2017 | Trial Higher School Certificate Examination

Killara HIGH SCHOOL

MATHEMATICS EXTENSION 2

2 August 2017

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black pen.
- NESA approved calculators may be used.
- **Commence each new question in a new booklet.** Write on both sides of the paper.
- A reference sheet is provided.
- In Question 11–16 show relevant mathematical reasoning and/or calculations
- At the conclusion of the examination, bundle the booklets used in the correct order **including your reference sheet** within this paper and hand to examination supervisor.

Total Marks: 100

Section 1 – 10 marks (pages 3 - 6)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section 2 – 90 marks (pages 7 - 14)

- Attempt Questions 11 – 16
- Allow about 2 hours and 45 minutes for this section

NESA NUMBER: .... **# BOOKLETS USED**: .....

Marker's use only.

QUESTION	1-10	11	12	13	14	15	16	**Total**
MARKS	$\overline{}$ 10	15	15	15	15	15	15	100

This task constitutes 40% of the HSC Course Assessment

Section I

10 marks
Attempt Question 1 to 10
Allow approximately 15 minutes for this section

Mark your answers on the answer grid provided (labelled as page 15).

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1. The circle $|z - 3 - 2i| = 2$ is intersected exactly twice by the line given by:

(A) $Im(z) = 0$

(B) $|z - i| = |z + 1|$

(C) $Re(z) = 5$

(D) $|z - 3 - 2i| = |z - 5|$

2. If $z_1 + z_2 + z_3 = 0$ and $|z_1| = |z_2| = |z_3| = 1$, then $z_1^2 + z_2^2 + z_3^2$ equals:

(A) -3

(B) $-\dfrac{1}{3}$

(C) 0

(D) 3

3. The graph of $y = \dfrac{1}{ax^2 + bx + c}$ has asymptotes at $x = 5$ and $x = -3$.

Given the graph has only one stationary point with y-value of $-\dfrac{1}{8}$, it follows that:

(A) $a = \dfrac{1}{2},\ b = 1$ and $c = -\dfrac{15}{2}$

(B) $a = \dfrac{1}{2},\ b = -1$ and $c = -\dfrac{15}{2}$

(C) $a = 1,\ b = 2$ and $c = -15$

(D) $a = 1,\ b = -2$ and $c = -15$

4. $\displaystyle\int \sqrt{1+\sin x}\, dx$ equals:

(A) $-2\sqrt{1-\sin x}+C$

(B) $\sin\left(\dfrac{x}{2}\right)+\cos\left(\dfrac{x}{2}\right)+C$

(C) $\cos\left(\dfrac{x}{2}\right)-\sin\left(\dfrac{x}{2}\right)+C$

(D) $2\sqrt{1-\sin x}+C$

5. In how many ways can 30 distinct toys be divided into 10 packets?

(A) 10^{30}

(B) 30^{10}

(C) $\dfrac{30!}{(3!)^{10}}$

(D) $\dfrac{30!}{10!\times(3!)^{10}}$

6. If $e^x+e^y=2$ then $\dfrac{dy}{dx}$ is:

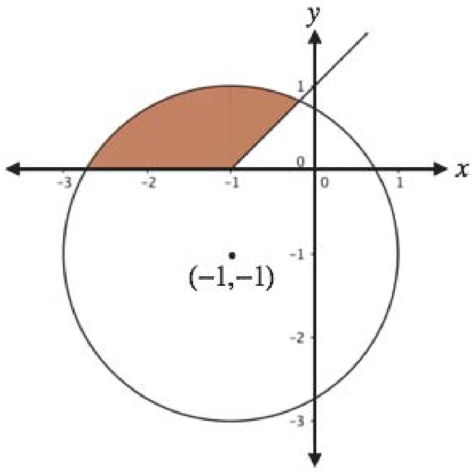
(A) e^{x-y}

(B) $-e^{x-y}$

(C) e^{y-x}

(D) $-e^{y-x}$

7. Consider the following shaded region on the Argand diagram.



Which of the following inequations would represent the region?

(A) $|z-1-i| \leq 2$ and $\dfrac{\pi}{4} \leq arg(z+1) \leq \pi$

(B) $|z+1+i| \leq 2$ and $\dfrac{\pi}{4} \leq arg(z+1) \leq \pi$

(C) $|z-1-i| \leq 2$ and $0 \leq arg(z+1) \leq \dfrac{\pi}{4}$

(D) $|z+1+i| \leq 2$ and $0 \leq arg(z+1) \leq \dfrac{\pi}{4}$

8. The equations of the directrices of the ellipse $\dfrac{x^2}{9} + y^2 = 1$ are:

(A) $x = \pm\dfrac{1}{2\sqrt{2}}$

(B) $x = \pm\dfrac{9}{2\sqrt{2}}$

(C) $x = \pm 3$

(D) $x = \pm\dfrac{2\sqrt{2}}{9}$

9. The region bounded by the curves $y = x^2$ and $y = x^3$ in the first quadrant is rotated about the y-axis. Which integral could be used to find the volume of the solid of revolution formed?

(A) $V = \pi \int_0^1 \left(y^{\frac{1}{3}} - y^{\frac{1}{2}} \right) dy$

(B) $V = \pi \int_0^1 \left(y^{\frac{1}{2}} - y^{\frac{1}{3}} \right) dy$

(C) $V = \pi \int_0^1 \left(y^{\frac{2}{3}} - y \right) dy$

(D) $V = \pi \int_0^1 \left(x^4 - x^6 \right) dx$

10. The points $P(a\cos\theta,\, b\sin\theta)$ and $Q(a\cos\phi,\, b\sin\phi)$ lie on the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ and the chord PQ subtends a right angle at (0,0). Which of the following is the correct expression?

(A) $\tan\theta\tan\phi = -\dfrac{b^2}{a^2}$

(B) $\tan\theta\tan\phi = -\dfrac{a^2}{b^2}$

(C) $\tan\theta\tan\phi = \dfrac{b^2}{a^2}$

(D) $\tan\theta\tan\phi = -\dfrac{a^2}{b^2}$

Section II

90 marks
Attempt Questions 11 to 16
Allow approximately 2 hours and 45 minutes for this section.

Write your answers in the writing booklets supplied. Additional writing booklets are available.
Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 Marks) Use a SEPARATE writing booklet **Marks**

(a) Find $\displaystyle\int \frac{\sin x}{\cos^3 x}\,dx$ **2**

(b) By splitting the integral find $\displaystyle\int \frac{3x+1}{x^2+2x+3}\,dx$ **3**

(c) Find $\displaystyle\int x\,\tan^{-1}(x)\,dx$ **3**

(d) Find $\displaystyle\int \frac{2\,dx}{x^3+x^2+x+1}$ **3**

(e) Use the substitution $t = \tan\frac{x}{2}$ to find the exact value of **4**

$$\int_0^{\frac{\pi}{2}} \frac{dx}{5+4\cos x}$$

Question 12 (15 Marks) Use a SEPARATE writing booklet **Marks**

(a) If $p^2 = 24 - 70i$, express p in the form $a + bi$, where a and b are real. **2**

(b) Given that p and q are real and also that $1 - 4i$ is a root of the equation:

$$x^2 + (p + i)x + (q - 5i) = 0$$

i. Find the values of p and q. **2**

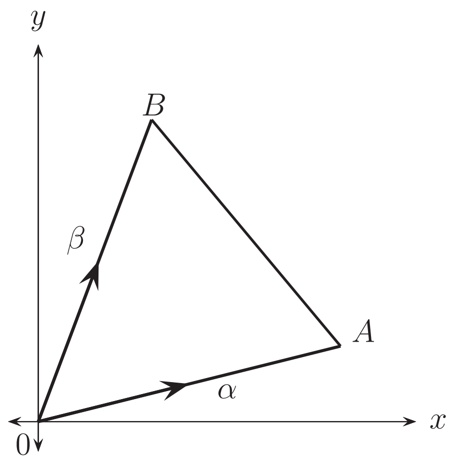
ii. Find the other root of the equation. **2**

(c) The complex number $z = x + iy$ is such that $\dfrac{z - 8i}{z - 6}$ is purely imaginary.

i. Find the locus of the point P representing z. **2**

ii. Sketch the locus of x on an Argand diagram. **2**

(d)



The diagram show the equilateral triangle OAB in the complex plane. O is the origin and the points A and B represent the complex numbers α and β respectively.

Let $\mu = \cos\dfrac{\pi}{3} + i\sin\dfrac{\pi}{3}$.

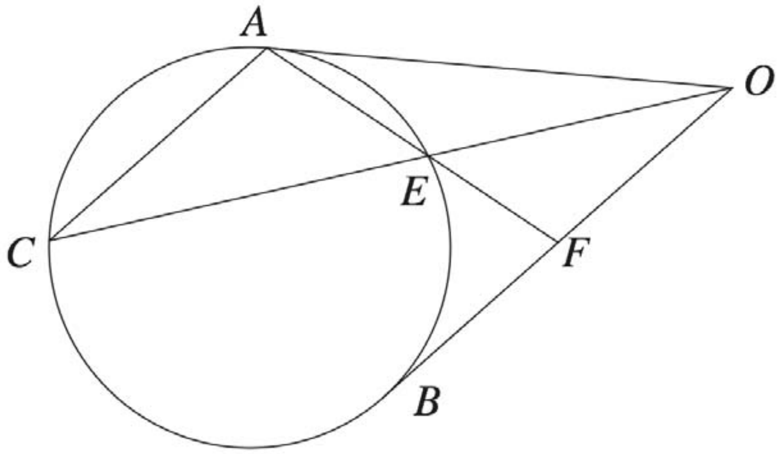
i. Write down the complex number $\overrightarrow{BA}$. **1**

ii. Show that $\alpha = \mu(\alpha - \beta)$. **2**

iii. Prove that $\alpha^2 + \beta^2 = \alpha\beta$. **2**

Question 13 (15 Marks) Use a SEPARATE writing booklet **Marks**

(a) Two tangents OA and OB are drawn from a point O to a given circle. Through A a chord AC is drawn parallel to the other tangent OB. OC meets the circle at E.



Copy the diagram into your answer booklet

i. Prove that the triangles AFO and EFO are similar. **3**

ii. Hence show that $OF^2 = AF \times EF$. **1**

iii. Hence prove that AE extended bisects OB. **2**

(b) Consider the function $f(x) = (5-x)(x+1)$. On separate axes sketch, using $\frac{1}{3}$ of a page, showing all important features, the graphs of:

i. $y = f(|x|)$ **1**

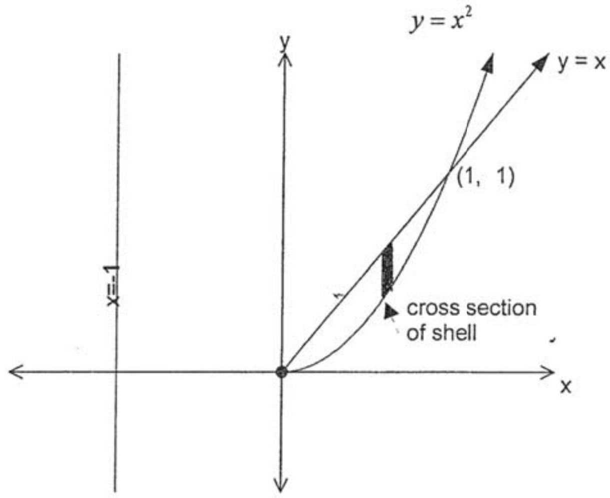
ii. $y = \dfrac{1}{f(x)}$ **1**

iii. $y^2 = f(x)$ **1**

(c) If $x^3 + 3mx + n = 0$ has a double root, prove that $n^2 = -4m^3$ **2**

Question 13 continues on page 10

(d) **4**



Use the method of **cylindrical shells** to calculate the volume of the solid formed when the area bounded by $y = x$ and $y = x^2$ is rotated about the line $x = -1$.

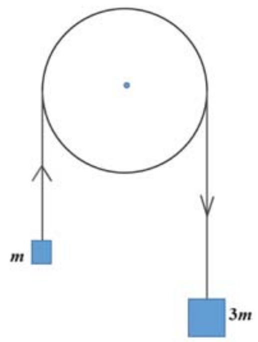
Question 14 (15 Marks) Use a SEPARATE writing booklet **Marks**

(a) Factorise $P(x) = 2(x+1)^2(2x+1)+1$. **2**

(b) Prove by mathematical induction that for all integer values of n, **3**

$$\tan^{-1}\frac{1}{2\times 1^2}+\tan^{-1}\frac{1}{2\times 2^2}+\cdots+\tan^{-1}\frac{1}{2n^2}=\frac{\pi}{4}-\tan^{-1}\frac{1}{2n+1}$$

(c) Particles of mass m and $3m$ kilograms are connected by a light inextensible string which passes over a smooth fixed pulley. The string hangs vertically on each side, as shown in the diagram.



The particles are released from rest and move under the influence of gravity. The air resistance on each particle is kv Newtons, when the speed of the particles is $v\ ms^{-1}$ and the acceleration due to gravity is $g\ ms^{-2}$ and is taken as positive throughout the question and is assumed to be constant. k is a positive constant.

i. Draw diagrams to show the forces acting on each particle. **2**

ii. Show that the equation of motion is: **2**

$$\ddot{x}=\frac{mg-kv}{2m}$$

iii. Find the terminal velocity V or maximum speed of the system stating your answer in terms of m, g and k. **1**

iv. Prove that the time elapsed since the beginning of the motion is given by: **3**

$$t=\frac{2m}{k}\ln\left|\frac{mg}{mg-kv}\right|$$

v. If the bodies attain a velocity equal to half of the terminal speed, show by using the results in iii. and iv. that the time elapsed is equal to $\frac{V}{g}\ln 4$, where V is the terminal velocity. **2**

Question 15 (15 Marks) Use a SEPARATE writing booklet **Marks**

(a) Given that $I_n = \int x^n e^{2x}\,dx$

i. Show that $I_n = \dfrac{x^n e^{2x}}{2} - \dfrac{n}{2} I_{n-1}$. **3**

ii. Use the above result to find $\int x^2 e^{2x}\,dx$. **2**

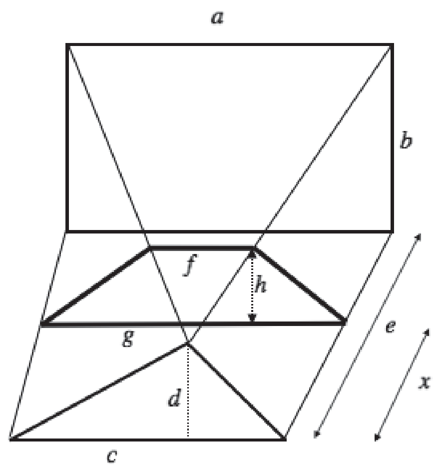
(b) If α, β and γ are the roots of $x^3 + 5x^2 - 2x - 3 = 0$:

i. Find $\alpha^2 + \beta^2 + \gamma^2$. **1**

ii. Find $\alpha^3 + \beta^3 + \gamma^3$. **2**

iii. Find the equation whose roots are $\dfrac{1}{\alpha}$, $\dfrac{1}{\beta}$ and $\dfrac{1}{\gamma}$. **2**

(c) As shown in the diagram below, a solid has parallel vertical ends and the base is horizontal. One end is a rectangle with length a and breadth b and the other end is a scalene triangle with base c and height d. The parallel ends are at a distance e apart. A typical slice is taken parallel to the ends at a distance x from the triangular end.



i. Let the top of the trapezium be f, the base of the trapezium be g and the height of the trapezium be h. Show that: **3**

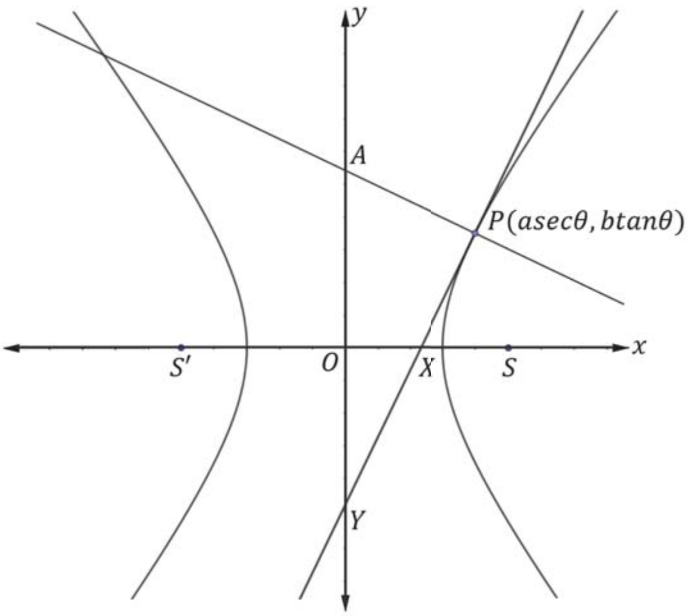
$$g = c + \left(\frac{a-c}{e}\right)x$$

and hence write down similar expressions for both f and h in terms of x.

ii. Hence find the volume of the solid. **2**

Question 16 (15 Marks) Use a SEPARATE writing booklet **Marks**

(a) The point $P(a\sec\theta,\, b\tan\theta)$ lies on the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ and S and S' are the foci.



The equations of the tangent and normal at P are as given below:

Tangent: $\dfrac{x\sec\theta}{a} - \dfrac{y\tan\theta}{b} = 1$

Normal: $\dfrac{ax}{\sec\theta} + \dfrac{by}{\tan\theta} = a^2 + b^2$ DO NOT PROVE THIS

Copy the diagram into your writing booklet.
The tangent at P intersects the x–axis at X and the y–axis at Y.

i. Show that $\dfrac{PX}{PY} = \sin^2\theta$ **3**

ii. Deduce that if P is an extremity of a latus rectum, then $\dfrac{PX}{PY} = \dfrac{e^2-1}{e^2}$. **1**

Let the normal at $P(a\sec\theta,\, b\tan\theta)$ intersect the y–axis at A.

iii. Prove that $\triangle ASY$ is right angled at S **2**

iv. Explain why $\angle PSA = \angle PS'A$ **2**

Question 16 continues on page 14

(b) Newton's method may be used to determine numerical approximations to find the value of $\sqrt[3]{2}$. This can be done by finding the real roots of the equation $x^3 - 2 = 0$.

Let $x_1,\ x_2,\ x_3, \ldots, x_n, \ldots$ be the series of estimators obtained by iterative applications of Newton's method.

i. Taking x_n as the first root, use Newton's method to show that: **1**

$$x_{n+1} = \frac{2}{3}\left(x_n + \frac{1}{(x_n)^2}\right)$$

ii. Show algebraically that: **2**

$$x_{n+1} - \sqrt[3]{2} = \frac{\left(x_n - \sqrt[3]{2}\right)^2\left(2x_n + \sqrt[3]{2}\right)}{3(x_n)^2}$$

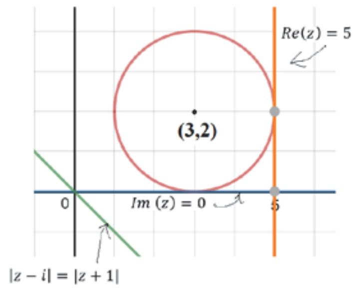
iii. Given that $x_n > \sqrt[3]{2}$ show that: **2**

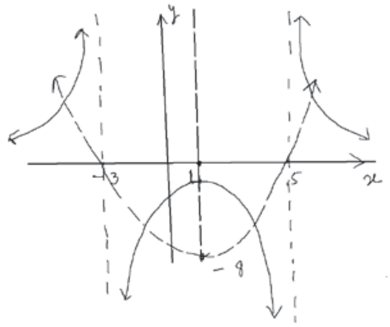
$$x_{n+1} - \sqrt[3]{2} < \left(x_n - \sqrt[3]{2}\right)^2$$

iv. Show that x_{12} and $\sqrt[3]{2}$ agree to at least 267 decimal places. **2**

End of Examination ☺

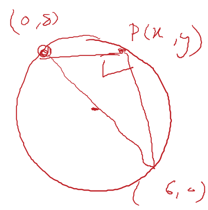
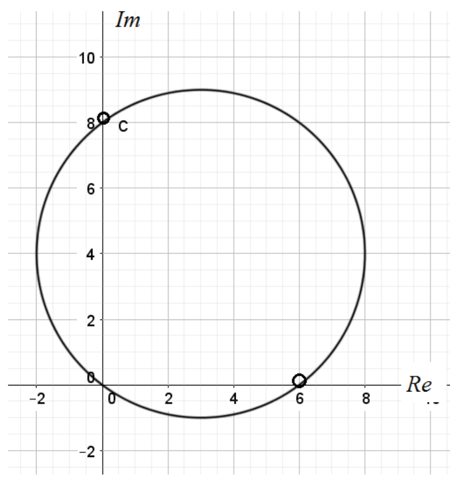
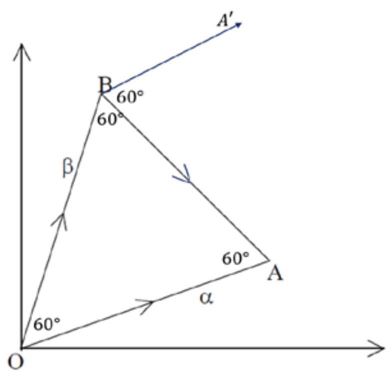
KHS Extension 2 Trial Marking Scheme

<table>
<tr><td colspan="4">Multiple Choice</td></tr>
<tr><td>1.</td><td>


By elimination, A and C has only one solution and C does not have any.
Hence, the solution is D</td><td>D</td><td></td></tr>
<tr><td>2.</td><td>
$(z_1 + z_2 + z_3)^2 \quad = 0$
$z_1^2 + z_2^2 + z_3^2 + 2(z_1z_2 + z_1z_3 + z_2z_3) = 0$
$z_1^2 + z_2^2 + z_3^2 + 2z_1z_2z_3\left(\frac{1}{z_3} + \frac{1}{z_2} + \frac{1}{z_1}\right) = 0$
$z_1^2 + z_2^2 + z_3^2 + 2z_1z_2z_3(\overline{z_1} + \overline{z_2} + \overline{z_3}) = 0$
$z_1^2 + z_2^2 + z_3^2 + 2z_1z_2z_3\left(\overline{z_1 + z_2 + z_3}\right) = 0$
$z_1^2 + z_2^2 + z_3^2 + 2z_1z_2z_3(0) = 0$
$z_1^2 + z_2^2 + z_3^2 = 0$</td><td>C</td><td></td></tr>
<tr><td>3.</td><td>
$y = \frac{1}{ax^2 + bx + c}$
Asymptotes at $x = 5$ and $x = -3$
$\therefore \frac{1}{ax^2 + bx + c} = \frac{1}{k(x-5)(x+3)}$


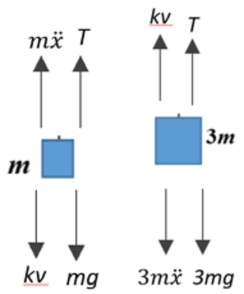
Hence, when $x = 1, y = -\frac{1}{8}$
$\frac{1}{k(1-5)(1+3)} = -\frac{1}{8}$
$\therefore -16k = -8$
$\therefore k = \frac{1}{2}$
$\frac{1}{\frac{1}{2}(x-5)(x+3)} = \frac{1}{\frac{1}{2}(x^2-2x-15)}$</td><td>B</td><td></td></tr>
</table>

	Hence, $a = \frac{1}{2}, \quad b = -1$ and $c = -\frac{15}{2}$		
4.	$\int \sqrt{1+sinx}\,dx$ $= \int \frac{\sqrt{1+sinx}\sqrt{1-sinx}}{\sqrt{1-sinx}}\,dx$ $= \int \frac{cosx}{\sqrt{1-sinx}}\,dx$ $= -2\sqrt{1-sinx} + C$	A	
5.	30 distinct toys need to be equally divided into 10 packets. Number of toys in each packet = $\frac{30}{10} = 3$ Since packets do not have distinct identity, we can consider that all groups are identical (not distinct). i.e., we need to divide 30 distinct toys into 10 identical groups containing 3 toys each. $\therefore$ Total no. of ways = $\frac{30!}{10!\times(3!)^{10}}$	**D**	
Question 12			
a)(i)	$\rho^2 = 24 - 70i$ $= 7^2 - 2\times7\times5i + (5i)^2$ $= (7-5i)^2$ $\therefore \rho = \pm(7-5i)$ $x^2 - y^2 = 24$ $x^2 + y^2 = 74$ Solve for x and y. $x^2+y^2 = \sqrt{(x^2-y^2)^2+(2xy)^2}$ $= \sqrt{24^2+70^2}$ $= 74$	1 mark: attempts to express $x^2 - y^2 = 24$ and $xy = 35$ 1 mark: gives the answer in the form $\pm(a+ib)$ using their x and y.	Many students used the more laborious method to solve this problem You need to learn easier methods
b(i)	$x^2 + (p+i)x + (q-5i) = 0$ $1-4i$ is a root. Substitute $1-4i$ $\therefore (1-4i)^2 + (p+i)(1-4i) + q - 5i = 0$ $-15 - 8i + p - 4pi + i + 4 + q - 5i = 0$ Equating real and imaginary parts, $-15 + p + 4 + q = 0$ $\therefore p + q = 11 \quad (1)$ $-8 - 4p + 1 - 5 = 0$ $\therefore p = -3$ $\therefore p = 14$, q = 14	1 mark: substitutes $1-4i$ into the equation and separates real and imaginary parts 1 mark: Solves for p and q.	Some students had difficulty in separating real and imaginary parts
(ii)	$x^2 + (-3+i)x + (14-5i) = 0$ Root sum $1 - 4i + \beta = 3 - i$ $\therefore \beta = 2 + 3i$	2 marks: correct answer from correct working 1 mark: minor error in calculations	Students who used sums and product of roots method were more successful. The coefficients are not real. Hence the conjugate can't be a root

c)(i)	$\frac{z-8i}{z-6}$ is purely imaginary $$\frac{z-8i}{z-6}\times\frac{\overline{z-6}}{\overline{z-6}}$$ $$\frac{z-8i}{z-6}\times\frac{\bar{z}-6}{\overline{z-6}}$$ $$\frac{z\bar{z}-6z-8i\bar{z}+48i}{\lvert z-6\rvert^2}$$ $$\frac{x^2+y^2-6(x+iy)-8i(x-iy)+48i}{(x-6)^2+y^2}$$ Is purely imaginary. $$\therefore x^2+y^2-6x-8y=0$$ $$x^2-6x+9+y^2-8y+16=25$$ $$(x-3)^2(y-4)^2=25$$ This is a circle with centre (3,4) and radius 5. $\arg\left(\frac{z-8i}{z-6}\right)=\pm\pi/2$. This means the line joining z, P(x,y) to A(0,8) and (6,0) subtends 90° at P. ie AB is the diameter of the circle	2 marks: Correct answer from correct working 1 mark: minor error	Again, use easier methods compared to the algebra-heavy approach. 
(ii)		2 marks: correct diagram 1 mark: (0,8) and (6,0) are not excluded	Students need to exclude (0,8) and (6,0) as z cannot be on the extremities of the diameter.
d)(i)	$\overrightarrow{BA}=\alpha-\beta$	1 mark: correct vector representation	
(ii)		1 mark: correct diagram 1 mark: Expresses $\overrightarrow{BA'}$ in terms of $\overrightarrow{BA}$ and hence the result	Drawing the diagram is quite essential in this case. Students should draw BA' and explain showing $\overrightarrow{BA'}=\overrightarrow{OA}$

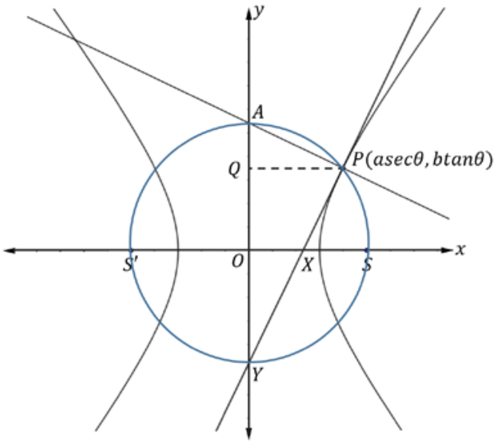
	$\mu\,\overrightarrow{BA} = \overrightarrow{BA'} = \alpha$ $\mu\,(\alpha - \beta) = \alpha$ from (i)		
(iii)	Now, $\beta = \mu\alpha$ $\therefore \dfrac{\alpha}{\beta} = \dfrac{\alpha - \beta}{\alpha}$ using (ii) or $\alpha^2 = \alpha\beta - \beta^2$ $\alpha^2 + \beta^2 = \alpha\beta$	1 mark: Expresses $\overrightarrow{OB}$ in terms of $\overrightarrow{OA}$ 1 mark: writes two expressions for μ and equates to get the result	In this question, you need to prove the result; substituting into it is not sufficient. The idea is to eliminate μ . this must be understood by the students first. $\beta = \mu\alpha$ $\mu(\alpha - \beta) = \alpha$

Question 14			
a)(i)	$P(x) = 2(x+1)^2(2x+1)+1$ $= 4x^3 + 10x^2 + 8x + 3$ $P\left(-\frac{3}{2}\right) = 0$ Hence $2x+3$ is a factor $\begin{array}{c\|cccc} -\frac{3}{2} & 4 & 10 & 8 & 3 \\ & & -6 & -6 & -3 \\ \hline & 4 & 4 & 2 & 0 \end{array}$ $4x^3 + 10x^2 + 8x + 3 =$ $(2x+3)(2x^2+2x+1)$	1 mark: correct linear factor 1 mark: divides by the linear factor and hence the factored form Or use comparison of coefficients	Badly done. Many students expanded the polynomial, but did not bother to factorise it.
(ii)	Step 1 For n = 1, needs to prove $\tan^{-1}\frac{1}{2\times1^2} = \frac{\pi}{4} - \tan^{-1}\frac{1}{2\times1+1}$ Consider $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3}$ $tan\left(\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3}\right)=$ $\frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\times\frac{1}{3}}$ = 1 $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3} = \frac{\pi}{4}$ Hence, the result is true for n = 1. Step 2 Assume the result is true for n = k. Hence, $s_k = \tan^{-1}\frac{1}{2\times1^2} + \tan^{-1}\frac{1}{2\times2^2} + \cdots + \tan^{-1}\frac{1}{2k} = \frac{\pi}{4} - \tan^{-1}\frac{1}{2k+1}$ To prove the result is true for n = k+1, we need to prove that Step 3 $s_k + \tan^{-1}\frac{1}{2(k+1)^2} =$ $= \frac{\pi}{4} - \tan^{-1}\frac{1}{2k+1} + \tan^{-1}\frac{1}{2(k+1)^2}$ Ie. we need to prove	3 marks: correct proof 2 marks: Step 1 proves accurately. must demonstrate the use of tan(A+B) formula, writes the results for n=k and n=k+1 and makes significant progress. 1 mark: proves the result for n = 1 (not necessarily using tan(A+B) and writes the result for n=k and n=k+1	

	$\tan^{-1}\frac{1}{2(k+1)^2} - \tan^{-1}\frac{1}{2k+1} = \tan^{-1}\frac{1}{2k+3}$ Consider, $\tan\left(\tan^{-1}\frac{1}{2(k+1)^2} - \tan^{-1}\frac{1}{2k+1}\right)$ $= \frac{\frac{1}{2(k+1)^2} - \frac{1}{2k+1}}{1 - \frac{1}{2(k+1)^2} \times \frac{1}{2k+1}}$ $= -\frac{2k^2+2k+1}{2(k+1)^2(2k+1)+1}$ $= -\frac{2k^2+2k+1}{(2k+3)(2k^2+2k+1)} = \frac{-1}{2k+3}$ Hence, if S_k is true, then S_{k+1} is true. Hence, using principle of mathematical induction, the result holds good.		
b)(i)	 	1 mark each: Correct free body diagrams	Students need to draw separate force diagrams for each particle
(ii)	On the body of mass m kg: $m\ddot{x} = T - kv - mg$ (1) On the body of mass *3m* kg: $3m\ddot{x} = 3mg - kv - T$ (2) 1 mark From (1), $T = m\ddot{x} + kv + mg$ Sub. In (2), $3m\ddot{x} = 3mg - kv - m\ddot{x} - kv - mg$ $4m\ddot{x} = 2mg - 2kv$ $\ddot{x} = \frac{2mg-2kv}{4m}$ $\ddot{x} = \frac{mg-kv}{2m}$	1 mark: writes the correct force equations for each particle. 1 mark: Eliminates T and proves the result (the key part of the question is to realise that the tension in the string is the same throughout the string)	In Mathematics, force equations for each body must be written before deriving the results -All results must be proved, <u>not</u> use formulae Many students resorted to fudging their answer

(iii)	For terminal velocity, $\ddot{x} = 0$ $mg - kv = 0$ $v = \frac{mg}{k}$ Hence, $V = \frac{mg}{k}$	1 mark: sets $\ddot{x} = 0$ and makes v subject. (must show working)	Well done
(iv)	From (ii), $\frac{dv}{dt} = \frac{mg-kv}{2m}$ $dt = \frac{2m}{mg - kv}dv$ $\int_0^t dt = \int_0^v \frac{2m}{mg - kv}dv$ $t = -\frac{2m}{k}\int_0^v \frac{1\times -k}{mg - kv}dv$ $= -\frac{2m}{k}[ln(mg - kv)\ ]_0^v$ $= -\frac{2m}{k}\ln\left(\frac{mg - kv}{mg}\right)$ $= \frac{2m}{k}\ln\left\|\frac{mg}{mg - kv}\right\|$ Alternate working: $\frac{dv}{dt} = \frac{mg-kv}{2m}$ $dt = \frac{2m}{mg - kv}dv$ $\int dt = \int \frac{2m}{mg - kv}dv$ $t = -\frac{2m}{k}\int \frac{1\times -k}{mg - kv}dv$ $= -\frac{2m}{k}[ln(mg - kv)] + C$ When t = 0, $v = 0$. $0 = -\frac{2m}{k}[ln(mg)] + C$ $C = \frac{2m}{k}[ln(mg)]$ $\therefore t = -\frac{2m}{k}[ln(mg - kv)] + \frac{2m}{k}[ln(mg)]$ Ie. $t = \frac{2m}{k}\ln\left\|\frac{mg}{mg - kv}\right\|$ $= -\frac{2m}{k}\ln\left(\frac{mg - kv}{mg}\right)$	3 marks: correct proof 2 marks: Correctly integrates, but minor error in evaluating C, the constant of integration and hence incorrect result. 1 mark: expresses $\ddot{x} = \frac{mg-kv}{2m}$ as $\frac{dv}{dt} = \frac{mg-kv}{2m}$ and attempts to integrate.	Many students did not bother to use the result given in (ii) to use in this question.
(v)	$t = \frac{2m}{k}\ln\left\|\frac{mg}{mg - kv}\right\|$ When $v = \frac{mg}{2k} = \frac{V}{2}$, $t = \frac{2m}{k}\ln\left\|\frac{mg}{mg - k\times\frac{mg}{2k}}\right\|$	1 mark: Substitutes $v = \frac{mg}{2k}$ into the expression for time t in (iv) and simplifies	Well done

	$= \dfrac{2m}{k}\ln\left\|\dfrac{mg}{\frac{mg}{2}}\right\|$ $= \dfrac{2m}{k}\ln 2$ But $\frac{m}{k} = \frac{V}{g}$ $t = \dfrac{2V}{g}\ln 2$ $= \dfrac{V}{g}2\ln 2$ $= \dfrac{V}{g}\ln 4$	1 mark: expresses the result in terms of the escape velocity *V* and proves the resiult	

Question 16			
a)(i)	 Tangent is $\frac{x\sec\theta}{a} - \frac{y\tan\theta}{b} = 1$ Y – intercept: $x = 0$ $y = -\frac{b}{\tan\theta}$ $Y\left(0, -\frac{b}{\tan\theta}\right)$ and $Q(0, b\tan\theta)$ In ΔOXY and ΔOPQ, $\frac{XY}{PY} = \frac{OY}{OQ} = \frac{\frac{b}{\tan\theta}}{\frac{b}{\tan\theta} + \tan\theta}$ $= \frac{b}{b + b\tan^2\theta}$ $= \cos^2\theta$ ① $\frac{PY}{XY} = \frac{PX + XY}{XY} = \sec^2\theta$ $= \frac{PX}{XY} + 1 = \sec^2\theta$ $\therefore \frac{PX}{XY} = \tan^2\theta$ ② ①×② gives $\frac{PX}{PY} = \sin^2\theta$	1 mark: calculating the coordinates of X and Q and writing the similarity ratio 1 mark: for developing ① 1 mark: for developing ② and hence the final result.	Again, use easier methods than resorting to long trigonometric derivations
(ii)	If SP is the latus rectum, then $a\sec\theta = ae$ $\therefore \sec\theta = e$ $\therefore \frac{PX}{PY} = \frac{e^2 - 1}{e^2}$	1 mark: for correct proof with working	
(iii)	y-intercept of normal: $\frac{ax}{\sec\theta} + \frac{by}{\tan\theta} = a^2 + b^2$ $\frac{ax}{\sec\theta} + \frac{by}{\tan\theta} = a^2 + b^2$		Well done.

	$x = 0, \quad \dfrac{by}{tan\theta} = a^2 + b^2$ $by = (a^2 + b^2)tan\theta$ $= a^2e^2 \ tan\theta$ $\therefore \ y = \dfrac{a^2e^2 \ tan\theta}{b}$ $A\left(0, \dfrac{a^2e^2 \ tan\theta}{b}\right)$ and $S(ae, 0)$ $m_{SA} = \dfrac{0 - \dfrac{a^2e^2 \ tan\theta}{b}}{ae - 0}$ $-\dfrac{ae \ tan\theta}{b}$ $m_{SY} = \dfrac{0 + \dfrac{b}{tan\theta}}{ae - 0} = \dfrac{b}{aetan\theta}$ $m_{SA} \times m_{SY} = \dfrac{aetan\theta}{b} \times \dfrac{b}{aetan\theta}$ $= -1$ $SA \perp SY$ $\quad \Delta ASY$ is right-angled at S.	1 mark: calculates either of the gradients 1 mark: proves the result	
(iv)	$\angle APY = \angle ASY = 90°$ from (iii) and angle between tangent and normal equals 90° Hence APSY is a cyclic quadrilateral and by symmetry S' also should lie on it. $\angle ASP = \angle AS'P$ (angles in the same segment of arc AP are equal. 1 mark	1 mark: Proves that APSY are con-cyclic 1 mark: proves the result with reasoning	
b)(i)	$f(x) = x^3 - 2$ $f'(x) = 3x^2$ $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$ $x_{n+1} = x_n - \dfrac{x_n{}^3 - 2}{3x_n{}^2}$ $= \dfrac{3x_n{}^3 - x_n{}^3 + 2}{3x_n{}^2}$ $= \dfrac{3x_n{}^3 + 2}{3x_n{}^2}$ $= \frac{2}{3}\left(x_n + \frac{1}{x_n^2}\right)$	1 mark: correct application of Newton's method to $x^3 - 2 = 0$ and proves the result.	Well done
(ii)	$\dfrac{\left(x_n - \sqrt[3]{2}\right)^2 (2x_n + \sqrt[3]{2})}{3x_n{}^2}$ $= \dfrac{\left(x_n{}^3 - 2\sqrt[3]{2}x_n + 2^{\frac{2}{3}}\right)(2x_n + \sqrt[3]{2})}{3x_n{}^2}$ $= \dfrac{\begin{pmatrix} 2x_n{}^3 - 4\sqrt[3]{2}x_n{}^2 + 2^{\frac{5}{3}}x_n + \\ \sqrt[3]{2}x_n{}^3 - 2^{\frac{5}{3}}x_n + 2 \end{pmatrix}}{3x_n{}^2}$	2 marks: Correct proof 1 mark: significant progress to the result	

	$= \frac{2}{3}x_n - \sqrt[3]{2} + \frac{2}{3{x_n}^2}$ $= \frac{2}{3}\left(x_n - \frac{1}{{x_n}^2}\right) - \sqrt[3]{2}$ $= x_{n+1} - \sqrt[3]{2}$		
(iii)	$x_n > \sqrt[3]{2} \Rightarrow x_n - \sqrt[3]{2} > 0$ $x_{n+1} - \sqrt[3]{2} = \frac{(x_n - \sqrt[3]{2})^2(2x_n + \sqrt[3]{2})}{3{x_n}^2}$ $< \frac{(x_n - \sqrt[3]{2})^2(2x_n + x_n)}{3{x_n}^2}$ $\boldsymbol{1\ mark}$ $= \frac{(x_n - \sqrt[3]{2})^2}{x_n}$ $< (x_n - \sqrt[3]{2})^2$ as $x_n > \sqrt[3]{2} > 1$ Hence $\frac{1}{x_n} < 1$ **1 mark** $\therefore x_{n+1} - \sqrt[3]{2} < (x_{n+1} - \sqrt[3]{2})^2$	1 mark: substitutes $\sqrt[3]{2}$ with x_n and gives the inequality 1 mark: proves $\frac{1}{x_n} < 1$ and gives the required result	
(iv)	Using (iii) $x_2 - \sqrt[3]{2} < (x_1 - \sqrt[3]{2})^2$ Applying \$x_1 = 2$, $x_2 - \sqrt[3]{2} < (2 - \sqrt[3]{2})^2$ $x_3 - \sqrt[3]{2} < (x_2 - \sqrt[3]{2})^2$ $< (2 - \sqrt[3]{2})^4 = (2 - \sqrt[3]{2})^{2^2}$ $x_4 - \sqrt[3]{2} < (x_3 - \sqrt[3]{2})^2$ $< (2 - \sqrt[3]{2})^8 = (2 - \sqrt[3]{2})^{2^3}$: $x_{12} - \sqrt[3]{2} < (x_3 - \sqrt[3]{2})^2$ $< (2 - \sqrt[3]{2})^8 = (2 - \sqrt[3]{2})^{2^{11}}$ = 1.9118×10^{-268} = 0.000000….019118… (267 zeroes) x_{12} and $\sqrt[3]{2}$ agrees to 267 decimal places	2 mark: correct proof 1 mark: for making significant progress in developing the sequence.	

Question 11 (15 Marks)

(a) **1 mark** Makes a substitution or equivalent working
1 mark Correct result

$$\int \frac{\sin x}{\cos^3 x}\,dx$$

$$\text{Let } u = \cos x \implies du = -\sin x\,dx$$

$$\begin{aligned}
\therefore \int \frac{\sin x}{\cos^3 x}\,dx &= -\int \frac{du}{u^3} \\
&= \frac{1}{2}u^{-2} + C \\
&= \frac{1}{2\cos^2 x} + C \\
\text{alternatively} &= \frac{\tan^2 x}{2} + C
\end{aligned}$$

(b) **1 mark** Splits the integral correctly
1 mark Evaluates the log part correctly for their split
1 mark Correctly evaluates the $\tan^{-1}$ part for their integral

$$\begin{aligned}
\int \frac{3x+1}{x^2+2x+3}\,dx &= \int \left(\frac{3(2x+2)}{2(x^2+2x+3)} - \frac{2}{x^2+2x+3}\right)\,dx \\
&= \frac{3\log(x^2+2x+3)}{2} - 2\int \frac{1}{(x+1^2)+2}\,dx \\
&= \frac{3\log(x^2+2x+3)}{2} - \sqrt{2}\tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right) + C
\end{aligned}$$

(c) **1 mark** Substituting into parts formula correctly
1 mark Evaluating $\tan^{-1}$ component correctly
1 mark Correctly evaluating $\int \frac{x^2}{2(x^2+1)}\,dx$

$$\int x\,\tan^{-1}(x)\,dx$$

$$u = \tan^{-1} x \qquad v = \frac{x^2}{2}$$

$$u' = \frac{1}{x^2+1} \qquad v' = x$$

$$\begin{aligned}
\therefore \int x\,\tan^{-1}(x)\,dx &= \frac{x^2}{2}\tan^{-1} x - \int \frac{x^2}{2(x^2+1)}\,dx \\
&= \frac{x^2}{2}\tan^{-1} x - \frac{1}{2}\int 1 - \frac{1}{(x^2+1)}\,dx \\
&= \frac{x^2}{2}\tan^{-1} x - \frac{1}{2}\left(x - \tan^{-1}(x)\right) \\
&= \frac{1}{2}\left[\left(\left(x^2+1\right)\tan^{-1}(x) - x\right] + C
\end{aligned}$$

(d) **1 mark** Correctly factorising and expressing in partial fractions
1 mark Splitting into three distinct parts
1 mark Correctly evaluating all parts

$$\begin{aligned}
\frac{2}{x^3+x^2+x+1} &= \frac{2}{x^2(x+1)+1(x+1)}\\
&= \frac{2}{(x^2+1)(x+1)}\\
&\equiv \frac{ax+b}{x^2+1}+\frac{c}{x+1}\\
\therefore (ax+b)(x+1)+c(x^2+1) &= 2\\
\text{Put } x &= -1\\
\therefore 2c = 2 &\implies c=1\\
\text{Now } ax^2+ax+bs+b+x^2+1 &= 2\\
\therefore (a+1)x^2+x(a+b)+b+1 &= 2\\
a+1=0 \qquad &\implies a=-1\\
a+b=0 \qquad &\implies b=1\\
\therefore \int \frac{2}{x^3+x^2+x+1}\,dx &= \int \frac{1-x}{x^2+1}+\frac{1}{x+1}\,dx\\
&= -\int \frac{x-1}{x^2+1}\,dx+\int\frac{1}{x+1}\,dx\\
&= -\int \frac{x}{x^2+1}\,dx+\int\frac{1}{x^2+1}\,dx+\log(x+1)\\
&= -\frac{1}{2}\log(x^2+1)=\tan^{-1}x=\log(x+1)+C
\end{aligned}$$

Markers Comment: Most did this very well - a few errors when people skipped steps or failed to recognise standard integrals.

(e) **2 marks** Changing the limits and finding dx in terms of t

1 mark Correct substitution leading to $\int_0^1 \frac{2\,dt}{t^2+9}$

1 mark Correctly evaluating their integral

$$\int_0^{2\pi} \frac{dx}{5+4\cos x}\,dx$$

$$t = \tan x2 \implies \cos x = \frac{1-t^2}{1+t^2}$$

$$\therefore x = \tan^{-1}(2t)$$

$$\therefore dx = \frac{2\,dt}{1+t^2}$$

$$x = 0 \implies t = 0$$

$$x = \frac{\pi}{2} \implies t = 1$$

$$\therefore \int_0^{2\pi} \frac{dx}{5+4\cos x}\,dx = \int_0^1 \frac{\frac{2\,dt}{1+t^2}}{5+4\left[\frac{1-t^2}{1+t^2}\right]}$$

$$= \int_0^1 \frac{\frac{2\,dt}{1+t^2}}{\frac{5(1+t^2)+4(1-t^2)}{1+t^2}}$$

$$= \int 0^1 \frac{2\,dt}{t^2+9}$$

$$= 2\left[\frac{1}{3}\tan^{-1}\left(\frac{t}{3}\right)\right]_0^1$$

$$= \frac{2}{3}\tan^{-1}\left(\frac{1}{3}\right)$$

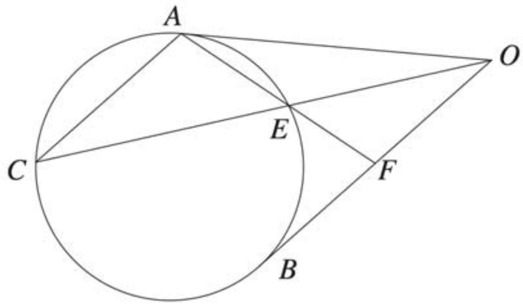
Markers Comment: Very well done.

Question 13 (15 Marks)

(a) i. **3 marks** Correct proof with all correct reasons using correct terminology

2 marks Correct proof with 2 correct reasons

1 mark Significant progress towards proof with correct reasons



$\angle OAF = \angle ACO$ (the angle between a tangent and a chord through the point of contact is equal to the angle in the alternate segment)

$\angle OAF = \angle ACO = \angle COB$ (alternate angles $AC \parallel OB$)

In $\triangle AFO$and $\triangle EFO$

$\angle AFO = \angle EFO$ (common)

$\angle OAF = \angle EOF$ (proven above)

$\therefore \angle AOF = \angle OEF$ (angle sum of a triangle)

$\therefore \triangle FAO \,|||\, \triangle FOE$ (equiangluar)

Markers Comment: Generally quite well done but many still not being precise enough with proof - don't take the risk of losing marks

ii. **1 mark** Correct ratios with correct reasoning

$$\frac{FO}{FE} = \frac{FA}{OF} \quad \text{(corresponding sides in congruent triangles)}$$

$$\therefore OF^2 = EF \times AF$$

Markers Comment: Well done - but reason often not given

iii. **2 marks** Correct solution with correct reasoning

1 mark Significant progress but incorrect reasoning

But $AF \times FE = FB^2$ (The square of the length of the tangent fro external point is equal to the product o intercepts of the secant passing through this po

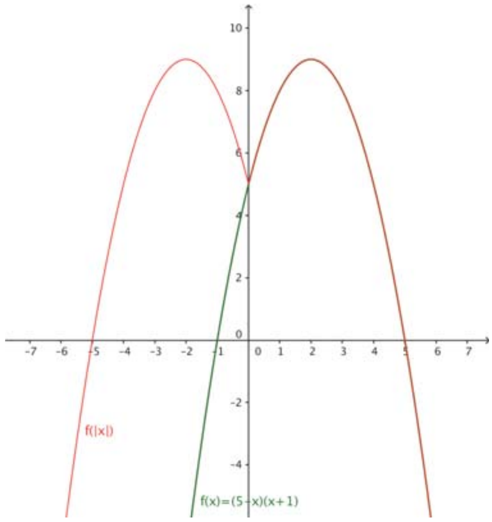
$$\therefore OF^2 = FB^2$$
$$\therefore OF = FB$$
$$\therefore AE \text{ extended bisects } OB$$

Markers Comment: Very poorly done - this is the forgotten circle geometry theorem - Learn it!

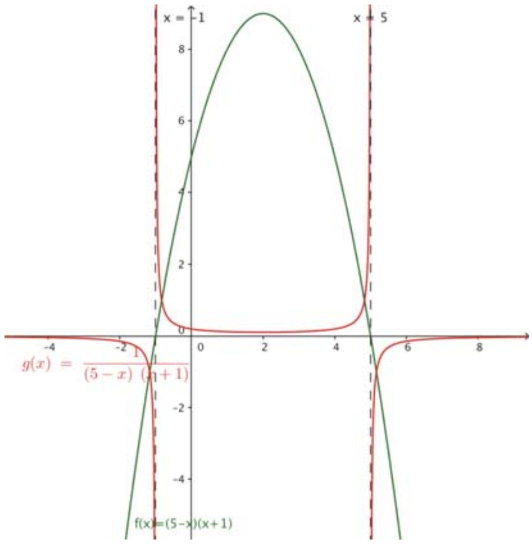
(b) For each of the graphs

1 mark Correct accurate shape with relevant intercepts and asymptotes labeled

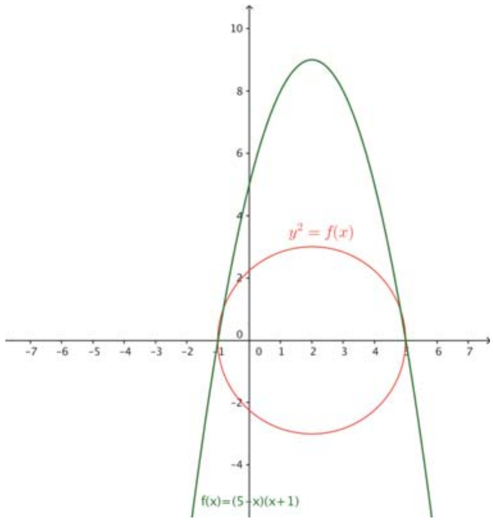
i.



ii.



iii.



Markers Comment: Most were fairly well done although some students still not accurate enough with their graphs

(c) **1 mark** Uses the double root property to establish a solution
1 mark Logical reasoning leading to the desired result

$$x^3 + 3x + n = 0$$

If has a double root then :

$$3x^2 + 3m = 0 \qquad \Longrightarrow x = \pm\sqrt{-m}$$

$$\therefore \left(\sqrt{-m}\right)^3 + 3m\left(\sqrt{-m}\right) + n = 0$$

$$\therefore -\sqrt{-m} + 3m\sqrt{-m} = -n$$

$$\therefore 2m\sqrt{-m} = -n$$

$$\therefore -4m^3 = n^2$$

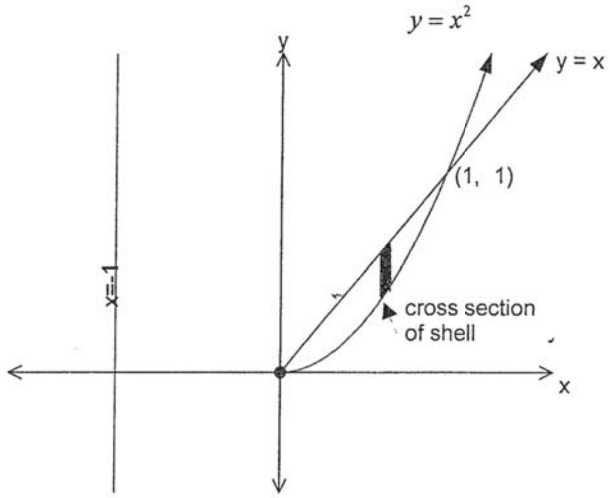
Markers Comment: Generally well done - no need to use complex numbers!

(d) **1 mark** Finds correct radius and height

1 mark Establishing the integral $V = 2\pi \int_0^1 (1+x)(x-x^2)\, dx$

1 mark Correct primitive function for their integral

1 mark Correct solution for their integral



$$\Delta V \cong 2\pi r h \delta x$$

Radius of a typical shell $= x + 1$ Height of a typical shell $= x - x^2$

$$\begin{aligned}
\therefore V &= 2\pi \int_0^1 (1+x)(x-x^2)\, dx \\
&= 2\pi \int_0^1 (x - x^3)\, dx \\
&= 2\pi \left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1 \\
&= 2\pi \left[\frac{1}{2} - \frac{1}{3}\right] \\
&= \frac{\pi}{2} units^3
\end{aligned}$$

Markers Comment: Generally very well done - those that made mistakes did so because they skipped steps

Question 15 (15 Marks)

(a) i. **1 mark** Splitting into parts correctly

1 mark Substituting into parts formula correctly

1 mark Resolving the parts formula to get desired reduction formula

$$I_n = \int x^n e^{2x}\, dx$$

$$u = x^n \qquad v = \frac{1}{2}e^{2x}$$
$$u' = nx^{n-1} \qquad v' = e^{2x}$$

$$\begin{aligned}\therefore I_n &= (x^n)\left(\frac{1}{2}e^{2x}\right) - \int \left(nx^{n-1}\right)\left(\frac{1}{2}e^{2x}\right)\, dx \\ &= \frac{x^n e^{2x}}{2} - \frac{n}{2}\int x^{n-1}e^{2x}\, dx \\ &= \frac{x^n e^{2x}}{2} - \frac{n}{2}I_{n-1}\end{aligned}$$

Markers Comment: Generally well done - some were not explicit enough with their parts integration and made errors.

ii. **1 mark** Getting to $\frac{1}{2}x^2e^{2x} - \left[\frac{1}{2}xe^{2x} - \frac{1}{2}I_0\right]$ or similar place

1 mark Correctly using reduction formula to obtain final result

$$\begin{aligned}I_2 &= \int x^2 e^{2x}\, dx \\ &= \frac{1}{2}x^2e^{2x} - I_1 \\ &= \frac{1}{2}x^2e^{2x} - \left[\frac{1}{2}xe^{2x} - \frac{1}{2}I_0\right] \\ &= \frac{1}{2}x^2e^{2x} - \frac{1}{2}xe^{2x} + \frac{1}{2}\int x^0e^{2x}\, dx \\ &= \frac{1}{2}x^2e^{2x} - \frac{1}{2}xe^{2x} + \frac{1}{4}e^{2x} + C \\ &= \frac{e^{2x}}{4}\left(2x^2 - 2x + 1\right) + C\end{aligned}$$

Markers Comment: Skipping steps lead to sign errors - show all working

(b) i. **1 mark** Correct result with working

$$x^3 + 5x^2 - 2x - 3 = 0$$
$$\therefore \sum \alpha^2 = (\alpha+\beta+\gamma)^2 - 2(\alpha+\beta+\gamma)$$
$$= 25 - 2(-2)$$
$$= 29$$

Markers Comment: Well done

ii. **1 mark** Generates the expression $\sum \alpha^3 + 15\sum \alpha^2 - 8\sum \alpha - 9 = 0$ or similar equivalent progress towards solutions

1mark substitutes in correctly to obtain desired result

Substitute the roots into the polynomial

$$\therefore \alpha^3 + 5\alpha^2 - 2\alpha - 3 = 0$$
$$\beta^3 + 5\beta^2 - 2\beta - 3 = 0$$
$$\gamma^3 + 5\gamma^2 - 2\gamma - 3 = 0$$
$$\therefore \sum \alpha^3 + 5\sum \alpha^2 - 2\sum \alpha - 9 = 0$$
$$\therefore \sum \alpha^3 = 9 - 5\sum \alpha^2 + 2\sum \alpha$$
$$= 9 - 5(29) + 2(-5)$$
$$= -146$$

Markers Comment: Some students made very careless errors trying to skip steps - some were very inefficent

iii. **1 mark** Attempts to substitute in roots and manipulate or equivalent significant progress producing the expression $\left(x - \frac{1}{\alpha}\right)\left(x - \frac{1}{\beta}\right)\left(x - \frac{1}{\gamma}\right) = 0$

1mark Obtains desired solution

If the equation has roots $\dfrac{1}{\alpha}, \dfrac{1}{\beta}$ and $\dfrac{1}{\gamma}$ then

$$\left(x - \frac{1}{\alpha}\right)\left(x - \frac{1}{\beta}\right)\left(x - \frac{1}{\gamma}\right) = 0$$
$$\therefore \left(\frac{1}{x} - \alpha\right)\left(\frac{1}{x} - \beta\right)\left(\frac{1}{x} - \gamma\right) = 0$$
$$\therefore \text{Equation is: } \frac{1}{x^3} + \frac{5}{x^2} - \frac{2}{x} - 3 = 0$$
$$\therefore 1 + 5x - 2x^2 - 3x^3 = 0$$
$$\therefore 3x^3 + 2x^2 - 5x - 1 = 0$$

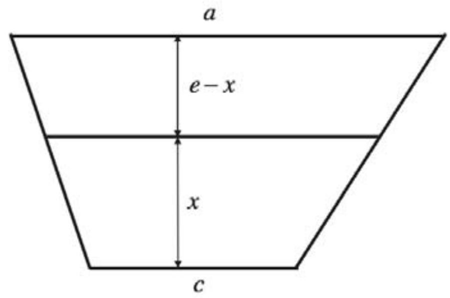
Markers Comment: Generally well done - but solution must be a polynomial in x

(c) i

2 marks Correctly deriving expression for g with appropriate reasoning.

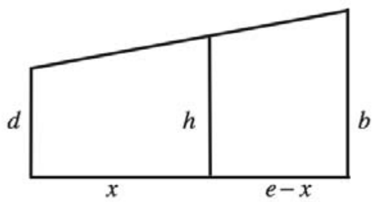
1 mark Corresponding expression for h and f

The base of the solid is a trapezium and it's area is the sum of the two trapezia.



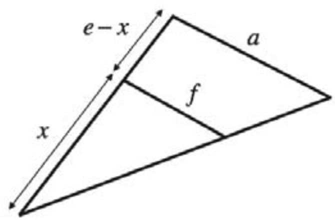
$$\therefore \frac{e}{2}(a+c) = \frac{x}{2}(g+c) + \frac{(e-x)}{2}(a+g)$$
$$\therefore e(a+c) = x(g+c) + (e-x)(a+g)$$
$$\therefore \cancel{ae} + ce = \cancel{gx} + cx + \cancel{ae} \cancel{- gx}$$
$$\therefore ge = ce + ax - cx \qquad \Longrightarrow g = c + \frac{a-c}{e}x$$

Similarly (using the same reason as part i.)



$$h = d + \left(\frac{b-d}{e}\right)x$$

and



$$f = \left(\frac{a}{e}\right)x$$

Markers Comment: Not very well done - many tried to use linear expressions without defining where any axes are - lots of fudging to get desired result. Most only got 1 mark

ii. **1 mark** Correct expression for $V(x)$

1 mark Significant progress towards the final solution

$$\text{Now } A(x) = \left\{c + \frac{(2a-c)}{e}x\right\}\left\{\frac{d + \frac{(b-d)}{e}x}{2}\right\}$$

$$\begin{aligned}
\therefore V(x) &= \int_0^e \left\{\frac{cd}{2} + \left(\frac{bc+2ad-2cd}{2e}\right)x + \left(\frac{2a-c}{2e}\right)\left(\frac{b-d}{e}\right)x^2\right\}.dx \\
&= \left[\left\{\frac{cd}{2}x + \left(\frac{bc+2ad-2cd}{2e}\right)\frac{x^2}{2} + \left(\frac{2a-c}{2e}\right)\left(\frac{b-d}{e}\right)\frac{x^3}{3}\right\}\right]_0^e \\
&= \left\{\frac{cd}{2}e + \left(\frac{bc+2ad-2cd}{2e}\right)\frac{e^2}{2} + \left(\frac{2a-c}{2e}\right)\left(\frac{b-d}{e}\right)\frac{e^3}{3}\right\} \\
&= \frac{e}{12}\{6cd + 3bc + 6ad - 4cd + 4ab - 4ad - 2bc + 2cd\} \\
&= \frac{e}{12}\{2cd + bc + 2ad + 4ab\}
\end{aligned}$$

Alternatively solve by Simpson's rule using $A_1 = \frac{1}{2}cd$, $A_2 = \left(\frac{a}{2} + \frac{a+c}{2}\right)\left(\frac{\frac{b+d}{2}}{2}\right)$ and $A_3 = ab$ which gives:

$$V = \frac{e}{6}\left\{\frac{1}{2}cd + 4\left(\frac{a}{2} + \frac{a+c}{2}\right)\left(\frac{\frac{b+d}{2}}{2}\right) + 2ab\right\}$$

yielding the same result.

Markers Comment: Quite poorly done - it seems as if many were intimidated by a simple integral - no one fully simplified and no one thought to use Simpson's rule since it is quadratic - much easier!