HURLSTONE AGRICULTURAL HIGH SCHOOL

EXTENSION 2 MATHEMATICS

2013

YEAR 12

TRIAL (TASK 4) EXAMINATION

EXAMINERS ~ S. GEE, G. HUXLEY

GENERAL INSTRUCTIONS

- Reading Time – 5 minutes.
- Working Time – 3 hours.
- Attempt **all** questions.
- Each question in part B is worth 15 marks.
- **All** necessary working should be shown in every question in Part B.
- This paper contains Ten(10) multiple choice questions in Part A and Six(6) free response questions in Part B, totalling 14 pages including title page, multiple choice answer sheet and standard integral table.
- Board approved calculators and MathAids may be used.
- **Each free response question is to be started in a new answer booklet**. Write the question number and your student number at the top of each answer booklet.
- You **must** hand in the multiple choice answer sheet as well as an answer booklet for **each question** even if a question has not been attempted.
- This examination must **NOT** be removed from the examination room

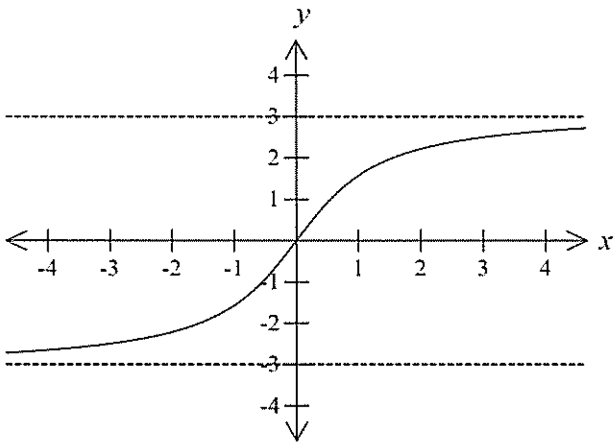
STUDENT NAME:______________________

TEACHER: ______________________

Part A Multiple Choice (Complete on the answer sheet provided)

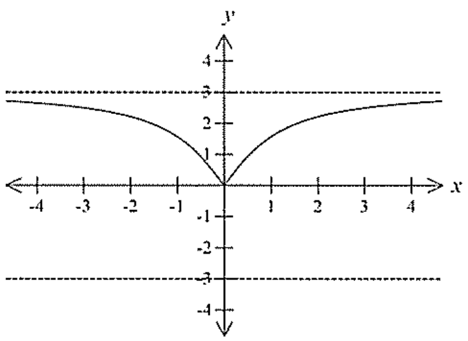
QUESTION 1

The diagram shows the graph of the function $y = f(x)$.

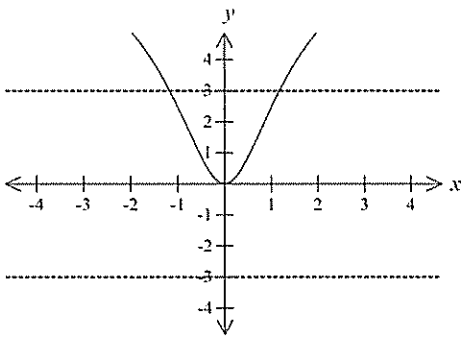


Which of the following is the graph of $y = \sqrt{f(x)}$?

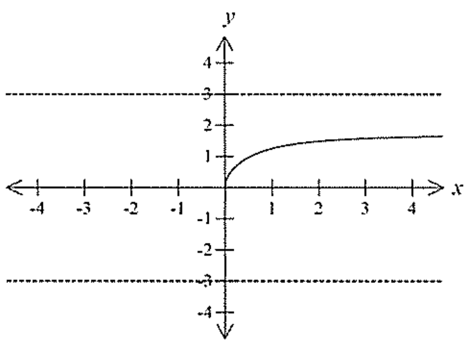
(A)



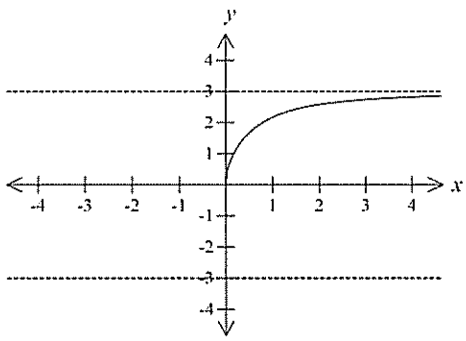
(B)



(C)



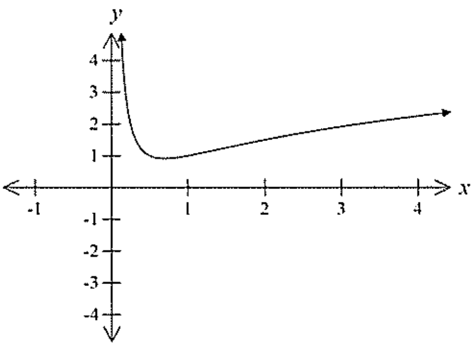
(D)



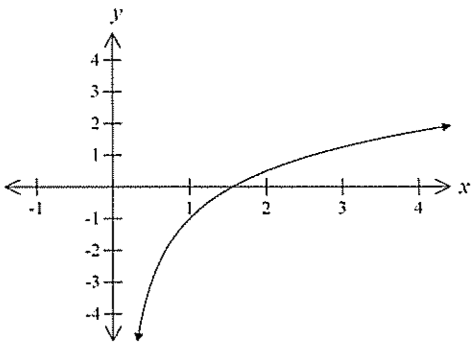
QUESTION 2

Which of the following is the sketch of $y = \log_2 x + \frac{1}{x}$?

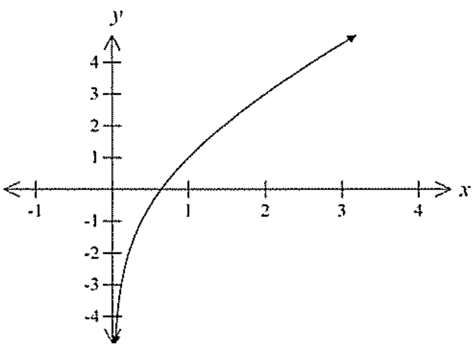
(A)



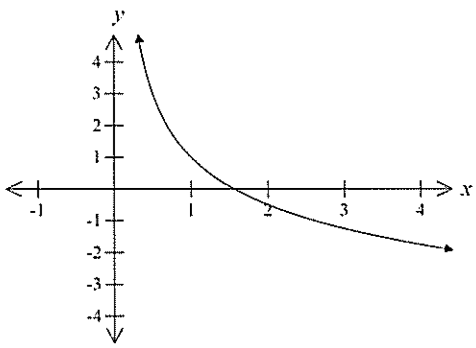
(B)



(C)

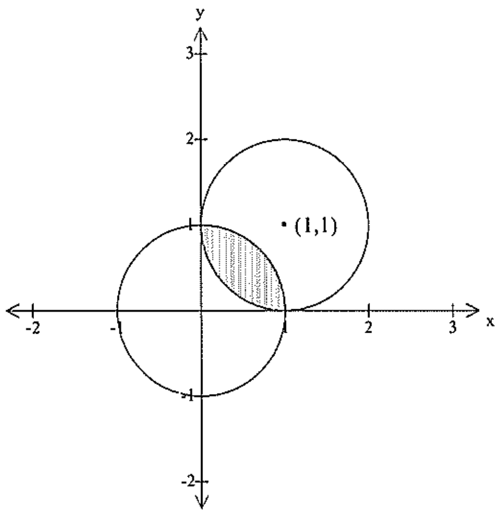


(D)



QUESTION 3

Consider the Argand diagram below.



Which inequality could define the shaded area?

(A) $|z| \leq 1$ and $|z-(1-i)| \geq 1$

(B) $|z| \leq 1$ and $|z-(1+i)| \geq 1$

(C) $|z| \leq 1$ and $|z-(1-i)| \leq 1$

(D) $|z| \leq 1$ and $|z-(1+i)| \leq 1$

QUESTION 4

It is given that $3+i$ is a root of $P(z)=z^3+az^2+bz+10$ where a and b are real numbers. Which expression factorises $P(z)$ over the real numbers?

(A) $(z-1)(z^2+6z-10)$

(B) $(z-1)(z^2-6z-10)$

(C) $(z+1)(z^2+6z+10)$

(D) $(z+1)(z^2-6z+10)$

QUESTION 5

How many distinct permutations of the letters of the word ‘ATTAINS’ are possible in a straight line when the word begins and ends with the letter T?

(A) 60

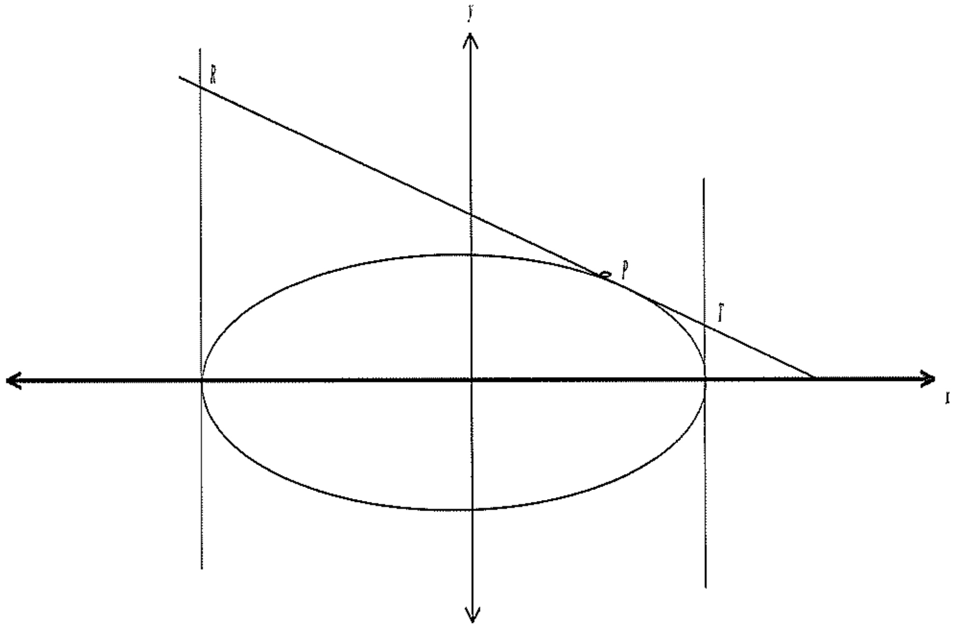
(B) 120

(C) 360

(D) 1260

QUESTION 6

The point $P(a\cos\theta, b\sin\theta)$ lies on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ where $a>b>0$.



What is the equation of the tangent at P?

(A) $\frac{ax}{\cos\theta}-\frac{by}{\sin\theta}=a^2-b^2$

(B) $\frac{ax}{\sec\theta}+\frac{by}{\tan\theta}=a^2+b^2$

(C) $\frac{x}{a}\sec\theta-\frac{y}{b}\tan\theta=1$

(D) $\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1$

QUESTION 7

What is the value of $\int_0^1 \frac{e^x}{1+e^x}dx$?

(A) $\log_e(1+e)$

(B) 1

(C) $\log_e\left(\frac{1+e}{2}\right)$

(D) $\log_e\frac{e}{2}-2$

QUESTION 8

The equation $24x^3 - 12x^2 - 6x + 1$ has roots α, β and γ.
What is the value of α if $\alpha = \beta + \gamma$?

(A) $-\frac{1}{2}$

(B) $\frac{1}{4}$

(C) $\frac{1}{2}$

(D) 1

QUESTION 9

Consider the hyperbola with the equation $\frac{x^2}{144} - \frac{y^2}{25} = 1$.
What are the equations of the directrices?

(A) $x = \pm\frac{13}{144}$

(B) $x = \pm\frac{13}{25}$

(C) $x = \pm\frac{25}{13}$

(D) $x = \pm\frac{144}{13}$

QUESTION 10

The polynomial $P(x) = x^4 + ax^2 + bx + 28$ has a double root at $x = 2$.
What are the values of a and b?

(A) $a = -11$ and $b = -12$

(B) $a = -5$ and $b = -12$

(C) $a = -11$ and $b = 12$

(D) $a = -5$ and $b = 12$

QUESTION 11 (Commence a new answer booklet) **Marks**

(a) For the complex number $w = 1 - i\sqrt{3}$:

(i) Find $|w|$ and $\arg(w)$ **2**

(ii) Express $\overline{w}, w^2, \frac{1}{w}$ and $\sqrt{w}$ in the form $a + ib$.
Plot them on the Argand diagram. **5**

(b) Describe and sketch the locus of the point z such that $|z+3i| + |z-3i| = 10$ **2**

(c) (i) Show that $(1-2i)^2 = -3-4i$. **1**

(ii) Hence solve the equation $x^2 - 5x + (7+i) = 0$ **2**

(d) Let $OABC$ be a square on the Argand diagram where O is the origin.
The points A and C represent the complex numbers z and iz respectively.

(i) Find the complex number represented by B. **1**

(ii) The square is now rotated through $45°$ in an anticlockwise direction about O to $OA'B'C'$. Find the complex number represented by A'. **2**

QUESTION 12 (Commence a new answer booklet) **Marks**

(a) For the curve, $f(x) = \dfrac{4x}{1+x^2}$,

(i) Prove the curve $y = f(x)$ has a relative minimum at A (–1,–2), a relative maximum at B (1,2) and a point of inflexion at O (0,0). Sketch $y = f(x)$ **3**

(ii) The cubic curve $g(x) = ax^3 + bx^2 + cx + d$ also has a relative minimum at A (–1,–2) and a relative maximum at B (1,2).

(α) Obtain values of the coefficients a, b, c, and d. **2**

(β) Deduce that O is also a point of inflexion on $y = g(x)$ **1**

(iii) Prove that the two curves $y = f(x)$ and $y = g(x)$ have only the three points A, B and O in common. **2**

(b) Given $h(x) = 6 + x - x^2$
Draw neat sketches, on separate diagrams, of at least one third of a page for

(i) $y = h(x)$ **1**

(ii) $y = |h(x)|$ **1**

(iii) $y = \sqrt{h(x)}$ **1**

(iv) $y^2 = h(x)$ **1**

(v) $y = \dfrac{1}{h(x)}$ **1**

(vi) $y = e^{h(x)}$ **1**

(vii) $y = \log_2 h(x)$ **1**

QUESTION 13 (Commence a new answer booklet) **Marks**

(a) Consider the ellipse E whose equation is $\frac{x^2}{4}+\frac{y^2}{2}=1$.

(i) Show that the equation of E may be written in the parametric form

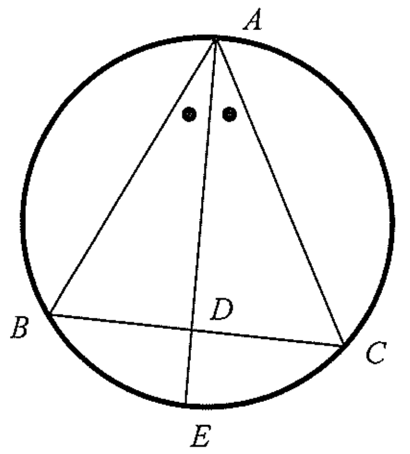
$$x = 2\cos\theta$$
$$y = \sqrt{2}\sin\theta$$
1

(ii) Assuming that the perimeter of E is given by the formula

$p = 2\int_0^{\pi}\left[\sqrt{\left(\frac{dx}{d\theta}\right)^2+\left(\frac{dy}{d\theta}\right)^2}\right]d\theta$ show that $p = 2\sqrt{2}\int_0^{\pi}\left[\sqrt{2-\cos^2\theta}\right]d\theta$ **2**

(iii) Use five evenly spaced function values from $\theta = 0$ to $\theta = \pi$ and Simpson's rule to estimate p correct to two decimal places. **2**

(b)



In the diagram, the bisector AD of $\angle BAC$ has been extended to intersect the circle ABC at E.

(i) Prove that the triangles ABE and ADC are similar. **2**

(ii) Show that $AB.AC = AD.AE$. **1**

(iii) Prove that $AD^2 = AB.AC - BD.DC$. **2**

(c) Given $a_n = \sqrt{2+a_{n-1}}$ for integers $n \geq 1$, and that $a_0 = 1$,
use the process of mathematical induction to prove for $n \geq 1, \sqrt{2} < a_n < 2$. **3**

(d) A woman travelling along a straight flat road passes three points at intervals of 200 metres. From these points she observes the angle of elevation of the top of the hill to the left of the road to be respectively $30°, 45°$ and again $45°$.
Find the height of the hill to the nearest metre. **2**

QUESTION 14 (Commence a new answer booklet) **Marks**

(a) $P\left(pt, \dfrac{t}{p}\right)$ and $Q\left(qt, \dfrac{t}{q}\right)$ are two points on the rectangular hyperbola: $xy = t^2$, where p and q are constants.

(i) Show that the gradient of PQ is $\dfrac{-1}{pq}$ **1**

(ii) Show that the gradient of the tangent to the hyperbola at P is $\dfrac{-1}{p^2}$ **1**

(iii) Hence, or otherwise, determine an expression for q in terms of p that will Make PQ a normal to the hyperbola at P. **2**

(b) For the hyperbola $5x^2 - 4y^2 = 20$

(i) Find the eccentricity and the co-ordinates of the foci. **2**

(ii) Find the equations of the asymptotes. **1**

(c) When the polynomial $P(x)$ is divided by $(x+2)(x-3)$ the remainder is $4x+1$.

What is the remainder when $P(x)$ is divided by $(x+2)$? **2**

(d) (i) Show that $2+i$ is a zero of $x^3 - 11x + 20 = 0$. **2**

(ii) Hence, or otherwise, solve $x^3 - 11x + 20 = 0$. **2**

(e) The equation $x^3 + 2x - 1 = 0$ has roots α, β, δ.

Find the polynomial equation with roots $\dfrac{1}{\alpha}, \dfrac{1}{\beta}, \dfrac{1}{\delta}$. **2**

QUESTION 15 (Commence a new answer booklet) **Marks**

(a) Evaluate the integral $\int_{-2}^{2} x\sqrt{4-x^2} - \sqrt{4-x^2}\,dx$ **2**

(b) Using integration by parts, evaluate: $\int_{0}^{1} x\tan^{-1}x \;dx$ **3**

(c) (i) Find the real numbers a, b, c such that $\dfrac{5}{x^2(2-x)} \equiv \dfrac{ax+b}{x^2} + \dfrac{c}{2-x}$ **2**

(ii) Hence, or otherwise, find $\int \dfrac{20}{x^2(2-x)}\,dx$ **3**

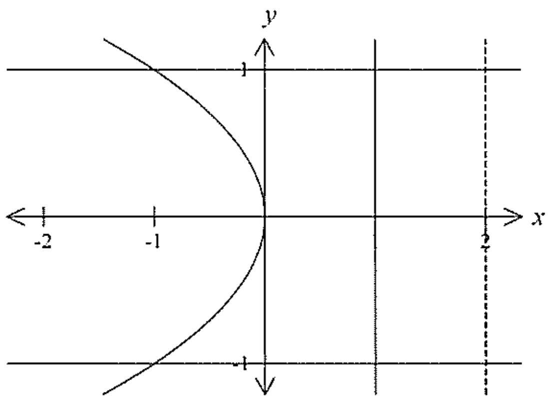
(d) Let $I_n = \int_{1}^{2} (\ln x)^n \;dx$, where n is a positive integer.

(i) Show that $I_n = 2(\ln 2)^n - nI_{n-1}$ **2**

(ii) Hence evaluate $I_4 = \int_{1}^{2} (\ln x)^4 \;dx$. Write your answer in exact form. **3**

QUESTION 16 (Commence a new answer booklet) **Marks**

(a) (i) The region bounded by the lines $x=1$, $y=1$, $y=-1$ and by the curve $x=-y^2$ is rotated through $360°$ about the line $x=2$ to form a solid.



By considering summation of slices in the shape of circular discs, show that the correct expression for the volume of the solid generated is:

$$V=\int_{-1}^{1}\pi\left(y^4+4y^2+4\right)dy$$ **3**

(ii) Hence or otherwise, evaluate the volume of the solid of revolution. **2**

(b) (i) Draw a neat sketch of the region enclosed by the curve $y=4x-x^2$, the x – axis, and the lines $x=1$ and $x=3$. Include in your diagram all points of intersection. **2**

(ii) Use the method of cylindrical shells to find the volume of the solid generated when the area in part (i) is rotated about the y-axis. **3**

(c) (i) On a number plane, draw the region bounded by: the curve $y^2=4x$ and the lines $x+y=0$ and $x=4$. **1**

(ii) A solid is generated using the region in (i) as a base. There are square cross-sectional slices perpendicular to the x – axis. Each has a side with one end-point on the line $x+y=0$ and the other on the curve $y^2=4x$.

(α) Show that the area of a cross-section is given by:

$$A(x)=4x+x^2+4x^{\frac{3}{2}}.$$ **2**

(β) Hence find the volume of the solid formed. **2**

END OF EXAMINATION

STUDENT NUMBER:........................ ....

HURLSTONE AGRICULTURAL HIGH SCHOOL

2013 TRIAL EXAMINATION
YEAR 12
EXTENSION 2
MATHEMATICS

PART A Answers

1	ⓐ	ⓑ	ⓒ	ⓓ
2	ⓐ	ⓑ	ⓒ	ⓓ
3	ⓐ	ⓑ	ⓒ	ⓓ
4	ⓐ	ⓑ	ⓒ	ⓓ
5	ⓐ	ⓑ	ⓒ	ⓓ
6	ⓐ	ⓑ	ⓒ	ⓓ
7	ⓐ	ⓑ	ⓒ	ⓓ
8	ⓐ	ⓑ	ⓒ	ⓓ
9	ⓐ	ⓑ	ⓒ	ⓓ
10	ⓐ	ⓑ	ⓒ	ⓓ

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1} + C, \quad n \neq -1; \; x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x + C, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + C, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax + C, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax + C, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax + C, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax + C, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + C, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a} + C, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right) + C, \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right) + C$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Question No.1-13 Solutions and Marking Guidelines

Outcomes Addressed in this Questions

E2	chooses appropriate strategies to construct arguments and proofs in both concrete and abstract settings
E3	uses the relationship between algebraic and geometric representations of complex numbers and of conic sections
E4	uses efficient techniques for the algebraic manipulation required in dealing with questions such as those involving conic sections and polynomials.
E6	combines the ideas of algebra and calculus to determine the important features of the graphs of a wide variety of functions
E7	uses the techniques of slicing and cylindrical shells to determine volumes
E8	applies further techniques of integration, including partial fractions, integration by parts and recurrence formulae, to problems
E9	communicates abstract ideas and relationships using appropriate notation and logical argument

Outcome | **Solutions** | **Marking Guidelines**

Outcome: E3

1 C 2 A 3 D 4 D 5 A 6 D 7 C 8 B 9 D 10 B

Question 11

a)

i) $w = 1-\sqrt{3}i$

$|w| = \sqrt{1^2+\left(-\sqrt{3}\right)^2} = 2$

$\arg(w) = \tan^{-1}\left(\frac{-\sqrt{3}}{1}\right) = \frac{-\pi}{3}$

Marking Guidelines: 2 marks both correct answers; 1mark one correct answer

ii $\overline{w} = 1+\sqrt{3}i$

$w^2 = (1-\sqrt{3}i)^2 = -2-2\sqrt{3}i$

$\frac{1}{w} = \frac{1}{1-\sqrt{3}i}\times\frac{1+\sqrt{3}i}{1+\sqrt{3}i} = \frac{1+\sqrt{3}i}{4}$

$\sqrt{w} = \sqrt{1-\sqrt{3}i}$

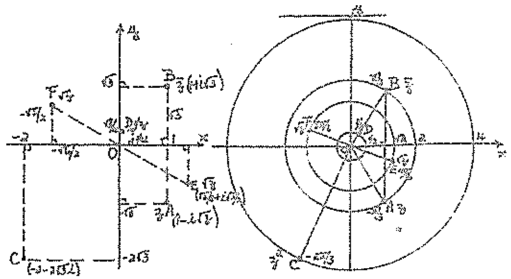
$1-\sqrt{3}i = \left(a^2-b^2\right)+2iab$

$a^2-b^2 = 1,\ 2ab = -\sqrt{3}$

$a = \frac{\pm\sqrt{6}}{2},\ b = \frac{\pm\sqrt{2}}{2}$

$\sqrt{w} = \frac{\sqrt{6}-\sqrt{2}i}{2}, \frac{-\sqrt{6}+\sqrt{2}i}{2}$

Marking Guidelines: 5 marks correct method leading to correct solutions with correct graph; 4 marks (1) incorrect answer; 3 marks (2) incorrect answers or error on graph; 2 marks substantial method leading to some correct solutions; 1markelementary progress towards the correct solution



b)

$|z+3i|+|z-3i| = 10$

is an ellipse major axis y axis under the locus definition $PA+PB = 2b$ in this case.

using geometry or $a^2 = b^2\left(1-e^2\right)$ and a focus $be = 3$

$b = 5, a = 4$

$\therefore$ ellipse has equation $\frac{x^2}{16}+\frac{y^2}{25} = 1$

Marking Guidelines: 2 marks correct method leading to correct solution; 1mark substantial progress towards the correct solution

c)

i) $(1-2i)^2 = 1-4i+4i^2$

$= 1-4i-4 = -3-4i$

Marking Guidelines: 1mark correct solution

ii) $x = \frac{5\pm\sqrt{25-4(7+i)}}{2}$

$x = \frac{5\pm\sqrt{-3-4i}}{2}$

$x = \frac{5\pm(1-2i)}{2}$

$x = 3-i, 2+i$

Marking Guidelines: 2 marks correct method leading to correct solution; 1mark substantial progress towards the correct solution

d)

i) $OB = OA+OC = z+iz = z(1+i)$ (parallelogram of vectors)

Marking Guidelines: 1mark correct solution

ii) $z = |z|\arg(z)$ and $|z| = |z'|$

$z' = |z'|\left(\cos\left(\theta+\frac{\pi}{4}\right)+i\sin\left(\theta+\frac{\pi}{4}\right)\right)$

$z' = |z|\left(\cos\theta\cos\frac{\pi}{4}-\sin\theta\sin\frac{\pi}{4}+i\left(\sin\theta\cos\frac{\pi}{4}+\cos\theta\sin\frac{\pi}{4}\right)\right)$

$z' = \frac{|z|}{\sqrt{2}}(\cos\theta-\sin\theta+i\sin\theta+i\cos\theta)$

$z' = \frac{|z|}{\sqrt{2}}(\cos\theta+i\sin\theta+i(\cos\theta+i\sin\theta))$

$z' = \frac{1}{\sqrt{2}}(|z|(\cos\theta+i\sin\theta)+i|z|(\cos\theta+i\sin\theta))$

$z' = \frac{1}{\sqrt{2}}(z+iz) = \frac{1+i}{\sqrt{2}}z$

Marking Guidelines: 2 marks correct method leading to correct solution; 1mark substantial progress towards the correct solution

E6

Question 12

a) i

$$y=\frac{4x}{1+x^2}$$

$$\frac{dy}{dx}=4\left[\frac{(1+x^2)\times 1-x\times 2x}{(1+x^2)^2}\right]=\frac{4(1-x^2)}{(1+x^2)^2}$$

stationary points occur when $\frac{dy}{dx}=0$

ie. $\frac{4(1-x^2)}{(1+x^2)^2}=0$

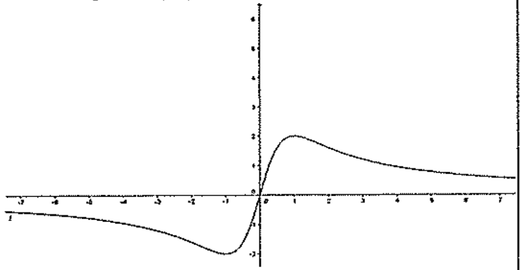
$(1-x^2)=0, x=\pm 1, y=\pm 2$ relative max at $(1,2)$, relative min at $(-1,-2)$

$$\frac{d^2y}{dx^2}=\frac{-8x(3-x^2)}{(1+x^2)^3}$$

possible points of inflexion occur when $x=0,\pm\sqrt{3}$

Testing concavity change in the second differential yields inflectional points at $(0,0)$, $(\sqrt{3},\sqrt{3})$ and $(-\sqrt{3},-\sqrt{3})$

2 marks correct method leading to correct solution
1mark substantial progress towards the correct solution



1mark correct graph

ii

$g(x)=ax^3+bx^2+cx+d$

$g'(x)=3ax^2+2bx+c$

substitutig thestationarypointsintoboth theabove equation:

$-a+b-c+d=-2$

$a+b+c+d=2$

$3a-2b+c=0$

$3a+2b+c=0$ andsolvingsimultaneuslygives

$a=-1,\ b=0,\ c=3$ and $d=0$

$g(x)=-x^3+3x$

$g'(x)=-3x^2+3$

$g''(x)=-6x$

$g''(0)=0$ and $g''(0^+)<0$, and $g''(0^-)>0$ then $(0,0)$ is a pointof inflexion

2 marks correct method leading to correct solution
1mark substantial progress towards the correct solution

1 mark correct method leading to correct solution to point of inflexion

iii

$$\frac{4x}{1+x^2}=-x^3+3x$$

$4x=(-x^3+3x)(1+x^2)$

$x^5-2x^3+x=0$

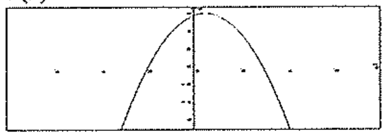
$x(x^4-2x^2+1)=x(x^2-1)^2=0$

$x=-1,0,1$ giving 3 unique solutions which correspond to A, B and O

2 marks correct method leading to correct solution
1mark substantial progress towards the correct solution

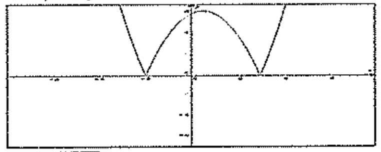
(b)

$h(x)=6+x-x^2$



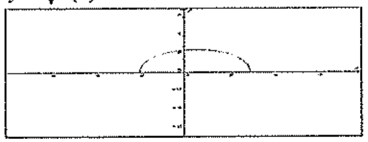
1mark correct graph

$y=|h(x)|$



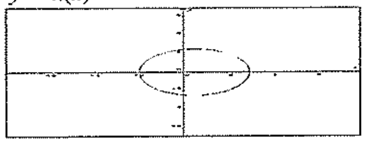
1mark correct graph

$y=\sqrt{h(x)}$



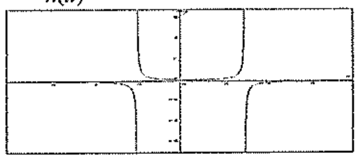
1mark correct graph

$y^2=h(x)$



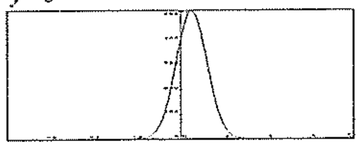
1mark correct graph

$y=\frac{1}{h(x)}$



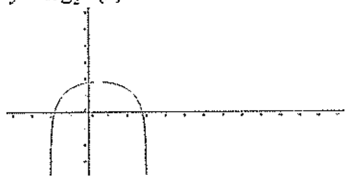
1mark correct graph

$y=e^{h(x)}$



1mark correct graph

$y=\log_2 h(x)$



1mark correct graph

E2/E9

Question 13

a)

i)

$\frac{x^2}{4}+\frac{y^2}{2}=1,\ x=2\cos\theta, y=\sqrt{2}\sin\theta$

if the $\frac{(2\cos\theta)^2}{4}+\frac{\left(\sqrt{2}\sin\theta\right)^2}{2}=1$ then E can be written as

$x=2\cos\theta, y=\sqrt{2}\sin\theta$ in parametric form.

$\frac{4\cos^2\theta}{4}+\frac{2\sin^2\theta}{2}=\cos^2\theta+\sin^2\theta=1$

1mark correct solution

ii)

$x=2\cos\theta \qquad y=\sqrt{2}\sin\theta$

$\frac{dx}{d\theta}=-2\sin\theta \quad \frac{dy}{d\theta}=\sqrt{2}\cos\theta$

$p=2\int_0^{\pi}\left(\sqrt{\left(\frac{dx}{d\theta}\right)^2+\left(\frac{dy}{d\theta}\right)^2}\right)d\theta$

$p=2\int_0^{\pi}\left(\sqrt{(-2\sin\theta)^2+\left(\sqrt{2}\cos\theta\right)^2}\right)d\theta$

$p=2\int_0^{\pi}\left(\sqrt{4\sin^2\theta+2\cos^2\theta}\right)d\theta$

$p=2\int_0^{\pi}\left(\sqrt{2\sin^2\theta+2\cos^2\theta+2\sin^2\theta}\right)d\theta$

$p=2\int_0^{\pi}\left(\sqrt{2+2\sin^2\theta}\right)d\theta$

$p=2\sqrt{2}\int_0^{\pi}\left(\sqrt{1+\sin^2\theta}\right)d\theta=2\sqrt{2}\int_0^{\pi}\left(\sqrt{1+1-\cos^2\theta}\right)d\theta$

$p=2\sqrt{2}\int_0^{\pi}\left(\sqrt{2-\cos^2\theta}\right)d\theta$

2 marks correct method leading to correct solution
1mark substantial progress towards the correct solution

iii)

if $f(\theta)=\sqrt{2-\cos^2\theta}$

$f(0)=\sqrt{2-\cos^2 0}=1$

$f\left(\frac{\pi}{4}\right)=\sqrt{2-\cos^2\frac{\pi}{4}}=\sqrt{\frac{3}{2}}$

$f\left(\frac{\pi}{2}\right)=\sqrt{2-\cos^2\frac{\pi}{2}}=\sqrt{2}$

$f\left(\frac{3\pi}{4}\right)=\sqrt{2-\cos^2\frac{\pi}{4}}=\sqrt{\frac{3}{2}}$

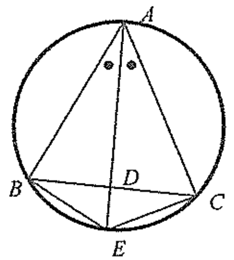
$f(\pi)=\sqrt{2-\cos^2\pi}=1$

$\therefore\int_0^{\pi}\left(\sqrt{2-\cos^2\theta}\right)d\theta=\frac{\frac{\pi}{2}}{6}\left(1+\sqrt{2}+4\times\sqrt{\frac{3}{2}}\right)+\frac{\frac{\pi}{2}}{6}\left(1+\sqrt{2}+4\times\sqrt{\frac{3}{2}}\right)$

$\therefore p\approx 2\sqrt{2}\times\left[\frac{\pi}{12}\left(1+\sqrt{2}+4\times\sqrt{\frac{3}{2}}\right)+\frac{\pi}{12}\left(1+\sqrt{2}+4\times\sqrt{\frac{3}{2}}\right)\right]\approx 10\cdot 83$

2 marks correct method leading to correct solution
1mark substantial progress towards the correct solution

b)



(i) Prove that the triangles ABE and ADC are similar.

In ΔABE and ΔADC

$\angle BAE=\angle DAC$ (AD bisects $\angle BAC$ by data)

$\angle AEB=\angle ACD$ (angles subtended by common chord AB)

$\therefore \Delta ABE\ ///\ \Delta ADC$ (equiangular)

2 marks correct method, with reasons, leading to correct solution
1mark substantial progress, with reasons, towards the correct solution

(ii) Show that $AB.AC=AD.AE$.

Since corresponding sides of similar triangles are in the same ratio then

$\frac{AB}{AD}=\frac{AE}{AC}=\frac{BE}{DC}$

$\therefore AB\times AC=AD\times AE$

1 mark correct method, with reasons, leading to correct solution

(iii) Prove that $AD^2=AB.AC-BD.DC$.

In ΔABD and ΔCED

$\angle ADB=\angle CDE$ (vertically opposite angles)

$\angle ADB=\angle CDE$ (angles subtended by common chord AC)

$\therefore \Delta ABD\ ///\ \Delta CED$ (equiangular)

$\therefore\frac{AD}{CD}=\frac{BD}{ED}$ or

$BD\times DC=AD\times DE$

$AB\times AC=AD\times AE$ (from (*ii*))

$AB\times AC-BD\times DC=AD\times AE-AD\times DE$

$=AD(AE-DE)$

$=AD\times AD=AD^2$

2 marks correct method, with reasons, leading to correct solution
1mark substantial progress, with reasons, towards the correct solution

c)

Prove true for n = 1

$ie \sqrt{2} < a_n < 2$ for $n \geq 1$ given $a_0 = 1$ and $a_n = \sqrt{2 + a_{n-1}}$

$a_n = \sqrt{2 + a_{n-1}}$, then $a_1 = \sqrt{2 + a_0} = \sqrt{2+1} = \sqrt{3}$

$\sqrt{2} < \sqrt{3} < \sqrt{4}$

$\therefore \sqrt{2} < a_1 < 2$ and true for $n = 1$

Assume true for $n = k$ ie $a_k = \sqrt{2 + a_{k-1}}$

Assume true for $n = k+1$ ie $a_{k+1} = \sqrt{2 + a_k}$

from $\sqrt{2} < a_k < 2$

$2 + \sqrt{2} < 2 + a_k < 2 + 2$

$\sqrt{2 + \sqrt{2}} < \sqrt{2 + a_k} < \sqrt{4}$

since $2 < 2 + \sqrt{2}$, then $\sqrt{2} < \sqrt{2 + \sqrt{2}}$ and hence

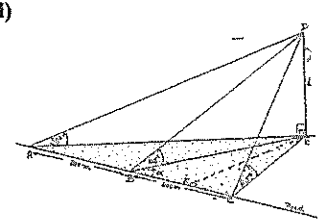
$\sqrt{2} < \sqrt{2 + \sqrt{2}} < \sqrt{2 + a_k} < 2$

but $a_{k+1} = \sqrt{2 + a_k}$, and $\sqrt{2} < a_{k+1} < 2$

$\therefore$ if true for is true $n = k$ is true for $n = k+1$ and by the pro
of mathematical induction true for all positive integral n.

3 marks correct method leading to correct solution
2 marks substantial progress towards the correct solution
1 mark elementary progress towards the correct solution

d)



$EA = h\cot 30°, BE = h\cot 45°, CE = h\cot 45°$

$$\cos\angle ECB = \frac{h^2 + 200^2 - h^2}{2 \times 200 \times h} = \frac{100}{h}$$

$$\cos\angle ECA = \frac{h^2 + 400^2 - \left(\sqrt{3}h\right)^2}{2 \times 400 \times h} = \frac{80000 - h^2}{400h}$$

$\angle ECB = \angle ECA \therefore \cos\angle ECB = \cos\angle ECA$

$$\frac{100}{h} = \frac{80000 - h^2}{400h}$$

$40000 = 80000 - h^2$

$h^2 = 40000$

$h = 200m$

2 marks correct method leading to correct solution
1mark substantial progress towards the correct solution

Mathematics Extension 2 Solutions and Marking Guidelines HSC Trial 2013

Question 14

Outcome	Solutions	Marking Guidelines
E4 (a)	(i) $M_{pq}=\dfrac{\frac{t}{q}-\frac{t}{p}}{qt-pt}=\dfrac{pt-qt}{pq}\times\dfrac{1}{qt-pt}=\dfrac{-1}{pq}$	**1 mark:** Sufficient working shown.
	(ii) Lots of choices: e.g: $y'=\dfrac{-t^2}{p^2t^2}$ or $y'=\dfrac{-y}{x}$ or $M_T=\lim\limits_{q\to p}\dfrac{-1}{pq}$ all leading to $M_T=\dfrac{-1}{p^2}$	**1 mark:** Sufficient working shown.
	(iii) $\dfrac{-1}{pq}\times\dfrac{-1}{p^2}=-1 \rightarrow \quad q=\dfrac{-1}{p^3}$ *Full marks required q to be the subject.*	**2 marks:** Correct solution. **1 mark:** Relationship between gradients.
E4 (b)	(i) $e=\dfrac{3}{2}$ Foci = $(\pm 3,0)$	**2 marks:** Both correct **1 mark:** One correct.(foci CFPA)
	(ii) $y=\pm\dfrac{\sqrt{5}}{2}x$	**1 mark:** Correct lines CFPA.
E4 (c)	$P(x)=(x+2)(x-3)Q(x)+4x+1$ $\therefore P(-2)=0+4(-2)+1$ $=-7$ Remainder when division by $(x+2)=-7$	**2 marks:** Correct solution **1 mark:** Considerable progress.
E4 (d)	(i) Substitute $x=2+i$: $(2+i)^3-11(2+i)+20=(2+i)[(2+i)^2-11]+20$ $=(2+i)(4i-8)+20$ $=8i-16-4-8i+20$ $=0$	**2 marks:** Correct solution, with sufficient working. **1 mark:** Considerable progress.
	(ii) Real coefficients, so complex conjugate pairs are roots. Sum of roots = 0 $x=2+i, 2-i, -4$ are roots.	**2 marks:** Correct solution **1 mark:** Considerable progress.
E4 (e)	$\left(\dfrac{1}{x}\right)^3+2\left(\dfrac{1}{x}\right)-1=0$ has roots $\dfrac{1}{\alpha},\dfrac{1}{\beta},\dfrac{1}{\gamma}$ Required polynomial is: $x^3-2x^2-1=0$ *or equivalent.*	**2 marks:** Correct solution **1 mark:** Considerable progress.

Mathematics Extension 2 Solutions and Marking Guidelines HSC Trial 2013

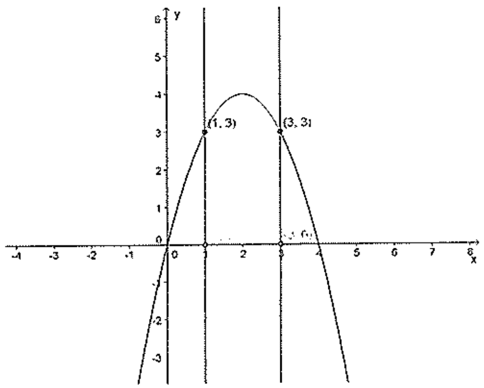
Question 15

Outcome	Solutions	Marking Guidelines
E8 (a)	$\int_{-2}^{2} x\sqrt{4-x^2}-\sqrt{4-x^2}\,dx$ $=0-\int_{-2}^{2}\sqrt{4-x^2}\,dx$ $=-2\pi$ *units* First integral = 0 because it is an odd function. Second integral is the area of a semi-circle with radius = 2 *2 very simple marks if you recognised the forms. Lots of time spent by those who didn't. Several people forgot the pesky negative sign, probably because they were thinking of positive area.*	**2 marks:** Correct answer **1 mark:** Considerable progress.
E8 (b)	$\int_0^1 x\tan^{-1}x\,dx=\left.\dfrac{1}{2}x^2\tan^{-1}x\right]_0^1-\int_0^1\dfrac{1}{2}x^2\left(\dfrac{1}{1+x^2}\right)dx$ $=\dfrac{\pi}{8}-\dfrac{1}{2}\int_0^1 1-\dfrac{1}{1+x^2}\,dx$ $=\dfrac{\pi}{8}-\dfrac{1}{2}\left[x-\tan^{-1}x\right]_0^1$ $=\dfrac{\pi}{4}-\dfrac{1}{2}$ *Evaluating the integral was the tricky bit…*	**3 marks:** Correct answer with sufficient working **2 marks: :** Considerable progress. **1 mark:** Some progress.
E8 (c)	(i) $5\equiv(ax+b)(2-x)+cx^2$ $\therefore a=c=\dfrac{5}{4} \quad b=\dfrac{5}{2}$ *Since a and c were equal, finding both was only worth 1 mark out of 2.*	**2 marks:** All 3 correct. **1 mark:** Either *b* correct or *a* and *c* correct.

	(ii) $\int \frac{20}{x^2(2-x)}dx = 4\int \frac{5}{x^2(2-x)}dx$ $= \int \frac{5x+10}{x^2} + \frac{5}{2-x}dx$ $= 5\left(\ln x - \frac{2}{x} - \ln\lvert 2-x\rvert\right)dx$ *The absolute value signs weren't essential for the marks.*	**3 marks:** Correct answer with sufficient working **2 marks: :** Considerable progress. **1 mark:** Some progress.
E8 (d)	(i) $I_n = \int_1^2 1(\ln x)^n\,dx = x(\ln x)^n\Big]_1^2 - \int_1^2 x\frac{n}{x}(\ln x)^n\,dx$ $= 2(\ln 2)^n - 0 - \int_1^2 n(\ln x)^{n-1}\,dx$ $= 2(\ln 2)^n - I_{n-1}$ *Line 1 is very important. Several people lost a mark by not showing the full operation.*	**2 marks:** Correct solution, with sufficient working. This includes the 2nd integral on the 1st line that shows how the factor of x is divided out. **1 mark:** Considerable progress.
	(ii) $I_0 = 1$ $I_1 = 2(\ln 2) - I_0$ $I_2 = 2(\ln 2) - 2I_1$ $I_3 = 2(\ln 2) - 3I_2$ $I_4 = 2(\ln 2) - 4I_3$ $I_4 = 2(\ln 2)^4 - 8(\ln 2)^3 + 24(\ln 2)^2 - 48\ln 2 + 24$ *The lowest success rate occurred when the calculation was attempted as one big expression.*	(ii) **3 marks:** Correct solution, with sufficient working. **2 marks:** 1 error in working that is followed through consistently when calculating all variables. **1 mark:** Partial progress.

Mathematics Extension 2 Solutions and Marking Guidelines **HSC Trial 2013**

Question 16

Outcome	Solutions	Marking Guidelines
E7 (a)	(i)Using a boundary of $x = 2$. Curve is $x = -y^2$ Radius $= 2 - x = 2 + y^2$ $A(y) = \pi(2+y^2)^2 \quad \to \Delta V = \pi(2+y^2)^2\,\Delta y$ $V = \lim_{\Delta y \to 0}\sum_{-1}^{1} \pi(4+4y^2+y^4)\Delta y$ $= \pi\int_{-1}^{1}(4+4y^2+y^4)dy$ *Students handled the confusing question well. Working that was crossed out was still marked, and full marks could be gained by at least 3 different answers. The only problem was due to some students subtracting y^2 instead of adding.*	**3 marks:** Correct solution **2 marks:** Considerable progress **1 mark:** Partial progress.
	(ii) $V = \pi\int_{-1}^{1}(4+4y^2+y^4)dy$ $= \pi\left[\frac{y^5}{5} + \frac{4y^3}{3} + 4y\right]_{-1}^{1}$ $= \frac{166\pi}{15}\,units^3$ *Excellent exam technique. Most people used the given answer in part (i) and got full marks for this part.*	**2 marks:** Correct answer CFPA **1 mark:** Considerable progress.
E7 (b)	(i) 	**1 mark:** Correct, neat diagram, showing all points of intersection, and shading.

(β)

$$V=\lim_{\Delta x\to 0}\sum_{x=0}^{4} 4x+4x\sqrt{x}+x^2\ \Delta x$$

$$=\int_0^4 4x+4x^{\frac{3}{2}}+x^2\,dx$$

$$=2x^2+\frac{8x^{\frac{5}{2}}}{5}+\frac{x^3}{3}2x^2+\frac{8x^{\frac{5}{2}}}{5}+\frac{x^3}{3}\Bigg]_0^4$$

$$=\frac{1568}{15}$$

It was very pleasing to see the number of people who got full marks for this part, even though (α) was incorrect.

(β) **3 marks:** Correct solution
2 marks: 1 error in working that is followed through consistently when calculating all variables.
1 mark: Partial progress.

(ii)Radius of shell = x. Height of shell ...

$$\therefore A(x)=2\pi x\left(4x-x^2\right)\rightarrow \qquad \Delta V=2\pi\left(4x^2-x^3\right)\Delta x$$

$$\rightarrow V=2\pi\int_1^3 4x^2-x^3\,dx$$

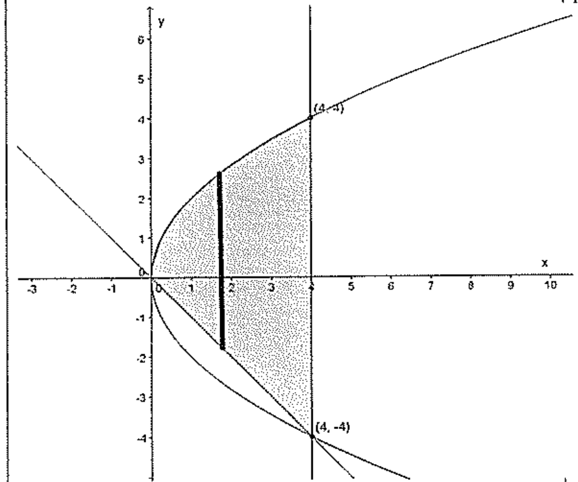
$$=2\pi\left[\frac{4x^3}{3}-\frac{x^4}{4}\right]_1^3=\frac{88\pi}{3}$$

Some people thought that every cylinder had a height of 3. This really changed the question and made it difficult to award any marks, because there was no progress towards the correct path of solution.

2 marks: 1 error in working that is followed through consistently when calculating all variables.
1 mark: Partial progress.

E7 (i)

(c)



1 mark: Correct, neat diagram, showing all points of intersection, and shading.

(ii) (α) Thickness of slice = Δx

So, if $y^2=4x$, $y=2\sqrt{x}$ is the component above the x-axis.

If $x+y=0$, $y=-x$ is the component below the axis.

Sidelength = $2\sqrt{x}-(-x)=2\sqrt{x}+x$

$$A(x)=\left(2\sqrt{x}+x\right)^2=4x+4x\sqrt{x}+x^2$$

(α) 2 marks: Correct solution, with sufficient working.
1 mark: Considerable progress.