Student Number:	
Class:	

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION 2018

MATHEMATICS EXTENSION 2

General Instructions:

- Reading Time: 5 minutes.
- Working Time: 3 hours.
- Write in black pen.
- Board approved calculators & templates may be used
- A Standard Integral Sheet is provided.
- In Question 11 - 16, show all relevant mathematical reasoning and/or calculations.
- Marks may not be awarded for careless or badly arranged working.

Total Marks 100

Section I: 10 marks

- Attempt Question 1 – 10.
- Answer on the Multiple Choice answer sheet provided.
- Allow about 15 minutes for this section.

Section II: 90 Marks

- Attempt Question 11 - 16
- Answer on lined paper provided. Start a new page for each new question.
- Allow about 2 hours & 45 minutes for this section.

The answers to all questions are to be returned in separate *stapled* bundles clearly labelled Question 11, Question 12, etc. Each question must show your Candidate Number.

Section A MULTIPLE CHOICE

(Use the Multiple choice answer sheet for questions 1 – 10)

One mark will be awarded for each correct answer.

Question 1

After using the method of slicing to find a volume, the final calculation requires the evaluation of this integral

$$\int_{-8}^{8} \sqrt{64 - x^2} \; dx$$

The value of this integral is :

A) 0 B) 32π C) 64π D) 128π

Question 2

If ω is a complex root of the equation $z^3 = 1$, then the value of

$$\frac{1}{5+6\omega+5\omega^2} - \frac{1}{3+3\omega+2w^2} \text{ is}$$

A) -1 B) $-\frac{1}{2}$ C) $\frac{1}{2}$ D) 1

Question 3

If the polynomial $P(x)$ is divided by $(x^2 + 25)$ then the remainder is $3x + 4$.
What is the remainder when $P(x)$ is divided by $(x + 5i)$?

A) $15 - 4i$ B) $15 + 4i$ C) $4 - 15i$ D) $4 + 15i$

Question 4

The gradient of the tangent to the curve $x^3 + y^3 - 8xy + 7 = 0$ at the point (1,2) is:

A) $\frac{4}{13}$ B) Undefined C) $\frac{13}{4}$ D) $\frac{3}{2}$

Question 5

A projectile is fired from the origin at a fixed angle θ with a velocity V. If this velocity is doubled, while the angle of projection remains the same, then the range of the projectile across horizontal ground will:

A) Remain unchanged.

B) Increase by a factor of $\sqrt{2}$.

C) Increase by a factor of 2 .

D) Increase by a factor of 4.

Question 6

Which of the following loci is identical to the locus $|z| = 2$, $\text{Im}(z) > 0$?

A) $z\bar{z} = 4$, $\text{Re}(z) > 0$

B) $\text{Arg}\left(\frac{z-2}{z+2}\right) = \frac{\pi}{2}$

C) $\text{Re}\left(\frac{z+2}{z-2}\right) = 0$

D) $\text{Arg}\left(\frac{z+2}{z-2}\right) = \frac{\pi}{2}$

Question 7

What is the solution of the inequality $\frac{x(5-x)}{x-4} \geq -3$?

A) $2 \leq x < 4$ or $x \geq 6$

B) $4 < x \leq 5$ or $x \leq 1$

C) $4 < x \leq 6$ or $x \leq 2$

D) $1 \leq x < 4$ or $x \geq 5$

Question 8

$f(x)$ is a continuous, differentiable function over $a \leq x \leq b$ and $g(x)$ is a continuous, differentiable function over $c \leq x \leq d$. Which one of the following integrals is always greater than, or equal to, the other choices?

A) $\int_a^b f(x)\,dx + \int_c^d g(x)\,dx$

B) $\left|\int_a^b f(x)\,dx\right| + \left|\int_c^d g(x)\,dx\right|$

C) $\int_a^b |f(x)|\,dx + \int_c^d |g(x)|\,dx$

D) $\left|\int_a^b f(x)\,dx + \int_c^d g(x)\,dx\right|$

Question 9

A particle is moving along a straight line so that initially its displacement is $x=1$, its velocity $v=2$ and its acceleration is $a=4$.

Which of the following is a possible equation describing the motion of the particle?

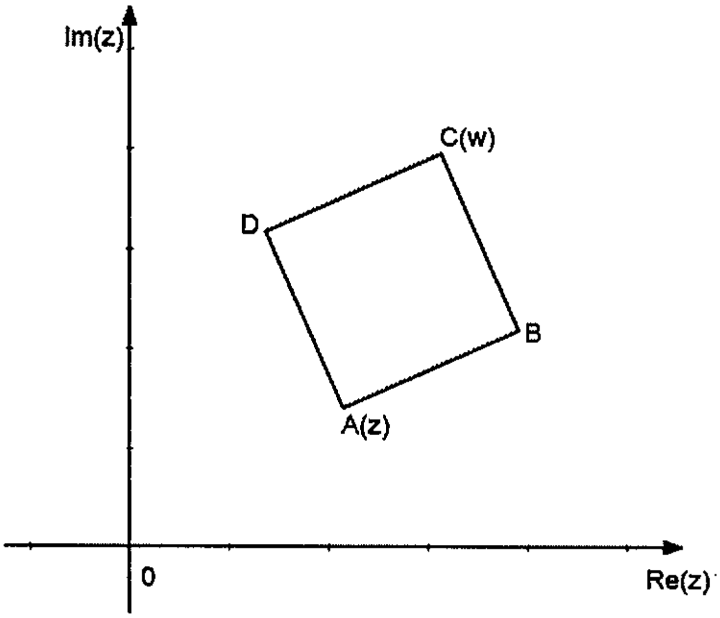
A) $v^2 = 4(x^2 - 2)$

B) $v = 2 + 4\log_e x$

C) $v = 2\sin(x-1) + 2$

D) $v = x^2 + 2x + 4$

Question 10



In the diagram, $ABCD$ is a square in the first quadrant as shown. If A represents the complex number z and C represents the complex number w, then B represents which of the following complex numbers?

A) $\frac{z-w}{2} - \frac{i(z+w)}{2}$

B) $\frac{z-w}{2} + \frac{i(z+w)}{2}$

C) $\frac{z+w}{2} - \frac{i(z-w)}{2}$

D) $\frac{z+w}{2} + \frac{i(z-w)}{2}$

End of Section A

Section B

In this section you should include all necessary reasoning and calculations.

Question 11 (Start each new question on a separate sheet of paper.) **Marks**

(a) By using partial fractions, or otherwise, evaluate $\int_{-2}^{-1} \frac{18dx}{x^2(x+3)}$ **3**

(b) Evaluate $\int_0^{\frac{\pi}{4}} \sec^4 x \ dx$ **3**

(c) (i) Show that $\frac{d}{dx}(x^2 \tan^{-1} x) = 2x\tan^{-1} x + 1 - \frac{1}{1+x^2}$ **2**

(ii) Hence find the exact value of $\int_0^{\sqrt{3}} x\tan^{-1} x \ dx$ **2**

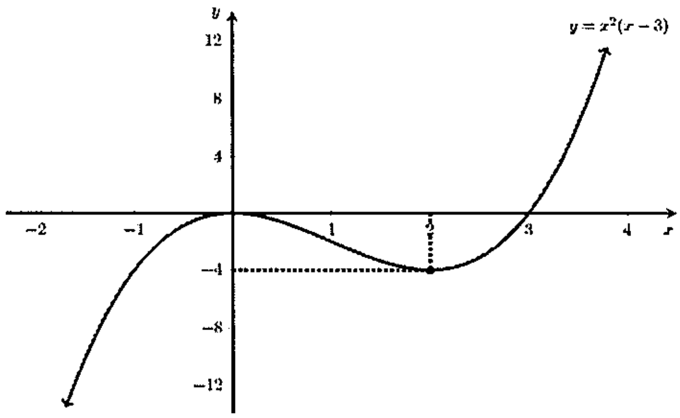
(d) $z_1 = -2 + 2i$ and $z_2 = -1 + i\sqrt{3}$ are two complex numbers.

(i) Express z_1 , z_2 and $\frac{z_1}{z_2}$ in modulus/argument form. **3**

(ii) Find the smallest positive integer n such that $\frac{{z_1}^n}{{z_2}^n}$ is purely imaginary and, for this value of n, write down the value of $\frac{{z_1}^n}{{z_2}^n}$ in the form bi where b is a real number. **2**

Question 12 (Start each new question on a separate sheet of paper.) **Marks**

(a)



A sketch of the curve $y = x^2(x-3)$ is shown above.

On separate sets of axes sketch the graphs of:

(i) $y^2 = x^2(x-3)$ **2**

(ii) $y = \frac{1}{x^2(x-3)}$ **2**

(iii) $y = \frac{1}{x^2(|x|-3)}$ **1**

(b) Consider the ellipse $4x^2 + 3y^2 = 48$.

(i) Find the eccentricity of the ellipse. **1**

(ii) Write down the coordinates of the foci and the equations of the directrices. **2**

(iii) Sketch the ellipse, showing the above features and the intercepts on the coordinate axes. **1**

(c) (i) Prove that, for all real a and b,

$$a^2 + b^2 \geq 2ab$$ **1**

(ii) Prove that, for any positive, real x and y,

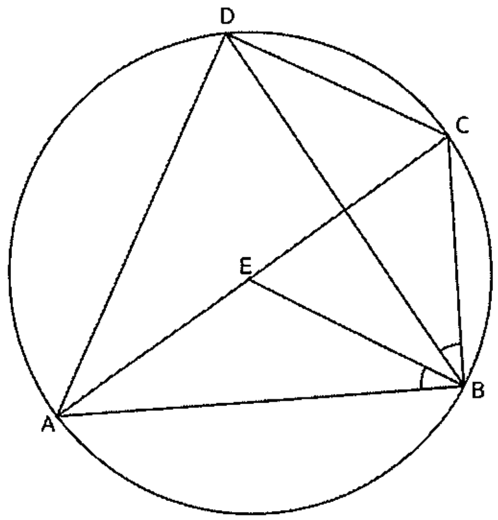
$$\frac{x}{y} + \frac{y}{x} \geq 2$$ **1**

(iii) Prove by induction, or otherwise, that

$$(x_1 + x_2 + x_3 + \cdots + x_{n-1} + x_n)\left(\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \cdots + \frac{1}{x_{n-1}} + \frac{1}{x_n}\right) \geq n^2$$

where the x_i are all real and positive. **4**

Question 13 (Start each new question on a separate sheet of paper.) **Marks**



(a) The figure shows a cyclic quadrilateral $ABCD$ with diagonals AC and BD. E is a point on AC such that angle ABE equals angle DBC.

(i) Prove that triangle ABE is similar to triangle DBC. **2**

(ii) Prove that triangle ABD is similar to triangle EBC. **2**

(iii) Hence prove Ptolemy's Theorem, which, in this context, states

$$AB \times CD + AD \times BC = AC \times BD$$ **3**

(b) Sketch the curve $y = \frac{(x-2)(x+1)}{5-x}$ over the domain $\{x: -5 < x < 15\}$ showing the axis intercepts and the relationship of the curve to its oblique asymptote. (You are given that there are turning points near $x = 0.75$ and $x = 9.25$ but you need not evaluate their values exactly.) **3**

(c) A jar contains w white and r red jelly beans. Three jellybeans are taken at random from the jar and eaten.

(i) Write down an expression, in terms of w and r, for the probability that the three jellybeans were all white. (Leave in factorised form.) **1**

It was observed that, if the jar had initially contained $(w + 1)$ white and r red jellybeans, then the probability that the 3 eaten jellybeans were white would have been double that in part (i).

(ii) Show that $r = \frac{(w-2)(w+1)}{5-w}$ **2**

(iii) Using part (b) or otherwise, determine all possible values of w and r. **2**

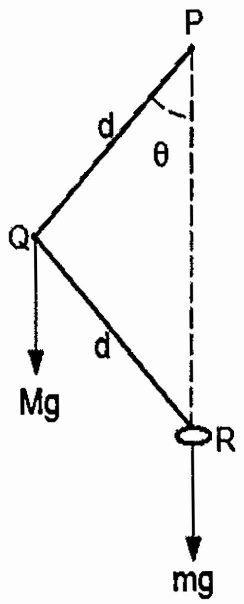
Question 14 (Start each new question on a separate sheet of paper.) **Marks**

(a) Consider the polynomial $p(x) = ax^4 + bx^3 + cx^2 + dx + f$ where a, b, c, d and f are integers. Suppose α is a non-zero integer such that $p(\alpha) = 0$.

(i) Prove that α divides f. **1**

(ii) Show that the polynomial $q(x) = 4x^4 - x^3 + 3x^2 + 2x - 3$ does not have an integer zero. **2**

(b) Two light rigid rods PQ and QR, each of length d metres, are smoothly jointed at Q and the rod PQ is smoothly jointed at the fixed point P. The joint at Q has a mass of M kg attached.



A small ring of mass m kg is smoothly jointed to QR at R and can slide on a smooth vertical rod which also passes through P.

The system rotates about the vertical causing the ring to rise up the vertical rod.

In equilibrium the system is rotating at ω rads/sec and the rods form an angle θ with the vertical (as shown).

(i) Copy the diagram and, by resolving forces at points Q and R, show that $M\omega^2 d = (2mg + Mg)\sec\theta$ **4**

(ii) Now suppose that $M = 4m$ and that a safety switch operates when $\theta = \pi/3$. **1**

Show that this occurs when $\omega = \sqrt{\frac{3g}{d}}$

(c) The complex number z satisfies the condition $|z - 8| = 2\,\mathrm{Re}(z - 2)$.

(i) Write down the equation of the locus in its more usual Cartesian form, noting any restrictions that apply. **3**

(ii) Sketch the locus on an Argand diagram and find the possible values of Arg z. **3**

(iii) Write down the value of $|z + 8| - |z - 8|$ for any point on the curve. **1**

Question 15 (Start each new question on a separate sheet of paper.) **Marks**

(a) The area bounded by the parabola $y = x(3 - x)$ and the x axis is rotated about the line $x = 4$. Use the method of cylindrical shells to find the volume of the solid formed. **4**

(b) Let α, β and γ be the roots of the equation $x^3 - 3x^2 + 5 = 0$.

(i) Find a cubic polynomial equation with integer coefficients whose roots are α^2, β^2 and γ^2. **2**

(ii) Find the value of $\alpha^4 + \beta^4 + \gamma^4$. **2**

(c) $P(3p, 3/p)$ and $Q(3q, 3/q)$ are two points on the rectangular hyperbola $xy = 9$. The equation of the chord PQ is $pqy + x = 3(p + q)$ (DO NOT PROVE THIS)

The chord PQ meets the x and y axes at G and H respectively.

(i) Find the coordinates of M, the midpoint of PQ. **1**

(ii) Show that $PH = QG$ **2**

(iii) If the chord PQ always passes through the point $(0,2)$, find the locus of M, proving any restrictions. **4**

Question 16 (Start each new question on a separate sheet of paper.) **Marks**

(a) A particle P, of unit mass, is thrown vertically downwards in a resistive medium where the resistive force is proportional to speed. Thus, taking downwards as positive, the equation of motion is $\ddot{x} = g - kv$ where $k > 0$.

The particle is thrown, with an initial speed U, from a point T which is d units above a fixed point O which should be taken as the origin. So the initial conditions are : when $t = 0,\ v = U\ and\ x = -d$

(i) Show that , during the subsequent motion of P,

$$v = \frac{g}{k} - \left(\frac{g-kU}{k}\right)e^{-kt}$$

(You may assume that $U < \frac{g}{k}$) **2**

(ii) Integrate again to show that

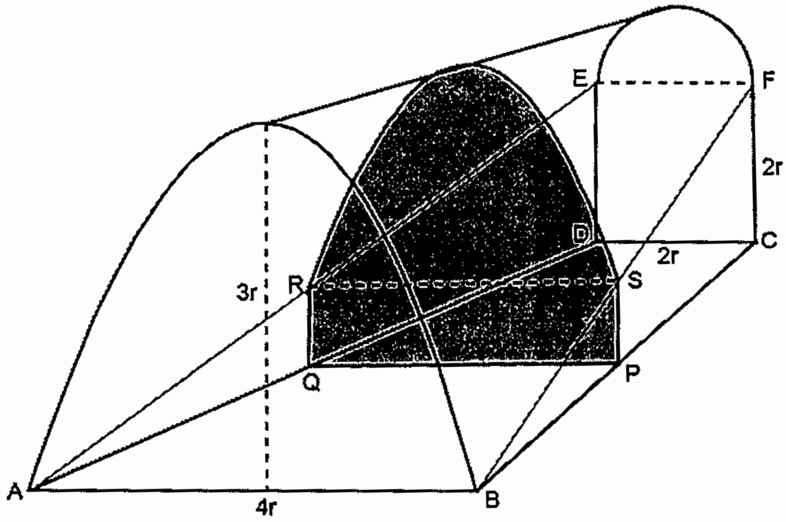
$$x = \frac{gt-kd}{k} + \left(\frac{g-kU}{k^2}\right)(e^{-kt} - 1).$$ **2**

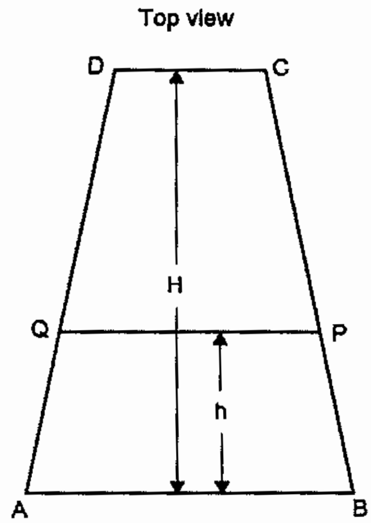
(iii) An identical particle Q is dropped from rest at O at the same instant that P is thrown down. Use the above results to write down expressions for v and x as functions of time for Q. **2**

(iv) The particles P and Q collide. Find when the collision occurs and the difference in speeds as the particles collide. **3**

Question 16 is continued on the next page...

(b)





A tunnel has an isosceles trapezium $ABCD$ as its base with AB of side length 4r and CD of side length $2r$. The tunnel is of length H and the height along the centre line is constant at $3r$. The cross section perpendicular to the floor at AB is a semi-ellipse with AB as the minor axis and with a semi-major axis of length $3r$. The cross section perpendicular to CD is a square $CDEF$ surmounted by a semicircle of radius r.

A typical internal cross-section (as shown) comprises a rectangle surmounted by a semi-ellipse. AED and BCF are thence plane triangles.

(You may assume that the area of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is πab square units.)

(i) Show that the tunnel cross-sectional area at a perpendicular distance h from AB is given by :

$$\frac{r^2}{2H^2}(2H-h)(3\pi H+(8-2\pi)h)$$ **4**

(ii) Hence find the volume of the tunnel. **2**

END OF EXAMINATION

MULTIPLE CHOICE

1. B 2. A 3. C 4. C 5. D 6. B 7. C 8. C 9. C 10. D

1 Semicircle rad $= 8$. $\pi 8^2/2$

2. $1+\omega+\omega^2=0$ $\quad \frac{1}{\omega}-\frac{1}{-\omega^2}=\frac{1}{\omega}+\frac{1}{\omega^2}=\frac{\omega+\omega^2}{\omega^3}=-1$

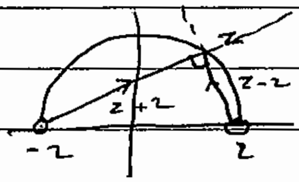
3. $P(x) = Q(x)(x+5i)(x-5i) + 3x+4$

$P(-5i) = \qquad -15i+4$

4. $3x^2+3y^2y'-8y-8xy'=0 \qquad y'=\frac{8y-3x^2}{3y^2-8}=\frac{16-3}{12-8}$

5. Range $=\frac{V^2\sin 2\theta}{g}$

6.

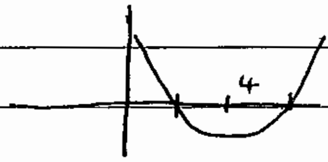


7. $\underline{x>4}$ $\quad x(5-x)\geq -3(x-4)$

$0\geq x^2-8x+12$

$0\geq (x-6)(x-2)$

$4< x\leq 6$



$x<4 \quad 0\leq (x-6)(x-2)$

$x\leq 2$.

8. C

9. a) $v^2<0$ ✗ b) $v\frac{dv}{dx}=(2+4\ln x)\frac{4}{x}=8$ when $x=1$ ✗ d) $v(1)\neq 1$ $\quad\therefore$ c ✓

10. $OB = OA + AB$

$= OA + AC\,\frac{1}{\sqrt{2}}\,\mathrm{cis}(-45^\circ)$

$= z+(w-z)\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}-\frac{i}{\sqrt{2}}\right) = z+(w-z)\left(\frac{1}{2}-\frac{i}{2}\right)$

$= w\left(\frac{1}{2}-\frac{i}{2}\right) \quad z\left(1-\frac{1}{2}+\frac{i}{2}\right) = (z+w)\frac{1}{2}+(z-w)\frac{i}{2}.$

MATHEMATICS Extension 2: Question 11

Suggested Solutions	Marks	Marker's Comments
(a) $\dfrac{18}{x^2(x+3)} = \dfrac{A}{x^2} + \dfrac{B}{x} + \dfrac{C}{x+3}$ $A(x+3) + Bx(x+3) + Cx^2 \equiv 18$ $x=-3 \quad \underline{C=2}$ $x=0 \quad 3A=18 \quad \underline{A=6}$ $x=1 \quad 4A+4B+C=18$ $24+4B+2=18$ $4B=-8$ $\underline{B=-2}$	1	Other methods also valid $\dfrac{A}{x^2} + \dfrac{B}{x+3}$ is not correct
$\displaystyle\int_{-2}^{-1} \frac{6}{x^2} - \frac{2}{x} + \frac{2}{x+3}\,dx$		$\underline{NB}$ $\displaystyle\int -\frac{6}{x^2} \neq -\frac{1}{3x^3}$
$= \left[-\dfrac{6}{x} - 2\ln\lvert x\rvert + 2\ln\lvert x+3\rvert\right]_{-2}^{-1}$ or $\underline{-\ln x^2}$	1	$\lvert x\rvert$ critical as $\ln(-2)$ not defined
$= (6+2\ln 2) - (3 - 2\ln 2 + 0)$ $= \underline{3 + 4\ln 2}$	1	careless errors with negative
b) $\displaystyle\int_0^{\pi/4} \sec^4 x\,dx = \int_0^{\pi/4} \sec^2 x \sec^2 x\,dx$ $\displaystyle= \int_0^{\frac{\pi}{4}} (1+\tan^2 x)\sec^2 x\,dx$	1	

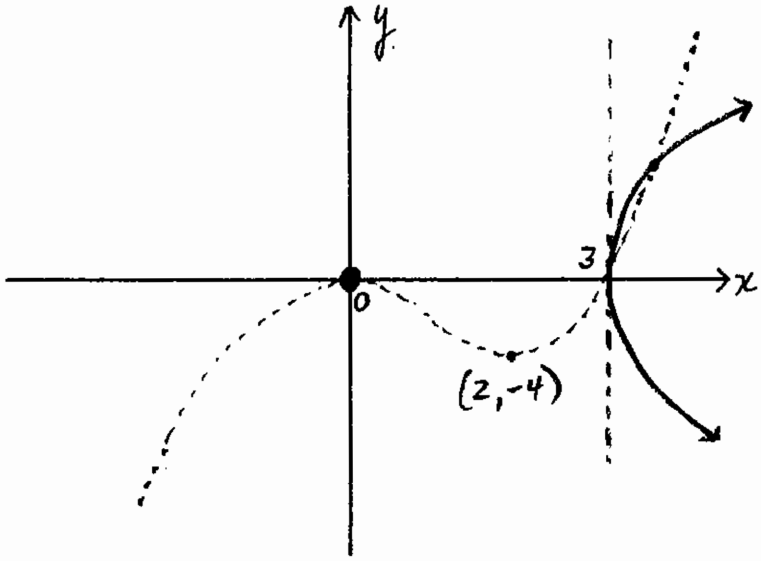
MATHEMATICS Extension 2 : Question 1		
Suggested Solutions	**Marks**	**Marker's Comments**
let $u = \tan x$ $\quad \frac{du}{dx} = \sec^2 x$		students made this more difficult by using integration by parts.
$x = \frac{\pi}{4}$ $\quad u = 1$		
$x = 0$ $\quad u = 0$		
$\int_0^1 (1+u^2)\,du$	1	
$= \left[u + \frac{u^3}{3}\right]_0^1$		
$= \frac{4}{3}$	1	

MATHEMATICS Ext 2: Question ~~8~~ 11

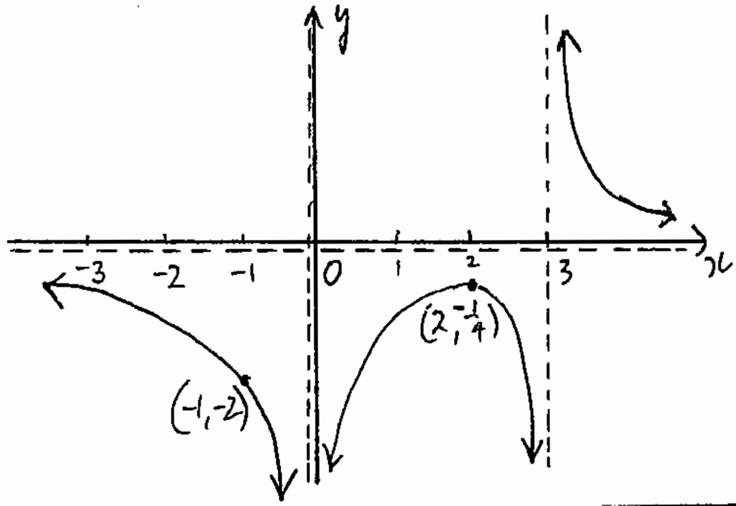
Suggested Solutions	Marks	Marker's Comments
(c) (i) $\frac{d}{dx}\left(x^2\tan^{-1}x\right) = 2x\tan^{-1}x + x^2 \times \frac{1}{1+x^2}$ (product rule)		
(ii) $\int_0^{\sqrt{3}} x\tan^{-1}x = \frac{1}{2}\left[x^2\tan^{-1}x\right]_0^{\sqrt{3}} - \frac{1}{2}\int_0^{\sqrt{3}} 1\,dx + \frac{1}{2}\int_0^{\sqrt{3}} \frac{dx}{1+x^2}$	1	
$= \frac{1}{2}\left[x^2\tan^{-1}x - x + \tan^{-1}x\right]_0^{\sqrt{3}}$		
$= \frac{1}{2}\left[3 \times \frac{\pi}{3} - \sqrt{3} + \frac{\pi}{3} - 0\right]$	1	
$= \frac{2\pi}{3} - \frac{\sqrt{3}}{2}$		
(d) $z_1 = 2\sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$	1	Students who wrote cis lost a mark. A number of students found point in wrong quadrant
$z_2 = 2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$	1	
$\frac{z_1}{z_2} = \sqrt{2}\,\text{cis}\left(\frac{3\pi}{4} - \frac{2\pi}{3}\right)$ (de Moivre's)		
$= \sqrt{2}\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}$	1	

MATHEMATICS Extension 2 : Question5		
Suggested Solutions	**Marks**	**Marker's Comments**
(ii) $\left(\frac{z_1}{z_2}\right)^n = (\sqrt{2})^n\left(\cos\frac{n\pi}{12} + i\sin\frac{n\pi}{12}\right)$ (De Moivres)		
Purely imaginary when $\frac{n\pi}{12} = \frac{\pi}{2}$ ie $n=6$ ~~need~~	1	needed to show working
$\therefore \left(\frac{z_1}{z_2}\right)^6 = (\sqrt{2})^6 i\sin\frac{\pi}{2}$ $= 8i$ $b=8$.	1	

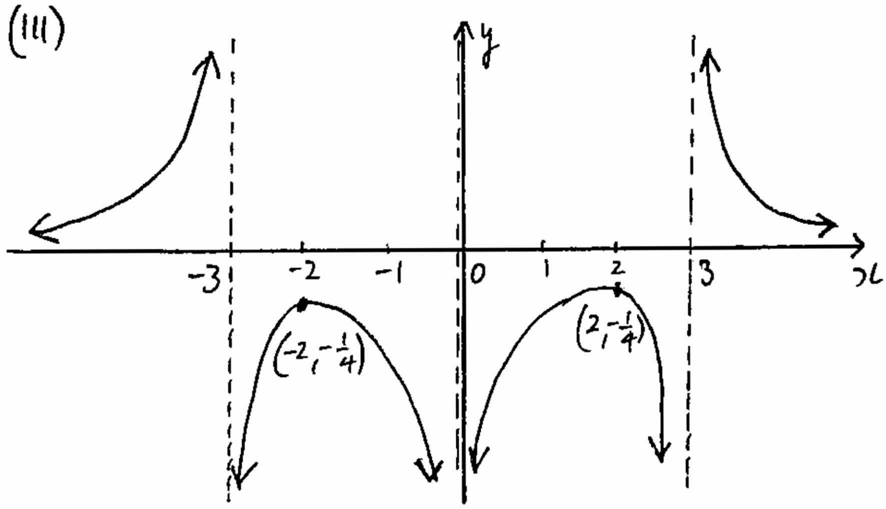
JRAHS : TRIALS 2018 **MATHEMATICS Extension 2: Question** 12

<table>
<tr><th>Suggested Solutions</th><th>Marks</th><th>Marker's Comments</th></tr>
<tr><td>a) (i)

</td><td>1

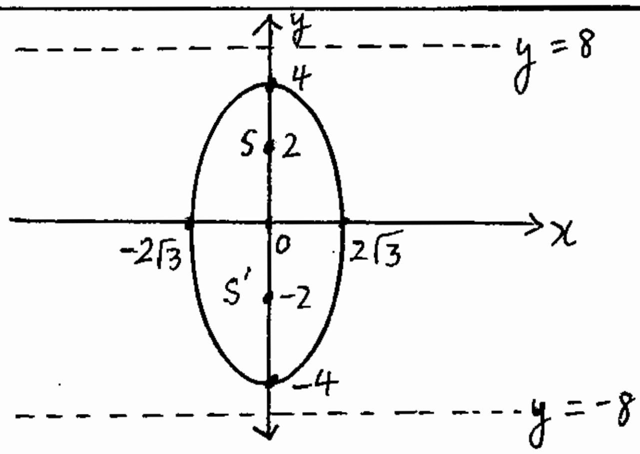
1</td><td>Correct domain and range

For vertical tangent at $x = 3$
(Note $\frac{dy}{dx} = \frac{3x^2 - 6x}{2y}$
- hence at $y = 0$ undefined)</td></tr>
<tr><td>(ii)

</td><td>1

1</td><td>For asymptotes and shape

Coordinates of turning point i.e. $(2, -\frac{1}{4})$</td></tr>
<tr><td>(iii)

</td><td>1</td><td>For $y = f(|x|)$
$= \frac{1}{x^2(|x| - 3)}$</td></tr>
</table>

JRAHS : TRIALS 2018 **MATHEMATICS Extension 2: Question** 12

Suggested Solutions	Marks	Marker's Comments
b) (i) $4x^2+3y^2=48$ $\Rightarrow \dfrac{x^2}{12}+\dfrac{y^2}{16}=1$ which is of the form $\dfrac{x^2}{b^2}+\dfrac{y^2}{a^2}=1$ with $a>b$ [Vertical ellipse] Hence $b^2=a^2(1-e^2)$ $12=16(1-e^2)$ $\therefore e=\frac{1}{2}$, $0<e<1$	1	<u>Note</u> for transverse <u>ellipse</u>: $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$ $a>b$ and $b^2=a^2(1-e^2)$ For correct $e>0$
(ii) Foci at $(0,\pm ae)$ $(0,\pm 4\times\frac{1}{2})$ $(0,\pm 2)$	1	<u>Note</u> for horizontal <u>ellipse</u>, Foci at $(\pm ae, 0)$ For Foci
directrix at $y=\pm\dfrac{a}{e}$ $=\pm\dfrac{4}{\frac{1}{2}}$ $=\pm 8$	1	<u>Note</u> for horizontal <u>ellipse</u>, directrix at $x=\pm a/e$ For directrix
(iii) 	1	For Foci, directrix and intercepts

JRAHS : TRIALS 2018 MATHEMATICS Extension 2: Question 12		
Suggested Solutions	**Marks**	**Marker's Comments**
c) (i) For $a, b \in \mathbb{R}$, $(a-b)^2 \geq 0$ $\Rightarrow a^2 - 2ab + b^2 \geq 0$ $\therefore a^2 + b^2 \geq 2ab$ Method II: Consider a^2+b^2-2ab Method III: Assume $a^2+b^2 \leq 2ab$ and arrive at a contradiction	1	For correct proof
(ii) Let $a = \sqrt{x/y}$ and $b = \sqrt{y/x}$ where $x, y > 0$ Since $a^2+b^2 \geq 2ab$, from (i) then $\left(\sqrt{x/y}\right)^2 + \left(\sqrt{y/x}\right)^2 \geq 2\sqrt{\frac{x}{y}} \cdot \sqrt{\frac{y}{x}}$ $\Rightarrow \frac{x}{y} + \frac{y}{x} \geq 2$ Method II: From (i) $x^2+y^2 \geq 2xy$ $\Rightarrow \frac{x^2+y^2}{xy} \geq 2$ if $x, y > 0$ and $xy > 0$ $\Rightarrow \frac{x^2}{xy} + \frac{y^2}{xy} \geq 2$ i.e. $\frac{x}{y} + \frac{y}{x} \geq 2$ Method III: Consider $\frac{x}{y} + \frac{y}{x} - 2$ and proceed as usual	1	For correct proof

JRAHS : TRIALS : 2018 **MATHEMATICS Extension 2: Question** 12

Suggested Solutions	Marks	Marker's Comments
(iii) To prove $(x_1+x_2+\cdots+x_n)\left(\frac{1}{x_1}+\frac{1}{x_2}+\cdots+\frac{1}{x_n}\right) \geq n^2$		for $n \geq$ [?] $x_i \in \mathbb{R}^+$
	1	For Base case
	1	For Assumption
	1	For using $A.B \geq k^2$
	1	For use of part (ii)

Proof: Base case: Prove true for $n=1$

$LHS = x_1 \times \frac{1}{x_1} = 1$

$RHS = 1^2 = 1$

$\therefore LHS \geq RHS$

$\therefore$ true for $n=1$

Assume true for $n = k \in \mathbb{Z}^+$

i.e. $(x_1+x_2+\cdots+x_k)\left(\frac{1}{x_1}+\frac{1}{x_2}+\cdots+\frac{1}{x_k}\right) \geq k^2$ ---> $A.B \geq k^2$

Prove statement true for $n = k+1$

i.e. to show/prove

$$\left[(x_1+\cdots+x_k)+x_{k+1}\right]\left[\left(\frac{1}{x_1}+\cdots+\frac{1}{x_k}\right)+\frac{1}{x_{k+1}}\right] \geq (k+1)^2$$

Now

$LHS = \left[A + x_{k+1}\right]\left[B + \frac{1}{x_{k+1}}\right]$ where $A = (x_1+x_2+\cdots+x_k)$, $B = \left(\frac{1}{x_1}+\frac{1}{x_2}+\cdots+\frac{1}{x_k}\right)$

$= AB + A\cdot\frac{1}{x_{k+1}} + B.x_{k+1} + \frac{x_{k+1}}{x_{k+1}}$

By assumption

$\geq k^2 + \left[\left(\frac{x_1}{x_{k+1}}+\frac{x_{k+1}}{x_1}\right)+\left(\frac{x_2}{x_{k+1}}+\frac{x_{k+1}}{x_2}\right)+\cdots+\left(\frac{x_k}{x_{k+1}}+\frac{x_{k+1}}{x_k}\right)\right]+1$

$\geq k^2 + [2+2+\cdots+2]+1$

k-times ---- by part (ii)

$= k^2+2k+1$

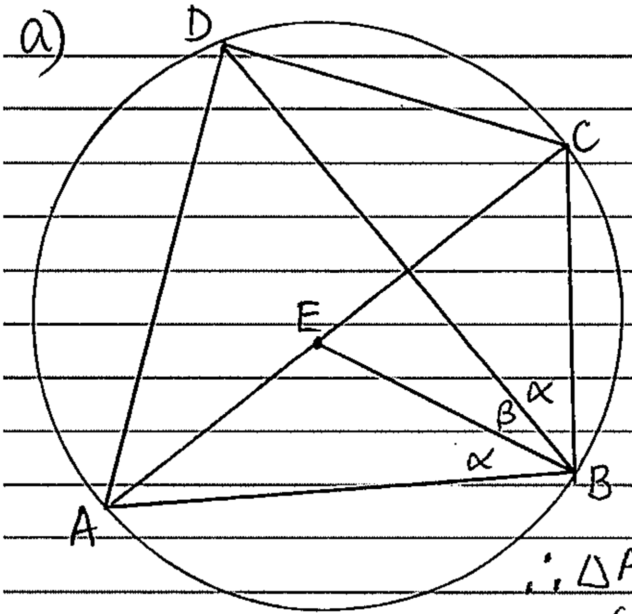
$= (k+1)^2 = RHS$

Hence by PMI, the statement is true for all $n \in \mathbb{Z}^+$

Note: • clear evidence for "appearance" of $+1$

• " " for use of $\frac{x}{y}+\frac{y}{x}$

MATHEMATICS Extension 2: Question 13

Suggested Solutions	Marks	Marker's Comments
a) 		
i) In $\triangle ABE$ & $\triangle DBC$ $\angle ABE = \angle CBD$ (given) $\angle BAE = \angle BDC$ (angles at the circumference standing on arc BC are equal)	1	2 pairs of equal angles Need "at the circumference" Please check your wording.
$\therefore \triangle ABE \parallel\!\mid \triangle DBC$ (equiangular)	1	
ii) In $\triangle ABD$ & $\triangle EBC$ $\angle ABD = \angle EBC$ (Both are sum of given equal angles and $\angle EBD$) $\angle ADB = \angle ECB$ (angles at the circumference standing on the same arc AB are equal)	1	2 pairs of equal angles
$\therefore \triangle ABD \parallel\!\mid \triangle EBC$ (equiangular)	1	
iii) $\frac{AB}{DB} = \frac{AE}{DC}$ (corresponding sides of similar triangles ABE and DBC are in the same ratio)	1	Check wording of reason
Similarly in $\triangle ABD$ and $\triangle EBC$ $\frac{BC}{BD} = \frac{EC}{AD}$	1	
$\therefore AB \times CD = AE \times BD$ ① and $AD \times BC = EC \times BD$ ② ① + ② $AB \times CD + AD \times BC = BD(AE + EC)$ $= BD \times AC$ as AEC is a straight line	1	combining

MATHEMATICS Extension 2: Question 13

Suggested Solutions	Marks	Marker's Comments

b) $y = \dfrac{(x-2)(x+1)}{5-x} = \dfrac{x^2-x-2}{5-x}$

$$\begin{array}{r} -x-4 \\ x-5\,\overline{)\,-x^2+x+2} \\ \underline{-x^2+5x} \\ x+2 \\ \underline{-4x+20} \\ -18 \end{array}$$

$\therefore y = -x-4 - \dfrac{18}{x-5}$

$\therefore$ Asymptotes are $y=-x-4$, $x=5$

x-intercepts are 2 and -1

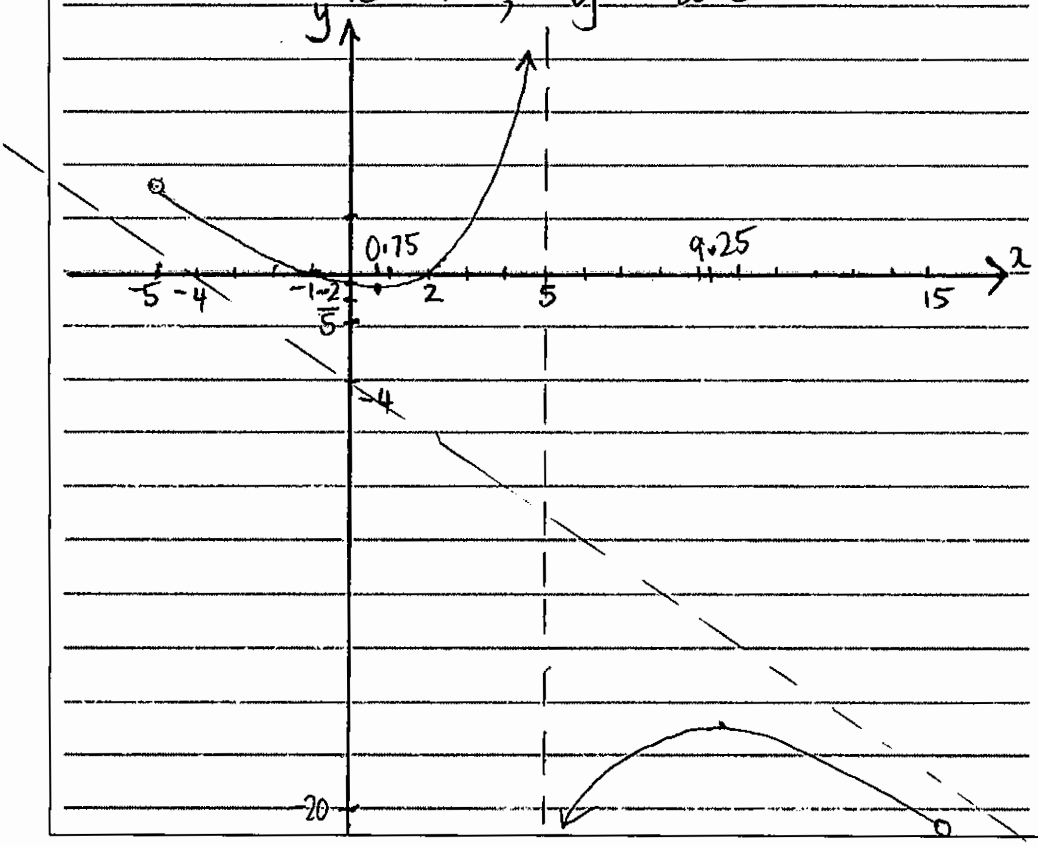
y-intercept is $-2/5$

When $x = 0.75$, $y \doteqdot -0.5$ } turning points

$x = 9.25$, $y \doteqdot -17.5$

endpoints: $x=-5$, $y = 2.8$

$x = 15$, $y = -20.8$

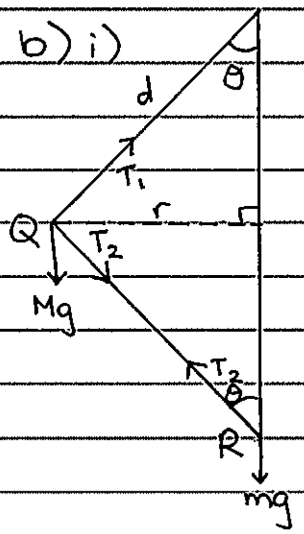


Marks	Marker's Comments
1	Asymptotes
1	x-intercepts y-intercept 1 extrema x-values of TPs must have 5 out of these 6.
1	shape

MATHEMATICS Extension 2: Question 13

Suggested Solutions	Marks	Marker's Comments
c) i) $\left(\frac{w}{w+r}\right)\left(\frac{w-1}{w+r-1}\right)\left(\frac{w-2}{w+r-2}\right)$ OR, $\frac{{}^{w}C_3}{{}^{w+r}C_3}$	1	
ii) $\left(\frac{w+1}{w+r+1}\right)\left(\frac{w}{w+r}\right)\left(\frac{w-1}{w+r-1}\right) = 2\left(\frac{w}{w+r}\right)\left(\frac{w-1}{w+r-1}\right)\left(\frac{w-2}{w+r-2}\right)$	1	correct equation from your (i)
$\therefore \frac{w+1}{w+r+1} = \frac{2(w-2)}{w+r-2}$		
$(w+1)(w+r-2) = 2(w-2)(w+r+1)$		
$w^2+rw-2w+w+r-2 = 2w^2+2wr+2w-4w-4r-4$		
$r(-w+5) = w^2-w-2$	1	simplification
$r = \frac{w^2-w-2}{-w+5}$		
$= \frac{(w-2)(w+1)}{5-w}$		
iii) From graph in b) $r, w > 0$ and integers	1	Statement including "integers".
$\therefore w = 3$ or 4 only		
When $w=3$, $r=2$ $w=4$, $r=10$	1	final answers
		Only one mark given if only one pair found
Note: $r \neq 0$ as this would give $w=2$, but 3 jellybeans are taken $\therefore w>2$.		

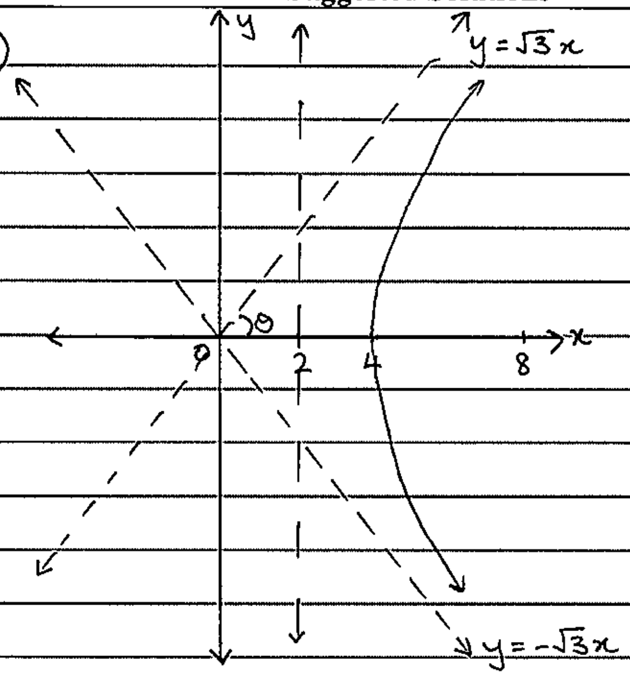
MATHEMATICS Extension 2: Question..14..

Suggested Solutions	Marks	Marker's Comments
a) i) $p(x) = ax^4 + bx^3 + cx^2 + dx + f$		
$p(\alpha) = 0$		
$\therefore\ a\alpha^4 + b\alpha^3 + c\alpha^2 + d\alpha + f = 0$		
$a\alpha^4 + b\alpha^3 + c\alpha^2 + d\alpha = -f$		must mention integers.
$\alpha(a\alpha^3 + b\alpha^2 + \alpha + d) = -f$		
since a, b, c, d, f and α are integers, α divides f.	1	
ii) $q(x) = 4x^4 - x^3 + 3x^2 + 2x - 3$		
Suppose there is an integer zero, α. Then α divides -3, ie. $\alpha = \pm 1$ or ± 3	1	listing all possible α
$q(1) = 4(1)^4 - (1)^3 + 3(1)^2 + 2(1) - 3 = 5$		
$q(-1) = 4(-1)^4 - (-1)^3 + 3(-1)^2 + 2(-1) - 3 = 3$		
$q(3) = 4(3)^4 - (3)^3 + 3(3)^2 + 2(3) - 3 = 327$		
$q(-3) = 4(-3)^4 - (-3)^3 + 3(-3)^2 + 2(-3) - 3 = 369$		
None are 0.	1	showing none are zero.
$\therefore q(x)$ does not have an integer zero.		
b) i)  At R: $T_2\cos\theta = mg$ ①	1	
At Q: vertically: $T_1\cos\theta - T_2\cos\theta = Mg$ ②	1	
horizontally: $T_1\sin\theta + T_2\sin\theta = M\omega^2 r$		
since $\sin\theta = \frac{r}{d}$, $r = d\sin\theta$		
$\therefore T_1\sin\theta + T_2\sin\theta = M\omega^2 d\sin\theta$		
$\therefore T_1 + T_2 = M\omega^2 d$ ③	1	
sub. ① into ②		
$T_1\cos\theta - mg = Mg$		

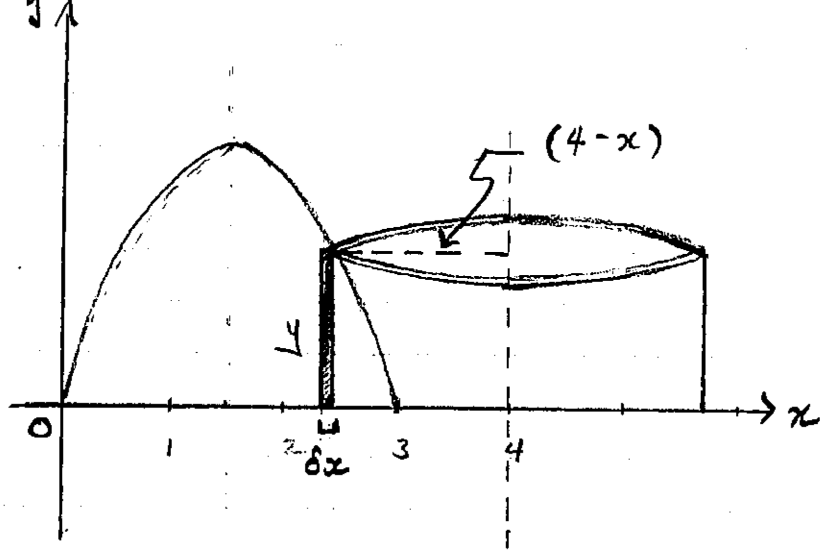
MATHEMATICS Extension 2: Question 14

Suggested Solutions	Marks	Marker's Comments
$\therefore T_1\cos\theta = Mg + mg$		
$T_1 = (Mg+mg)\sec\theta$ ④		
from ①: $T_2 = mg\sec\theta$ ⑤		
Sub ④ and ⑤ into ③		
$\therefore (Mg+mg)\sec\theta + mg\sec\theta = M\omega^2 d$	1	for working out.
$\therefore (Mg+mg+mg)\sec\theta = M\omega^2 d$		
$\therefore M\omega^2 d = (2mg+Mg)\sec\theta$		
ii) $M = 4m \quad \theta = \frac{\pi}{3}$		
$4m\omega^2 d = (2mg+4mg)\sec\frac{\pi}{3}$		
$4m\omega^2 d = (6mg)(2)$	1	must show substitution
$4m\omega^2 d = 12mg$		
$\omega^2 = \dfrac{12mg}{4md}$		
$= \dfrac{3g}{d}$		
$\therefore \omega = \sqrt{\dfrac{3g}{d}}, \ \omega > 0$		
c) i) $\|z-8\| = 2\mathrm{Re}(z-2)$		
let $z = x+iy$		
$\|x+iy-8\| = 2\mathrm{Re}(x+iy-2)$		
$\sqrt{(x-8)^2+y^2} = 2(x-2)$	1	
$(x-8)^2+y^2 = 4(x-2)^2$		
$x^2-16x+64+y^2 = 4(x^2-4x+4)$		
$x^2-16x+64+y^2 = 4x^2-16x+16$		
$48 = 3x^2-y^2$		
$\therefore \dfrac{x^2}{16} - \dfrac{y^2}{48} = 1$	1	
$\|z-8\| \geq 0 \quad \therefore 2\mathrm{Re}(z-2) \geq 0$		
$2(x-2) \geq 0$		
$x-2 \geq 0$		
$\therefore x \geq 2$	1	

MATHEMATICS Extension 2: Question 14

Suggested Solutions	Marks	Marker's Comments
ii) 	1 1	shape asymptotes
$a=4$, $b=4\sqrt{3}$		
asymptotes: $y=\pm\frac{b}{a}x$		
$=\pm\sqrt{3}x$		
$y=\sqrt{3}x$		
$m=\sqrt{3}$		if no restriction in part (i)
$\therefore \tan\theta=\sqrt{3}$		then $-\frac{\pi}{3}<\text{Arg} z<\frac{\pi}{3}$
$\theta=\frac{\pi}{3}$		and Arg z for
$\therefore -\frac{\pi}{3}<\text{Arg } z<\frac{\pi}{3}$	1	the other branch.
iii) $\|z+8\|-\|z-8\|=2a$		needed for 1 mark.
$=8$	1	

MATHEMATICS: Question 15

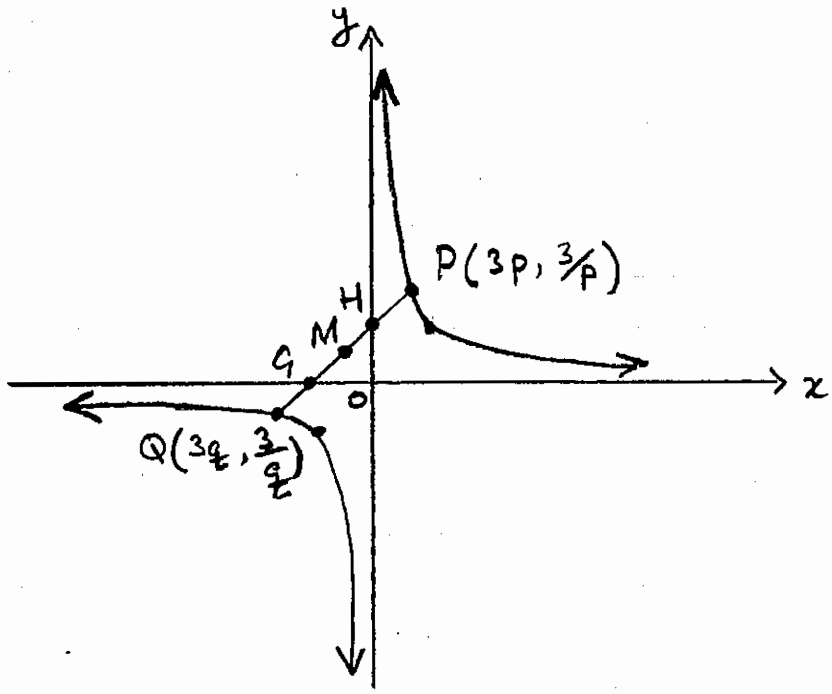
Suggested Solutions EXT 2	Marks	Marker's Comments
(a) 		Handled well.
Element of volume: $\delta V \simeq 2\pi(4-x)\,y\,\delta x$	1	
So $V = \lim\limits_{\delta x \to 0} \sum\limits_{x=0}^{3} 2\pi(4-x)\,y\,\delta x$		
$= \int_0^3 2\pi\,(4-x)\,y\,dx$		
$= 2\pi \int_0^3 (4-x)\cdot x\,(3-x)\,dx$		
$= 2\pi \int_0^3 x^3 - 7x^2 + 12x\,dx$		
$= 2\pi \left[\frac{x^4}{4} - \frac{7x^3}{3} + 6x^2 \right]_0^3$	1	
$= 2\pi \left[\frac{81}{4} - 63 + 54 \right]$		
$= \frac{45\pi}{2}$ units^3.	1	

MATHEMATICS: Question 15		
Suggested Solutions EXT 2	Marks	Marker's Comments
(b)(i) Let $\alpha^2 = x$ be solution to cubic polynomial (w/ integer coefficients). Then $\alpha = \pm\sqrt{x}$.	1	Reasoned well.
We have, by construction, $\alpha^3 - 3\alpha^2 + 5 = 0$		
so, $(\pm\sqrt{x})^3 - 3(\pm\sqrt{x})^2 + 5 = 0$		
$\pm x\sqrt{x} - 3x + 5 = 0$		
$\Rightarrow (\pm x\sqrt{x})^2 = (3x-5)^2$		
i.e. $x^3 = 9x^2 - 30x + 25$		
$\therefore x^3 - 9x^2 + 30x - 25 = 0$	1	
has solutions $\alpha^2, \beta^2, \gamma^2$ if α, β, γ solutions to $x^3 - 3x^2 + 5 = 0$ //		

MATHEMATICS: Question 15		
Suggested Solutions EXT 2	Marks	Marker's Comments
(ii) We have that $(\alpha^2+\beta^2+\gamma^2)^2 = \alpha^4+\beta^4+\gamma^4$ $+2(\alpha^2\beta^2+\beta^2\gamma^2+\gamma^2\alpha^2)$	1	Handled well.
From cubic found in part (i), $\sum_1 \alpha^2 = 9 \qquad \sum_1 \alpha^2\beta^2 = 30$, so $\quad 9^2 = \sum_1 \alpha^4 + 2(30)$ i.e. $\alpha^4+\beta^4+\gamma^4 = 81-60$ $= 21$.	1	

MATHEMATICS: Question 15		
Suggested Solutions EXT 2	Marks	Marker's Comments

(c)



(i) $M = \left(\dfrac{3(p+q)}{2}, \dfrac{3(p+q)}{2pq} \right)$ — **1**

(ii) Given $H = \left(0, \dfrac{3(p+q)}{pq}\right)$

$G = \left(3(p+q), 0\right)$

substituting $x=0$, $y=0$ in eqn of chord respectively, we have — **1**

One mark for some appropriate reasoning.

$$PH^2 - QG^2 = \left[(3p)^2 + \left(\frac{3}{p} - \frac{3(p+q)}{pq}\right)^2\right]$$

$$-\left[\left(3(p+q) - 3q\right)^2 + \left(3/q\right)^2\right]$$

MATHEMATICS: Question 15		
Suggested Solutions EXT2	Marks	Marker's Comments
$= \left[(3p)^2 + \left(-\frac{3}{q}\right)^2\right]$ $- \left[(3p)^2 + \left(\frac{3}{q}\right)^2\right]$		One mark for correct conclusion.
$= 0$		
so $PH^2 = QG^2 \rightarrow PH = QG$ $(PH, QG > 0)$ //	1	
		Could also have argued this geometrically, but so/n presented here is that which majority of candidature presented.

MATHEMATICS: Question 15

Suggested Solutions EXT 2	**Marks**	**Marker's Comments**
(iii) Locus:		Deriving locus, handled well.
chord passes through $(0,2)$ always, so, from eq^n of chord:		
$pq(2)+0 = 3(p+q)$		
i.e. $2pq = 3(p+q)$ —— ①	1	First mark for constraint.
Let $x = \frac{3}{2}(p+q)$, $y = \frac{3(p+q)}{2pq}$		
By ①, $\frac{3(p+q)}{2pq} = 1$, so		
$y = 1$	1	Next mark for correct locus.
for all allowable x.		
Restrictions:		Determining restrictions, not handled that well.
By ①, $q = \frac{3p}{2p-3}$, so		
$x = \frac{3}{2}\left(p + \frac{3p}{2p-3}\right)$		
$\Rightarrow 3p^2 - 2px + 3x = 0$	1	One mark awarded for making a reasonable start, obtaining one/part of restriction.
Real values of p exist here only for $\Delta \geq 0$.		

MATHEMATICS: Question 15		
Suggested Solutions EXT2	**Marks**	**Marker's Comments**

Now,
$$\Delta = (-2x)^2 - 4(3)(3x)$$
$$= 4x^2 - 36x$$

Require $4x^2 - 36x \geq 0$

$\rightarrow \quad x(x-9) \geq 0$

$\rightarrow \quad x \leq 0 \quad$ or $\quad x \geq 9$

<u>But</u>, checking endpoints:

① if $x = 0$, then $\frac{3}{2}(p+q) = 0$

so $p = -q$.

Also require constraint

$$2pq = 3(p+q)$$

be satisfied; then

$$2p(-p) = 0 \Rightarrow p = 0$$

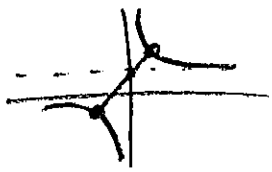
so $q = 0$ also.

This can't happen since then P, Q would be undefined (e.g. "$(0, \infty)$")

MATHEMATICS: Question 15		
Suggested Solutions EXT 2	Marks	Marker's Comments
(ii) if $x = 9$, then $$9 = \frac{3}{2}(p+q)$$ so $q = 6 - p$ Checking constraint: $$2p(6-p) = 3(p+(6-p))$$ $\rightarrow$ $p^2 - 6p + 9 = 0$ $\rightarrow$ $(p-3)^2 = 0$ so $p = 3$, implying $q = 3$, too. But then PQ does not form a line segment ... and so no midpoint.		

MATHEMATICS: Question 15 EXT2		
Suggested Solutions	Marks	Marker's Comments
Now, if $x > 9$, then $\frac{3p^2}{2p-3} > 9$ so $3p^2(2p-3) > 9(2p-3)$ so $(2p-3)(p-3)^2 > 9$ $\rightarrow \quad p > 3/2$ But $q = \frac{3p}{2p-3}$ so $q > 0$ if $p > 3/2$ ... but the P, Q are on same branch & $\therefore$ don't generate a chord passing through $(0,2)$. So $\boxed{x > 9 \text{ not allowed}}$ On the other hand, if $x < 0$, then $(2p-3)(p-3)^2 < 0$ $\rightarrow \quad p < 3/2$		

MATHEMATICS: Question 15

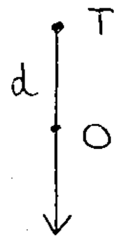
Suggested Solutions EXT2	**Marks**	**Marker's Comments**
If $0 < p < 3/2$, then $q = \dfrac{3p}{2p-3} < 0$, so P, Q on separate branches – OK. If $p < 0$, then $q = \dfrac{3p}{2p-3} > 0$ and, so P, Q again on separate branches.  No chord through $(0,2)$ if $x > 0$  Chord through $(0,2)$ if $x < 0$. Hence, the locus is $y = 1$ for all real $x < 0$.	1	Final mark for correct restriction.

C:\Users\sguest12\Desktop\Suggested Mk solns template.doc

Ext 2 MATHEMATICS: Question 16

Suggested Solutions	Marks	Marker's Comments

a) i



$\downarrow$ positive

$$\ddot{x} = g - kv \quad \text{(given)}$$

$$\frac{dv}{dt} = g - kv$$

$$\int_U^v \frac{1}{g-kv}\,dv = \int_0^t dt$$

$$\frac{-1}{k}\Big[\ln(g-kv)\Big]_U^v = \Big[t\Big]_0^t$$ — (1) After Integrating

$$\frac{-1}{k}\Big[\ln|g-kv| - \ln|g-kU|\Big] = t$$

$$\frac{-1}{k}\Big[\ln(g-kv) - \ln(g-kU)\Big] = t \quad (g > kv)$$

$$\frac{-1}{k}\ln\left(\frac{g-kv}{g-kU}\right) = t$$

$$\ln\left(\frac{g-kv}{g-kU}\right) = -kt$$

$$\frac{g-kv}{g-kU} = e^{-kt}$$

$$g - kv = (g-kU)e^{-kt}$$

$$kv = g - (g-kU)e^{-kt}$$

$$v = \frac{g}{k} - \frac{(g-kU)}{k}e^{-kt}$$ — (1)

MATHEMATICS Extension 1 : Question……..

Suggested Solutions	Marks	Marker's Comments
ii $\dfrac{dx}{dt} = \dfrac{g}{k} - \left(\dfrac{g-kV}{k}\right)e^{-kt}$		
$\displaystyle\int_{-d}^{x} dx = \int_0^t \frac{g}{k} - \left(\frac{g-kV}{k}\right)e^{-kt}\,dt$		
$\Big[x\Big]_{-d}^{x} = \left[\dfrac{gt}{k} + \left(\dfrac{g-kV}{k^2}\right)e^{-kt}\right]_0^t$	1	For Integrating
$x+d = \dfrac{gt}{k} + \left(\dfrac{g-kV}{k^2}\right)e^{-kt} - 0 - \left(\dfrac{g-kV}{k^2}\right)$		
$x+d = \dfrac{gt}{k} + \left(\dfrac{g-kV}{k^2}\right)e^{-kt} - \left(\dfrac{g-kV}{k^2}\right)$		
$x = \dfrac{gt}{k} - d + \left(\dfrac{g-kV}{k^2}\right)\left(e^{-kt} - 1\right)$		
$x = \dfrac{gt-kd}{k} + \left(\dfrac{g-kV}{k^2}\right)\left(e^{-kt} - 1\right)$	1	
iii Substitute $V=0$, $d=0$		
$\therefore V_Q = \dfrac{g}{k}\left(1-e^{-kt}\right)$	1	
$x_Q = \dfrac{gt}{k} + \dfrac{g}{k^2}\left(e^{-kt} - 1\right)$	1	

MATHEMATICS Extension 2: Question……..

Suggested Solutions	Marks	Marker's Comments
iv) Collision $\Longrightarrow x_P = x_Q$		
$\therefore \dfrac{gt-kd}{k} + \left(\dfrac{g-kU}{k^2}\right)\left(e^{-kt}-1\right) = \dfrac{gt}{k} + \dfrac{g}{k^2}\left(e^{-kt}-1\right)$		(1) Equating x_P and x_Q
$-d + \dfrac{ge^{-kt}}{k^2} - \dfrac{g}{k^2} - \dfrac{Ue^{-kt}}{k} + \dfrac{U}{k} = \dfrac{ge^{-kt}}{k^2} - \dfrac{g}{k^2}$		
$-d - \dfrac{Ue^{-kt}}{k} + \dfrac{U}{k} = 0$		
$\dfrac{Ue^{-kt}}{k} = \dfrac{U}{k} - d$		
$Ue^{-kt} = U - kd$		
$e^{-kt} = 1 - \dfrac{kd}{U}$		
$\therefore -kt = \ln\left(1 - \dfrac{kd}{U}\right)$		
$t = \dfrac{-1}{k}\ln\left(\dfrac{U-kd}{U}\right)$		(1)
$\therefore V_P = \dfrac{g}{k} - \left(\dfrac{g-kU}{k}\right)e^{\ln\left(\frac{U-kd}{U}\right)}$		
$= \dfrac{g}{k} - \left(\dfrac{g-kU}{k}\right)\left(\dfrac{U-kd}{U}\right)$		
$V_Q = \dfrac{g}{k}\left(1 - e^{\ln\left(\frac{U-kd}{U}\right)}\right)$		
$= \dfrac{g}{k}\left(\dfrac{kd}{U}\right)$		
$= \dfrac{gd}{U}$		

MATHEMATICS: Question		
Suggested Solutions	**Marks**	**Marker's Comments**
$\therefore V_P - V_Q = \frac{g}{k} - \left(\frac{g-kU}{k}\right)\left(\frac{U-kd}{U}\right) - \frac{gd}{U}$		1
$= \left(\frac{g}{k} - \frac{g}{k} + U + \frac{gd}{U} - kd\right) - \frac{gd}{U}$		* Substitution of t correctly.
$= U - kd$		* Showing the subtraction $V_P - V_Q$
b) $\because$ SP is linear in h.		* Express independent of Exp & Log
Let $SP = mh + b$, when $h=0$, $SP=0$, $\therefore b=0$; $h=H$, $SP=2r$		
$\therefore 2r = mH$ $\therefore m = \frac{2r}{H}$		
$\therefore SP = \frac{2rh}{H}$		1 height of rectangle
PQ is also linear in h.		
Let $PQ = nh + p$		
when $h=0$, $PQ = 4r$ $\therefore p = 4r$		
when $h=H$, $PQ = 2r$		
$\therefore 2r = nH + 4r$; $n = \frac{-2r}{H}$		
$\therefore PQ = 4r - \frac{2rh}{H}$		1 base of rectangle

MATHEMATICS Extension 1 : Question……..		
Suggested Solutions	**Marks**	**Marker's Comments**
$\therefore A_{\text{rectangle}} = \frac{2rh}{H}\left(4r - \frac{2rh}{H}\right)$		
$A_{\text{semi-ellipse}} = \frac{\pi}{2} \times \left(3r - \frac{2rh}{H}\right)\left(2r - \frac{rh}{H}\right)$	1	correctly substituting into semi-ellipse.
$\therefore A_{\text{cross-section}} = \frac{2rh}{H}\left(4r - \frac{2rh}{H}\right) + \frac{\pi}{2}\left(3r - \frac{2rh}{H}\right)\left(2r - \frac{rh}{H}\right)$		
$= \frac{2r^2h}{H^2}(4H - 2h) + \frac{\pi r^2}{2}\left(3 - \frac{2h}{H}\right)\left(2 - \frac{h}{H}\right)$		
$= \frac{4r^2h}{H^2}(2H - h) + \frac{\pi r^2}{2H^2}(3H - 2h)(2H - h)$		
$= \frac{r^2}{2H^2}(8h)(2H-h) + \frac{r^2}{2H^2}(3\pi H - 2h\pi)(2H-h)$		
$= \frac{r^2}{2H^2}(2H-h)(8h + 3\pi H - 2h\pi)$		
$= \frac{r^2}{2H^2}(2H-h)(3\pi H + (8-2\pi)h)$	1	
iii) $V_{\text{slice}} = \frac{r^2}{2H^2}(2H-h)(3\pi H + (8-2\pi)h)\,\delta h$		
$V_{\text{solid}} = \lim_{\delta h \to 0} \sum_{0}^{H} \frac{r^2}{2H^2}(2H-h)(3\pi H + (8-2\pi)h)\,\delta h$		
$= \frac{r^2}{2H^2}\int_0^H (2H-h)(3\pi H + (8-2\pi)h)\,dh$		
$= \frac{r^2}{2H^2}\int_0^H \left(6\pi H^2 + 2H(8-2\pi)h - 3\pi hH - (8-2\pi)h^2\right) dh$	1	Expand into an expression that can be integrated

MATHEMATICS Extension 2: Question……..		
Suggested Solutions	Marks	Marker's Comments

$$= \frac{r^2}{2H^2}\left[6\pi H^2 h + \frac{H(8-2\pi)h^2}{2} - \frac{3}{2}\pi H h^2 - \frac{(8-2\pi)h^3}{3}\right]_0^H$$

$$= \frac{r^2}{2H^2}\left[6\pi H^3 + (8-2\pi)H^3 - \frac{3}{2}\pi H^3 - \frac{(8-2\pi)}{3}H^3\right]$$

$$= \frac{r^2 H}{2}\left[6\pi - 2\pi - \frac{3}{2}\pi + \frac{2\pi}{3} + 8 - \frac{8}{3}\right]$$

$$= \frac{r^2 H}{2}\left[\frac{19\pi}{6} + \frac{16}{3}\right]$$

$$= \frac{r^2 H}{12}(19\pi + 32)\ u^3$$ 1

Some students were awarded 1/2 if they got either the $\frac{19\pi}{12}$ or $\frac{32}{12}$ term in their final answer. (provided it wasn't acquired by accident).