Student Number:	
Class:	

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION 2014

MATHEMATICS EXTENSION 2

General Instructions:

- Reading Time: 5 minutes.
- Working Time: 3 hours.
- Write in black pen.
- Board approved calculators & templates may be used
- A Standard Integral Sheet is provided.
- In Question 11 - 16, show all relevant mathematical reasoning and/or calculations.
- Marks may not be awarded for careless or badly arranged working.

Total Marks 100

Section I: 10 marks

- Attempt Question 1 – 10.
- Answer on the Multiple Choice answer sheet provided.
- Allow about 15 minutes for this section.

Section II: 90 Marks

- Attempt Question 11 - 16
- Answer on lined paper provided. Start a new page for each new question.
- Allow about 2 hours & 45 minutes for this section.

The answers to all questions are to be retured in separate *stapled* bundles clearly labelled Question 11, Question 12, etc. Each question must show your Candidate Nnmber.

Section I

10 marks Attempt Questions 1–10
Allow about 15 minutes for this section
Use the multiple-choice answer sheet for Questions 1 -10.

1. If $z = 1 + 2i$ and $w = 3 - i$, what is the value of $z - \overline{w}$?

(A) $3i - 2$

(B) $4 + 3i$

(C) $i - 2$

(D) $4 + i$

2. Which of the following is an expression for? $\int \frac{\cos^3 x + \sin^3 x}{\cos x - \sin x} dx$

(A) $x + \frac{1}{2}\cos 2x + c$

(B) $x - \frac{1}{2}\cos 2x + c$

(C) $x + \frac{1}{2}\sin^2 x + c$

(D) $x - \frac{1}{2}\sin^2 x + c$

3. The equation of the tangent to $xy^3 + 2y = 4$ at the point (2, 1) is

(A) x+8y=10

(B) x-8y=10

(C) x+8y=-10

(D) x-8y=-10

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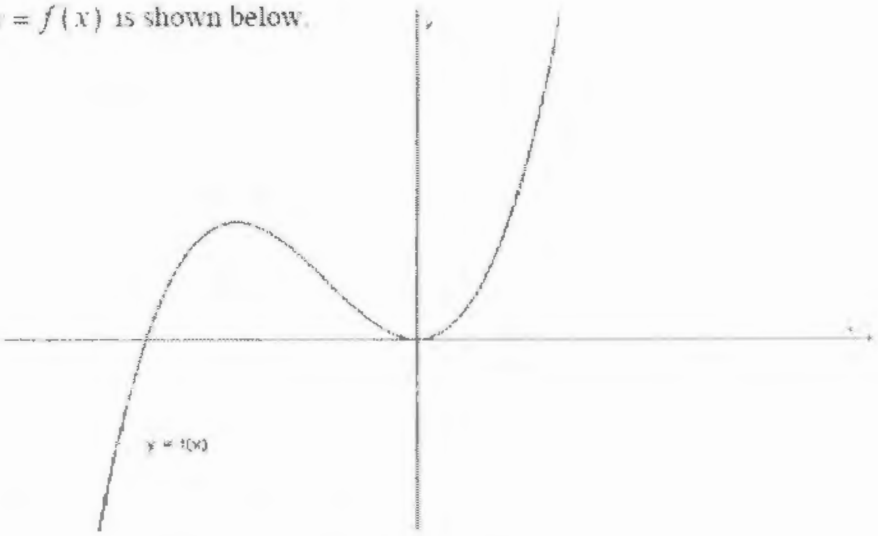
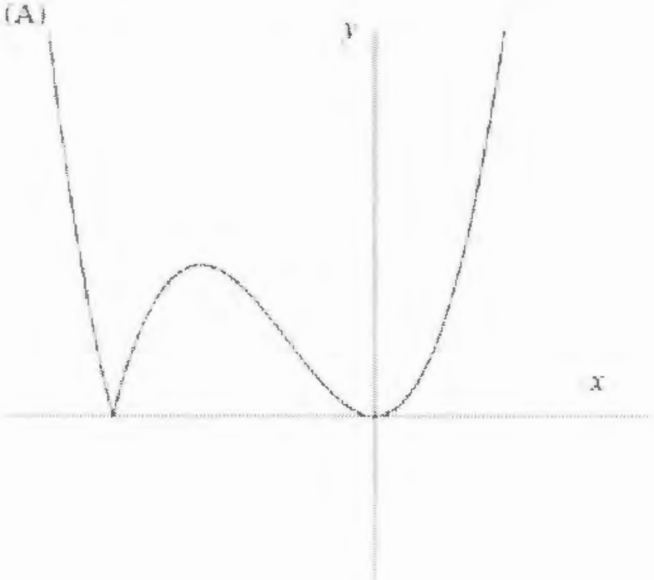
4.

The graph of $y = f(x)$ is shown below.

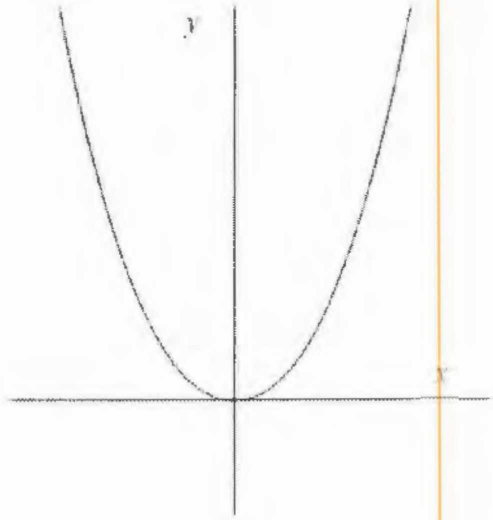


Which of the following graphs best represents $y = f(|x|)$?

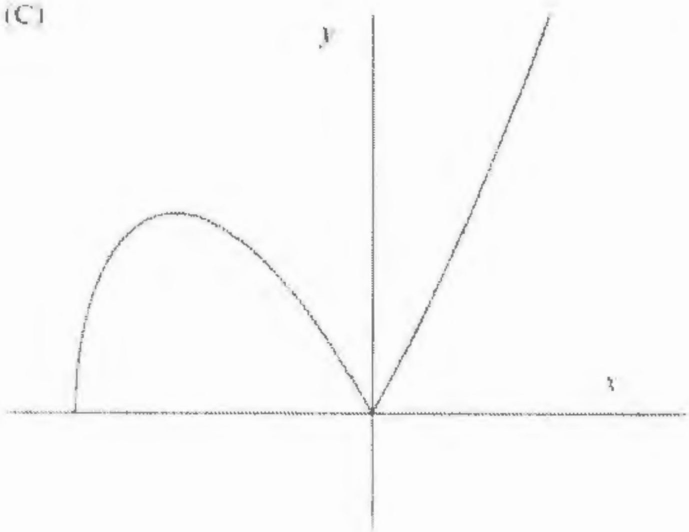
(A)



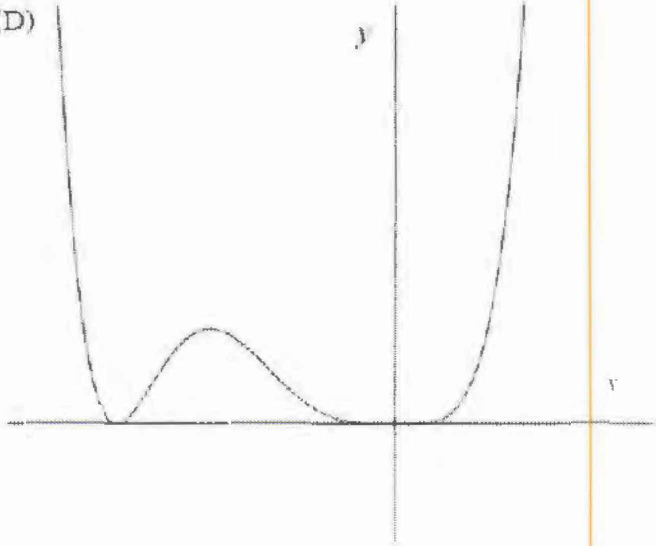
(B)



(C)



(D)



5. The point $P(a\cos\theta, b\sin\theta)$ lies on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ where a>b>0

What is the equation of the normal at P?

(A) $\frac{ax}{\cos\theta}-\frac{by}{\sin\theta}=a^2-b^2$

(B) $\frac{ax}{\cos\theta}+\frac{by}{\sin\theta}=a^2+b^2$

(C) $\frac{x}{a}\sec\theta-\frac{y}{b}\tan\theta=1$

(D) $\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1$

6. What is the multiplicity of the root x=1 of the equation $3x^5-5x^4+5x-3=0$

(A) 1

(B) 2

(C) 3

(D) 4

7. Without evaluating the integrals which one of the following will give an answer of zero?

(A) $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{\cos^3\theta+1}{\cos^2\theta}\,d\theta$

(B) $\int_{-1}^{1}(x^2-1)(1-x^2)^3\,dx$

(C) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\sin^7 x\cos x\,dx$

(D) $\int_{-2}^{2}|x^2-4|\,dx$

8. The base of a solid is the circle $x^2 + y^2 = 1$. Every cross section of the solid taken perpendicular to the x axis is a right angled, isosceles triangle with its hypotenuse lying in the base of the solid. Which of the following is an expression for the volume V of the solid?

(A) $\int_{-1}^{1}(1 - x^2)\,dx$

(B) $2\int_{-1}^{1}(1 - x^2)\,dx$

(C) $4\int_{-1}^{1}(1 - x^2)\,dx$

(D) $\frac{1}{2}\int_{-1}^{1}(1 - x^2)\,dx$

9. A particle of mass m is moving horizontally in a straight line. Its motion is opposed by a force of magnitude mk $(v+v^2)$ Newtons when its speed is v ms^{-1} (where k is a positive constant). At time t seconds the particle has displacement x metres from a fixed point O on the line and velocity v ms^{-1}. Which of the following is an expression for x in terms of v?

(A) $\frac{1}{k}\int\frac{1}{1+v}dv$

(B) $\frac{1}{k}\int\frac{1}{v(1+v)}dv$

(C) $-\frac{1}{k}\int\frac{1}{v(1+v)}dv$

(D) $-\frac{1}{k}\int\frac{1}{1+v}dv$

10 Four digit numbers are formed from the digits 1, 2, 3 and 4.
Each digit is used once only. The sum of all the numbers that can be formed is?

(A) 266,640

(B) 66,660

(C) 44,440

(D) 6,666

Section II

90 marks Attempt Questions 11–16

Allow about 2 hours and 45 minutes for this section

Answer each question on a SEPARATE sheet of paper. Extra paper is available.
In Questions 11–16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11

a) Find $\int \frac{dx}{\sqrt{6x - x^2}}$ 2

b) Show that $\int_2^4 \frac{dx}{x\sqrt{x-1}} = \frac{\pi}{6}$ 3

c) Find $\int e^x \sin x \, dx$ 3

d) Draw a diagram to illustrate the locus of the points z on the Argand diagram such that:

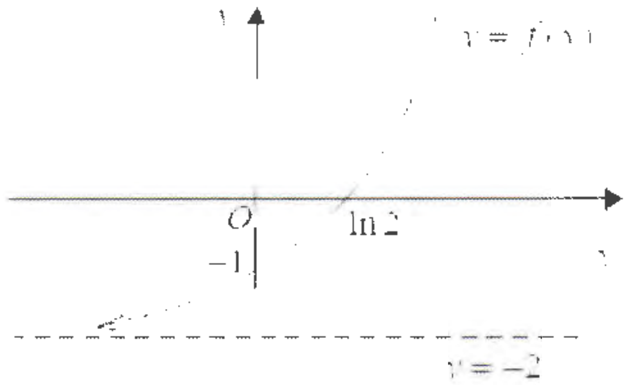
(i) $|z - \bar{z}| \leq 1$ and $|z - 1| \leq 2$ 2

(ii) $Arg\left\{\frac{z-1}{z+1}\right\} = \frac{\pi}{4}$ 2

e) Show by geometrical means or otherwise that if z_1 and z_2 are complex numbers such that $|z_1| = |z_2|$, then $\frac{z_1 + z_2}{z_1 - z_2}$ is purely imaginary. 3

Question 12 (Start a new page)

(a) The diagram below shows the graph of $f(x) = e^x - 2$



On separate diagrams sketch the following graphs, in each case showing the intercepts on the axes and the equations of the asymptotes.

(i) $y = (f(x))^2$ **1**

(ii) $y = \log_e f(x)$ **1**

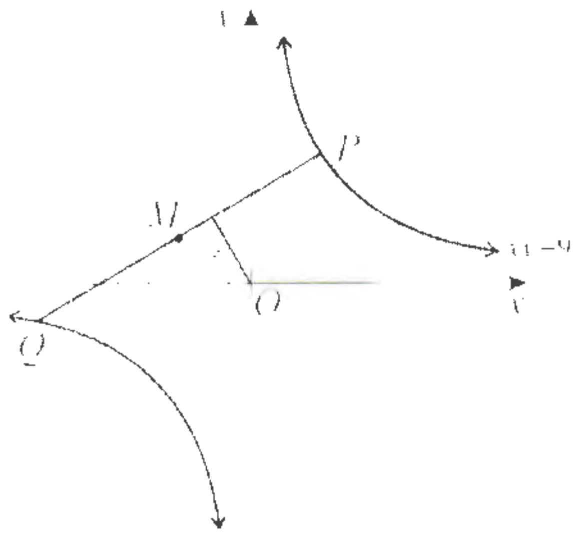
(iii) $y = \dfrac{1}{f(x)}$ **2**

(iv) $y^2 = |f(x)|$ **2**

(b) (i) Show that $4x^2 + 9y^2 + 16x + 18y - 11 = 0$ represents an ellipse. **1**

(ii) Find the eccentricity and hence the coordinates of its foci and the equation of its directrices. **2**

(c)



In the diagram above, $P\left(3p, \frac{3}{p}\right)$ and $Q\left(3q, \frac{3}{q}\right)$ are variable points on the rectangular hyperbola $xy = 9$. The perpendicular distance from the origin to the chord PQ is $\sqrt{5}$ units. Let M be the midpoint of the chord PQ.

(i) Show that the chord PQ has the equation $x+pqy=3(p+q)$ **2**

(ii) Using the perpendicular distance formula, or otherwise, show that $9(p+q)^2 = 5(1+p^2q^2)$ **1**

(iii) Show that the locus of M has the Cartesian equation **3**

$$y^2 = \frac{5x^2}{4x^2-5}$$

Question 13 (Start a new page)

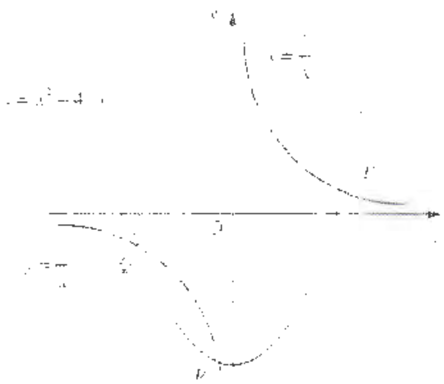
(a) (i) Use the substitution $x = 10\sqrt{2}\sin\theta$ to show that $\int_{-10}^{10} \sqrt{200 - x^2}\, dx = 100 + 50\pi$ 3

(ii) Use a geometrical argument to verify the result in part (i) 1

(iii) A mould for a model railway tunnel is made by rotating the region bounded by the curve $y = \sqrt{200 - x^2}$ and the x-axis between the lines x=-10 and x=10 through 180° about the line x=100 (where all the measurements are in cm).
Use the method of cylindrical shells to show that the volume Vcm^3 of the tunnel is given by $\pi \int_{-10}^{10} (100 - x)\sqrt{200 - x^2}\, dx$. 4

Hence find the volume of the tunnel in m^3 correct to 2 significant figures.

(b)



The curves $y = x^2 - 4$ and $y = \frac{1}{x}$ intersect at the points P, Q, R where $x = \alpha, x = \beta, x = \gamma$

(i) Show that α, β, γ are the roots of the equation $x^3 - 4x - 1 = 0$ 1

(ii) Find a polynomial equation with integer coefficients which has roots $\alpha^2, \beta^2, \gamma^2$. 2

(iii) Find a polynomial equation with integer coefficients which has roots $\frac{1}{\alpha^2}, \frac{1}{\beta^2}, \frac{1}{\gamma^2}$. 2

(iv) Hence find the numerical value of $OP^2 + OQ^2 + OR^2$ 2

Question 14 **(Start a new page)**

(a) Figure 1 below shows a scale model of the volcano Mt Snaefellsjökull
The base of the model is elliptical in shape with the axes 60cm by 40cm reducing uniformly to a circle of radius 12cm at the top.
The hollow core of the model has circular cross sections with a circle of radius 6cm at the base rising uniformly to a circle of radius 12cm at the top.
The model is 24cm high.
Figure 2 shows the top view of the cross sectional area of the volcano

Figure 1

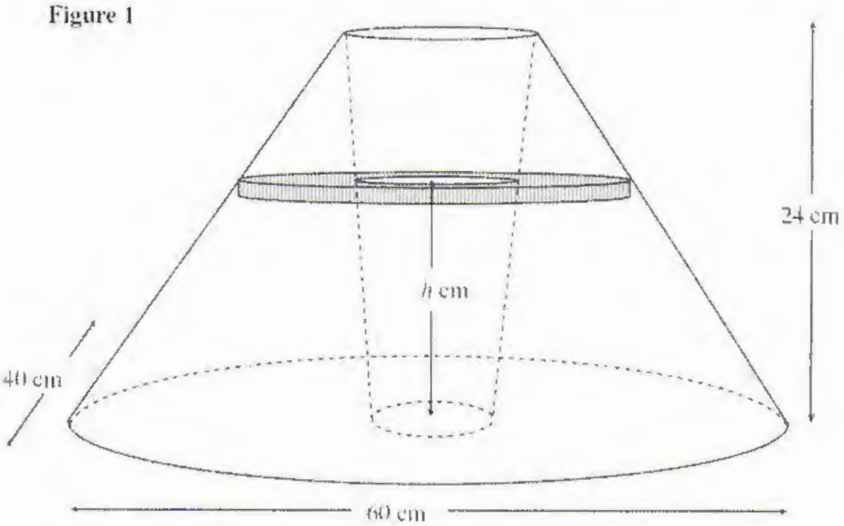
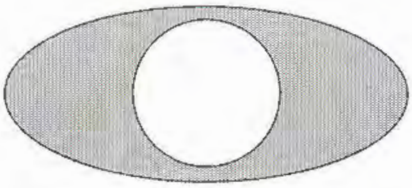


Figure 2



(i) Show that at height h, the length of the semi-major axis is given by **2**

$$a = 30 - \frac{3}{4}h$$

(ii) Show that the cross sectional slice at height h is given by **3**

$$A = \frac{\pi}{16}(9024 - 448h + 3h^2)$$

You can assume the area of an ellipse with semi-major axis a and semi-minor axis b is given by πab

(iii) Find the volume of the scale model of Mt Snaefellsjökull **1**

Question 14 continued on next page

Question 14 continued

(b) (i) Show that if n is an even positive integer, then **2**

$$(1+x)^n + (1-x)^n = 2\sum_{k=0}^{n/2}\binom{n}{2k}x^{2k}$$

(ii) An alphabet consists of three letters A, B and C

(I) Show that the number of words of 4 letters containing exactly 2B's is **1**

$$\binom{4}{2}\times 2^2$$

(II) Hence, or otherwise, show that if n is an even positive integer then the number of words of n letters with zero or an even number of B's is given by **2**

$$\frac{1}{2}(3^n+1)$$

(c) Three men Bill, Garry and Jason observe a vertical tower.
Bill stands due North of the tower, and sees its top at an angle of elevation of $\alpha°$
Garry stands due East of the tower, and sees its top at an angle of elevation of $\beta°$
Jason stands on a line from Bill to Garry, exactly half way between them.

If Jason observes the top of the tower at an angle of elevation of $\theta°$.

Show that $\cot\theta° = \frac{1}{2}\sqrt{\cot^2\alpha° + \cot^2\beta°}$

Question 15 (Start a new page)

(a) Let $I_n = \int_1^2 \left(1-\frac{1}{x}\right)^n dx$ for $n = 1,2,3,.......$

(i) Show that $\frac{1}{n+1}I_{n+1} = \frac{1}{n}I_n - \frac{1}{n(n+1)2^n}$ for $n = 1,2,3,.....$ **3**

(ii) Hence show that $\frac{1}{n+1}I_{n+1} = I_1 - \sum_{r=1}^{n}\frac{1}{r(r+1)2^r}$ **2**

(i) Show that $\sum_{r=1}^{n}\frac{1}{r(r+1)2^r} = (1-\log_e 2) - \frac{1}{n+1}I_{n+1}$ and hence find the limiting sum of the series $\frac{1}{1\text{x}2\text{x}2^1} + \frac{1}{2\text{x}3\text{x}2^2} + \frac{1}{3\text{x}4\text{x}2^3} + ...$ **3**

(b) Let α be the complex root of the polynomial $z^7 = 1$ with the smallest positive argument.

Let $\theta = \alpha + \alpha^2 + \alpha^4$ and $\phi = \alpha^3 + \alpha^5 + \alpha^6$

(i) Show that $\theta + \phi = -1$ and $\theta\phi = 2$ **3**

(ii) Write a quadratic equation whose roots are θ *and* ϕ **2**

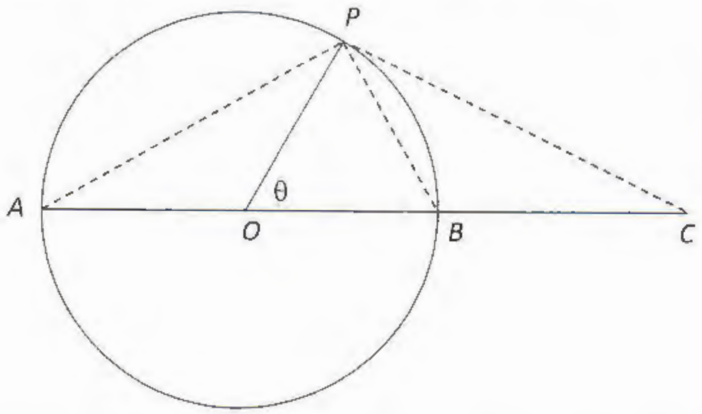
Hence show that $\theta = -\frac{1}{2} + \frac{i\sqrt{7}}{2}$ and $\phi = -\frac{1}{2} - \frac{i\sqrt{7}}{2}$

(iii) Show that $\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} - \cos\frac{\pi}{7} = -\frac{1}{2}$ **2**

Question 16 (Start a new page)

(a) The diagram below shows a point P rotating in a circle of radius 1 metre, whose centre is at O.

AB is a diameter produced to C such that $OC = 2$ metres.



NOT TO SCALE

The angular velocity of P about O is given by $\dot{\theta} = \pi$ rad/sec. ($\theta = \angle POB$)

(i) Find the angular velocity of P about A and about B. 2

(ii) If $\angle PCO = \alpha$, show that $\sin(\alpha + \theta) = 2\sin\alpha$. 1

(iii) Hence find the angular velocity of P about C at the instant when $\theta = \frac{\pi}{2}$. 4

(b) The circular bend on a bike track has a constant radius of 20 metres and is banked at a constant angle of 30° to the horizontal. A bicycle rider can safely negotiate the bend if the maximum sideways thrust F, up or down the slope is at most one-tenth of the normal reaction N. By resolving the forces vertically and horizontally, show that the range of speeds V, correct to two decimal places and in metres per second, at which the bend can be safely negotiated, is $9.50 \le V \le 11.99$. Take $g = 10\text{m/s}^2$. 8

Multiple Choice Answers

1) C 2) D 3) A 4) B 5) A 6) C 7) C 8) A 9) D 10) B

① $1+2i-(3+i)$

$= -2+i$ (c)

② $\int \frac{(\cos x+\sin x)(\cos^2 x-\cos x\sin x+\sin^2 x)}{\cos x+\sin x}$

$= \int 1-\cos x\sin x \, dx$

$= x-\frac{1}{2}\sin^2 x+c$ (D)

③ $xy^3+2y=4$.

$y^3+x\times 3y^2\frac{dy}{dx}+2\frac{dy}{dx}=0$

$x=2$ $y=1$

$1+6\frac{dy}{dx}+2\frac{dy}{dx}=0$

$\frac{dy}{dx}=\frac{-1}{8}$

$y-1=\frac{-1}{8}(x-2)$

$x+8y=10$ (A)

④ part B.

⑤ $\frac{dx}{d\theta}=-a\sin\theta$ $\frac{dy}{d\theta}=b\cos\theta$

$\frac{dy}{d\theta}=\frac{b\cos\theta}{-a\sin\theta}$.

$y-b\sin\theta=\frac{a\sin\theta}{b\cos\theta}(x-a\cos\theta)$

$b\cos\theta y-b^2\cos\theta\sin\theta$

$=a\sin\theta x-a^2\sin\theta\cos\theta$.

$\therefore \frac{ax}{\cos\theta}-\frac{by}{\sin\theta}=a^2-b^2$ $(-\sin\theta\cos\theta)$

⑥ $P'(x)=15x^4-20x^3+5$

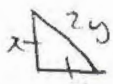
$P''(x)=60x^3-60x$

$P'(1)=0$ $P''(1)=0$

$P'''(x)=180x-60$ $P'''(1)\neq 0$

part C

⑦ A even B even C odd D even

⑧  $\frac{x}{2y}=\cos 45^{\circ}$

$x=\frac{2y}{\sqrt{2}}$

area $=\frac{1}{2}\times\frac{2y}{\sqrt{2}}\times\frac{2y}{\sqrt{2}}$.

$\therefore \int_{-1}^{1}(1-x^2)\,dx$ A

⑨ $m\ddot{x}=-mk(v+v^2)$

$x=-\frac{1}{k}\int\frac{1}{1+v}$

⑩ $1000+100+10+1=1111$

3! ways of arranging 2, 3, 4

$3!(1111)(1+2+3+4)$

$=66660$

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MATHEMATICS Extension 2: Question 11

Suggested Solutions	Marks	Marker's Comments
(a) $\int \frac{dx}{\sqrt{6x-x^2}}dx = \int \frac{dx}{\sqrt{9-(x^2-6x+9)}}$	1	1 mark off for each error
$= \int \frac{dx}{\sqrt{9-(x-3)^2}}$		
$= \sin^{-1}\frac{x-3}{3} + C$	1	
(b) let $u=\sqrt{x-1}$		
$\frac{du}{dx} = \frac{1}{2\sqrt{x-1}}$ when $x=2$, $u=1$; when $x=4$, $u=\sqrt{3}$	1	
now $u^2 = x-1$		
$\therefore x = u^2+1$		
$\therefore 2\int_1^{\sqrt{3}} \frac{du}{u^2+1} = 2\left[\tan^{-1}u\right]_1^{\sqrt{3}}$	1	
$= 2\left[\tan^{-1}\sqrt{3} - \tan^{-1}1\right]$	1	
$= 2\left(\frac{\pi}{3}-\frac{\pi}{4}\right)$		
$= 2\times\frac{\pi}{12}$		
$= \frac{\pi}{6}$		
(c) let $u=\sin x$, $\frac{dv}{dx}=e^x$		
$\frac{du}{dx}=\cos x$, $v=e^x$		
$\therefore I = uv - \int v\,du$		
$I = e^x\sin x - \int e^x\cos x\,dx$	1	
now let $u=\cos x$, $\frac{dv}{dx}=e^x$		
$\frac{du}{dx}=-\sin x$, $v=e^x$		
$I = e^x\sin x - \left[e^x\cos x + \int e^x\sin x\,dx\right]$	1	
$I = e^x\sin x - e^x\cos x - \int e^x\sin x\,dx$		
but $I = \int e^x\sin x\,dx$		
$\therefore 2I = e^x\sin x - e^x\cos x$	1	
$I = \frac{1}{2}\left(e^x\sin x - e^x\cos x\right)$		

MATHEMATICS Extension 2: Question 11

Suggested Solutions	Marks	Marker's Comments

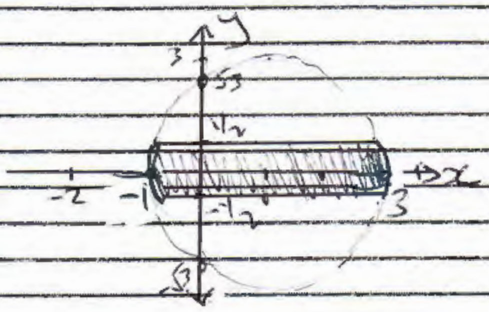
(d)(i) $|z-\bar{z}| \leq 1$ and $|z-1| \leq 2$

$(1,0)$ radius 2.

$|x+yi-(x-yi)| \leq 1$

$|2iy| \leq 1$

$|y| \leq \frac{1}{2}$



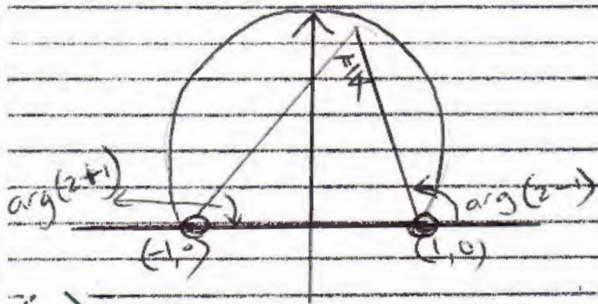
* 1 off for each error.

1 for circle (lightly drawn)

1 for correct region shaded. (most students forgot $y \geq -\frac{1}{2}$).

(ii) $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{4}$

$\arg(z-1) - \arg(z+1) = \frac{\pi}{4}$

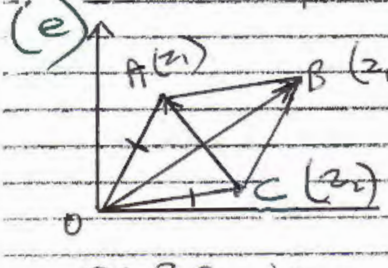


major arc of a circle standing on $(-1,0)$ and $(1,0)$

1 for open circles

1 for major arc (and both lines and $\frac{\pi}{4}$ → lightly)

(e)



$\vec{OB} = z_1 + z_2$

$CA = z_1 - z_2$

$OA = OC \quad (|z_1| = |z_2|)$

OABC is a parallelogram

$OA = OC$ (as $|z_1| = |z_2|$)

∴ OABC is a rhombus (parm with a pair of adjacent sides equal.) — 1

OB and AC intersect at 90° (diagonals of a rhombus intersect at 90°) — 1

$\therefore \arg(z_1+z_2) - \arg(z_1-z_2) = \pm\frac{\pi}{2}$

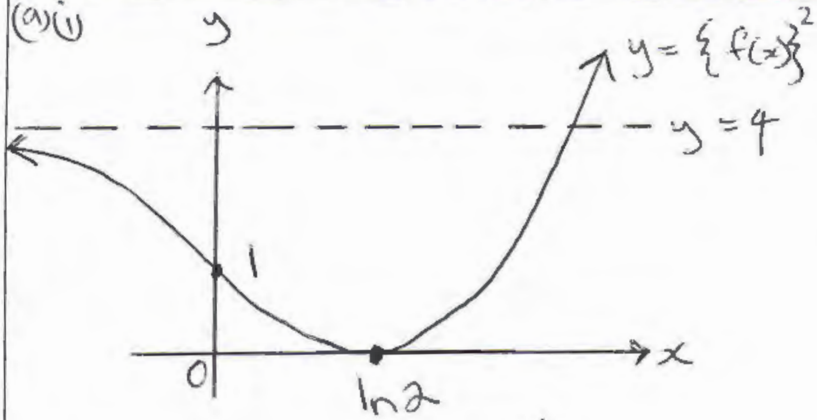
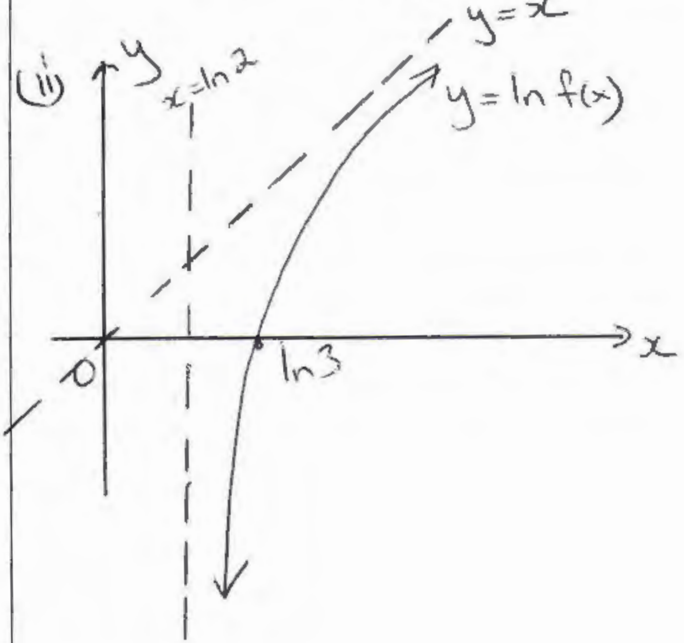
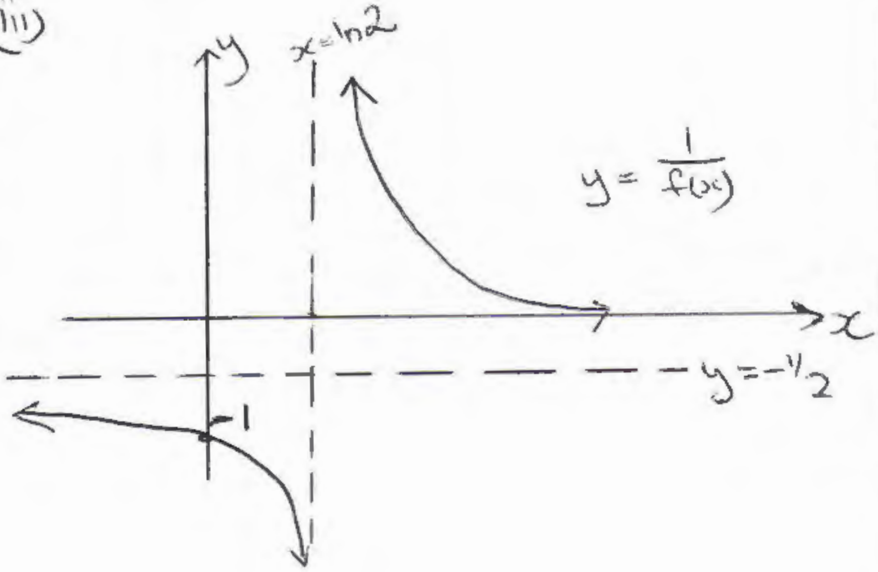
$\arg\left(\frac{z_1+z_2}{z_1-z_2}\right) = \pm\frac{\pi}{2}$

$\therefore \frac{z_1+z_2}{z_1-z_2}$ is purely imaginary. — 1

Or could discuss rotation by 90°.

or it could have been done algebraically

MATHEMATICS Extension 2: Question 12

Suggested Solutions	Marks Awarded	Marker's Comments
(a)(i) 	1	- if you left off an intercept then = 0 mks
(ii) 	1	- a lot of students forgot the asymptote $y=x$, - if you left off the x-intercept then you lost the mark.
(iii) 	1 1	both asymptotes shape + intercept * as soon as you forgot one thing you lost the mark.

MATHEMATICS Extension 2: Question 12 cont.

<table>
<tr><th>Suggested Solutions</th><th>Marks Awarded</th><th>Marker's Comments</th></tr>
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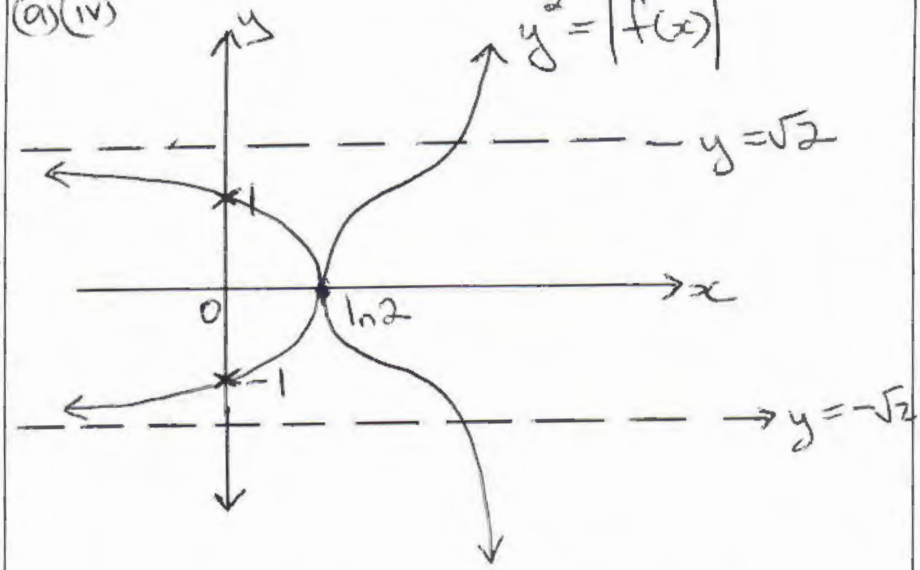
(a)(iv)



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1

1

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all 3 intercepts + horizontal asymptote

shape of both graphs.

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<tr><td>

b(i) $4(x^2+4x+4)+9(y^2+2y+1)=11+16+9$

$4(x+2)^2+9(y+1)^2=36$

$\frac{(x+2)^2}{9}+\frac{(y+1)^2}{4}=1$

which is in the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

and so it is an ellipse

</td><td>1</td><td></td></tr>
<tr><td>

(ii) $b^2=a^2(1-e^2)$

$4=9(1-e^2)$

$e^2=\frac{5}{9}$

$e=\frac{\sqrt{5}}{3}$ as $(0<e<1)$

</td><td>1</td><td></td></tr>
<tr><td>

directrices: $x=-2\pm\frac{9}{\sqrt{5}}$

$\frac{a}{e}=\frac{3}{\sqrt{5}/3}=9/\sqrt{5}$

focus: $(-2+\sqrt{5},-1)$ and $(-2-\sqrt{5},-1)$

$ae=3\times\sqrt{5}/3=\sqrt{5}$

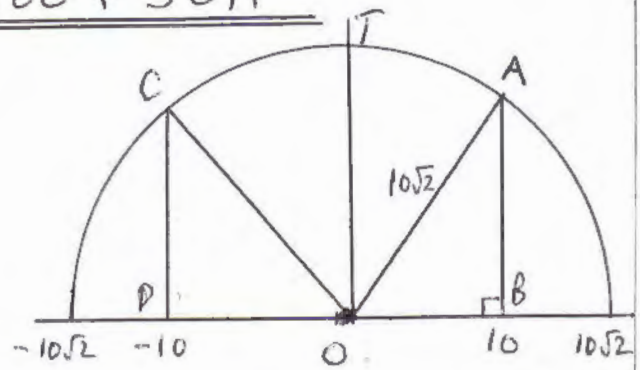
</td><td>1</td><td>had to get <u>both</u> correct to get the 2nd mark.</td></tr>
</table>

MATHEMATICS Extension 2: Question 12

Suggested Solutions	Marks Awarded	Marker's Comments
c)(i) Equation of the chord PQ $m_{PQ} = \frac{y_2-y_1}{x_2-x_1} = \frac{3/p - 3/q}{3p-3q} = \frac{3(q-p)}{pq(p-q)}$ $= \frac{-1}{pq} \quad p \neq q$. ∴ Equation of PQ $y - y_1 = m(x-x_1)$ $y - 3/p = -\frac{1}{pq}(x-3p)$ $pqy - 3q = -x + 3p$ $x + pqy = 3(p+q)$	2	① gradient ① Equation with correct working
(ii) For perpendicular distance $d = \frac{\lvert ax+by+c \rvert}{\sqrt{a^2+b^2}}$ Point $(0,0)$ line $x + pqy - 3(p+q) = 0$ $d = \sqrt{5}$ $\sqrt{5} = \frac{\lvert 0(1) + 0(pq) - 3(p+q) \rvert}{\sqrt{1+(pq)^2}}$ square b.s. $5 = \frac{9(p+q)^2}{1+p^2q^2}$ $5(1+p^2q^2) = 9(p+q)^2$	1	Must show perpendicular distance formula and substitution. showing zeros.
(iii) Midpoint PQ $\left(\frac{3(p+q)}{2}, \frac{3/p + 3/q}{2} \right)$ $x = \frac{3(p+q)}{2}$, $y = \frac{3(p+q)}{2pq}$ $p+q = \frac{2x}{3}$, $y = \frac{x}{pq}$, $pq = \frac{x}{y}$ sub into $5(1+p^2q^2) = 9(p+q)^2$ from part (ii) $5\left(1+\frac{x^2}{y^2}\right) = 9\left(\frac{2x}{3}\right)^2$ $1 + \frac{x^2}{y^2} = \frac{4x^2}{5}$ $\frac{x^2}{y^2} = \frac{4x^2}{5} - 1$ $\frac{x^2}{y^2} = \frac{4x^2-5}{5}$ ∴ $y^2 = \frac{5x^2}{4x^2-5}$.	3	① Midpoint ① $pq = \frac{x}{y}$ (or similar substitution) ① correct answer with working

* All substitutions needed to be sequential and easily followed

2014 TRIAL X2 MATHEMATICS: Question 13. p 1 of 4

Suggested Solutions	Marks	Marker's Comments
a) Let $x = 10\sqrt{2}\sin\theta$		
$\therefore \dfrac{dx}{d\theta} = 10\sqrt{2}\cos\theta$		
" $dx = 10\sqrt{2}\cos\theta\, d\theta$ "		
When $x = 10$, $\sin\theta = \frac{1}{\sqrt{2}}$, $\theta = \frac{\pi}{4}$ $x = -10$, $\sin\theta = -\frac{1}{\sqrt{2}}$, $\theta = -\frac{\pi}{4}$		Generally well done.
$\therefore \displaystyle\int_{-10}^{10} \sqrt{200 - x^2}\, dx = \int_{-\pi/4}^{\pi/4} \sqrt{200 - 200\sin^2\theta}\; 10\sqrt{2}\cos\theta\, d\theta$	1	Some people used even symmetry to reduce work at substitution.
$= 200 \displaystyle\int_{-\pi/4}^{\pi/4} \cos^2\theta\, d\theta$		
$= 100 \displaystyle\int_{-\pi/4}^{\pi/4} 1 + \cos 2\theta\, d\theta$	1	
$= 100 \left[\theta + \dfrac{\sin 2\theta}{2}\right]_{-\pi/4}^{\pi/4}$		
$= 100\left[\dfrac{\pi}{4} + \dfrac{1}{2} - \left(-\dfrac{\pi}{4} - \dfrac{1}{2}\right)\right]$		
$= 100\left(\dfrac{\pi}{2} + 1\right)$		
$= \underline{\underline{100 + 50\pi}}$	1	
b) 		A diagram was fairly essential to explain this answer.

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Suggested Solutions	Marks	Marker's Comments

Integral represent the area $B\hat{A}CD$

i.e. $\underline{\underline{\Delta ABO + \Delta OCD + \text{Sector } OCA}}$

By Pythagoras, $AB = 10$.

$\therefore$ Area ΔABO = Area $\Delta OCD = \frac{1}{2} \times 10 \times 10$

$= \underline{50.}$

ΔCAB is isosceles and $\pi/2$ at B

$\therefore \angle AOC = 2\angle AOT = \pi/2$

$\therefore$ Area sector is $\frac{(10\sqrt{2})^2}{2}\frac{\pi}{2} = \underline{\underline{50\pi}}$

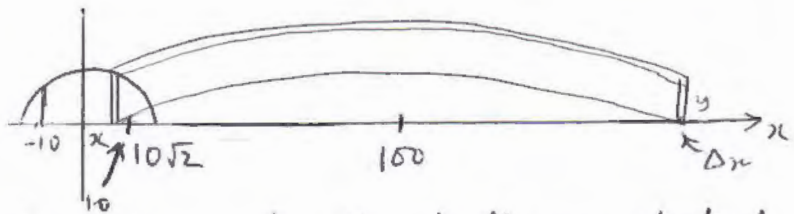
$\therefore$ Integral equals $50 + 50 + 50\pi$

$= \underline{\underline{100 + 50\pi}}$ — **1**

Marker's comments: Only 1 mark so all issues had to be addressed.

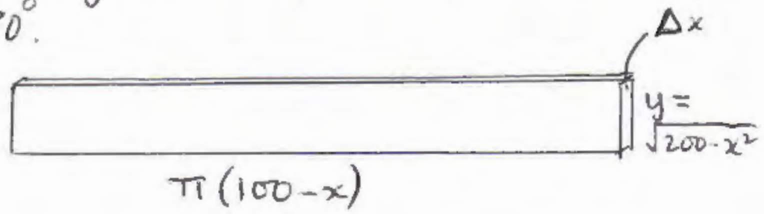
Alternative version possible with rectangle + segment.

c)



N.B. Only a half shell as rotated by 180°. — **1**

Marker's comments: One mark for recognising the half cylinder (180°)



$\Delta V \doteqdot \pi(100-x)\,y\,\Delta x$

$= \pi(100-x)\sqrt{200-x^2}\,\Delta x$ — **1**

$\therefore V = \lim_{\Delta x \to 0} \sum_{x=-10}^{10} \pi(100-x)\sqrt{200-x^2}\,\Delta x$

$= \pi\int_{-10}^{10} 100\sqrt{200-x^2}\,dx - \pi\int_{-10}^{10} x\sqrt{200-x^2}\,dx$

But the second integral is of

Marker's comments: for setup i.e. diagram(s) $\Delta V \doteqdot$ $V = \lim_{\Delta x \to 0} \sum$ etc.

Too many lost this mark.

Suggested Solutions	Marks	Marker's Comments
an odd function between symmetric limits. i.e. equals 0. $\therefore V = 100\pi \int_{-10}^{10} \sqrt{200-x^2}\,dx$ $= 100\pi(100+50\pi)$ using (i) $= 80763.94\ldots \text{ cm}^3$ $= \underline{\underline{0.081 \text{ m}^3}}$ (to 2 S.F.)	 1 1	 Too many people lost this last mark. Highlight potential traps when reading question.
b) i) Points of intersection of the two graphs will provide the roots α, β & γ. (as the x values of intersection) Substitute $y = \frac{1}{x}$ into $y = x^2 - 4$ $\frac{1}{x} = x^2 - 4$ $1 = x^3 - 4x \quad (x \neq 0)$ i.e. $\underline{\underline{x^3 - 4x - 1 = 0}}$ ii) We need $y = x^2 \Rightarrow x = \sqrt{y}$ Sub. into equation $y\sqrt{y} - 4\sqrt{y} - 1 = 0$ $\sqrt{y}(y-4) = 1$ $y(y^2 - 8y + 16) = 1$ (Square both sides) $\underline{\underline{y^3 - 8y^2 + 16y - 1 = 0}}$	 1 1 1	Substitution alone was NOT enough. The mention of the word "intersection" or equivalent guaranteed the mark. No real need to change back to x's.

2014 TRIAL X2 **MATHEMATICS: Question** 13.

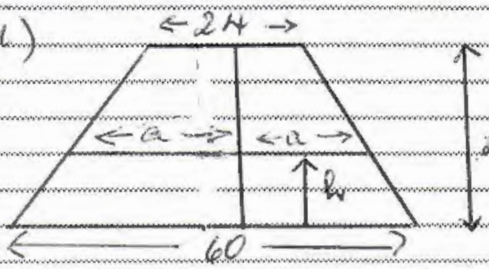
Suggested Solutions	Marks	Marker's Comments
iii) From eqn in (ii), Substitute $z = \frac{1}{y} \Rightarrow y = \frac{1}{z}$	1	Many people started from (i) with $z = \frac{1}{\sqrt{x}}$. First mark for equivalent point as in (ii).
$\therefore \frac{1}{z^3} - \frac{8}{z^2} + \frac{16}{z} - 1 = 0$		
$\underline{\underline{z^3 - 16z^2 + 8z - 1 = 0}}$	1	
iv) The co-ordinates of P are given by $\left(\alpha, \frac{1}{\alpha}\right)$, say.		
Then $OP^2 = \alpha^2 + \frac{1}{\alpha^2}$ (Pythagoras)		
Similarly $OQ^2 = \beta^2 + \frac{1}{\beta^2}$, say		
$OR^2 = \gamma^2 + \frac{1}{\gamma^2}$, say.		
$\therefore OP^2 + OQ^2 + OR^2 = (\alpha^2 + \beta^2 + \gamma^2) + \left(\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2}\right)$	1	
$= 8 + 16$		
(Sum of roots in equations from (ii) & (iii))		
$= \underline{\underline{24}}$ (units)	1	

EXT 2.

MATHEMATICS: Question 14

Suggested Solutions	Marks	Marker's Comments

a) (i)



h	0	24
a	30	12

Linear Relationship

$a = mh + b$

$m = \frac{30-12}{0-24} = \frac{3}{4}$

When $h = 0 \quad a = 30$

$a = -\frac{3}{4}h + 30$

$a = 30 - \frac{3}{4}h$

Marks: ②

Marker's comments: ① reasoning. ① answer with correct working. Could also be done with similar triangles.

(ii) Similarly for minor axis

$b = \left(\frac{12-20}{24-0}\right)h + 20$

h	0	24
b	20	12

$= 20 - \frac{1}{3}h$

Inner Radius

$R = \frac{12-6}{24-0}h + 6$

h	0	24
R	6	12

$= \frac{1}{4}h + 6$

Area is $\pi ab - \pi R^2$

$= \pi\left[ab - R^2\right]$

$= \pi\left[20 - \frac{1}{3}h\right]\left[30 - \frac{3}{4}h\right] - \left[6 + \frac{1}{4}h\right]^2$

$= \pi\left[\frac{60-h}{3}\right]\left[\frac{120-3h}{4}\right] - \left[\frac{24+h}{4}\right]^2$

$= \frac{\pi}{16}\left[9024 - 448h + 3h^2\right]$

Marks: ③

Marker's comments: ① $b = 20 - \frac{h}{3}$. ① $R = \frac{1}{4}h + 6$. ① correct answer with working

(iii) $\delta V = A\,\delta h$

$V = \lim_{\delta h \to 0} \sum_{0}^{24} \frac{\pi}{16}\left[9024 - 448h + 3h^2\right]\delta h$

$= \frac{\pi}{16}\int_0^{24}\left[9024 - 448h + 3h^2\right]dh$

$= \pi/16\left[9024h - 224h^2 + h^3\right]_0^{24}$

$= \frac{\pi}{16}\left[9024(24) - 224(24)^2 + (24)^3\right]$

$= 6336\pi \qquad$ Volume is $6336\pi\ \text{cm}^3$

Marks: ①

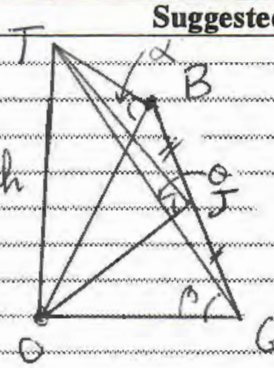
Marker's comments: Correct answer.

MATHEMATICS: Question 14

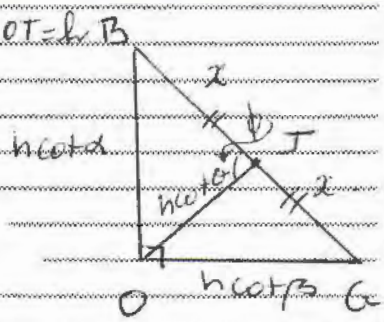
Suggested Solutions	Marks	Marker's Comments
(b) (i) To show $(1+x)^n + (1-x)^n = 2\sum_{k=0}^{n/2} \binom{n}{2k} x^{2k}$	2	
LHS $= {}^nC_0 x^0 + {}^nC_1 x^1 + {}^nC_2 x^2 - - - - {}^nC_n x^n +$		① for expansions
${}^nC_0(-x)^0 + {}^nC_1(-x)^1 + {}^nC_2(-x)^2 - (-1)^n\, {}^nC_n(-x)^n$		
$(-1)^n = 1$ when n is even		① cancelling odd powers, adding even power
$= 2\left[{}^nC_0 x^0 + {}^nC_2 x^2 + {}^nC_4 x^4 - - - - {}^nC_n\right]$		
$= 2\left[{}^nC_{2\times(0)} x^{2\times(0)} + {}^nC_{2\times(1)} x^{2\times(1)} + {}^nC_{2\times(2)} x^{2\times(2)} - - - {}^nC_{2\times(\frac{n}{2})} x^{2\times(\frac{n}{2})}\right]$ *		and* showing new series is only even powers and $\frac{n}{2}$ terms.
$= 2\left[\sum_{k=0}^{n/2} {}^nC_{2k}\, x^{2k}\right]$		
(ii) 2 B's and 2 other letters (A or C)		
_ _ _ _ 4 letter word.		
choose 2 places for 2 B's $= {}^4C_2$	1	Correct description
other spaces 2 × 2 choices $= 2^2$		
Total is ${}^4C_2 \times 2^2$		
(iii) Even number of B's in even number of letters. (A or C)	2	
zero B's no. of words $= {}^nC_0\, 2^n$		① for full explanation of series
2 B's " " $= {}^nC_2 \times 2^{n-2}$		
4 B's " " ${}^nC_4 \times 2^{n-4}$		
etc "		
$n-2$ B's " ${}^nC_{n-2}\, 2^2$		
all B's $= {}^nC_n\, 2^0$		① sub $x=2$ into eqn in (i).
$\therefore$ Total $= {}^nC_0 2^n + {}^nC_2 2^{n-2} + {}^nC_4 2^{n-4} \dots {}^nC_n 2^0$		
But ${}^nC_r = {}^nC_{n-r}$		
$\therefore$ Total $= {}^nC_n 2^n + {}^nC_{n-2} 2^{n-2} - \cdots {}^nC_4 2^4 + {}^nC_2 2^2 + {}^nC_0 2^0$		
$= \sum_0^{n/2} {}^nC_{2k}\, 2^{2k} = \frac{1}{2}\left\{[1+2]^n + [1-2]^n\right\}$ from (i) $x = 2$.		
$= \frac{1}{2}\left[3^n + (-1)^n\right]$		
$= \frac{1}{2}\left[3^n + 1\right]$ as $(-1)^n = 1$ as n is even		

MATHEMATICS: Question 14

Suggested Solutions	Marks	Marker's Comments
	4	



Let $OT = h$



$\tan\alpha = \dfrac{OT}{OB}$ $\qquad$ $\tan\beta = \dfrac{OT}{OG}$

$h\cot\alpha = OB$ $\qquad$ $h\cot\beta = OG$

$\tan\theta = \dfrac{OT}{OJ}$ $\qquad$ $OJ = h\cot\theta$

Many methods

① as $BJ = JG$ and $\angle BOG = 90^\circ$

BG is a diameter of circle through points B, O and G

$\therefore BJ = JG = OJ$ (radii of circle)

$OJ = \frac{1}{2}BG = h\cot\theta$

By Pythagoras: $OB^2 + OG^2 = BG^2$

$BG^2 = h^2\cot^2\alpha + h^2\cot^2\beta$

$\therefore h\cot\theta = \dfrac{h}{2}\sqrt{\cot^2\alpha + \cot^2\beta}$

$\cot\theta = \dfrac{1}{2}\sqrt{\cot^2\alpha + \cot^2\beta}$

Marker's comments:
① stating $OJ = \frac{1}{2}BG$.
① with reasons (a proof)
① Pythagoras statemen
① correct answer with working

② Alternative Let $\angle BJO = \phi$ and $BJ = JG = x$.

In $\triangle OJB$

$h^2\cot^2\alpha = h^2\cot^2\theta + x^2 - 2h\cot\theta\, x\cos\phi$

In $\triangle OJG$

$h^2\cot^2\beta = h^2\cot^2\theta + x^2 - 2h\cot\theta\, x\cos(180-\phi)$

$\cos(180-\phi) = -\cos\phi$

ADD EQUATIONS

$h^2(\cot^2\alpha + \cot^2\beta) = 2h^2\cot^2\theta + 2x^2$

But $4x^2 = h^2\cot^2\alpha + h^2\cot^2\beta$ (Pythagoras)

$\therefore h^2(\cot^2\alpha + \cot^2\beta) = 2h^2\cot^2\theta + \frac{1}{2}(h^2\cot^2\alpha + h^2\cot^2\beta)$

$\therefore 2\cot^2\theta = \frac{1}{2}(\cot^2\alpha + \cot^2\beta)$

$\cot\theta = \frac{1}{2}\sqrt{\cot^2\alpha + \cot^2\beta}$

Marker's comments:
② cos Rule statements.
① Pythagoras. statement.
① answer with correct working

a) i) $I_n = \int_1^2 \left(1-\frac{1}{x}\right)^n dx$ for $n = 1, 2, 3, \ldots$

$= \int_1^2 \frac{(x-1)^n}{x^n} dx$ Let $u = x^{-n}$ $u' = \frac{-n}{x^{n+1}}$

$v' = (x-1)^n$ $v = \frac{(x-1)^{n+1}}{n+1}$

$= uv - \int v\,du$

$= \left.\frac{(x-1)^{n+1}}{(n+1)x^n}\right]_1^2 - \int_1^2 \frac{(x-1)^{n+1}}{n+1} \cdot \frac{-n}{x^{n+1}} dx$ 1 m

$= \frac{1}{(n+1)2^n} + \frac{n}{n+1}\int_1^2 \frac{(x-1)^{n+1}}{x^{n+1}} dx$ 1 m

$I_n = \frac{1}{2^n(n+1)} + \frac{n}{n+1} I_{n+1}$ 1 m

$\therefore \frac{I_{n+1}}{n+1} = \frac{I_n}{n} - \frac{1}{n(n+1)2^n}$ #

Other alternatives
$u = \left(1-\frac{1}{x}\right)^{n+1}$ $dv = dx$
$du = (n+1)\left(1-\frac{1}{x}\right)^n\left(\frac{1}{x^2}\right)dx$, $v = x$
many mistakes here

There are other alternatives

ii) $\frac{I_n}{n} = \frac{I_{n-1}}{n-1} - \frac{1}{(n-1)n\cdot 2^{n-1}}$ (from i replace $n+1$ by n)

$\frac{I_{n-1}}{n-1} = \frac{I_{n-2}}{n-2} - \frac{1}{(n-2)(n-1)\,2^{n-2}}$

$\vdots$

1m
must show at least 3 expansions there

$\frac{I_2}{2} = \frac{I_1}{1} - \frac{1}{1.2.2^1}$

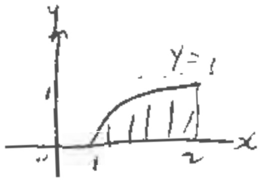
$\therefore \frac{I_{n+1}}{n+1} = I_1 - \left[\frac{1}{1.2.2} + \ldots + \frac{1}{(n-2)(n-1)2^{n-2}} + \frac{1}{n(n-1)2^{n-1}} + \frac{1}{n(n+1)2^n}\right]$ 1 m

$\frac{I_{n+1}}{n+1} = I_1 - \sum_{r=1}^{n} \frac{1}{r(r+1)2^r}$

(iii) $I_1 = \int_1^2 \left(1-\frac{1}{x}\right)dx = x - \ln(x)\Big]_1^2 = 2 - \ln 2 - (1 - \ln 1)$
$= 1 - \ln 2$ 1 m

iii) $\sum_{r=1}^{n} \frac{1}{r(r+1)2^r} = I_1 - \frac{I_{n+1}}{n+1} = 1 - \ln 2 - \frac{I_{n+1}}{n+1}$

$0 < \int_1^2 \left(1-\frac{1}{x}\right)^n dx < 1$



$0 < I_n < 1$ and

$I_{n+1} < I_n$

$\therefore 0 < I_{n+1} < 1$

$0 < \frac{I_{n+1}}{n+1} < \frac{1}{n+1}$

As $n \to \infty$ $\frac{I_{n+1}}{n+1} \to 0$

Hence limiting sum $\frac{1}{1.2\,2^1} + \frac{1}{2.3\,2^2} + \frac{1}{3.4\,2^3} + \dots$

$= \underline{1 - \ln 2}$

$0 < I_n < 1$

$I_{n+1} < I_n$

as $n \to \infty$ $\frac{I_{n+1}}{n+1} \to 0$

Any 2 get 1m

All 3 of the above with correct answer $1 - \ln 2$ will get 2m

b) $z^7 = 1$

$z^7 - 1 = (z-1)(1+z+z^2+z^3+\dots+z^6) = 0$

Since α is a complex root with smallest argument

$\alpha^7 - 1 = (\alpha-1)(1+\alpha+\alpha^2+\dots+\alpha^6) = 0$

$\alpha \neq 1$ $\therefore 1+\alpha+\alpha^2+\dots+\alpha^6 = 0$

must mention α is root $\alpha \neq -1$

must show this line in terms of α

1m

$\theta + \phi = (\alpha+\alpha^2+\alpha^4) + (\alpha^3+\alpha^5+\alpha^6)$

$= (1+\alpha+\alpha^2+\alpha^3+\alpha^4+\alpha^5+\alpha^6) - 1$

$= 0 - 1$

$= -1$

v. poorly done. alternative see later

1m well done

$\theta\phi = (\alpha+\alpha^2+\alpha^4)(\alpha^3+\alpha^5+\alpha^6)$

$= \alpha^4+\alpha^6+\alpha^7+\alpha^5+\alpha^7+\alpha^9+\alpha^7+\alpha^9+\alpha^{10}$

$(\alpha^7=1)$ $= \alpha^4+\alpha^6+1+\alpha^5+1+(\alpha^7)\alpha^2+1+(\alpha^7)\alpha+(\alpha^7)\alpha^3$

$\therefore \theta\phi = \alpha^4+\alpha^6+1+\alpha^5+1+\alpha^2+1+\alpha+\alpha^3$

$= (1+\alpha+\alpha^2+\alpha^3+\alpha^4+\alpha^5+\alpha^6)+2$

$= 0 + 2 = \underline{2}$

1m

(from above) $1+\alpha+\alpha^2+\dots+\alpha^6=0$

well done

Alternative to (i)

Let $z = r \operatorname{cis}\theta$

$z^7 = r^7 \operatorname{cis} 7\theta$ (De Moivre's Th)

$r^7 \operatorname{cis} 7\theta = 1 = 1^7 \operatorname{cis} 0$

$\therefore r = 1, \ \theta = \frac{2n\pi}{7} \quad n \in J$

Rts of $z^7 = 1$ are

$\operatorname{cis} 0 = 1, \ \operatorname{cis}\frac{2\pi}{7}(\alpha), \ \operatorname{cis}\frac{4\pi}{7}(\alpha^2), \ \operatorname{cis}\frac{6\pi}{7}(\alpha^3)$

$\operatorname{cis}\frac{8\pi}{7}(\alpha^4), \ \operatorname{cis}\frac{10\pi}{7}(\alpha^5) \ \operatorname{cis}\frac{12\pi}{7}(\alpha^6)$

Sum of Rts $= 1 + \operatorname{cis}\frac{2\pi}{7} + \operatorname{cis}\frac{4\pi}{7} + \operatorname{cis}\frac{6\pi}{7} + \operatorname{cis}\frac{8\pi}{7} + \operatorname{cis}\frac{10\pi}{7} + \operatorname{cis}\frac{12\pi}{7}$

$= 0$

i.e. $1 + \alpha + \alpha^2 + \alpha^3 + \alpha^4 + \alpha^5 + \alpha^6 = 0$

$(\alpha + \alpha^2 + \alpha^4) + (\alpha^3 + \alpha^5 + \alpha^6) = -1$

$\therefore \theta + \phi = -1$ #

(ii) $x^2 + x + 2 = 0$

$x = \frac{-1 \pm i\sqrt{7}}{2}$

$\operatorname{Im}(\theta) = \operatorname{Im}\left[\sin\frac{2\pi}{7} + \sin\frac{4\pi}{7} + \sin\frac{8\pi}{7}\right] > 0$

$\therefore \theta = \frac{-1 + i\sqrt{7}}{2}, \quad \phi = \frac{-1 - i\sqrt{7}}{2}$

(iii) $\operatorname{Re}(\theta) = \cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{8\pi}{7} = -\frac{1}{2}$

but $\cos\frac{8\pi}{7} = -\cos\frac{\pi}{7}$

since $\cos(\pi + \theta) = -\cos\theta$

$\therefore \cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} - \cos\frac{\pi}{7} = -\frac{1}{2}$

must Prove $\alpha^2, \alpha^3, \alpha^4, \alpha^5, \alpha^6$ are also roots

must relate $\alpha, \alpha^2, \alpha^3, \alpha^4, \alpha^5, \alpha^6$ together as roots

1m

1m

1m well done

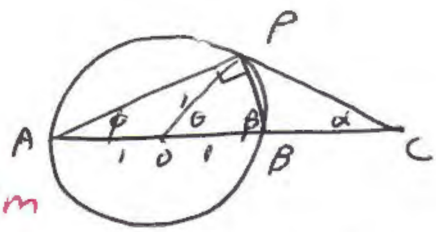
1m very few can explain why $\theta = \frac{-1 + i\sqrt{7}}{2}$

1m

must explain this

1m.

Ext 2 TRIAL 2014 Q16



a)i) $\theta = 2\phi$ (centre at centre twice angle at circumference)

$\dot{\theta} = 2\dot{\phi}$

$\dot{\phi} = \frac{\dot{\theta}}{2} = \frac{\pi}{2}$ rad/sec (angular vel about A) 1m

$\angle APB = \frac{\pi}{2}$ (angle of semi-circle)

$\beta = \pi - \frac{\pi}{2} - \phi$ (angle sum of ΔAPB)

$\dot{\beta} = -\dot{\phi} = -\frac{\pi}{2}$ rad/sec (angular vel about B) 1m

must give reasons

ii) In ΔOPC

$\angle OPC = \pi - (\theta + \alpha)$ (angle sum of triangle)

using Sine Rule $\frac{\sin\alpha}{OP} = \frac{\sin\angle OPC}{OC}$

$\frac{\sin\alpha}{1} = \frac{\sin(\pi - (\theta+\alpha))}{2}$ 1m

Since $\sin(\pi - A) = \sin A$

$\therefore 2\sin\alpha = \sin(\theta+\alpha)$ 1m

must mention ΔOPC & sine rule

many thought $\angle OPC = \frac{\pi}{2}$

iii) $(2\cos\alpha)\dot{\alpha} = \cos(\theta+\alpha)[\dot{\theta}+\dot{\alpha}]$

when $\theta = \frac{\pi}{2}$

$(2 \cdot \frac{2}{\sqrt{5}})\dot{\alpha} = \cos(\frac{\pi}{2}+\alpha][\pi+\dot{\alpha}]$

$\frac{4}{\sqrt{5}}\dot{\alpha} = -\sin\alpha(\pi+\dot{\alpha}]$ 1m

$\frac{4}{\sqrt{5}}\dot{\alpha} = -\frac{1}{\sqrt{5}}(\pi+\dot{\alpha})$ 1m

$5\dot{\alpha} = -\pi$

$\dot{\alpha} = -\frac{\pi}{5}$ rad/sec 1m

1m for sub all below

$\begin{cases} \sin\alpha = \frac{1}{\sqrt{5}} \\ \cos\alpha = \frac{2}{\sqrt{5}} \\ \dot{\theta} = \pi \end{cases}$

Alternatively using chain rule

$-\sin\theta\sin\alpha\frac{d\alpha}{d\theta} + \cos\alpha\cos\theta - \sin\alpha\sin\theta + \cos\theta\cos\alpha\frac{d\alpha}{d\theta} = 2\cos\alpha\frac{d\alpha}{d\theta}$ 1m

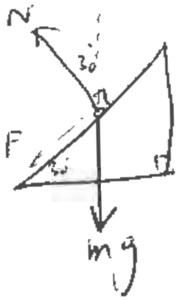
$\frac{d\alpha}{d\theta} = \frac{-\cos(\theta+\alpha)}{\cos\theta\cos\alpha - \sin\theta\sin\alpha - 2\cos\alpha} = \frac{-\cos(\frac{\pi}{2}+\alpha)}{\underbrace{\cancel{\cos\frac{\pi}{2}\cos\alpha}}_{0} - \sin\frac{\pi}{2}\sin\alpha - 2\cos\alpha}$ 1m

$\frac{d\alpha}{d\theta} = \frac{\sin\alpha}{-\sin\alpha - 2\cos\alpha} = \frac{\frac{1}{\sqrt{5}}}{-\frac{1}{\sqrt{5}} - \frac{4}{\sqrt{5}}} = -\frac{1}{5}$ 1m

$\frac{d\alpha}{dt} = \frac{d\alpha}{d\theta} \times \frac{d\theta}{dt} = -\frac{1}{5} \times \pi = -\frac{\pi}{5}$ rad/sec 1m

Q16

b)



Case 1 max vel F ↙

1 m { max vel F ↙ / min vel F ↑

1 m diagram with angles forces

Vertically

$N\cos 30° = 10m + F\sin 30°$

1 m vertical forces

$N\frac{\sqrt{3}}{2} = 10m + \frac{F}{2}$

$N\left(\frac{\sqrt{3}}{2} - \frac{1}{20}\right) = 10m$ (1)

1 m horizontal forces

Horizontally $\frac{mv^2}{20} = F\cos 30° + N\sin 30°$

1 m for sub in numerical value

$\frac{mv^2}{20} = F\frac{\sqrt{3}}{2} + \frac{N}{2}$

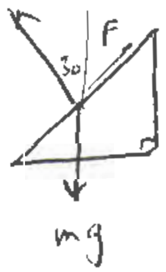
$N\left[\frac{1}{2} + \frac{\sqrt{3}}{20}\right] = \frac{mv^2}{20}$ (2)

(2) ÷ (1) $\frac{\frac{v^2 \cancel{m}}{20}}{10\cancel{m}} = \frac{\cancel{N}(10+\sqrt{3})}{\cancel{N}(10\sqrt{3}-1)}$ 1 m

$v^2 = 200\left(\frac{10+\sqrt{3}}{10\sqrt{3}-1}\right) = 143.77$

$v_{MAX} = 11.99$ m/s $(v>0)$

Case 2 min vel F ↑



Replace F by −F in the above calculation

$10m = N\left(\frac{\sqrt{3}}{2} + \frac{1}{20}\right]$

$\frac{mv^2}{20} = N\left(\frac{1}{2} - \frac{\sqrt{3}}{20}\right)$

} 1 m must have numerical values

(2) ÷ (1) $v^2 = 200\left(\frac{10-\sqrt{3}}{10\sqrt{3}+1}\right)$ 1 m

$v^2 = 90.2589\ldots$

$v_{min} = 9.5$ m/s $(v>0)$

$\therefore\ 9.5 \le v \le 11.99$