KAMBALA

Student Number: ________________

Trial HSC - Task 3
August 2013

Mathematics Extension 2

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may be used
- Answer questions 1 – 10 on the multiple choice answer sheet provided.
- Answer questions 11 – 16 in the booklets provided.
  Start each question in a new booklet.
- A table of standard integrals is provided at the back of this paper
- In Questions 11 – 16, show relevant mathematical reasoning and/or calculations

Total marks – 100

Section I – Pages 3 – 6
10 marks

- Attempt Questions 1 - 10
- Allow about 15 minutes for this section

Section II – Pages 7 – 14
90 marks

- Attempt Questions 11 - 16
- Allow about 2 hours 45 minutes for this section

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Section I

10 Marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10.

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1 Let $z = 3 + 2i$ and $w = 2 - i$.
What is the value of $\overline{z} + 3w$?

A. $9 + i$ B. $9 - i$ C. $9 - 3i$ D. $9 - 5i$

2 The equation $x^3 + 2xy - y^3 - 2 = 0$ defines y implicitly as a function of x.
What is the value of $\frac{dy}{dx}$ at the point $(1, 1)$?

A. 5 B. 1 C. $\frac{1}{5}$ D. -1

3 The equation $3x^3 + 2x^2 - 4x + 1 = 0$ has roots α, β and γ.
What is the value of $\frac{1}{\alpha^3 \beta^3 \gamma^3}$?

A. 27 B. -27 C. $\frac{1}{27}$ D. $-\frac{1}{27}$

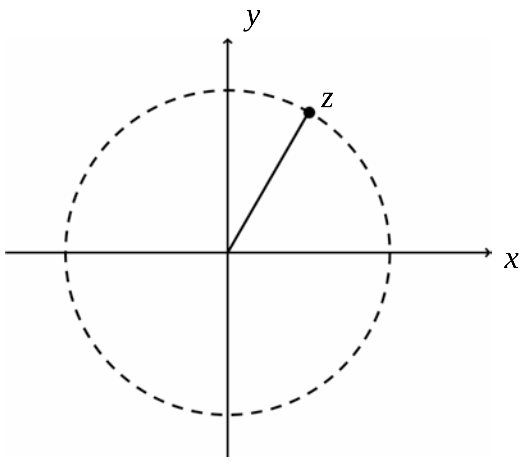
4 What is the eccentricity of the hyperbola $\frac{x^2}{6} - \frac{y^2}{10} = 1$?

A. $\frac{2\sqrt{10}}{5}$ B. $\frac{2\sqrt{6}}{3}$ C. $\frac{2\sqrt{3}}{3}$ D. $\frac{\sqrt{6}}{3}$

5 The polynomial $P(x) = 0$ has real coefficients. Three of the roots of $P(x)$ are $x = 4$, $x = 2 - i$ and $x = 1 + i$. What is the minimum degree of $P(x)$?

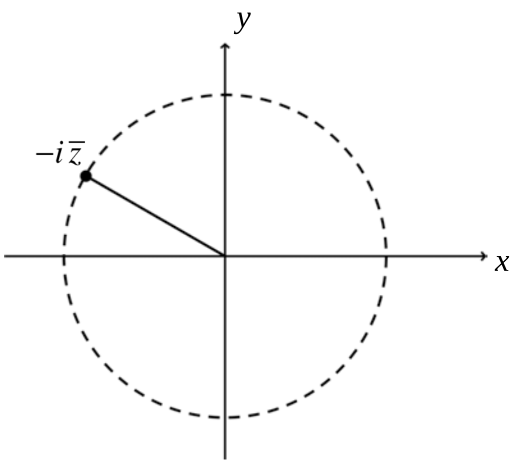
A. 3 B. 4 C. 5 D. 6

6 The complex number z is shown in the Argand diagram below.

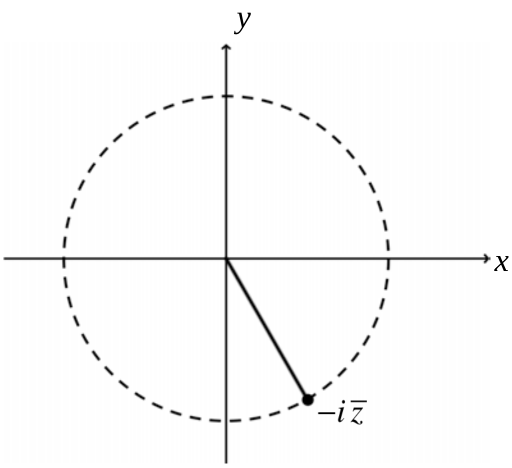


Which of the following best represents $-i\,\overline{z}$?

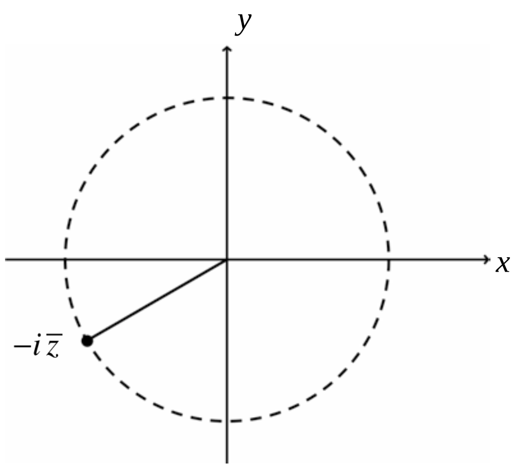
A.



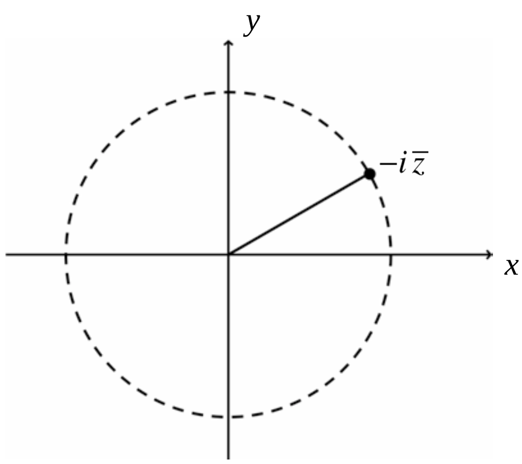
B.



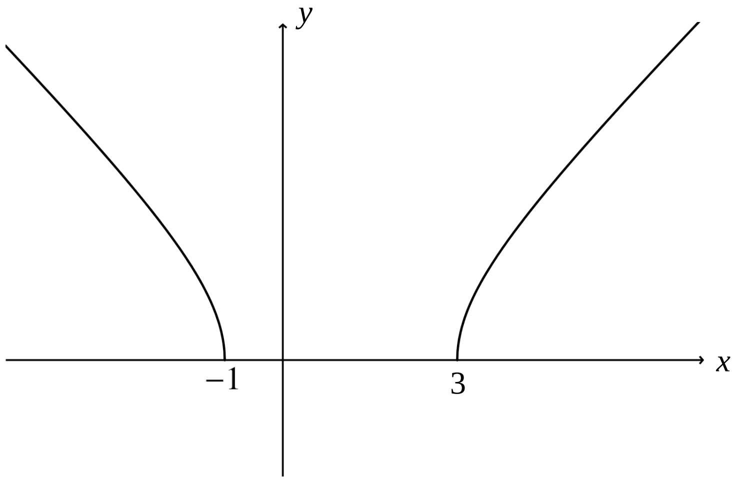
C.



D.

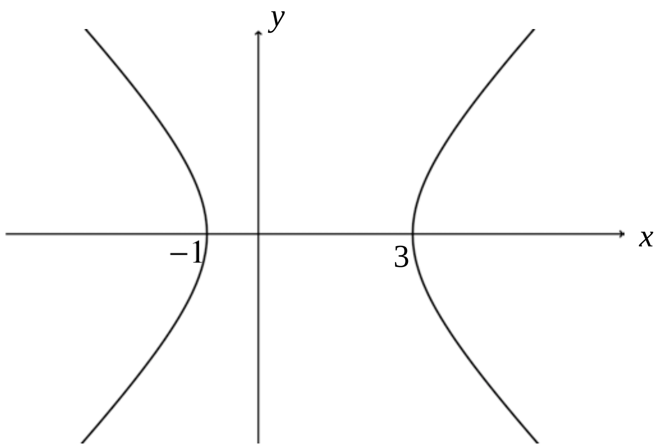


7 The graph of $y = \sqrt{f(x)}$ is shown below.

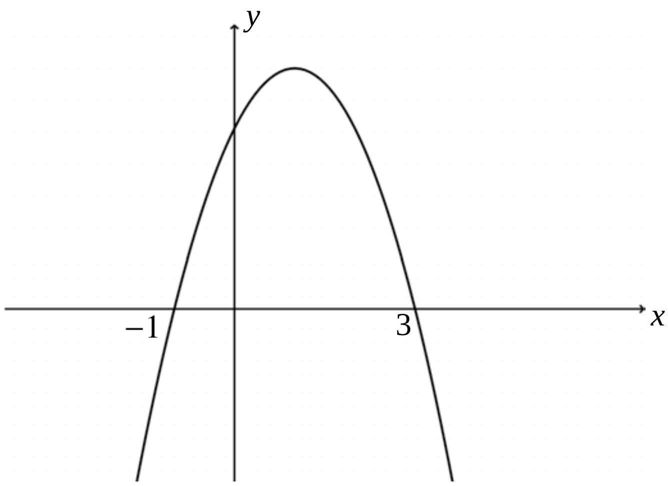


A possible graph of the function $y = f(x)$ is:

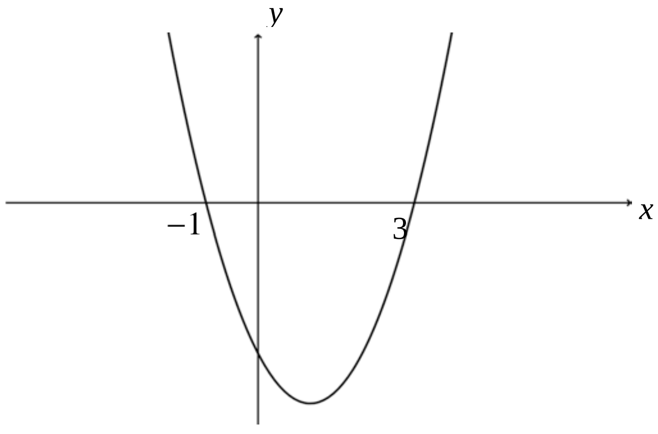
A.



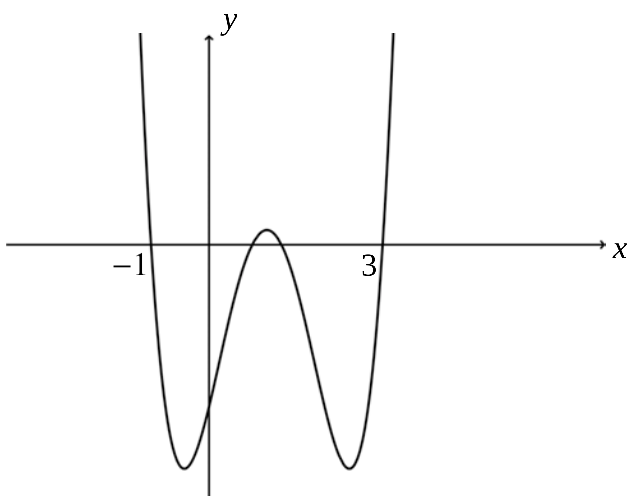
B.



C.



D.



8 If ω is an imaginary cube root of unity, then $\left(1+\omega-\omega^2\right)^7$ is equal to:

A. $-128\omega^2$ B. $128\omega^2$ C. -128ω D. 128ω

9 Let $y=\cos^{-1}e^x$. An expression for $\dfrac{dy}{dx}$ is given by:

A. $-\tan y$ B. $-\cot y$ C. $-\operatorname{cosec} y$ D. $-\sec y$

10 What are all the values of k for which the graph of $y=2x^3-6x^2+k$ will have three distinct x-intercepts?

A. $k > 0$ B. $k < 8$ C. $k = 0, 8$ D. $0 < k < 8$

End of Section I

Section II

60 Marks
Attempt Questions 11 – 16
Allow about 2 hours 45 minutes for this section

Answer each question in the booklets provided. Start each question in a new booklet.
In Questions 11 – 16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet.

(a) Express $\dfrac{2\sqrt{3}+i}{\sqrt{3}-i}$ in the form $x + iy$, where x and y are real. **2**

(b) Shade the region on the Argand diagram where the two inequalities **2**

$$|z+1| \le 1 \quad \text{and} \quad |z-2i| \ge 2$$

both hold.

(c) Given $z_1 = i\sqrt{2}$ and $z_2 = \dfrac{2}{1-i}$

(i) Express z_1 and z_2 in modulus-argument form. **2**

(ii) If $z_1 = wz_2$, find w in modulus-argument form. **1**

(iii) On an Argand diagram, plot the points *P*, *Q* and *R* , where *P* represents z_1, Q represents z_2 and *R* represents $(z_1 + z_2)$. **2**

(iv) Show that $\text{Arg}(z_1 + z_2) = \dfrac{3\pi}{8}$ and hence find the exact value of $\tan\dfrac{3\pi}{8}$. **2**

(d) Given that $2^{n+4} > (n+4)^2$ for all integral $n \ge 1$, show that $2^{3(a+2)} > 9(a+2)^2$. **2**

(e) Solve the equation $x^3 - 8x^2 - 5x + 84 = 0$, given that one of the roots is equal to the sum of the other two roots. **2**

Question 12 (15 marks) Use a SEPARATE writing booklet.

(a) Find $\int \frac{dx}{x^2 - 2x + 5}$. **2**

(b) Find $\int \ln x \, dx$. **2**

(c) Using the substitution $u = e^x$ and partial fractions, find $\int \frac{1}{e^x + 1} \, dx$. **3**

(d) Prove by the process of Mathematical Induction that $(1 + x)^n - nx - 1$ is divisible by x^2 for all integral $n \geq 2$. **3**

(e) Given that $f(x) = |x - 2| - 2$, sketch the graphs of the following showing the *x*- and *y*-intercepts. Use separate axes for each graph.

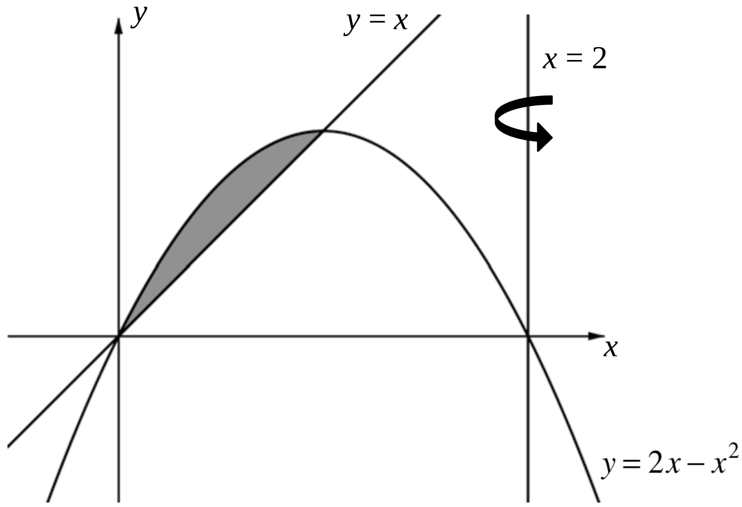
(i) $y = f(x)$ **1**

(ii) $y = [f(x)]^2$ **2**

(iii) $y = \ln[f(x)]$ **2**

Question 13 (15 marks) Use a SEPARATE writing booklet.

(a) The area between the two curves $y = 2x - x^2$ and $y = x$ is rotated about the line $x = 2$.



Using the method of cylindrical shells, calculate the exact volume of the solid of revolution formed. **3**

(b) (i) Find the centre and radius of the circle $x^2 + y^2 + 4x = 0$. **1**

(ii) Show that the line $y = mx + b$ will be a tangent to the circle if **2**

$$4(mb+1) = b^2$$

(iii) P is a point whose coordinates are $(k, 0)$. **2**
If P lies on the line $y = mx + b$ and is exterior to the circle, find possible values for k if the two tangents from P to the circle are perpendicular.

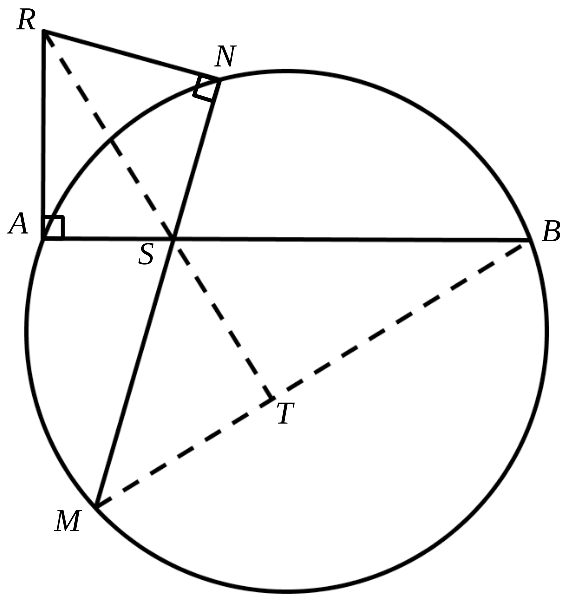
(c) (i) Show that $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$. **1**

(ii) Hence evaluate $\int_0^1 x^2\sqrt{1-x}\,dx$. **3**

Question 13 continues on page 10

Question 13 (continued)

(d) AB and MN are chords of a circle that intersect at S. R is a point external to the circle such that RA is perpendicular to the chord AB and RN is perpendicular to the chord MN.
The line RS is produced to T, a point lying on MB.



Copy or trace this diagram into your answer booklet.

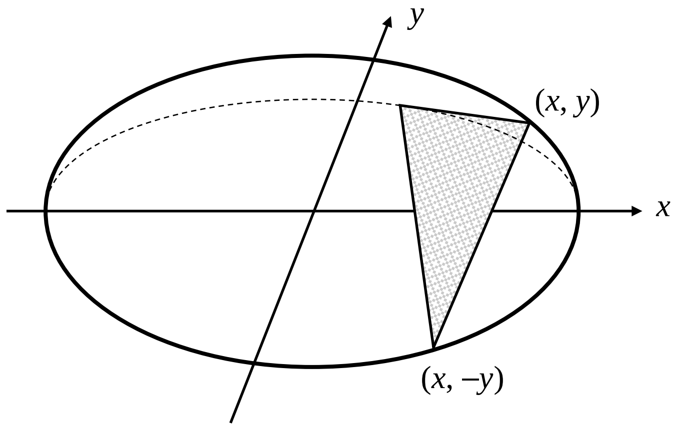
Prove that RT is perpendicular to MB. **3**

End of Question 13

Question 14 (15 marks) Use a SEPARATE writing booklet.

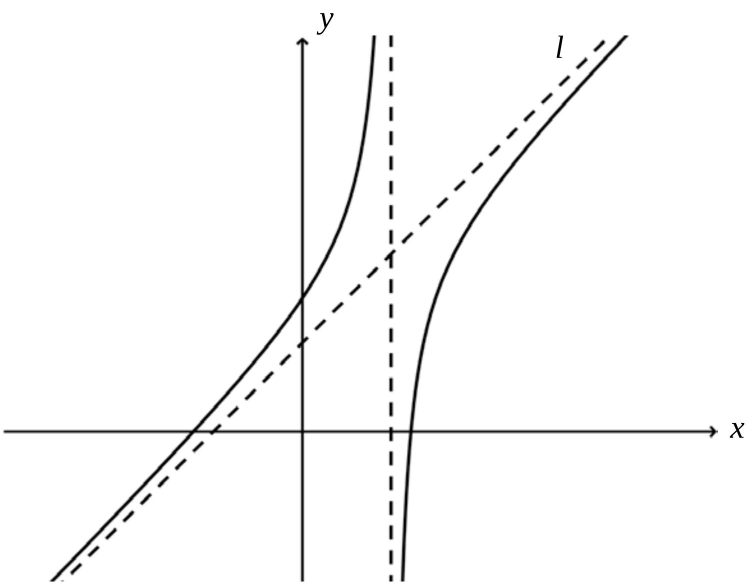
(a) The base of a certain solid is in the shape of an ellipse with equation $\dfrac{x^2}{16}+\dfrac{y^2}{9}=1$.

Sections parallel to the y-axis are equilateral triangles, with one side sitting in the base of the solid, as shown in the diagram below.



Find the exact volume of the shape. **4**

(b) The diagram shows the graph of $y=\dfrac{x^2-6}{x-2}$. The line l is an asymptote.



(i) Use the above graph to draw a one-third page sketch of the graph $y=\dfrac{x-2}{x^2-6}$. **2**

(ii) By writing $\dfrac{x^2-6}{x-2}$ in the form $mx+b+\dfrac{a}{x-2}$, find the equation of the line l. **2**

Question 14 continues on page 12

Question 14 (continued)

(c) (i) By considering the expansion of $(\cos\theta + i\sin\theta)^3$ and de Moivre's theorem, show that $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$. **1**

(ii) Deduce that $8x^3 - 6x - 1 = 0$ has solutions $x = \cos\theta$, where $\cos 3\theta = \frac{1}{2}$. **2**

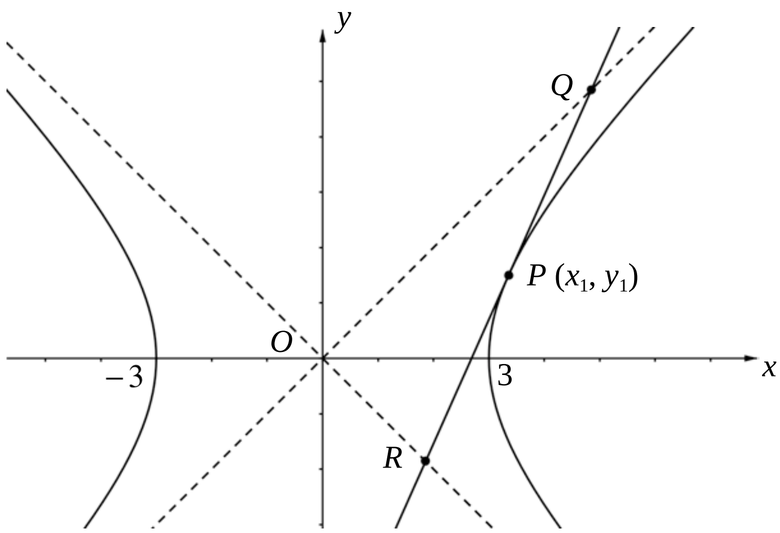
(iii) Find the roots of $8x^3 - 6x - 1 = 0$ in the form $\cos\theta$. **2**

(iv) Hence evaluate $\cos\left(\frac{\pi}{9}\right)\cos\left(\frac{2\pi}{9}\right)\cos\left(\frac{4\pi}{9}\right)$. **2**

End of Question 14

Question 15 (15 marks) Use a SEPARATE writing booklet.

(a) The hyperbola with equation $x^2 - y^2 = 9$ is shown in the diagram below. The point $P\left(x_1, y_1\right)$ lies on the hyperbola. The tangent to the hyperbola at P intersects the asymptotes of the hyperbola at points Q and R.



(i) Show that e, the eccentricity of the hyperbola, is equal to $\sqrt{2}$. **1**

(ii) Determine the coordinates of the foci, equations of the directrices and the equations of the asymptotes. **3**

(iii) Show that the equation of the tangent at P is **2**

$$yy_1 = xx_1 - 9$$

(iv) Prove that the area of triangle QOR is constant, where O is the origin. **3**

(b) The cubic function $y = x^3 - px + q$ has two turning points.

(i) Show that $p > 0$. **1**

(ii) The line $y = k$ intersects this cubic in three distinct points. **3**

Show that $q - \frac{2p}{3}\sqrt{\frac{p}{3}} < k < q + \frac{2p}{3}\sqrt{\frac{p}{3}}$.

(c) The equation $x^3 - x^2 - 3x + 5 = 0$ has roots α, β and γ. Find the equation whose roots are $(2\alpha + \beta + \gamma)$, $(\alpha + 2\beta + \gamma)$ and $(\alpha + \beta + 2\gamma)$. **2**

Question 16 (15 marks) Use a SEPARATE writing booklet.

(a) $T_1, T_2, T_3, \ldots$ are terms of an arithmetic sequence with common difference d. All terms in the sequence are positive.

(i) Show that $\dfrac{1}{\sqrt{T_{n-1}}+\sqrt{T_n}} = \dfrac{\sqrt{T_n}-\sqrt{T_{n-1}}}{d}$ for $n = 2, 3, 4, \ldots$ **2**

(ii) Hence or otherwise, show that **2**

$$\frac{1}{\sqrt{T_1}+\sqrt{T_2}} + \frac{1}{\sqrt{T_2}+\sqrt{T_3}} + \ldots + \frac{1}{\sqrt{T_{n-1}}+\sqrt{T_n}} = \frac{n-1}{\sqrt{T_1}+\sqrt{T_n}}$$

for $n = 2, 3, 4, \ldots$

(b) The equations of two conics are $3x^2+4y^2=48$ and $3x^2-y^2=3$.

(i) Show that these two conics have the same pair of foci. **1**

(ii) The point $\left(4\cos\theta,\, 2\sqrt{3}\sin\theta\right)$ lies on the ellipse for **all** values of θ. **2**
Find the four values of θ for which this point also lies on the other conic.

(iii) Show that the two conics intersect at right angles. **3**

(c) (i) Show that $\left(1-x^2\right)^{\frac{n-3}{2}} - \left(1-x^2\right)^{\frac{n-1}{2}} = x^2\left(1-x^2\right)^{\frac{n-3}{2}}$ **1**

(ii) Let $I_n = \displaystyle\int_0^1 \left(1-x^2\right)^{\frac{n-1}{2}}\,dx$ where $n = 1, 2, 3, \ldots$ **3**

Show that $n\,I_n = (n-1)I_{n-2}$ for $n = 2, 3, 4, \ldots$

(iii) Evaluate I_5. **1**

End of Paper

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \quad \text{if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x, \ x > 0$

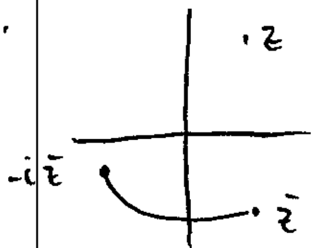
Qn	Solutions	Marks	Comments & Criteria
1	$z = 3 + 2i \quad w = 2 - i$ $\bar{z} = 3 - 2i \quad 3w = 6 - 3i$ $\therefore \bar{z} + 3w = 3 - 2i + 6 - 3i$ $= 9 - 5i$	D	
2.	$x^3 + 2xy - y^3 - 2 = 0$ $3x^2 + 2y + 2x\frac{dy}{dx} - 3y^2\frac{dy}{dx} = 0$ $(2x - 3y^2)\frac{dy}{dx} = -3x^2 - 2y$ $\frac{dy}{dx} = \frac{-3x^2 - 2y}{(2x - 3y^2)}$ At $(1, 1)$ $\frac{dy}{dx} = \frac{-3-2}{2-3}$ $= \frac{-5}{-1}$ $= 5$	A	
3.	$3x^3 + 2x^2 - 4x + 1 = 0$ $\alpha\beta\gamma = \frac{-1}{3}$ $(\alpha\beta\gamma)^3 = \frac{-1}{27}$ $\therefore \frac{1}{\alpha^3\beta^3\gamma^3} = -27$	B	
4.	$\frac{x^2}{6} - \frac{y^2}{10} = 1 \quad a^2 = 6$ $b^2 = 10$ $b^2 = a^2(e^2 - 1)$ $10 = 6(e^2 - 1)$		

$10 = 6e^2 - 6$

$e^2 = 16/6$ or $e^2 = 8/3$

$e = 2\sqrt{2}/3 = \frac{2\sqrt{6}}{3}$

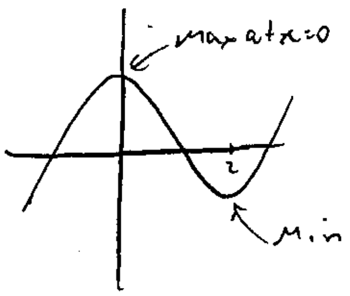
$e = \frac{4}{\sqrt{6}} = \frac{4\sqrt{6}}{6} = \frac{2\sqrt{6}}{3}$ B

Qn	Solutions	Marks	Comments & Criteria
5.	Roots $x=4$, $x=2-i$, $x=1+i$ As $P(x)$ has real coefficients, Complex roots occur in conjugate pairs $\therefore$ Other roots are $x=2+i$, $x=1-i$ $\therefore$ Min degree of $P(x)$ is 5.		$\underline{\underline{C}}$
6.	 $-i\bar{z}$ is a rotation through $\frac{\pi}{2}$ in a clockwise direction $\therefore$ $\underline{\underline{C}}$		
7.	Both A and C are possible, but A is <u>not</u> a function (fails vertical line test) $\therefore$ $f(x)$ must be C	$\underline{\underline{C}}$	
8	ω is a cube root of unity so a soln to $z^3=1$ i.e. $z^3-1=0$ Other roots are 1 and ω^2 $1+\omega+\omega^2=0$ (sum of roots) $\therefore 1+\omega=-\omega^2$ $(1+\omega-\omega^2)^7=(-2\omega^2)^7$ $=-128\,\omega^{14}$		

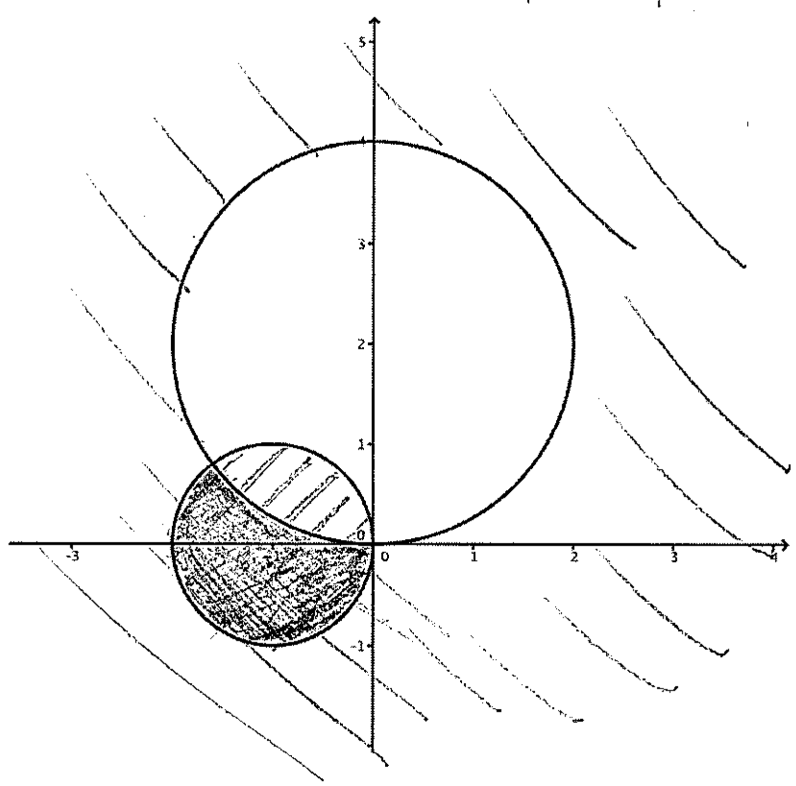
$=-128\,\omega^3.\omega^3.\omega^3.\omega^3.\omega^2$

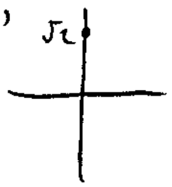
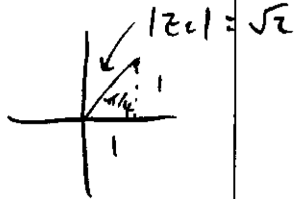
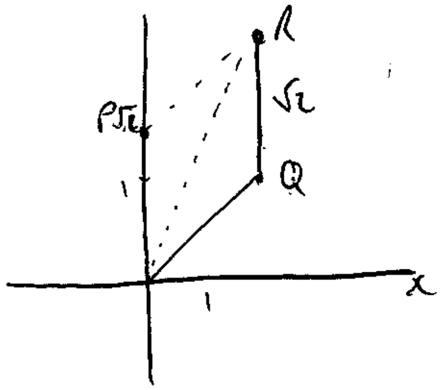
$=-128\,\omega^2$ as $\omega^3=1$ $\underline{A}$

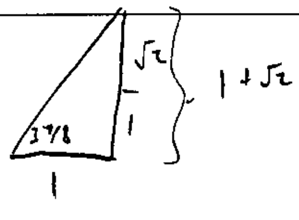
Kambala – 2013 – Year 12 – Ext 2 Trial HSC Examination **SOLUTIONS**

Qn	Solutions	Marks	Comments & Criteria
9.	$y = \cos^{-1} e^x$ $\therefore e^x = \cos y$ $\frac{dy}{dx}(-\sin y) = e^x$ $= \cos y$ $\therefore \frac{dy}{dx} = \frac{\cos y}{-\sin y}$ $= -\cot y$	B	
10.	$y = 2x^3 - 6x^2 + k$ $y' = 6x^2 - 12x$ $= 6x(x-2)$ $\therefore x = 0, 2$ are stat pts $\therefore$ Graph  At $x = 0$ $0 = 0 - 0 + k$ $k = 0$ At $x = 2$ $0 = 16 - 24 + k$ $k = 8$ $\therefore$ If k lies between 0 and 8 there are 3 roots $0 < k < 8$	D	

Qn	Solutions	Marks	Comments & Criteria
11	(a) $\dfrac{2\sqrt{3}+i}{\sqrt{3}-i} \times \dfrac{\sqrt{3}+i}{\sqrt{3}+i}$ $= \dfrac{(2\sqrt{3}+i)(\sqrt{3}+i)}{3+1}$ $= \dfrac{6+2\sqrt{3}i+\sqrt{3}i-1}{4}$ $= \dfrac{5+3\sqrt{3}i}{4}$ $= \dfrac{5}{4}+\dfrac{3\sqrt{3}}{4}i$		1 correct conjugate 1 simplified answer
	(b) $\lvert z+1\rvert \leq 1$ $\lvert z-(-1+0i)\rvert \leq 1$ Circle ctr $(-1,0)$, radius 1 unit $\lvert z-2i\rvert \geq 2$ $\lvert z-(0+2i)\rvert \geq 2$ Circle ctr $(0,2)$ radius 2 units		1 diagrams 1 regions correct



Qn	Solutions	Marks	Comments & Criteria
11	(c) $z_1 = i\sqrt{2}$ $\quad z_2 = \frac{2}{1-i} \times \frac{1+i}{1+i} = \frac{2+2i}{2} = 1+i$		
	(i)   $z_1 = \sqrt{2}\,\text{cis}\,\frac{\pi}{2}$ $\quad z_2 = \sqrt{2}\,\text{cis}\,\frac{\pi}{4}$		1 for z_1 1 for z_2
	(ii) $z_1 = w z_2$ i.e. $w = \frac{z_1}{z_2} = \frac{\sqrt{2}}{\sqrt{2}}\,\text{cis}\left(\frac{\pi}{2} - \frac{\pi}{4}\right) = \text{cis}\,\frac{\pi}{4}$		1 answer
	(iii) 		1 for P and Q 1 for R based on P and Q
	(iv) $\angle POQ = \frac{\pi}{2} - \frac{\pi}{4} = \pi/4$ $\angle QOR$ bisects $\angle POQ = \frac{\pi}{8}$ as OPRQ is a rhombus $\therefore \angle ROx = \frac{\pi}{4} + \frac{\pi}{8} = \frac{3\pi}{8}$		1 for showing



$\tan\frac{3\pi}{8} = \frac{1+\sqrt{2}}{1} = 1+\sqrt{2}$ $\qquad$ 1 for correct value based on diagram

Qn	Solutions	Marks	Comments & Criteria
11	(d) $2^{n+4} > (n+4)^2 \quad n \geq 1$		
	To show $2^{3(a+2)} > 9(a+2)^2$		
	i.e. $2^{3a+6} > 3^2(a+2)^2$		
	$2^{3a+6} > (3a+2)^2$		1 for progress
	Using result given $n+4 = 3a+6$		
	$n = 3a+2$	$9(a+2)^2 = 3^2(a+2)^2 = (3a+6)^2$	
	$\therefore$ If you let $n = 3a+2$		1 for substitution
	then $2^{3(a+2)} > 9(a+2)^2$	as req'd.	
	(e) $x^3 - 8x^2 - 5x + 84 = 0$		
	Let roots be $\alpha, \beta, (\alpha+\beta)$		
	Sum of roots: $\alpha + \beta + (\alpha+\beta) = \frac{-(-8)}{1}$		
	$2\alpha + 2\beta = 8$		
	$\alpha + \beta = 4 \Rightarrow \alpha = (4-\beta)$		
	Product $\alpha\beta(\alpha+\beta) = \frac{-84}{1}$		1 for sum/product or progress
	$(4-\beta)\beta(4-\cancel{\beta}+\cancel{\beta}) = -84$		
	$(4\beta-\beta^2)4 = -84 = 21$		
	$\beta^2 - 4\beta = 21$		
	$\beta^2 - 4\beta - 21 = 0$		
	$(\beta-7)(\beta+3) = 0$		
	$\therefore \beta = 7$ or -3		

$\therefore$ Roots $x = -3, 7, -3+7$

i.e. $x = -3, 4, 7$ | 1 correct soln

Qn	Solutions	Marks	Comments & Criteria
12	(a) $\int \frac{dx}{x^2-2x+5}$		
	$= \int \frac{dx}{x^2-2x+1+4}$		
	$= \int \frac{dx}{(x-1)^2+2^2}$	1	completing square
	$= \int \frac{dx}{2^2+(x-1)^2}$		
	$= \frac{1}{2}\tan^{-1}\left(\frac{x-1}{2}\right)+c$	1	correct integral
	(b) $\int \ln x \, dx$ $u=\ln x$ $v=x$		
	$= \int 1.\ln x \, dx$ $u'=\frac{1}{x}$ $v'=1$	1	for parts set-up
	$= x\ln x - \int \frac{1}{x}.x\,dx$		
	$= x\ln x - \int 1.dx$		
	$= x\ln x - x + c$	1	for correct answer
	(c) $\int \frac{1}{e^x+1}\,dx$ Let $u=e^x$		
	$\frac{du}{dx}=e^x$		
	$= \int \frac{1}{u+1}\cdot\frac{du}{u}$ $du=e^x dx$		
	$dx=\frac{du}{e^x}$	1	correct substitution set up
	$= \int \frac{1}{u(u+1)}\,du$		
	Now $\frac{1}{u(u+1)} = \frac{A}{u}+\frac{B}{u+1}$		
	$1 = A(u+1)+Bu$		
	Let $u=0 \Rightarrow A=1$		

$u=-1 \Rightarrow 1=-B$

$B=-1$ 1   for partial fractions

Qn	Solutions	Marks	Comments & Criteria
	$\therefore \int \frac{1}{u(u+1)}\,du$		
	$= \int \frac{1}{u}\,du - \int \frac{1}{u+1}\,du$		
	$= \ln(u) - \ln(u+1) + c$		
	But $u = e^x$		
	$\therefore I = \ln e^x - \ln(e^x+1) + c$		
	$= x - \ln(e^x+1) + c$		1 for answer
(d)	To prove $(1+x)^n - nx - 1$ is div		for x^2 for $n \geq 2$
	Show true for $n = 2$		
	$(1+x)^2 - 2x - 1$		
	$= 1 + 2x + x^2 - 2x - 1$		
	$= x^2$		
	$= 1.x^2 \quad \therefore$ Div by x^2		
	True for $n = 2$.		1 for $n = 2$
	Assume true for $n = k$		
	i.e. $(1+x)^k - kx - 1 = P(x).x^2$		for some polynomial $P(x)$
	i.e. $(1+x)^k = P(x).x^2 + kx + 1$		(*)
	Prove true for $n = k+1$		
	i.e. Show $(1+x)^{k+1} - (k+1)x - 1$ is		div by x^2
	$(1+x)^{k+1} - (k+1)x - 1$		
	$= (1+x)(1+x)^k - kx - x - 1$		
	$= (1+x)[P(x).x^2 + kx + 1] - kx - x - 1$		from (*) 1 for substn

$= P(x).x^2 + \cancel{kx} + \cancel{1} + P(x).x^3 + kx^2 + \cancel{x} - \cancel{kx} - \cancel{x} - \cancel{1}$

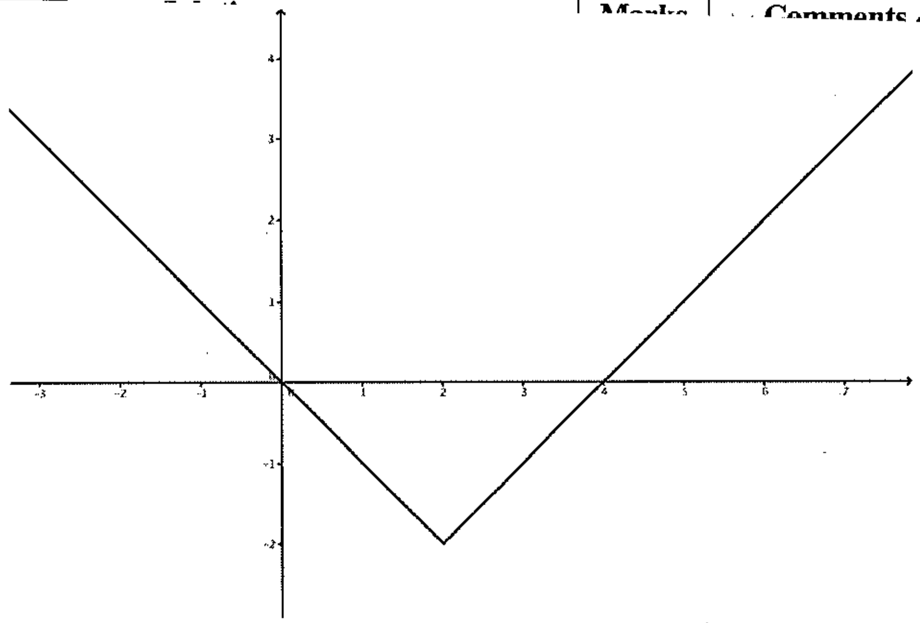
$= P(x)x^2 + P(x).x^3 + kx^2$

$= x^2[P(x) + P(x)x + k]$ which is div by x^2.

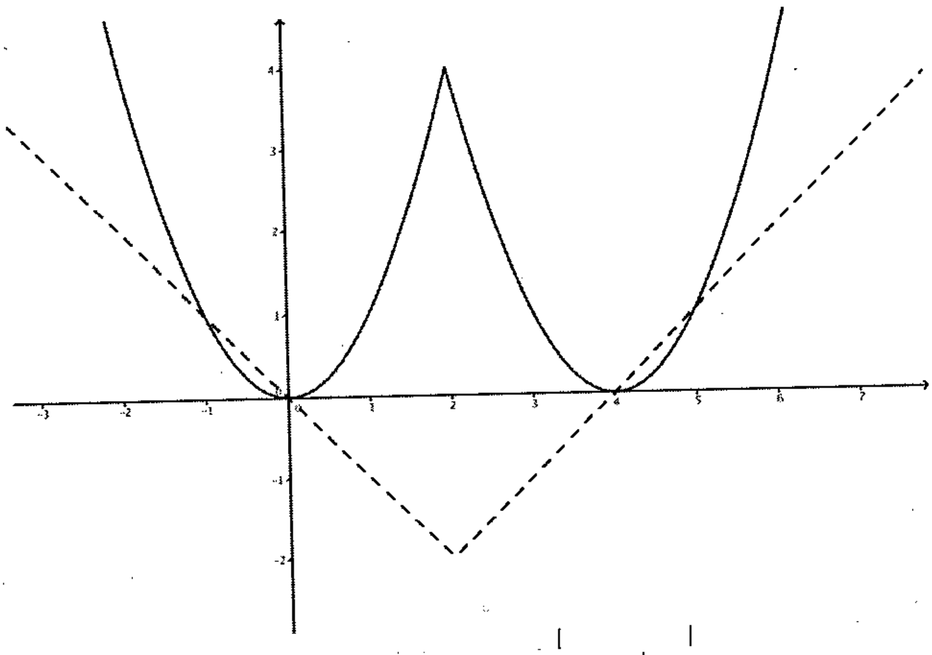
1 final proof

$\therefore$ By the process of Math Induction, this is proven for all integers $n \geq 2$.

Qn		Marks	Comments & Criteria

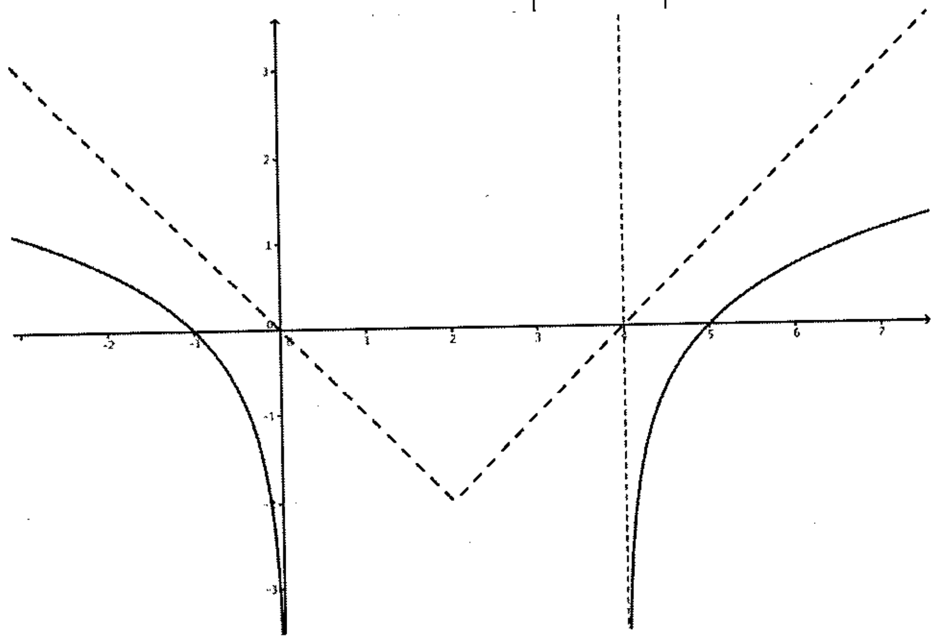


1



1 for shape

1 for point

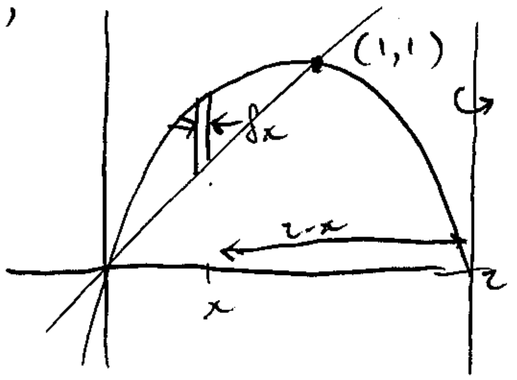


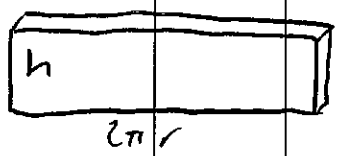
1 asympt

1 correct curve

Qn	Solutions	Marks	Comments & Criteria

13 (a)





δx thick

$$h = (2x - x^2) - x$$
$$= x - x^2$$
$$r = 2 - x$$

1 for correct h, r

$$\delta V = 2\pi r h\,\delta x$$
$$= 2\pi(2-x)(x-x^2)\,\delta x$$
$$= 2\pi(2x - 2x^2 - x^2 + x^3)\,\delta x$$
$$= 2\pi(2x - 3x^2 + x^3)\,\delta x$$
$$V = \sum_{x=0}^{1} 2\pi(2x - 3x^2 + x^3)\,\delta x$$
$$= \lim_{\delta x \to 0} \sum_{x=0}^{1} 2\pi(2x - 3x^2 + x^3)\,\delta x$$
$$V = 2\pi \int_0^1 (x^3 - 3x^2 + 2x)\,dx$$

1 for integral

$$= 2\pi\left[\frac{x^4}{4} - x^3 + x^2\right]_0^1$$
$$= 2\pi\left[\left(\frac{1}{4} - 1 + 1\right) - (0 - 0 + 0)\right]$$
$$= 2\pi \times \frac{1}{4}$$
$$= \frac{\pi}{2}\ u^2$$

1 correct answer

(b)(i)
$$x^2 + y^2 + 4x = 0$$
$$x^2 + 4x + 4 + y^2 = 4$$
$$(x+2)^2 + y^2 = 4$$
$\therefore$ Ctr $(-2, 0)$

radius 2 units

1

Qn	Solutions	Marks	Comments & Criteria
	(b)(ii) $y = mx + b \qquad (x+2)^2 + y^2 = 4$		
	i.e. $(x+2)^2 + (mx+b)^2 = 4$		
	$x^2 + 4x + \cancel{4} + m^2x^2 + 2mbx + b^2 - \cancel{4} = 0$		
	$(m^2+1)x^2 + (2mb+4)x + b^2 = 0$		1 for quadratic
	Tangent if $\Delta = 0$		
	i.e. $b^2 - 4ac = 0$		
	$(2mb+4)^2 - 4(m^2+1)(b^2) = 0$		
	$\cancel{4m^2b^2} + 16mb + 16 - \cancel{4m^2b^2} - 4b^2 = 0$		
	$16mb + 16 = 4b^2$		
	$4mb + 4 = b^2$		
	$4(mb+1) = b^2$ as req'd		1 for discriminant work. or similar
	(iii) $P(k, 0)$		
	$y = mx + b$		
	At P $\quad 0 = mk + b$		
	$b = -mk$		
	Sub into result from (ii)		
	$4[m(-mk) + 1] = (-mk)^2$		
	$-4m^2k + 4 = m^2k^2$		
	$m^2k^2 + 4m^2k - 4 = 0$		
	$(k^2+4k)m^2 - 4 = 0$		1 for progress
	Two possible values of m if tangents are perpendicular		
	i.e. $m_1 m_2 = -1 \Rightarrow$ product of roots $= -1$		
	$\therefore \dfrac{-4}{k^2+4k} = -1$		
	$k^2 + 4k = 4$		

$k^2 + 4k + 4 = 4 + 4$

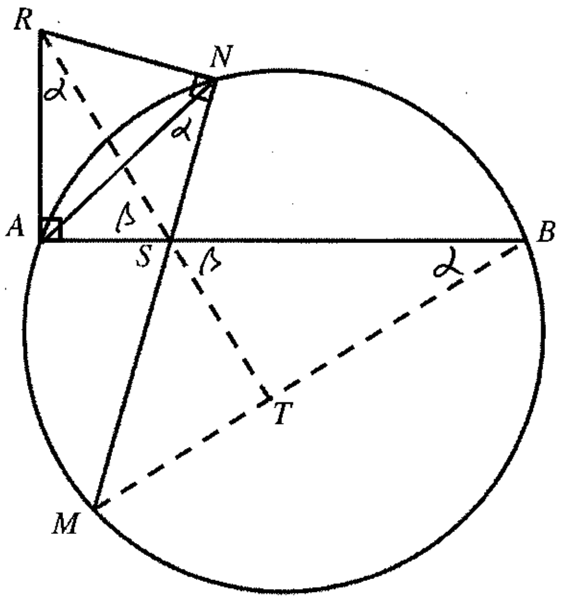
$(k+2)^2 = 8$

$k + 2 = \pm 2\sqrt{2}$

$\therefore k = -2 + 2\sqrt{2}, \ -2 - 2\sqrt{2}$ 1 for k values

Qn	Solutions	Marks	Comments & Criteria
13	(c) (i) $\int_0^a f(x)\,dx$ Let $u = a-x$, $x = a-u$		$\frac{du}{dx} = -1 \Rightarrow du = -dx$
	when $x = a$, $u = 0$; $x = 0$, $u = a$		
	$= \int_a^0 f(a-u)\,(-du)$		
	$= \int_0^a f(a-u)\,du$		1 for showing
	$= \int_0^a f(a-x)\,dx$ as integral is independent of variable		
	(ii) $\int_0^1 x^2\sqrt{1-x}\,dx$		
	$= \int_0^1 (1-x)^2\sqrt{1-(1-x)}\,dx$		1
	$= \int_0^1 (1-x)^2\sqrt{x}\,dx$		
	$= \int_0^1 (1-2x+x^2)\cdot x^{1/2}\,dx$		
	$= \int_0^1 \left(x^{1/2} - 2x^{3/2} + x^{5/2}\right)dx$		1 for rearranging integral
	$= \left[\frac{x^{3/2}}{3/2} - \frac{2x^{5/2}}{5/2} + \frac{x^{7/2}}{7/2}\right]_0^1$		
	$= \left[\frac{2}{3} - \frac{4}{5} + \frac{2}{7}\right] - [0 - 0 + 0]$		
	$= 16/105$		1 for answer

Qn	Solutions	Marks	Comments & Criteria
13	(d)		



To Prove $RT \perp MB$

RANS is a cyclic quadrilateral as opposite ∠'s supplementary

$\angle RAS + \angle RNS = 90° + 90° = 180°$

(α) $\angle ARS = \angle ANS$ (angles at circumference on same segment equal)

(α) $\angle ANS = \angle ABM$ (angles at circumference standing on same arc are equal)

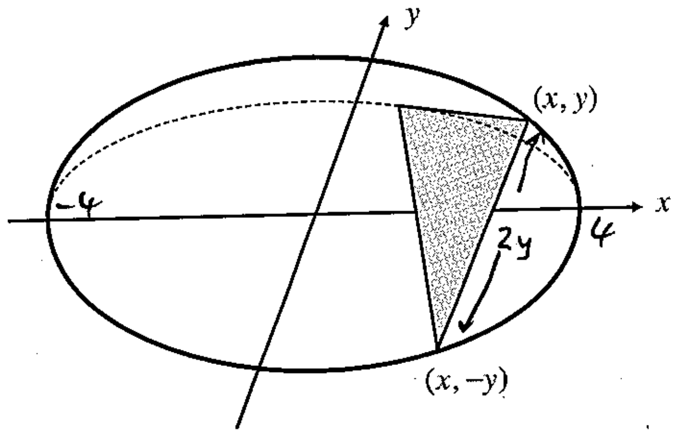
(β) $\angle RSA = \angle TSB$ (vertically opposite angles equal)

$\therefore \triangle RAS \sim \triangle STB$ (Equiangular)

$\therefore \angle STB = \angle RAS = 90°$ (Corresponding angles in similar triangles are equal)

$\therefore RT$ is perpendicular to MB.

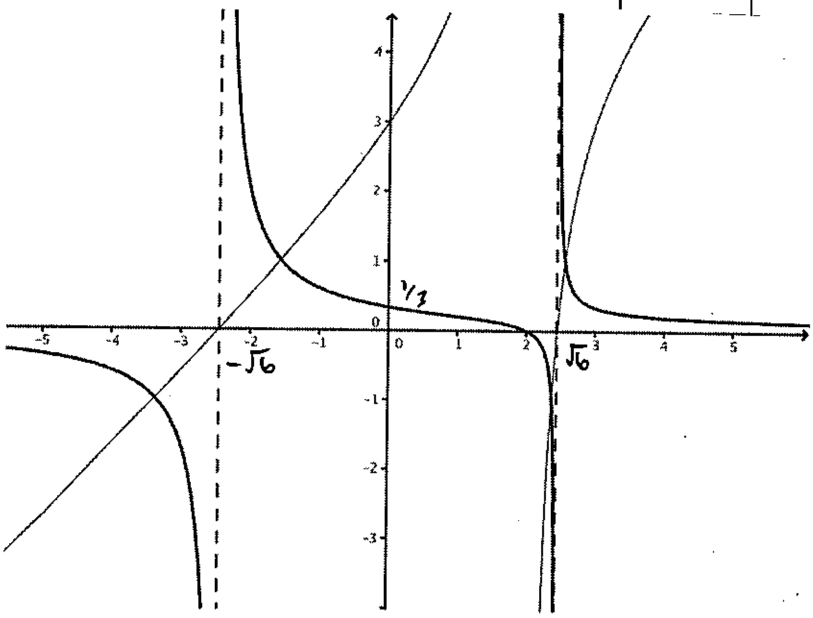
$\therefore$ Proven

Qn	Solutions	Marks	Comments & Criteria
14	(a) 		
	Triangle is equilateral with side length 2y		
	Area of $\Delta = \frac{1}{2}ab\sin C$ $= \frac{1}{2} \times 2y \times 2y \times \sin 60$ $= 2y^2 \times \frac{\sqrt{3}}{2}$ $= \sqrt{3}\,y^2$	1	Area Δ
	$V = \int_{-4}^{4} \sqrt{3}\,y^2\,dx$ $\frac{x^2}{16} + \frac{y^2}{9} = 1$ $= 2\sqrt{3}\int_0^4 9\left(1 - \frac{x^2}{16}\right) dx$ $y^2 = 9\left(1 - \frac{x^2}{16}\right)$	1	rearranging integral
	$= 18\sqrt{3}\int_0^4 \left(1 - \frac{x^2}{16}\right) dx$ $= 18\sqrt{3}\left[x - \frac{x^3}{48}\right]_0^4$	1	
	$= 18\sqrt{3}\left[\left(4 - \frac{64}{48}\right) - (0 - 0)\right]$ $= \cancel{18}^{6}\sqrt{3} \times \frac{8}{\cancel{3}}$ $= 48\sqrt{3}\ u^3$	1	Answer

Kambala – 2013 – Year 12 – Ext 2 Trial HSC Examination **SOLUTIONS**

Qn	Solutions	Marks	Comments & Criteria

14 (b) (i)



1 Asymptotes

1 shape

(ii)

$$\frac{x^2-6}{x-2} = mx+b+\frac{a}{x-2}$$

$$\begin{array}{r} x+2 \\ x-2\overline{)\,x^2-0x-6} \\ \underline{x^2-2x} \\ 2x-6 \\ \underline{2x-4} \\ -2 \end{array}$$

1 progress

$$\therefore \frac{x^2-6}{x-2} = x+2-\frac{2}{x-2}$$

$\therefore$ Eqn of ℓ : $y = x+2$

1 correct soln.

(c) (i) $(\cos\theta + i\sin\theta)^3$

$= \cos^3\theta + 3\cos^2\theta . i\sin\theta + 3\cos\theta . i^2\sin^2\theta + i^3\sin^3\theta$

$= \cos^3\theta - 3\cos\theta\sin^2\theta + i(3\cos^2\theta\sin\theta - \sin^3\theta)$

$= \cos^3\theta - 3\cos\theta(1-\cos^2\theta) + i(3\cos^2\theta\sin\theta - \sin^3\theta)$

$= \cos^3\theta - 3\cos\theta + 3\cos^3\theta + i(3\cos^2\theta\sin\theta - \sin^3\theta)$

$= 4\cos^3\theta - 3\cos\theta + i(3\cos^2\theta\sin\theta - \sin^3\theta)$

Qn	Solutions	Marks	Comments & Criteria
	$z = \text{cis}\,\theta$ $z^3 = (\text{cis}\,\theta)^3$ $= 1^3\,\text{cis}\,3\theta$ $z^3 = \cos 3\theta + i\sin 3\theta$ Equating Re parts $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$		1 for showing
	(ii) $8x^3 - 6x - 1 = 0$ Let $x = \cos\theta$ i.e. $8\cos^3\theta - 6\cos\theta - 1 = 0$ $2(4\cos^3\theta - 3\cos\theta) - 1 = 0$ $2\cos 3\theta - 1 = 0$ $2 \times \frac{1}{2} - 1 = 0$ where $\cos 3\theta = \frac{1}{2}$ $\therefore \cos\theta = x$ is a soln.	1	1
	(iii) $\cos 3\theta = \frac{1}{2}$ $3\theta = \cos^{-1}\left(\frac{1}{2}\right)$ $3\theta = \frac{\pi}{3} + 2n\pi$ for $n = 0, \pm 1, \pm 2$ etc $= \frac{\pi + 6n\pi}{3}$ $3\theta = \frac{\pi(6n+1)}{3}$ $\theta = \frac{\pi(6n+1)}{9}$ $\theta = \frac{\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}$ $\therefore$ Solns are $\cos\frac{\pi}{9}, \cos\left(\frac{5\pi}{9}\right), \cos\left(\frac{7\pi}{9}\right)$	1	1

Qn	Solutions	Marks	Comments & Criteria
	(iv) $\cos\frac{5\pi}{9} = -\cos\frac{4\pi}{9}$		
	$\cos\frac{7\pi}{9} = -\cos\frac{2\pi}{9}$		
	$\therefore \cos\frac{\pi}{9}.\cos\frac{5\pi}{9}.\cos\frac{7\pi}{9}$		
	$= \cos\frac{\pi}{9}\left(-\cos\frac{4\pi}{9}\right)\left(-\cos\frac{2\pi}{9}\right)$	1	for equating
	$= \cos\frac{\pi}{9}\cos\frac{2\pi}{9}\cos\frac{4\pi}{9}$		
	= product of roots.		
	$\alpha\beta\gamma = -\frac{(-1)}{8}$		
	$= \frac{1}{8}$		1 value
	$\therefore \cos\frac{\pi}{9}\cos\frac{2\pi}{9}\cos\frac{4\pi}{9} = \frac{1}{8}$		
15	(a,(i) $x^2 - y^2 = 9$		
	$\frac{x^2}{9} - \frac{y^2}{9} = 1$ or		As $a^2 = b^2$
	$b^2 = a^2(e^2-1)$		i.e $a = b$
	$9 = 9(e^2-1)$		$\therefore$ A rectangular hyperbola
	$1 = e^2 - 1$		$\therefore e = \sqrt{2}$
	$e^2 = 2$		
	$e = \sqrt{2}$ $\therefore$ A rectangular hyperbola		1
	(ii) $a = 3,\ b = 3$		
	Foci : $(\pm ae, 0)$		1
	$= (\pm 3\sqrt{2}, 0)$		
	Directrices : $x = \pm a/e$		
	$= \pm 3/\sqrt{2} = \pm\frac{3\sqrt{2}}{2}$		1
	Asymptotes $y = \pm x$ (as	$e = \sqrt{2}$)	

or $y = \pm\frac{bx}{a}$

$= \pm\frac{3x}{3}$ 1

$y = \pm x$

Qn	Solutions	Marks	Comments & Criteria
15	(iii)		

$$x^2 - y^2 = 9$$

$$2x - 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{2x}{2y} = \frac{x}{y}$$

1

At P, $m_T = \frac{x_1}{y_1}$

Eqn

$$y - y_1 = m(x - x_1)$$

$$y - y_1 = \frac{x_1}{y_1}(x - x_1)$$

$$y_1 y - y_1^2 = x_1 x - x_1^2$$

$$yy_1 = xx_1 - x_1^2 + y_1^2$$

$$= xx_1 - (x_1^2 - y_1^2)$$

$$yy_1 = xx_1 - 9$$

as $x^2 - y^2 = 9$ i.e. $x_1^2 - y_1^2 = 9$ 1

(iv) At Q $y = x$

$$\therefore xy_1 = xx_1 - 9$$

$$x = \frac{9}{x_1 - y_1}$$

$$\therefore y = \frac{9}{x_1 - y_1}$$

At R $y = -x$

$$-xy_1 = xx_1 - 9$$

$$x = \frac{9}{x_1 + y_1}$$

$$\therefore y = \frac{-9}{x_1 + y_1}$$

1

Area $\triangle QOR = \frac{1}{2}bh = \frac{1}{2} \cdot OQ \cdot OR$

$$OQ^2 = \left(\frac{9}{x_1 - y_1}\right)^2 + \left(\frac{9}{(x_1 - y_1)}\right)^2 = \frac{162}{(x_1 - y_1)^2}$$

$$OQ = \frac{9\sqrt{2}}{x_1 - y_1}$$

$$OR^2 = \left(\frac{9}{(x_1 + y_1)}\right)^2 + \left(\frac{-9}{(x_1 + y_1)}\right)^2 = \frac{162}{(x_1 + y_1)^2}$$

$$OR = \frac{9\sqrt{2}}{(x_1 + y_1)}$$

1

Qn	Solutions	Marks	Comments & Criteria

$\therefore$ Area $\triangle QOR$

$$= \frac{1}{\cancel{2}} \times \frac{9\sqrt{2}}{x_1 - y_1} \times \frac{9\sqrt{2}}{x_1 + y_1}$$

$$= \frac{81}{x_1^2 - y_1^2}$$

$$= \frac{81}{9}$$

$= 9u^2$ $\therefore$ a constant value. 1

(b) $y = x^3 - px + q$

(i) $y' = 3x^2 - p$

$= 0$ when $3x^2 = p$

$x = \pm\sqrt{\frac{p}{3}}$

$\therefore$ 2 turning pts when $p > 0$ 1

(ii) At $x = \sqrt{\frac{p}{3}}$

$$y = \left(\frac{\sqrt{p}}{\sqrt{3}}\right)^3 - p\sqrt{\frac{p}{3}} + q$$

$$= \frac{p}{3}\sqrt{\frac{p}{3}} - p\sqrt{\frac{p}{3}} + q$$

$$= -\frac{2p}{3}\sqrt{\frac{p}{3}} + q$$

$$= q - \frac{2p}{3}\sqrt{\frac{p}{3}}$$ 1

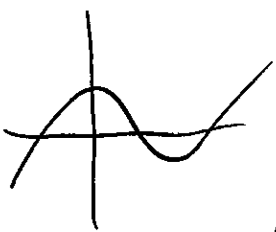
At $x = -\sqrt{\frac{p}{3}}$

$$y = \left(-\sqrt{\frac{p}{3}}\right)^3 + p\sqrt{\frac{p}{3}} + q$$

$$= -\frac{p}{3}\sqrt{\frac{p}{3}} + p\sqrt{\frac{p}{3}} + q$$

$$= \frac{2p}{3}\sqrt{\frac{p}{3}} + q$$

$$= q + \frac{2p}{3}\sqrt{\frac{p}{3}}$$ 1



$\therefore$ Min at $\left(\sqrt{\frac{p}{3}},\ q - \frac{2p}{3}\sqrt{\frac{p}{3}}\right)$

Max at $\left(-\sqrt{\frac{p}{3}},\ q + \frac{2p}{3}\sqrt{\frac{p}{3}}\right)$

For 3 intercepts, k lies between max and min

$\therefore q - \frac{2p}{3}\sqrt{\frac{p}{3}} < k < q + \frac{2p}{3}\sqrt{\frac{p}{3}}$ as req'd 1

<table>
<tr><th>Qn</th><th>Solutions</th><th>Marks</th><th>Comments & Criteria</th></tr>
<tr><td>15</td><td>(c) $x^3 - x^2 - 3x + 5 = 0$
Roots α, β, γ
Sum of roots $\alpha + \beta + \gamma = \frac{-(-1)}{1}$
$= 1$
Roots of eqn $2\alpha + \beta + \gamma,\ \alpha + 2\beta + \gamma,\ \alpha + \beta + 2\gamma$
Now $2\alpha + \beta + \gamma = \alpha + \beta + \gamma + \alpha$
$= 1 + \alpha$
$\therefore$ Roots of new eqn are $1+\alpha,\ 1+\beta,\ 1+\gamma$
$x = 1 + \alpha$
$\Rightarrow \alpha = x - 1$ etc
$\therefore$ Eqn $(x-1)^3 - (x-1)^2 - 3(x-1) + 5 = 0$
$x^3 - 3x^2 + 3x - 1 - x^2 + 2x - 1 - 3x + 3 + 5 = 0$
$\underline{x^3 - 4x^2 + 2x + 6 = 0}$</td><td></td><td>1
1</td></tr>
<tr><td>16</td><td>(a) $T_n = T_{n-1} + d$
(i) $\frac{1}{\sqrt{T_{n-1}} + \sqrt{T_n}} \times \frac{\sqrt{T_{n-1}} - \sqrt{T_n}}{\sqrt{T_{n-1}} - \sqrt{T_n}}$
$= \frac{\sqrt{T_{n-1}} - \sqrt{T_n}}{T_{n-1} - T_n}$
$= \frac{\sqrt{T_{n-1}} - \sqrt{T_n}}{T_{n-1} - (T_{n-1} + d)}$
$= \frac{\sqrt{T_{n-1}} - \sqrt{T_n}}{-d}$</td><td></td><td>Rationalising Denom. 1</td></tr>
</table>

$= \frac{\sqrt{T_n} - \sqrt{T_{n-1}}}{d}$ as req'd 1

Qn	Solutions	Marks	Comments & Criteria
16	(a)(ii) From (i) $\dfrac{1}{\sqrt{T_1}+\sqrt{T_2}} = \dfrac{\sqrt{T_2}-\sqrt{T_1}}{d}$ etc		
	$\therefore \dfrac{1}{\sqrt{T_1}+\sqrt{T_2}} + \dfrac{1}{\sqrt{T_2}+\sqrt{T_3}} + \dots + \dfrac{1}{\sqrt{T_{n-1}}+\sqrt{T_n}}$		
	$= \dfrac{\sqrt{T_2}-\sqrt{T_1}}{d} + \dfrac{\sqrt{T_3}-\sqrt{T_2}}{d} + \dots + \dfrac{\sqrt{T_n}-\sqrt{T_{n-1}}}{d}$		
	$= \dfrac{1}{d}\left[\cancel{\sqrt{T_2}} - \sqrt{T_1} + \cancel{\sqrt{T_3}} - \cancel{\sqrt{T_2}} + \cancel{\dots} + \sqrt{T_n} - \sqrt{T_{n-1}}\right]$ etc		|
	$= \dfrac{\sqrt{T_n}-\sqrt{T_1}}{d} \times \dfrac{\sqrt{T_n}+\sqrt{T_1}}{\sqrt{T_n}+\sqrt{T_1}}$		
	$= \dfrac{T_n - T_1}{d(\sqrt{T_n}+\sqrt{T_1})}$	Now $T_n =$ i.e.	$T_1 + (n-1)d$ $u_n = a + (n-1)d$ $\therefore T_n - T_1 = (n-1)d$
	$= \dfrac{(n-1)\cancel{d}}{\cancel{d}(\sqrt{T_n}+\sqrt{T_1})}$		
	$= \dfrac{n-1}{\sqrt{T_n}+\sqrt{T_1}}$		|
	$= \dfrac{n-1}{\sqrt{T_1}+\sqrt{T_n}}$ as req'd		

Qn	Solutions	Marks	Comments & Criteria
16	(b) $3x^2+4y^2=48$ $\quad$ $3x^2-y^2=3$ $\frac{x^2}{16}+\frac{y^2}{12}=1$ $\quad$ $x^2-\frac{y^2}{3}=1$ (i) Foci: Ellipse $b^2=a^2(1-e^2)$ $12=16(1-e^2)$ $12=16-16e^2$ $-4=-16e^2$ $e^2=\frac{1}{4}$ $e=\frac{1}{2}$ $\therefore$ Foci $(\pm ae,0)$ i.e. $(\pm 2,0)$ * $\therefore$ Same foci Hyperbola $b^2=a^2(e^2-1)$ $3=1(e^2-1)$ $3=e^2-1$ $e^2=4$ $e=2$ $\therefore ae=2$ Foci $:(\pm ae,0)$ $=(\pm 2,0)$ * $ae=4\times\frac{1}{2}=2$ (ii) $(4\cos\theta,\ 2\sqrt{3}\sin\theta)$ satisfies $3x^2-y^2=3$ $3(16\cos^2\theta)-12\sin^2\theta=3$ $16\cos^2\theta-4\sin^2\theta=1$ $16\cos^2\theta-4(1-\cos^2\theta)=1$ $16\cos^2\theta-4+4\cos^2\theta=5$ $20\cos^2\theta=5$ $\cos^2\theta=\frac{1}{4}$ $\cos\theta=\pm\frac{1}{2}$ $\therefore\theta=\frac{\pi}{3},\frac{2\pi}{3},\frac{4\pi}{3},\frac{5\pi}{3}$		| | |

At $\theta=\frac{\pi}{3}$ $\left(4\times\frac{1}{2},\ 2\sqrt{3}\times\frac{\sqrt{3}}{2}\right)=(2,3)$

By symmetry: Pts of intersection $(2,3)$ $(2,-3)$ $(-2,3)$ $(-2,-3)$

Qn	Solutions	Marks	Comments & Criteria

Q16 (b)(iii) Show intersect at right-angles

i.e. Show tangents intersect at right angles

$m_1 m_2 = -1$

$3x^2 + 4y^2 = 48$

$6x + 8y\dfrac{dy}{dx} = 0$

$\dfrac{dy}{dx} = -\dfrac{6x}{8y}$

$= \dfrac{-3x}{4y} \quad (m_1)$

$3x^2 - y^2 = 3$

$6x - 2y\dfrac{dy}{dx} = 0$

$\dfrac{dy}{dx} = \dfrac{6x}{2y}$

$= \dfrac{3x}{y}$

At $(2,3)$ $\quad m_1 = \dfrac{-6}{12} = -\dfrac{1}{2}$, $\quad m_2 = \dfrac{6}{3} = 2$ $\quad m_1 m_2 = -1$

At $(2,-3)$ $\quad m_1 = \dfrac{-6}{-12} = \dfrac{1}{2}$, $\quad m_2 = \dfrac{6}{-3} = -2$ $\quad m_1 m_2 = -1$

At $(-2,3)$ $\quad m_1 = \dfrac{6}{12} = \dfrac{1}{2}$, $\quad m_2 = \dfrac{-6}{3} = -2$ $\quad m_1 m_2 = -1$

At $(-2,-3)$ $\quad m_1 = \dfrac{6}{-12} = -\dfrac{1}{2}$, $\quad m_2 = \dfrac{-6}{-3} = 2$ $\quad m_1 m_2 = -1$

$\therefore$ Intersect at right angles

Qn	Solutions	Marks	Comments & Criteria
16	(c)(i) $(1-x^2)^{\frac{n-3}{2}} - (1-x^2)^{\frac{n-1}{2}} = x^2(1-x^2)^{\frac{n-3}{2}}$ LHS $= (1-x^2)^{\frac{n-3}{2}} - (1-x^2)^{\frac{n-1}{2}}$ $= (1-x^2)^{\frac{n-3}{2}}\left[1-(1-x^2)\right]$ $= (1-x^2)^{\frac{n-3}{2}}(1-1+x^2)$ $= x^2(1-x^2)^{\frac{n-3}{2}}$ $=$ RHS $\therefore$ Shown		1
	(ii) $I_n = \int_0^1 (1-x^2)^{\frac{n-1}{2}}\,dx$ $= \int_0^1 1.(1-x^2)^{\frac{n-1}{2}}\,dx$ $\therefore I_n = \left[(1-x^2)^{\frac{n-1}{2}}.x\right]_0^1 - \int_0^1 -(n-1)x(1-x^2)^{\frac{n-3}{2}}.x\,dx$ $= \left\{\left[\underbrace{(1-1)^{\frac{n-1}{2}}.1}_{0}\right] - \left[\underbrace{(1-0)^{\frac{n-1}{2}}.0}_{0}\right]\right\} + (n-1)\int_0^1 x^2(1-x^2)^{\frac{n-3}{2}}\,dx$ $= (n-1)\int_0^1 \left[(1-x^2)^{\frac{n-3}{2}} - (1-x^2)^{\frac{n-1}{2}}\right]dx$ from (i) $= (n-1)\int_0^1 (1-x^2)^{\frac{n-3}{2}} - (n-1)\underbrace{\int (1-x^2)^{\frac{n-1}{2}}\,dx}_{(I_n)}$ $\therefore I_n = (n-1)\underbrace{\int_0^1 (1-x^2)^{\frac{n-3}{2}}}_{(I_{n-2})} - (n-1)I_n$ $nI_n = (n-1)I_{n-2}$ as req'd		$u = (1-x^2)^{\frac{n-1}{2}}$ $u' = \frac{n-1}{2}(1-x^2)^{\frac{n-3}{2}}.(-2x)$ $= -(n-1)x(1-x^2)^{\frac{n-3}{2}}$ $v' = 1$ $v = x$ 1 1 1

Qn	Solutions	Marks	Comments & Criteria
16	(c) (iii) $nI_n = (n-1)I_{n-2}$ i.e. $I_n = \frac{(n-1)}{n} I_{n-2}$ $I_5 = \frac{4}{5} \times I_3$ $I_3 = \frac{2}{3} I_1$ $I_1 = \int_0^1 (1-x^2)^{\frac{1-1}{2}}\,dx$ $= \int_0^1 (1-x^2)^0\,dx$ $= \int_0^1 1\,.dx$ $= [x]_0^1$ $= 1$ $\therefore I_3 = \frac{2}{3}$ $I_5 = \frac{4}{5} \times \frac{2}{3}$ $= \frac{8}{15}$		1