+4x^2+24=0$

3 The derivative of the implicitly defined curve $x^2-2xy+4y^2=12$ is given by $\frac{dy}{dx}=\frac{x-y}{x-4y}$.

The x-values of the points on the curve where the tangents are vertical are:

(A) $x=\pm1$

(B) $x=\pm2$

(C) $x=\pm4$

(D) $x=1$

4 A hyperbola has asymptotes $y = \pm x$ and passes through the point $(3, 2)$.

The Cartesian equation of the hyperbola is:

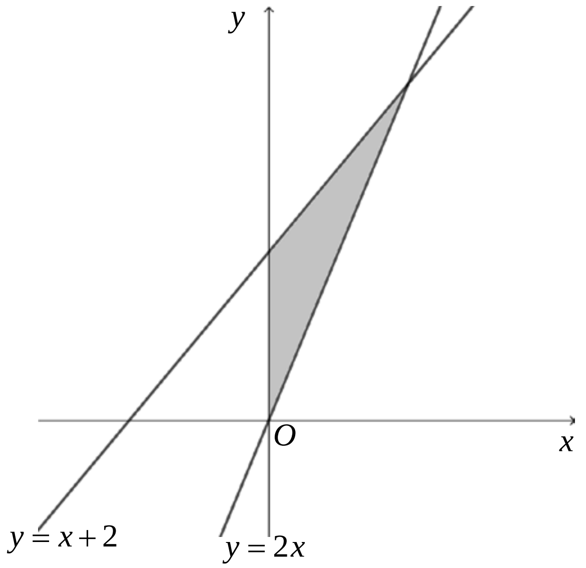
(A) $\frac{y^2}{3} - \frac{x^2}{3} = 1$

(B) $\frac{x^2}{3} - \frac{y^2}{3} = 1$

(C) $\frac{y^2}{5} - \frac{x^2}{5} = 1$

(D) $\frac{x^2}{5} - \frac{y^2}{5} = 1$

5 In the diagram below, the region bounded by the lines $y = x + 2$, $y = 2x$ and the y-axis is rotated about the line $y = 5$.



NOT TO SCALE

The volume of the solid of revolution is given by:

(A) $\pi \int_0^2 \left(16 - 14x + 3x^2\right) dx$

(B) $\pi \int_0^2 \left(14x - 16 - 3x^2\right) dx$

(C) $\pi \int_0^2 (5 - 2x)^2\, dx$

(D) $\pi \int_0^2 (3 - 2x)^2\, dx$

6 By taking logarithms of both sides, or otherwise, the derivative of $y = x^{\sin x}$ with respect to x is:

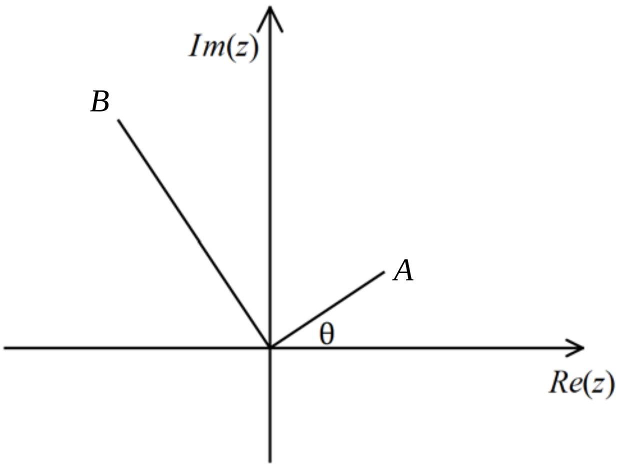
(A) $x^{\sin x - 1} \sin x$

(B) $x^{\sin x}\left(\cos x \log_e x + \dfrac{\sin x}{x}\right)$

(C) $x^{\sin x}\left(\cos x \log_e x\right)$

(D) $\cos x \log_e x + \dfrac{\sin x}{x}$

7 The points A and B in the diagram represent the complex numbers z_1 and z_2 respectively, where $|z_1| = 1$ and $\arg(z_1) = \theta$ and $z_2 = \sqrt{3} i z_1$.



NOT TO SCALE

$z_2 - z_1$ in modulus-argument form is:

(A) $2\left[\cos(120° + \theta) + i \sin(120° + \theta)\right]$

(B) $3\left[\cos(120° + \theta) + i \sin(120° + \theta)\right]$

(C) $2\left[\cos(120° - \theta) + i \sin(120° - \theta)\right]$

(D) $3\left[\cos(120° - \theta) + i \sin(120° - \theta)\right]$

8 The equation $2x^4 + 9x^3 + 6x^2 - 20x - 24 = 0$ has a root of multiplicity 3.
The root of multiplicity 3 is:

(A) $x = -2$

(B) $x = \frac{3}{2}$

(C) $x = -\frac{1}{4}$

(D) $x = -\frac{1}{2}$

9 The value of $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{dx}{\sin x \cos x}$ is:

(A) $\log_e \sqrt{2}$

(B) $\log_e \sqrt{3}$

(C) $\log_e \frac{1}{\sqrt{2}}$

(D) $\log_e \frac{1}{\sqrt{3}}$

10 The minimum value of $f\left(x\right) = x\cos^{-1}\left(\frac{x}{3}\right)$ is:

(A) 0

(B) -3

(C) $-\frac{3\pi}{2}$

(D) -3π

End of Section I

Section II

90 marks
Attempt Questions 11 – 16
Allow about 2 hours and 45 minutes for this section

Answer each question in a new writing booklet. Extra writing booklets are available.

In Questions 11 – 16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Start a new writing booklet

(a) Let $z = \dfrac{3-i}{1-2i}$.

(i) Express z in the form $a+ib$ where a and b are real. **1**

(ii) Hence, express z^4 in the form $a+ib$ where a and b are real. **2**

(b) On the Argand diagram, shade in the region containing all points representing the complex number z such that $\left|z-(1+i)\right| \le 1$ and $\operatorname{Re}(z) > 1$. **3**

(c) Find the square roots of $-5-12i$ in the form $a+ib$ where a and b are real. **2**

(d) If the roots of the equation $x^3+4x-2=0$ are α, β and γ, find the value of:

(i) $\alpha^2+\beta^2+\gamma^2$. **2**

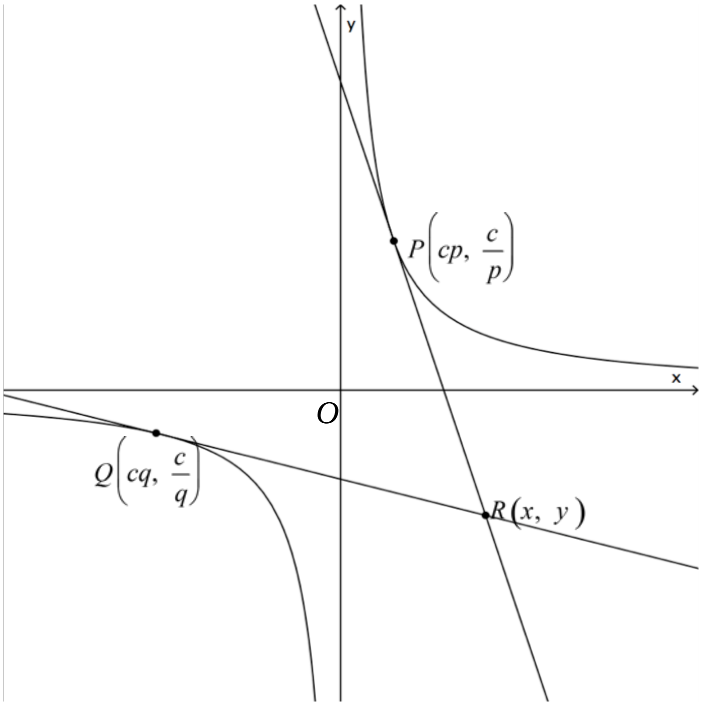
(ii) $\alpha^4+\beta^4+\gamma^4$. **2**

(e) Using the substitution $t = \tan\dfrac{x}{2}$, evaluate $\displaystyle\int_0^{\frac{\pi}{2}} \frac{dx}{3+5\cos x}$. **3**

Question 12 (15 marks) Start a new writing booklet

(a) $P\left(cp, \frac{c}{p}\right)$ and $Q\left(cq, \frac{c}{q}\right)$ are points on the rectangular hyperbola $xy = c^2$, $c > 0$.

Tangents drawn at P and Q intersect at $R(x, y)$.



NOT TO SCALE

(i) Show that the tangent at P has equation $x + p^2 y = 2cp$. **2**

(ii) Show that $R(x, y)$ has coordinates $x = \frac{2cpq}{p+q}$, $y = \frac{2c}{p+q}$. **2**

(iii) If P and Q move on the hyperbola such that $p^2 + q^2 = 2$, show that the Cartesian equation of the locus of R is $xy + y^2 = 2c^2$. **2**

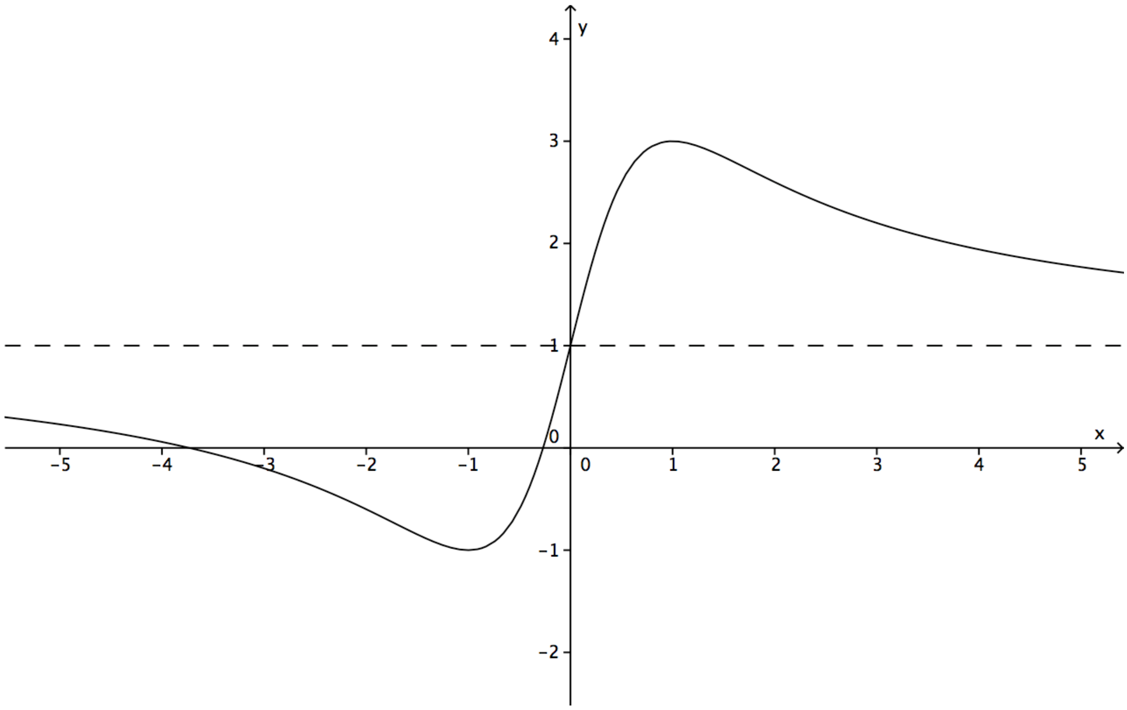
(b) (i) Find A and B such that $\cos x \equiv A(\cos x - 2\sin x) + B(\sin x + 2\cos x)$. **1**

(ii) Hence, find $\int \frac{\cos x}{\sin x + 2\cos x} dx$. **2**

Question 12 continues on page 9

Question 12 (continued)

(c) The diagram shows the graph of $y = f(x)$.



Draw a separate half-page diagram for each of the following functions, showing all asymptotes and intercepts.

(i) $y = \sqrt{f(x)}$ **2**

(ii) $y = \dfrac{1}{f(x)}$ **2**

(iii) $y = \cos^{-1}\left[f(x) \right]$ **2**

End of Question 12

Question 13 (15 marks) Start a new writing booklet

(a) By using a substitution, show that $\int_0^1 \frac{x}{(3x+1)^2}\,dx = \frac{2}{9}\log_e 2 - \frac{1}{12}$. **3**

(b) (i) Show that $\left(z - \frac{1}{z}\right)^5 = \left(z^5 - z^{-5}\right) - 5\left(z^3 - z^{-3}\right) + 10\left(z - z^{-1}\right)$. **2**

(ii) By letting $z = \cos\theta + i\sin\theta$, or otherwise, show that **2**

$32\sin^5\theta = 2\sin 5\theta - 10\sin 3\theta + 20\sin\theta$.

(iii) Hence, find the exact value of $\int_0^{\frac{\pi}{2}} \sin^5\theta\, d\theta$ **2**

(c) The polynomial $P(x) = 3x^3 + 4x^2 + 5x - 6$ has only one real zero α, where $|\alpha| < 1$. **2**

Factorise $P(x)$ over the complex field.

(d) A car of mass 800 kg travels along a straight horizontal road. The engine of the car produces a constant driving force of magnitude 2000N. At time t seconds, the speed of the car is v ms^{-1}. As the car moves, the total resistance to the motion of the car is of magnitude $\left(400 + 4v^2\right)$N. The car starts from rest.

(i) Show that the acceleration $a = 2 - \frac{v^2}{200}$. **1**

(ii) Show that $v = 20\left(\frac{e^{\frac{t}{5}} - 1}{e^{\frac{t}{5}} + 1}\right)$. **2**

(iii) Find the limiting speed of the car, giving reasons for your answer. **1**

End of Question 13

Question 14 (15 marks) Start a new writing booklet

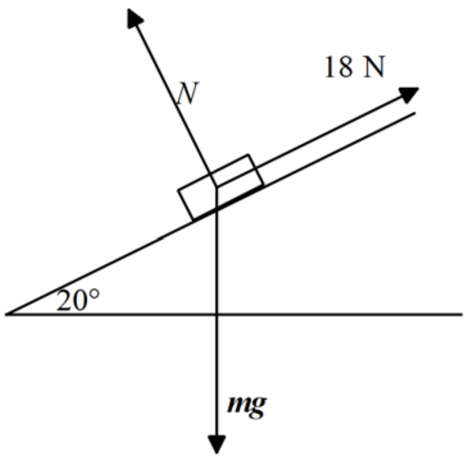
(a) Find $\int \frac{1}{e^x + 4e^{-x}}\,dx$. **2**

(b) It is given that $I_n = \int_0^4 x^n \sqrt{4-x}\,dx$, where $n \ge 0$, and $\int_0^4 \sqrt{4-x}\,dx = \frac{16}{3}$.

(i) Show that $I_n = \frac{8n}{2n+3} I_{n-1}$ for $n \ge 1$. **2**

(ii) Hence find I_2. **1**

(c) A box of mass 2 kg is pulled up a rough plane face by means of a light rope. The plane is inclined at an angle of $20°$ to the horizontal, as shown in the diagram below. The rope is parallel to the slope of the plane. The tension in the rope is 18N.
Use $g = 9.8\text{ ms}^{-2}$.



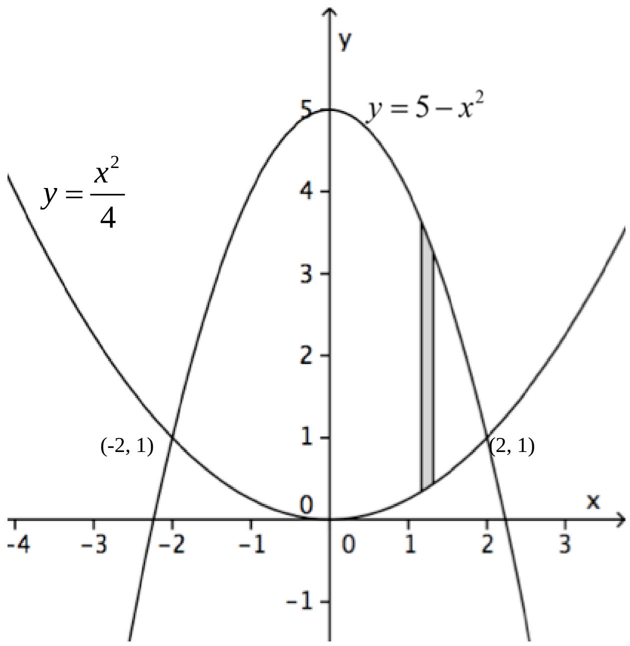
NOT TO SCALE

(i) Show that the normal reaction of the plane on the box is 18.4 N, correct to one decimal place. **1**

(ii) Given that the friction between the box and the plane is $\frac{3}{5}$ of the normal reaction, find the acceleration of the box. **2**

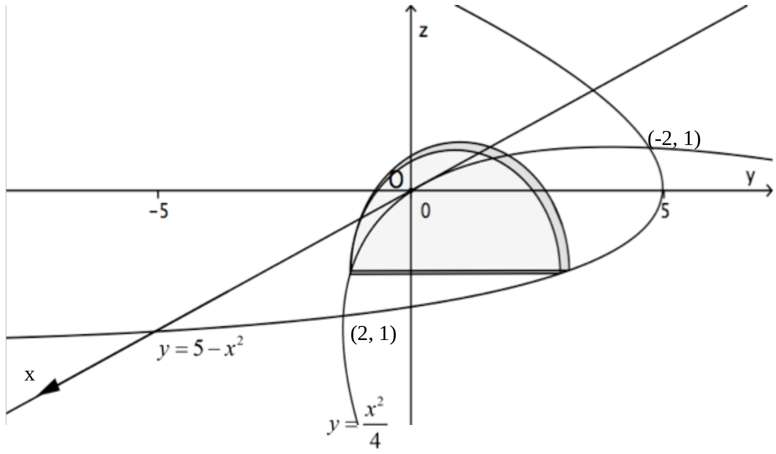
Question 14 continues on page 12

(d) The base of a solid is the region bounded by the curves $y=5-x^2$ and $y=\frac{x^2}{4}$, as shown in the diagram below. The two parabolas intersect at points $(2,1)$ and $(-2,1)$.



NOT TO SCALE

Cross sections by planes perpendicular to the x-axis are semi-circles with the diameter in the base, as shown in the diagram below.



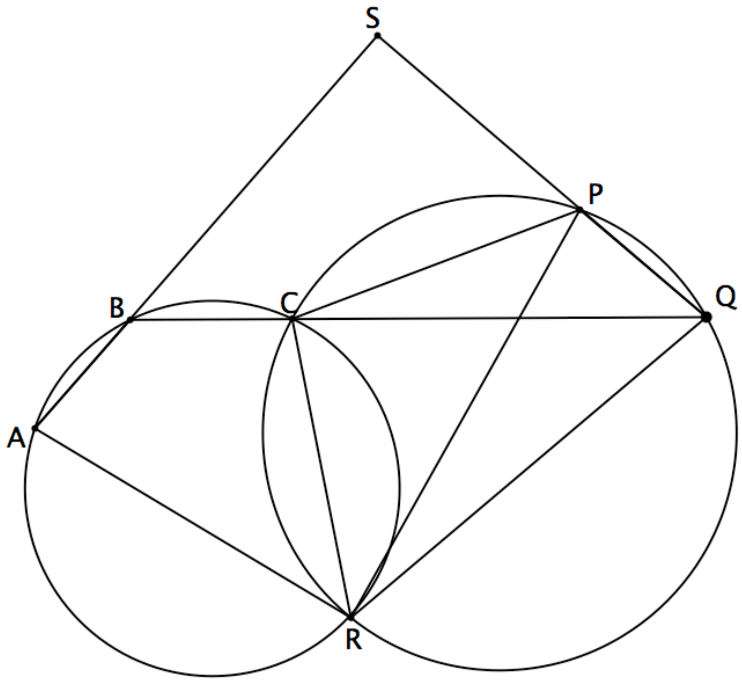
NOT TO SCALE

(i) Show that the area of the semi-circle is the expression $A=\frac{25\pi}{128}\left(4-x^2\right)^2$. **2**

(ii) Hence, find the volume of the solid. **2**

Question 14 continues on page 13

(e) In the diagram below, circles $ABCR$ and $CPQR$ intersect at C and R, and BCQ is a straight line. AB produced and QP produced meet at an external point S.



NOT TO SCALE

Copy or trace the diagram into your writing booklet.

(i) Prove that $\angle BAR = \angle QPR$. **2**

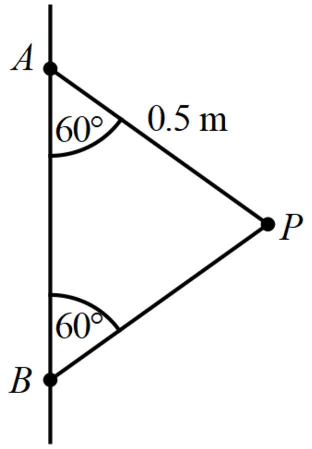
(ii) Prove that $ARPS$ is a cyclic quadrilateral. **1**

End of Question 14

Question 15 (15 marks) Start a new writing booklet

(a) The diagram below shows a particle *P* of mass *m* attached by two light strings to fixed points *A* and *B*, where *A* is vertically above *B*. The strings are both taut and *P* is moving in a horizontal circle with constant angular velocity $2\sqrt{3g}$ radians per second.

Both strings are 0.5 m in length and inclined at $60°$ to the vertical.



NOT TO SCALE

(i) If T_A is the tension in the string AP and T_B is the tension in the string BP, show that $T_A - T_B = 2mg$. **1**

(ii) Find the tensions, T_A and T_B, in the strings in terms of m and g. **2**

Question 15 continues on page 15

Question 15 (continued)

(b) A particle of mass m is falling vertically under gravity in a resisting medium. The particle is released from rest.

The speed v, in metres per second, of the particle at a distance x from rest is given by $v^2 = 2kg\left[1 - e^{\frac{-x}{k}}\right]$, where k is a positive constant.

(i) Show that the magnitude of resistance, of the medium is $\dfrac{mv^2}{2k}$. **2**

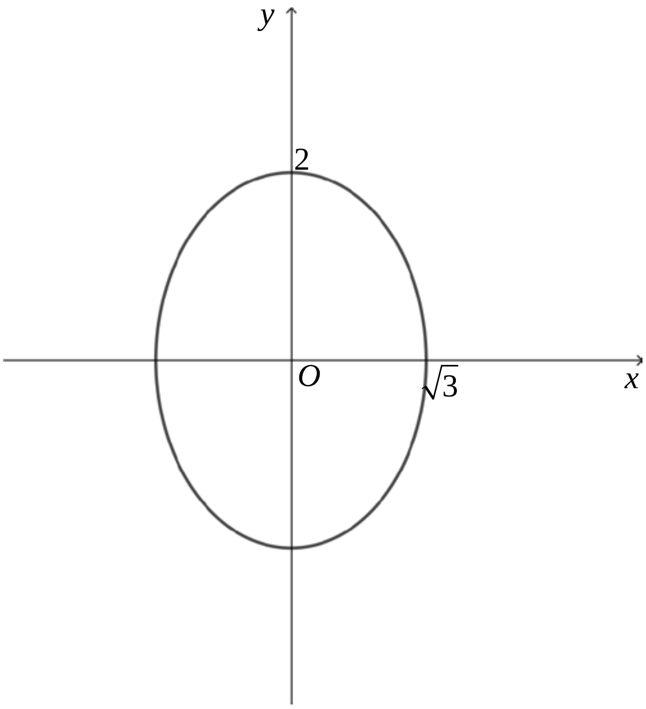
The particle is projected upwards in the same medium with speed $\sqrt{2kg}$.

(ii) Show that the maximum height reached by the particle above the point of projection is $k\log_e 2$ metres. **2**

(iii) Find the time taken to reach the maximum height above the point of projection. **2**

Question 15 continues on page 16

(c) The diagram of the ellipse E with equation is $\frac{x^2}{3}+\frac{y^2}{4}=1$ shown below.



NOT TO SCALE

The line $y=mx+4$, where $m>0$ is a tangent to the ellipse E at the point P.

(i) Find the value of m. **2**

(ii) Show that the coordinates of P are $\left(-\frac{3}{2},1\right)$. **1**

The normal at P crosses the y-axis at the point A.
The tangent at P crosses the y-axis at the point B.

(iii) Find the area of triangle APB. **3**

End of Question 15

Question 16 (15 marks) Start a new writing booklet

(a) One root of the cubic equation $z^3 + az + 10 = 0$, where a is real, is $1 + 2i$.

(i) Find all three roots of the equation and find the value of a. **2**

(ii) Plot all three roots on the Argand Diagram with a 1:1 horizontal : vertical axes. **1**

(iii) A point z moves in the complex plane such that it lies on the circumference of a circle that passes through the points representing the roots of the equation. **3**
Find the equation of the locus of z in the form $|z - z_1| = r$.

(b) It is known that n is a positive integer, where $n \geq 2$ and that a and b are real and positive.

(i) Prove that $\dfrac{a+b}{2} \geq \sqrt{ab}$. **1**

It is known that for positive real numbers $a_1, a_2, \ldots$ that $\sqrt[n]{a_1 \times a_2 \times \ldots \times a_n} \leq \dfrac{a_1 + a_2 + \ldots + a_n}{n}$.

(ii) Prove that $n! \leq \left(\dfrac{n+1}{2}\right)^n$. **2**

Question 16 continues on page 18

(c) It is known that $\sec\theta > \tan\theta$ for $0 \le \theta < \frac{\pi}{2}$.

(i) Sketch $y = \sec\theta$ and $y = \tan\theta$ for $0 \le \theta < \frac{\pi}{2}$ on the same number plane. **1**

(ii) Prove the identity $\sec\theta - \tan\theta = \dfrac{1}{\sec\theta + \tan\theta}$. **1**

(iii) Deduce from (i) and (ii) that $0 < \sec\theta - \tan\theta \le 1$ for $0 \le \theta < \frac{\pi}{2}$, giving clear reasons. **2**

(iv) Find the general solution to the equation $\sec\theta - \tan\theta = \frac{1}{2}$. **2**

End of Examination

Year 12 Mathematics Extension 2 Trial Examination Solutions 2018
Multiple Choice

Question 1

$\frac{x^2}{9}+\frac{y^2}{4}=1 \qquad a=3,\ b=2$

Eccentricity:

$b^2=a^2\left(1-e^2\right)$

$4=9\left(1-e^2\right)$

$e^2=\frac{5}{9}$

$e=\frac{\sqrt{5}}{3}\ (e>0)$

Foci:

$S=\left(\pm 3\times\frac{\sqrt{5}}{3},0\right)$

$=\left(\pm\sqrt{5},0\right)$

(B)

Question 2

$x^3+2x^2+3=0$

New equation:

$\left(\frac{x}{2}\right)^3+2\left(\frac{x}{2}\right)^2+3=0$

$\frac{x^3}{8}+\frac{2x^2}{4}+3=0$

$x^3+4x^2+24=0$

(D)

Question 3

$\frac{dy}{dx}=\frac{x-y}{x-4y}$

For vertical tangents, let

$x-4y=0$

$y=\frac{x}{4}$

Sub into curve:

$$x^2 - 2x\left(\frac{x}{4}\right) + 4\left(\frac{x}{4}\right)^2 = 12$$
$$x^2 - \frac{x^2}{2} + \frac{x^2}{4} = 12$$
$$16x^2 - 8x^2 + 4x^2 = 12\times 16$$
$$12x^2 = 12\times 16$$
$$x^2 = 16$$
$$x = \pm 4$$

(C)

Question 4

Note that $y = \pm x$ are asymptotes and hyperbola passes through $(3,2)$, that is $y < x$.

Therefore hyperbola is sideways.

$$y = \pm\frac{b}{a}x$$
$$b = \pm a$$
$$b^2 = a^2$$
$$\frac{x^2}{a^2} - \frac{y^2}{a^2} = 1$$

Sub $(3,2)$

$$\frac{9}{a^2} - \frac{4}{a^2} = 1$$
$$a^2 = 5$$
$$a = \sqrt{5}\,(a > 0)$$

Equation is $\frac{x^2}{5} - \frac{y^2}{5} = 1$ (D)

Question 5

Slices perpendicular to the axis of rotation form annular discs.

$R = 5 - 2x$

$$r = 5 - (x+2)$$
$$= 3 - x$$

$$\Delta V = \pi\left(R^2 - r^2\right)\Delta x$$
$$= \pi\left[(5-2x)^2 - (3-x)^2\right]\Delta x$$
$$= \pi\left[25 - 20x + 4x^2 - \left(9 - 6x + x^2\right)\right]\Delta x$$
$$= \pi\left[25 - 20x + 4x^2 - 9 + 6x - x^2\right]\Delta x$$
$$= \pi\left(3x^2 - 14x + 16\right)\Delta x$$

(A)

Question 6

$$y = x^{\sin x}$$

$$\ln y = \sin x \ln x$$

$$\frac{1}{y}\frac{dy}{dx} = \cos x \ln x + \sin x \times \frac{1}{x}$$

$$\frac{dy}{dx} = y\left(\cos x \ln x + \frac{\sin x}{x}\right)$$

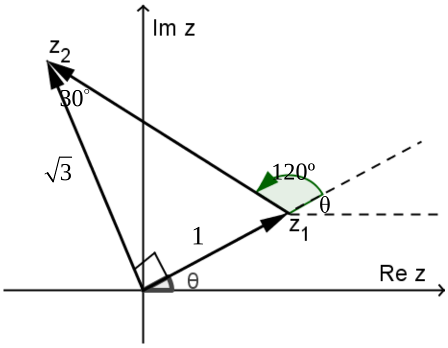
$$= x^{\sin x}\left(\cos x \ln x + \frac{\sin x}{x}\right)$$

(B)

Question 7

$$\arg(z_2 - z_1) = 90° + 30° + \theta$$

$$= 120° + \theta$$



(A)

$$|z_2 - z_1| = \sqrt{1^2 + \sqrt{3}^2}$$

$$= 2$$

Question 8

$$P(x) = 2x^4 + 9x^3 + 6x^2 - 20x - 24$$

$$P'(x) = 8x^3 + 27x^2 + 12x - 20$$

$$P''(x) = 24x^2 + 54x + 12$$

Let $P''(x) = 0$

$$24x^2 + 54x + 12 = 0$$

$$4x^2 + 9x + 2 = 0$$

$$x = \frac{-9 \pm \sqrt{81 - 32}}{8}$$

$$= \frac{-1}{4} \; or \; -2$$

$$P(-2) = 2(-2)^4 + 9(-2)^3 + 6(-2)^2 - 20(-2) - 24$$

$$= 0$$

(A)

Question 9

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{dx}{\sin x \cos x} = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\sec^2 x dx}{\sin x \cos x \sec^2 x}$$

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\sec^2 x}{\tan x} dx$$

$$= \left[\ln\left(\tan x\right)\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$$

$$= \ln\left(\tan\frac{\pi}{3}\right) - \ln\left(\tan\frac{\pi}{4}\right)$$

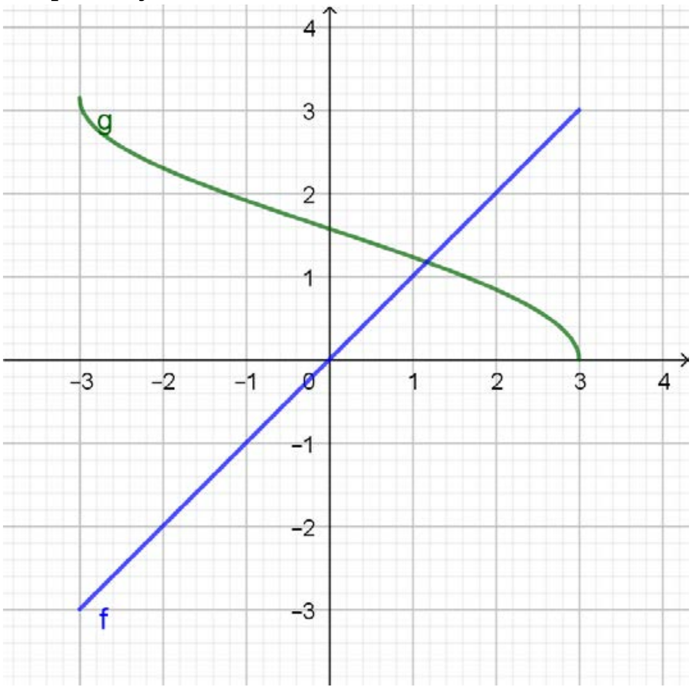
$$= \ln\sqrt{3} - \ln 1$$

$$= \ln\sqrt{3}$$

(B)

Question 10

Graphically:



Minimum will occur at $x = -3$

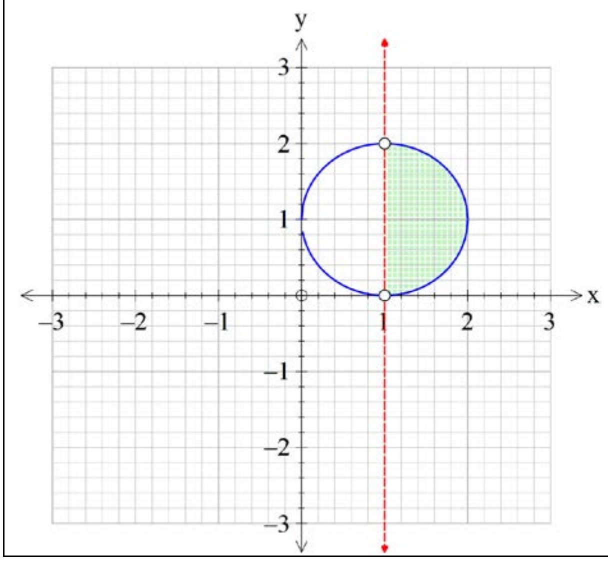
$$f\left(-3\right) = -3 \times \cos^{-1}\left(-1\right)$$

$$= -3 \times \pi$$

$$= -3\pi$$

(D)

Question 11

(a)	
(i) $z=\dfrac{3-i}{1-2i}$ $=\dfrac{(3-i)(1+2i)}{(1-2i)(1+2i)}$ $=\dfrac{3+6i-i-2i^2}{1+4}$ $=\dfrac{5+5i}{5}$ $=1+i$	
(ii) $z=\sqrt{2}\left(\cos\dfrac{\pi}{4}+i\sin\dfrac{\pi}{4}\right)$ $z^4=\left(\sqrt{2}\right)^4(\cos\pi+i\sin\pi)$ $=4(-1+0i)$ $=-4$	
(b) $\lvert z-(1+i)\rvert\le 1$ Circle centre $(1,1)$ radius 1 unit. $\mathrm{Re}(z)>1$ $x>1$ 	

(c) Let $(a+ib)=\sqrt{3}+i$ $a^2+2abi-b^2=-5-12i$ $a^2-b^2=-5$ ... (1) $ab=-6$ ...(2) $a=-\frac{6}{b}$ $\frac{36}{b^2}-b^2=-5$ $b^4-5b^2-36=0$ $(b^2-9)(b^2+4)=0$ $b=\pm3\,(b \text{ is real})$ When $\begin{array}{l} b=3, a=-2 \\ b=-3, a=2 \end{array}$ Therefore the square roots are $2-3i$ and $-2+3i$.	
(d) $x^3+4x-2=0$ (i) $(\alpha+\beta+\gamma)^2=(\alpha+\beta)^2+2(\alpha+\beta)\gamma+\gamma^2$ $=\alpha^2+2\alpha\beta+2\alpha\gamma+2\gamma\beta+\gamma^2$ $\therefore \alpha^2+\beta^2+\gamma^2=(\alpha+\beta+\gamma)^2-2(\alpha\beta+\alpha\gamma+\beta\gamma)$ $=(0)-2\times4$ $=-8$ OR $\alpha^3+4-2=0$ $\alpha^3=2-4\alpha$ $\alpha^2=\frac{2}{\alpha}-4$ $\therefore \sum\alpha^2=2\sum\frac{1}{\alpha}-4(3)$ $\therefore \sum\alpha^2=2\frac{\sum\alpha\beta}{\sum\alpha}-12$ $=2\left(\frac{4}{2}\right)-12$ $=-8$ (ii) $\alpha^3+4-2=0$ $\alpha^3=2-4\alpha$ $\alpha^4=2\alpha-4\alpha^2$ $\therefore \alpha^4+\beta^4+\gamma^4=2(\alpha+\beta+\gamma)-4(\alpha^2+\beta^2+\gamma^2)$ $=2\times0-4\times-8$ $=32$	

(e)	

$$\int_0^{\frac{\pi}{2}} \frac{dx}{3+5\cos x}, \quad \tan\frac{x}{2}=t$$

$$\frac{x}{2}=\tan^{-1}t$$

$$x=2\tan^{-1}t$$

$$\frac{dx}{dt}=\frac{2}{1+t^2}$$

$$dx=\frac{2}{1+t^2}\,dt$$

When $x=\frac{\pi}{2}, \quad t=1$

$x=0, \quad t=0$

$$=\int_0^1 \frac{1}{3+\frac{5(1-t^2)}{1+t^2}}\times\frac{2}{1+t^2}\,dt$$

$$=\int_0^1 \frac{1}{\frac{3(1+t^2)+5(1-t^2)}{1+t^2}}\times\frac{2}{1+t^2}\,dt$$

$$=\int_0^1 \frac{1+t^2}{3(1+t^2)+5(1-t^2)}\times\frac{2}{1+t^2}\,dt$$

$$=\int_0^1 \frac{2}{3+3t^2+5-5t^2}\,dt$$

$$=\int_0^1 \frac{2}{8-2t^2}\,dt$$

$$=\int_0^1 \frac{2}{2(4-t^2)}\,dt$$

$$=\int_0^1 \frac{1}{4-t^2}\,dt$$

$$=\int_0^1 \frac{1}{(2-t)(2+t)}\,dt$$

$$=\int_0^1 \frac{1}{2-t}+\frac{1}{2+t}\,dt$$

$$=\int_0^1 \frac{A}{2-t}+\frac{B}{2+t}\,dt$$

where $\frac{1}{(2-x)(2+x)}\equiv\frac{A}{2-x}+\frac{B}{2+x}$

$$1\equiv A(2+x)+B(2-x)$$

$x=2, \quad 1=4\text{A}$

$$\text{A}=\frac{1}{4}$$

when $x=-2, \quad 1=4B$

$$B=\frac{1}{4}$$

(e)cont. $= \frac{1}{4}\int_0^1 \frac{1}{2-t} + \frac{1}{2+t}\, dt$ $= \frac{1}{4}\int_0^1 -\frac{-1}{2-t} + \frac{1}{2+t}\, dt$ $= \frac{1}{4}\left[-\ln\lvert 2-t \rvert + \ln\lvert 2+t \rvert\right]_0^1$ $= \frac{1}{4}\left[\ln\left\lvert \frac{2+t}{2-t} \right\rvert\right]_0^1$ $= \frac{1}{4}\left[\ln\left\lvert \frac{3}{1} \right\rvert - \ln\left\lvert \frac{2}{2} \right\rvert\right]$ $= \frac{1}{4}\ln 3$	

Question 12

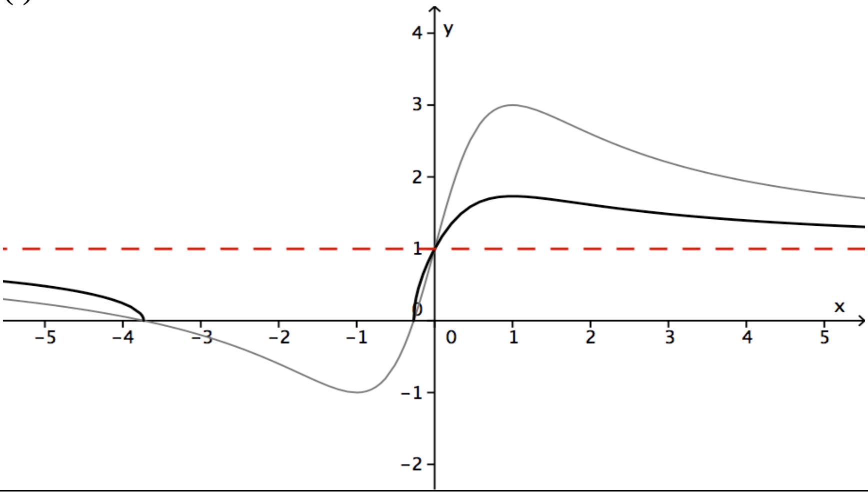
(a)	
(i) $xy = c^2$ Implicitly differentiating $y + x\frac{dy}{dx} = 0$ $\frac{dy}{dx} = -\frac{y}{x}$ At $P\left(cp, \frac{c}{p}\right)$, $\frac{dy}{dx} = -\frac{c}{p} \div cp$ $= -\frac{1}{p^2}$ Equation of tangent: $y - \frac{c}{p} = -\frac{1}{p^2}(x - cp)$ $p^2 y - cp = -x + cp$ $x + p^2 y = 2cp$	
(ii) Equation of tangent at *P*: $x + p^2 y = 2cp$ ... (1) $x + q^2 y = 2cq$ ...(2) (1)-(2): $y\left(p^2 - q^2\right) = 2cp - 2cq$ $y(p-q)(p+q) = 2c(p-q)$ $y = \frac{2c}{p+q}$ Sub into (1) $x + p^2 \times \frac{2c}{p+q} = 2cp$ $x = 2cp - \frac{2cp^2}{p+q}$ $= \frac{2cp^2 + 2cpq - 2cp^2}{p+q}$ $= \frac{2cpq}{p+q}$ $R = \left(\frac{2cpq}{p+q}, \frac{2c}{p+q}\right)$	

(iii) $x=\dfrac{2cpq}{p+q}$ $=pq\times\dfrac{2c}{p+q}$ $x=pqy$ $\dfrac{x}{y}=pq\ ...(2)$ $y=\dfrac{2c}{p+q}$ $p+q=\dfrac{2c}{y}...(4)$ $p^2+q^2=2$ $\left(p+q\right)^2-2pq=2$ $\left(\dfrac{2c}{y}\right)^2-2\times\dfrac{x}{y}=2$ $\dfrac{4c^2}{y^2}-\dfrac{2x}{y}=2$ $4c^2-2xy=2y^2$ $2c^2-xy=y^2$ $xy+y^2=2c^2$	

(b)	
(i) $\cos x = A(\cos x - 2\sin x) + B(\sin x + 2\cos x)$ $\cos x = A\cos x + 2B\cos x + B\sin x - 2A\sin x$ $\therefore A + 2B = 1$ $B = 2A$ $\therefore A + 4A = 1$ $A = \frac{1}{5}$ $B = \frac{2}{5}$	
(ii) $\cos x = \frac{1}{5}(\cos x - 2\sin x + 2\sin x + 4\cos x)$ $\int \frac{\cos x}{\sin x + 2\cos x} dx = \frac{1}{5}\int \frac{\cos x - 2\sin x + 2\sin x + 4\cos x}{\sin x + 2\cos x} dx$ $= \frac{1}{5}\int \left(\frac{\cos x - 2\sin x}{\sin x + 2\cos x} + \frac{2(\sin x + 2\cos x)}{\sin x + 2\cos x} \right) dx$ $= \frac{1}{5}\left[\ln(\sin x + 2\cos x) + 2x \right] + C$	

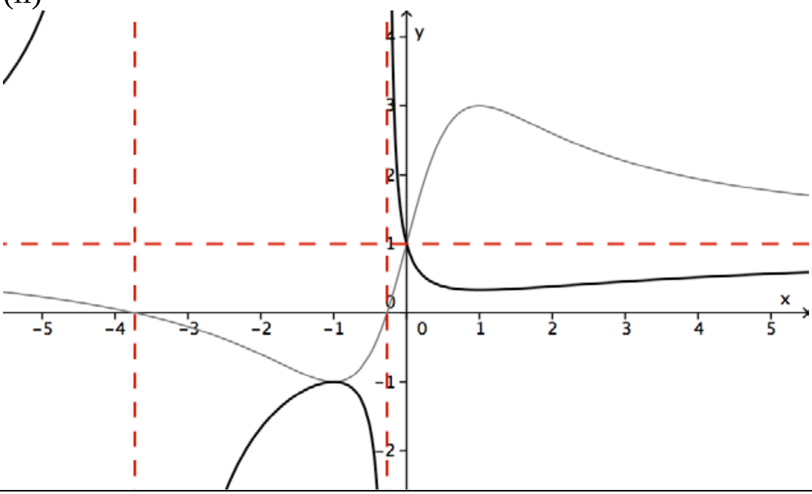
(c)	

(i)



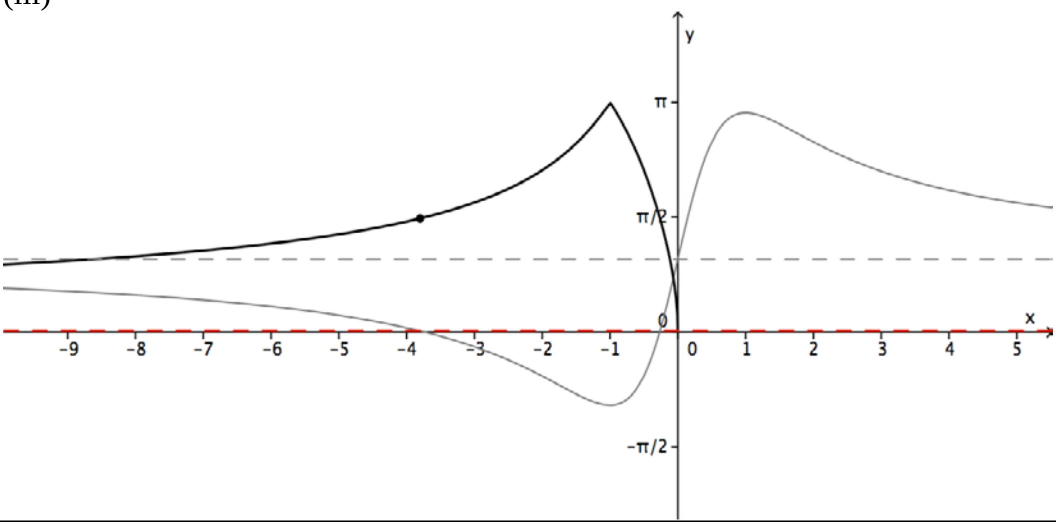
y
x
4
3
2
1
0
−1
−2
−5
−4
−3
−2
−1
0
1
2
3
4
5

(ii)



y
x
−5
−4
−3
−2
−1
0
1
2
3
4
5

(iii)



y
x
π
π/2
−π/2
0
−9
−8
−7
−6
−5
−4
−3
−2
−1
0
1
2
3
4
5

Question 13

(a)	
$I=\int_0^1 \frac{x}{(3x+1)^2}\,dx$ Let $u=3x+1$ When $x=0, u=1$. When $x=1, u=4$. $3x+1=u$ $3x=u-1$ $x=\frac{1}{3}(u-1)$ $u=3x+1$ $\frac{du}{dx}=3$ $dx=\frac{du}{3}$ $I=\frac{1}{3}\int_1^4 \frac{(u-1)}{3u^2}\,du$ $=\frac{1}{9}\int_1^4 \left(\frac{1}{u}-u^{-2}\right)du$ $=\frac{1}{9}\left[\ln u+\frac{1}{u}\right]_1^4$ $=\frac{1}{9}\left[\left(\ln 4+\frac{1}{4}\right)-(\ln 1+1)\right]$ $=\frac{1}{9}\left(\ln 2^2+\frac{1}{4}-1\right)$ $=\frac{1}{9}\left(2\ln 2-\frac{3}{4}\right)$ $=\frac{2}{9}\ln 2-\frac{1}{12}$	
(b)	
(i) $\left(z-\frac{1}{z}\right)^5=z^5-5\frac{z^4}{z}+10\frac{z^3}{z^2}-10\frac{z^2}{z^3}+5\frac{z}{z^4}-\frac{1}{z^5}$ $=z^5-5z^3+10z-\frac{10}{z}+\frac{5}{z^3}-\frac{1}{z^5}$ $=\left(z^5-z^{-5}\right)-5\left(z^3-z^{-3}\right)+10\left(z-z^{-1}\right)$	

(ii)

Let $z=\cos\theta+i\sin\theta$

$$z^{n}=\cos n\theta+i\sin n\theta$$

$$z^{-n}=\cos n\theta-i\sin n\theta$$

$$z^{n}-z^{-n}=2i\sin n\theta$$

From (i)

$$\left(z-\frac{1}{z}\right)^{5}=\left(z^{5}-z^{-5}\right)-5\left(z^{3}-z^{-3}\right)+10\left(z-z^{-1}\right)$$

$$\left(z-z^{-1}\right)^{5}=2i\sin 5\theta+10\times 2\sin\theta-5\times 2\sin 3\theta$$

$$\left(2i\sin\theta\right)^{5}=2i\sin 5\theta+20i\sin\theta-10i\sin 3\theta$$

$$2^{5}i^{5}\sin^{5}\theta=i\left(2\sin 5\theta+20\sin\theta-10\sin 3\theta\right)$$

$$32i\sin^{5}\theta=i\left(2\sin 5\theta-10\sin 3\theta+20\sin\theta\right)$$

$$32\sin^{5}\theta=2\sin 5\theta-10\sin 3\theta+20\sin\theta$$

(iii)

$$\int_{0}^{\frac{\pi}{2}}\sin^{5}\theta d\theta=\frac{1}{32}\int_{0}^{\frac{\pi}{2}}2\left(\sin 5\theta-5\sin 3\theta+10\sin\theta\right)$$

$$=\frac{1}{16}\left[-\frac{1}{5}\cos 5\theta+\frac{5}{3}\cos 3\theta-10\cos\theta\right]_{0}^{\frac{\pi}{2}}$$

$$=\frac{1}{16}\left[\left(-\frac{1}{5}\cos\frac{5\pi}{2}+\frac{5}{3}\cos\frac{3\pi}{2}-10\cos\frac{\pi}{2}\right)-\left(-\frac{1}{5}\cos 0+\frac{5}{3}\cos 0-10\cos 0\right)\right]$$

$$=\frac{1}{16}\left[\left(0+0-0\right)-\left(-\frac{1}{5}+\frac{5}{3}-10\right)\right]$$

$$=\frac{8}{15}$$

(c)

$$P\left(x\right)=3x^{3}+4x^{2}+5x-6$$

Zeros of $P\left(x\right)$ are in the form $\pm\frac{6}{3}=\pm\frac{1}{3},\pm\frac{2}{3},\quad \alpha<1$

$$P\left(\frac{2}{3}\right)=3\left(\frac{2}{3}\right)^{3}+4\left(\frac{2}{3}\right)^{2}+5\left(\frac{2}{3}\right)-6$$

$$=0$$

Therefore $\left(3x-2\right)$ is a factor of $P\left(x\right)$

By short division,

$$\left(3x^{3}+4x^{2}+5x-6\right)=\left(3x-2\right)\left(x^{2}+2x+3\right)$$

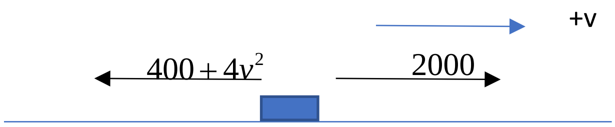
$$=\left(3x-2\right)\left(x^{2}+2x+1+2\right)$$

$$=\left(3x-2\right)\left(\left(x+1\right)^{2}-2i^{2}\right)$$

$$=\left(3x-2\right)\left(x+1-\sqrt{2}i\right)\left(x+1+\sqrt{2}i\right)$$

$$=\left(3x-2\right)\left(x-\left(-1+\sqrt{2}i\right)\right)\left(x-\left(-1-\sqrt{2i}\right)\right)$$

(d)



(i)

$$F = ma$$

$$2000-\left(400+4v^2\right)=800\ddot{x}$$

$$1600-4v^2 = 800\ddot{x}$$

$$\ddot{x}=2-\frac{v^2}{200}$$

(ii)

$$\frac{dv}{dt}=\frac{400-v^2}{200}$$

$$\frac{dt}{dv}=\frac{200}{\left(400-v^2\right)}$$

$$=\frac{5}{(20-v)}+\frac{5}{20+v}\quad [by\ \ partial\ \ fractions]$$

$$t=5\ln(20+v)-5(\ln 20-v)+C$$

$$=5\ln\left(\frac{20+v}{20-v}\right)+C$$

$$When\ v=0,$$

$$0=5\ln 1+C$$

$$C=0$$

$$t=5\ln\left(\frac{20+v}{20-v}\right)$$

$$\frac{t}{5}=\ln\left(\frac{20+v}{20-v}\right)$$

$$e^{\frac{t}{5}}=\frac{20+v}{20-v}$$

$$20e^{\frac{t}{5}}-ve^{\frac{t}{5}}=20+v$$

$$v\left(1+e^{\frac{t}{5}}\right)=20\left(e^{\frac{t}{5}}-1\right)$$

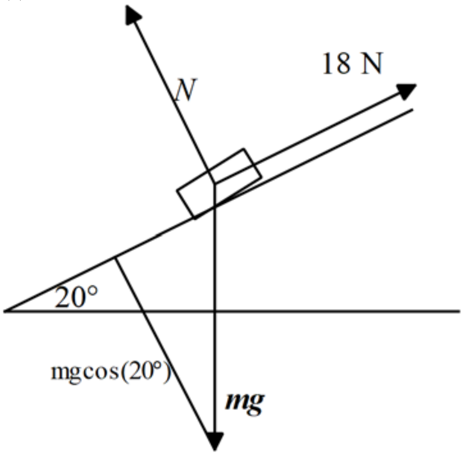
$$v=20\left(\frac{e^{\frac{t}{5}}-1}{e^{\frac{t}{5}}+1}\right)$$

(iii) As $t \to \infty$: $v = 20\left(\frac{e^{\frac{t}{5}}+1-1}{e^{\frac{t}{5}}+1}\right)$ $= 20\left(1-\frac{1}{e^{\frac{t}{5}}+1}\right)$ $\frac{1}{e^{\frac{t}{5}}+1} \to 0$ $v \to 20$ The limiting speed is 20 metres per second.	

Question 14

(a)	
$I=\int \frac{1}{e^x+4e^{-x}}\,dx$ $=\int \frac{1}{e^x+\frac{4}{e^x}}\,dx$ $=\int \frac{1}{\frac{e^{2x}+4}{e^x}}\,dx$ $=\int \frac{e^x}{e^{2x}+4}\,dx$ $=\int \frac{e^x}{\left(e^x\right)^2+\left(2\right)^2}dx$ $=\frac{1}{2}\tan^{-1}\frac{e^x}{2}+C$	
(b)	
(i) $I_n=\int_0^4 x^n\sqrt{4-x}\,dx$ $u=x^n \qquad v'=\left(4-x\right)^{\frac{1}{2}}$ $u'=nx^{n-1} \qquad v=\frac{-2}{3}\left(4-x\right)^{\frac{3}{2}}$ $I_n=\left[\frac{-2}{3}x^n\left(4-x\right)^{\frac{3}{2}}\right]_0^4+\frac{2}{3}n\int_0^4 x^{n-1}\left(4-x\right)^{\frac{3}{2}}\,dx$ $=\left[0-0\right]+\frac{2}{3}n\left[\int_0^4 x^{n-1}\left(4-x\right)\sqrt{4-x}\,dx\right]$ $=\frac{2}{3}n\left[4\int_0^4 x^{n-1}\sqrt{4-x}dx-\int_0^4 x^n\sqrt{4-x}\,dx\right]$ $=\frac{2}{3}n\left[4I_{n-1}-I_n\right]$ $I_n+\frac{2n}{3}I_n=\frac{8nI_{n-1}}{3}$ $I_n\left(1+\frac{2n}{3}\right)=\frac{8n}{3}I_{n-1}$ $I_n\left(\frac{2n+3}{3}\right)=\frac{8n}{3}I_{n-1}$ $I_n=\frac{8n}{2n+3}I_{n-1}$	

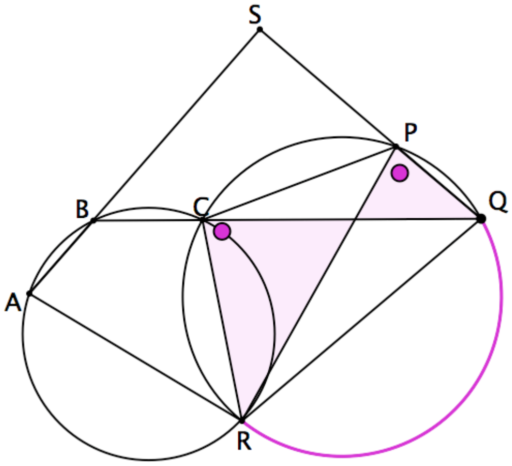
(b)	
(ii) $I_2 = \frac{8}{4+3} I_1$ $= \frac{8}{7}\left(\frac{8}{2+3} I_0\right)$ $= \frac{8}{7} \times \frac{8}{5} \times \frac{16}{3}$ $= \frac{1024}{105}$	

(c)	
(i)  Resolving forces perpendicular to the plane $N = mg\cos 20°$ $= 2 \times 9.8 \times \cos 20°$ $= 18.417...$ $= 18.4\text{N}$ (1dp)	
(ii) Resolving forces parallel to the plane: $2a = 18 - \frac{3}{5} \times 18.4 - 2 \times 9.8 \times \sin 20°$ $a = \frac{1}{2}\left(18 - \frac{3}{5} \times 18.4 - 2 \times 9.8 \times \sin 20°\right)$ $= 0.12829..$ $= 0.13 \text{ ms}^{-2}$	

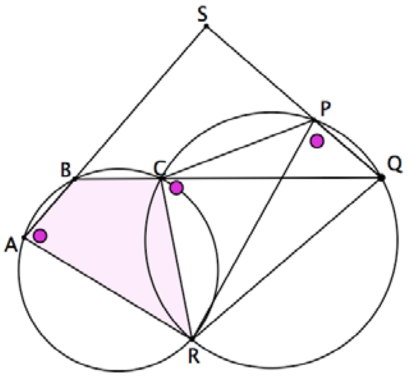
(d)	
(i) $PQ = y_1 - y_2$ and PQ is the diameter of the semi – circle. $2r = \left(5 - x^2\right) - \frac{x^2}{4}$ $2r = 5 - \frac{5x^2}{4}$ $2r = \frac{20 - 5x^2}{4}$ $2r = \frac{5\left(4 - x^2\right)}{4}$ $\therefore r = \frac{5\left(4 - x^2\right)}{8}$ units. $\therefore$ Area of slice: $A = \frac{1}{2}\pi r^2$ $= \frac{\pi}{2}\left[\frac{5\left(4 - x^2\right)}{8}\right]^2$ $= \frac{\pi}{2}\left[\frac{25\left(4 - x^2\right)^2}{64}\right]$ $\therefore A = \frac{25\pi\left(4 - x^2\right)^2}{128}$ unit^2	
(ii) Volume of slice: $\delta V = \frac{25\pi}{128}\left(4 - x^2\right)^2 \delta x$ $\therefore$ Volume of solid: $V = \lim_{\delta x \to 0} \sum_{-2}^{2} \frac{25\pi}{128}\left(4 - x^2\right)^2 \delta x$ $V = \frac{25\pi}{128}\int_{-2}^{2}\left(4 - x^2\right)^2 dx$ $= \frac{25\pi}{64}\int_{0}^{2}\left(4 - x^2\right)^2 dx$ $= \frac{25\pi}{64}\int_{0}^{2}\left(16 - 8x^2 + x^4\right) dx$ $= \frac{25\pi}{64}\left[16x - \frac{8x^3}{3} + \frac{x^5}{5}\right]_0^2$ $= \frac{25\pi}{64}\left[16(2) - \frac{8(2)^3}{3} + \frac{(2)^5}{5} - 0\right]$ $\therefore V = \frac{20\pi}{3}$ unit^3	

(e)

(i)



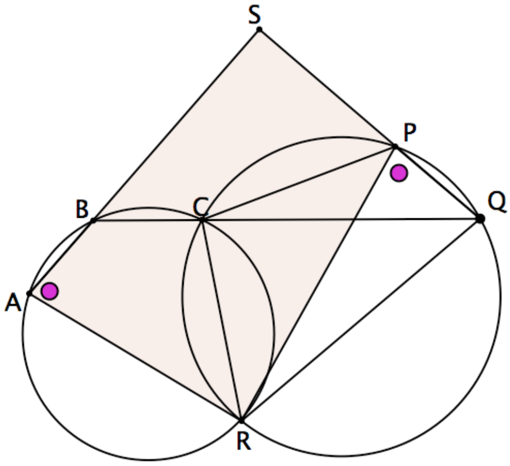
In circle $CPQR$, $\angle QPR = \angle QCR$ (angles standing on the same arc RQ subtend equal angles at the circumference)



In circle $ABCD$, $\angle BAR = \angle QCR$ (exterior angle of a cyclic quadrilateral equals the interior opposite angle)

$\therefore \angle BAR = \angle QPR$

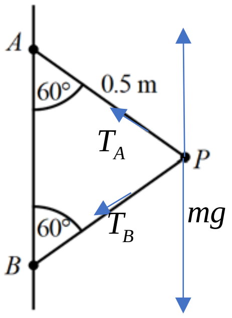
(ii)



From (i) $\angle BAR = \angle QPR$

$\therefore$ $ARPS$ is a cyclic quadrilateral (exterior angle of a cyclic quadrilateral equals the interior opposite angle)

Question 15

<table>
<tr><td>(a)</td><td></td></tr>
<tr><td>(i)


$\sin 60° = \frac{r}{0.5}$
$r = \frac{1}{2} \times \frac{\sqrt{3}}{2}$
$r = \frac{\sqrt{3}}{4}$ m

Vertically:
$T_A \cos 60° - T_B \cos 60° - mg = 0$
$T_A \times \frac{1}{2} - T_B \times \frac{1}{2} = mg$
$\frac{1}{2}(T_A - T_B) = mg$
$T_A - T_B = mg \quad ...(1)$</td><td></td></tr>
<tr><td>(ii)
Horizontally:
$T_A \sin 60° + T_B \sin 60° = mr\omega^2$
$\frac{\sqrt{3}}{2}(T_A + T_B) = m \times \frac{\sqrt{3}}{4} \times \left(2\sqrt{3g}\right)^2$
$\frac{\sqrt{3}}{2}(T_A + T_B) = \frac{\sqrt{3}m}{4} \times 12g$
$\frac{1}{2}(T_A + T_B) = \frac{m}{4} \times 12g$
$T_A + T_B = 6gm \quad ...(2)$

(1)+(2):
$2T_A = 8gm$
$T_A = 4gm$ N
$T_B = 2gm$ N</td><td></td></tr>
</table>

(b)	
(i) $F = m\ddot{x}$ $mg - R = m\ddot{x}$ $R = m\left(g - \ddot{x}\right)$ $= m\left(g - \frac{d}{dx}\left(kg\left(1 - e^{-\frac{x}{k}}\right)\right)\right)$ $= mg\left(1 - k \times \frac{e^{\frac{-x}{k}}}{k}\right)$ $= mg\left(1 - e^{-\frac{x}{k}}\right)$ $= \frac{m}{2k} \times 2kg\left(1 - e^{\frac{-x}{k}}\right)$ $= \frac{mv^2}{2k}$ OR $F = m\ddot{x}$ $mg - R = m\ddot{x}$ $\frac{mg - R}{m} = \ddot{x}$ $g - \frac{R}{m} = \frac{1}{2}\frac{dv^2}{dx}$ $= \frac{1}{2}\frac{d}{dx}\left[2kg\left(1 - e^{-\frac{x}{k}}\right)\right]$ $= \frac{d}{dx}\left[kg\left(1 - e^{-\frac{x}{k}}\right)\right]$ $= kg\left(\frac{1}{k}e^{\frac{-x}{k}}\right)$ $= ge^{\frac{-x}{k}}$ Since $v^2 = 2kg\left(1 - e^{\frac{-x}{k}}\right)$ $\frac{v^2}{2kg} = 1 - e^{\frac{-x}{k}}$ $e^{\frac{-x}{k}} = 1 - \frac{v^2}{2kg}$ $g - \frac{R}{m} = g - \frac{v^2 g}{2kg}$ $-\frac{R}{m} = -\frac{v^2}{2k}$ $\therefore R = \frac{mv^2}{2k}$	

(ii) $m\ddot{x} = -mg - \frac{mv^2}{2k}$ $\ddot{x} = -\left(g + \frac{v^2}{2k}\right)$ $v\frac{dv}{dx} = -\left(\frac{2kg + v^2}{2k}\right)$ $\frac{dv}{dx} = -\left(\frac{2kg + v^2}{2kv}\right)$ $\frac{dx}{dv} = -\left(\frac{2kv}{2kg + v^2}\right)$ $x = -k\int \frac{2v}{2kg + v^2}\,dv$ $= -k\ln\left(2kg + v^2\right) + C$ Sub $x = 0, v = \sqrt{2kg}$ $0 = -k\ln\left(2kg + 2kg\right) + C$ $C = k\ln 4kg$ $x = k\ln 4kg - k\ln\left(2kg + v^2\right)$ Let $v = 0$ $x = k\ln 4kg - k\ln 2kg$ $= k\ln\left(\frac{4kg}{2kg}\right)$ $= k\ln 2$	

(iii)

$$\ddot{x} = -\left(g + \frac{v^2}{2k}\right)$$

$$\frac{dv}{dt} = -\left(\frac{2kg + v^2}{2k}\right)$$

$$\frac{dt}{dv} = -2k\left(\frac{1}{\left(\sqrt{2kg}\right)^2 + v^2}\right)$$

$$t = \frac{-2k}{\sqrt{2kg}}\tan^{-1}\left(\frac{v}{\sqrt{2kg}}\right) + D$$

When $t = 0, v = \sqrt{2kg}$

$$0 = -\frac{2k}{\sqrt{2kg}}\tan^{-1}\left(\frac{\sqrt{2kg}}{\sqrt{2kg}}\right) + C$$

$$C = \frac{2k}{\sqrt{2kg}}\tan^{-1}1$$

$$C = \frac{2k}{\sqrt{2kg}} \times \frac{\pi}{4}$$

$$= \frac{k\pi}{2\sqrt{2kg}}$$

$$t = -\frac{2k}{\sqrt{2kg}}\tan^{-1}\left(\frac{v}{\sqrt{2kg}}\right) + \frac{\pi k}{2\sqrt{2kg}}$$

Let $v = 0$

$$t = \frac{\pi k}{2\sqrt{2kg}} \text{ seconds.}$$

(c)	
(i) $4x^2+3y^2=12 \ldots(1)$ $y=mx+4 \ldots(2)$ Sub (2) into (1): $4x^2+3\left[m^2x^2+8mx+16\right]=12$ $4x^2+3m^2x^2+24mx+48=12$ $x^2\left(4+3m^2\right)+24mx+36=0$ For tangent $\Delta=0$ $(24)^2-4\left(3m^2+4\right)\times 36=0$ $576m^2-432m^2-576=0$ $144m^2=576$ $m^2=4$ $m=2\;(m>0)$	
(ii) Sub $m=2$ into equation above $x^2\left(3\times 4+4\right)+48x+36=0$ $16x^2+48x+36=0$ $(2x+3)^2=0$ $x=\frac{-3}{2}$ When $x=\frac{-3}{2}$, $y=2\left(\frac{-3}{2}\right)+4$ $=-3+4$ $=1$ $P\left(\frac{-3}{2},1\right)$	
(iii) Gradient of tangent is 2 , gradient of normal is $-\frac{1}{2}$ Equation of normal is $y-1=\frac{-1}{2}\left(x+\frac{3}{2}\right)$ $2y-2=-x-\frac{3}{2}$ $4y-4=-2x-3$ $4y+2x-1=0$ $B(0,4)$ B	

A: Let $x = 0$ in normal $4y - 1 = 0$ $y = \frac{1}{4}$ $A\left(0, \frac{1}{4}\right)$ $A = \frac{1}{2}\left(4 - \frac{1}{4}\right) \times \frac{3}{2}$ $= \frac{45}{16}\ units^2$	

Question 16

(a)

(i)

The coefficients of $z^3 + az + 10 = 0$ are real.

Let the roots be α , $1-2i$, $1+2i$.

Product of the roots:

$\alpha(1-2i)(1+2i) = -10$

$5\alpha = -10$

$\alpha = -2$

Sum of roots two at a time:

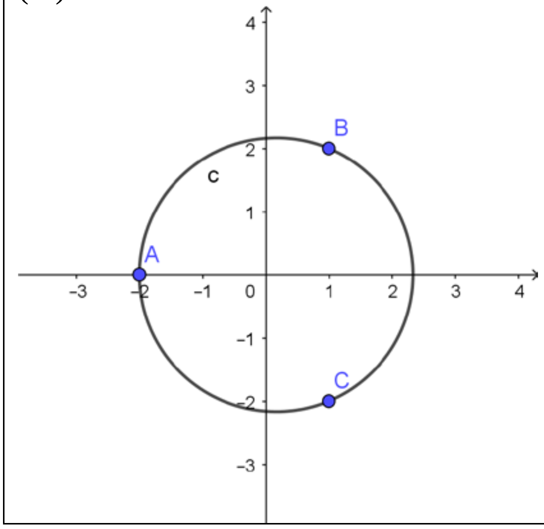
$\alpha(1-2i) + \alpha(1+2i) + (1+2i)(1-2i) = a$

$2\alpha + 5 = a$

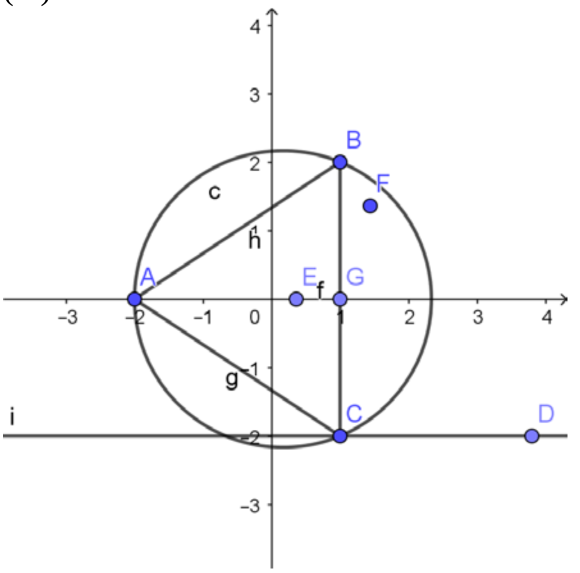
$a = 1$

$z^3 + az + 10 = 0$

(iii)



(iii)



By symmetry, centre must be on the x-axis.
Construct CD parallel to the x-axis.

$\arg\left(-2-(1-2i)\right)=\arg\left(-3+2i\right)$

$=\pi-\tan^{-1}\frac{2}{3}$

$\angle DCA=\pi-\tan^{-1}\frac{2}{3}$

$By\ co-\text{int}\,erior\ \angle's$

$\angle CAE=\pi-\left(\pi-\tan^{-1}\frac{2}{3}\right)$

$=\tan^{-1}\frac{2}{3}$

$\angle CEG=2\tan^{-1}\frac{2}{3}\left(\begin{array}{l}\text{angle at the centre is twice the}\\\text{angle at the circumference}\end{array}\right)$

In ΔCEG

$\sin\theta=\frac{2}{r}$

$r=\frac{2}{\sin\left(2\tan^{-1}\frac{2}{3}\right)}$

$=\frac{13}{6}$

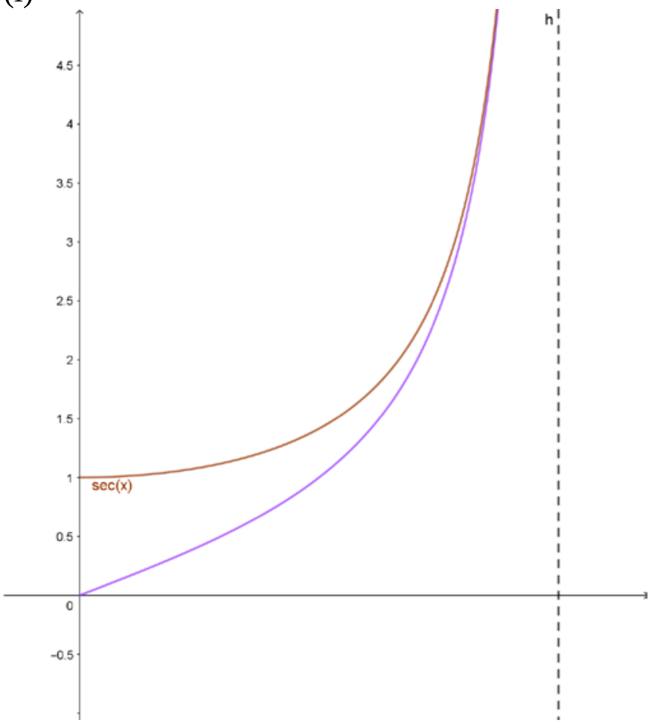
Therecentre center is $\left(\frac{1}{6},0\right)$

Equation of locus is $\left|z-\frac{1}{6}\right|=\frac{13}{6}$

(b)	
(i) $\left(\sqrt{a}-\sqrt{b}\right)^2 \geq 0$ $\left(\sqrt{a}\right)^2 - 2\sqrt{a}\sqrt{b} + \left(\sqrt{b}\right)^2 \geq 0$ $a+b \geq 2\sqrt{ab}$ $\frac{a+b}{2} \geq \sqrt{ab}$	
(ii) $n! = n\times(n-1)\times(n-2)\times...\times 2\times 1$ From what is given $\sqrt[n]{n\times(n-1)\times(n-2)\times...\times 2\times 1} \leq \frac{n+(n-1)+(n-2)+...+2+1}{n}$ $\sqrt[n]{n!} \leq \frac{1}{n}\times\frac{n}{2}(n+1)$ $\sqrt[n]{n!} \leq \frac{n+1}{2}$ $n! \leq \left(\frac{n+1}{2}\right)^n$	

(c)

(i)



(ii)

$$LHS = \sec\theta - \tan\theta$$

$$= \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}$$

$$= \frac{1-\sin\theta}{\cos\theta}$$

$$= \frac{1-\sin^2\theta}{\cos\theta(1+\sin\theta)}$$

$$= \frac{\cos^2\theta}{\cos\theta(1+\sin\theta)}$$

$$= \frac{\cos\theta}{1+\sin\theta}$$

$$= \frac{1}{\sec\theta(1+\sin\theta)}$$

$$= \frac{1}{\sec\theta + \frac{\sin\theta}{\cos\theta}}$$

$$= \frac{1}{\sec\theta+\tan\theta}$$

OR

$$LHS = \sec\theta - \tan\theta$$

$$= \sec\theta - \tan\theta \times \frac{\sec\theta+\tan\theta}{\sec\theta+\tan\theta}$$

$$= \frac{\sec^2\theta - \tan^2\theta}{\sec\theta+\tan\theta}$$

$$= \frac{1}{\sec\theta+\tan\theta}$$

(iii) Given that $\sec\theta > \tan\theta$ *for* $0 \le x < \frac{\pi}{2}$ $\sec\theta > \tan\theta$ $\sec\theta - \tan\theta > 0$ $\sec\theta - \tan\theta = \frac{1}{\sec\theta + \tan\theta}$ From graph $\sec\theta + \tan\theta \ge 1$ $1 \ge \frac{1}{\sec\theta + \tan\theta}$ $\therefore \sec\theta - \tan\theta \le 1$ $0 < \sec\theta - \tan\theta \le 1$ OR As graphically shown, $y_2 - y_1 = \sec\theta - \tan\theta$ and $\sec\theta - \tan\theta > 0$ for $0 \le \theta < \frac{\pi}{2}$ At $\theta = 0$, $\sec\theta - \tan\theta = 1$ i.e. the distance is maximum As $\theta \to \frac{\pi}{2}, (\sec\theta - \tan\theta) \to 0$ Hence $0 < \sec\theta - \tan\theta \le 1$.	
(iv) $\sec\theta - \tan\theta = \frac{1}{2} .... (1)$ $\frac{1}{\sec\theta + \tan\theta} = \frac{1}{2}$ $\sec\theta + \tan\theta = 2 ... (2)$ (1) + (2) $2\sec\theta = \frac{5}{2}$ $\sec\theta = \frac{5}{4}$ $\cos\theta = \frac{4}{5}$ $\theta = \cos^{-1}\frac{4}{5}$ $\theta = 2k\pi \pm \cos^{-1}\left(\frac{4}{5}\right)$	