**Total Marks – 120**
**Attempt Questions 1-8**
**All questions are of equal value**

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

---

**Question 1** (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) Show that $\displaystyle\int_1^e \frac{(\log_e x)^3\,dx}{x} = \frac{1}{4}$ **2**

(b) (i) Write $1-\dfrac{1}{1+x^2}$ as a single fraction. **1**

(ii) Use integration by parts to find $\int 2x\tan^{-1} x\,dx$ **2**

(c) Use the table of standard integrals to help evaluate $\displaystyle\int \frac{dx}{\sqrt{x^2-6x+25}}$ **2**

(d) (i) Find $A$ and $B$ such that $\dfrac{2x^2+7x-10}{(x-4)(x+1)^2} = \dfrac{A}{x-4}+\dfrac{B}{(x+1)^2}$ **2**

(ii) Hence find $\displaystyle\int \frac{2x^2+7x-10}{(x-4)(x+1)^2}\,dx$ **2**

(e) Use the substitution $t=\tan\dfrac{x}{2}$ to evaluate $\displaystyle\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \operatorname{cosec} x\,dx$ **4**

**Question 2** (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) Let $z=3-2i$ and $w=4+i$.
Find in the form $x+iy$,

(i) $z\bar{z}$ **1**

(ii) $\frac{1}{w}$ **1**

(b) On an Argand Diagram the point $A$ is represented by $z=1+i$ and the point $B$ is represented by $\frac{1}{z}$.

(i) Express $z$ in mod-arg form. **1**

(ii) Show clearly the points $A$ and $B$ on the Argand Diagram. **2**

(iii) Find the area of the triangle $OAB$ where $O$ is the origin.
Justify your answer carefully. **2**

(c) (i) Find the locus in the Argand Diagram satisfied by $z\bar{z}-2\operatorname{Re}(z)=0$ **2**

(ii) Draw a neat sketch of this locus. **1**

(iii) On the same diagram, draw the locus $\arg z=\frac{\pi}{4}$. **1**

(iv) Find the complex number satisfied by $\arg z=\frac{\pi}{4}$ and $z\bar{z}-2\operatorname{Re}(z)=0$. **2**

(d) If $\omega$ is a complex cube root of unity,

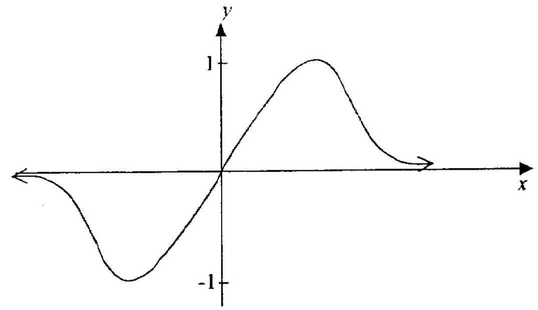
(i) Write down the value of $1+\omega+\omega^2$. **1**

(ii) Simplify $\omega^4+\omega^5+\omega^6$. **1**



**Question 3** (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) The diagram shows $y=f(x)$ which is an odd function.
There is a turning point at $(1,1)$.




Draw a separate sketch of each of the following graphs.
Use about one third of a page for each graph. Show all significant features.

(i) $y=f(-x)$ **2**

(ii) $y=\frac{1}{f(x)}$ **2**

(iii) $y=f(|x|)$ **2**

(iv) Draw $y=f(x)$ and $y=\sqrt{f(x)}$ on the same number plane. **2**

(v) $y=e^{f(x)}$ **2**

(vi) $y=(f(x))^2$ **2**

(vii) $y=f(x)\times\sin^{-1}x$ (show the coordinates of the endpoints) **3**

**Question 4** (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) If $\alpha$ is a root of $ax^4+bx^3+cx^2+dx+e=0$ where $a,b,c,d,e$ are real and $\alpha$ is complex, prove that $\overline{\alpha}$ is also a root. **3**

(b) (i) Given that $1-\sqrt{3}i$ is a root of $P(x)=0$ where $P(x)=x^4-2x^3+5x^2-2x+4$, write down two of the linear factors of $P(x)$. **2**

(ii) Hence factorise $P(x)$ completely into real factors. **2**

(c) (i) Show that the solutions of $z^6+z^3+1=0$ are contained in the solutions of $z^9-1=0$. **2**

(ii) Sketch the nine solutions of $z^9-1=0$ on an Argand Diagram. (about one third of a page in size) **2**

(iii) Mark clearly on your diagram, the six roots $z_1,z_2,z_3,z_4,z_5,z_6$ of $z^6+z^3+1=0$. **1**

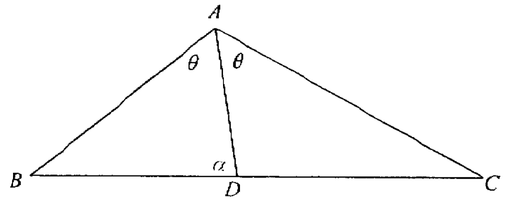
(iv) Show that the sum of the six roots of $z^6+z^3+1=0$ can be given by **3**

$$2\left(\cos\frac{2\pi}{9}+\cos\frac{4\pi}{9}-\cos\frac{\pi}{9}\right)$$



**Question 5** (15 marks) Use a SEPARATE writing booklet. **Marks**

(a)




In triangle $BAC$, $DA$ bisects $\angle BAC$.
$\angle BAD=\angle DAC=\theta$ and $\angle BDA=\alpha$.

(i) Use trigonometry to prove that $\frac{BD}{DC}=\frac{AB}{AC}$. **3**

(ii) If $\frac{AB}{AC}=r$, show that $\frac{\text{Area }\Delta ABD}{\text{Area }\Delta ADC}=r$ **2**

(b) $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is the equation of a hyperbola, and $P(x_1,y_1)$ is a point on the hyperbola.

(i) Write down the coordinates of the foci $S$, $S'$. **1**

(ii) Show that the equation of the tangent at $P(x_1,y_1)$ is given by $\frac{xx_1}{a^2}-\frac{yy_1}{b^2}=1$ **2**

(iii) Find $Q$, the point at which the tangent cuts the $x$-axis. **1**

(iv) Find the distances,

($\alpha$) $PS$ **1**

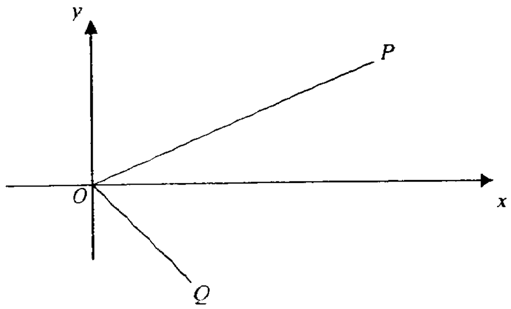
($\beta$) $PS'$ **1**

(v) Show that $\frac{PS'}{PS}=\frac{QS'}{QS}$ **2**

(vi) Using your proof in (a), what geometrical fact can you deduce in the triangle $PSS'$? **2**

**Question 6** (15 marks) Use a SEPARATE writing booklet. **Marks**

(a)




In the Argand Diagram above, point $P$ corresponds to complex number $z$.
The triangle $OPQ$ is a right-angled triangle and $OP=3OQ$.

(i) What is the complex number that corresponds to point $Q$? **1**

(ii) $QOPR$ is a rectangle. Write down the complex number that corresponds to $R$. **2**

(b) (i) Prove the identity $\cos(a-b)x-\cos(a+b)x=2\sin ax\sin bx$ **2**

(ii) Hence find $\int \sin 3x \sin 2x\,dx$ **2**

(c) If $u_1=8$, $u_2=20$ and $u_n=4u_{n-1}-4u_{n-2}$ for $n\geq 3$.

(i) Determine $u_3$ and $u_4$. **1**

(ii) Prove by induction that $u_n=(n+3)2^n$ for $n\geq 1$. **4**

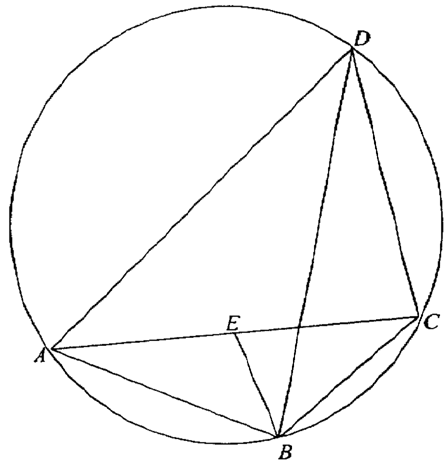
(d) If $ax^3+bx^2+d=0$ $(a,b,d\neq 0)$ has a double root, show that $27a^2d+4b^3=0$. **3**





**Question 7** (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) In the diagram $ABCD$ is a cyclic quadrilateral. $E$ is the point on $AC$ such that $\angle ABE=\angle DBC$.

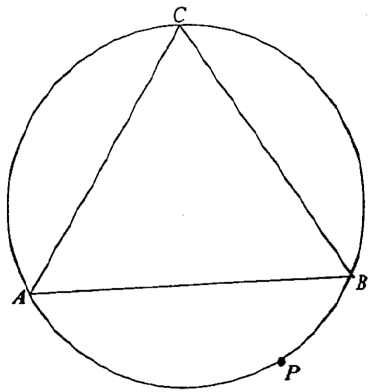



(i) Show that $\Delta ABE\,|||\,\Delta DBC$ and $\Delta ABD\,|||\,\Delta EBC$. **3**

(ii) Hence show that $(AB)(DC)+(AD)(BC)=(AC)(DB)$ **3**

(b) In the diagram $ABC$ is an equilateral triangle inscribed in a circle. $P$ is a point on the minor arc $AB$ of the circle. **2**

Use the result from (a) to show that $PC=PA+PB$.



Question 7 (continued)

(c) (i) Show that $\operatorname{cosec}2\theta + \cot 2\theta = \cot\theta$ for all real values of $\theta$. **2**

(ii) Use the result above to :

($\alpha$) find in surd form the values of $\cot\frac{\pi}{8}$ and $\cot\frac{\pi}{12}$. **2**

($\beta$) show without using calculators that

$$\operatorname{cosec}\frac{4\pi}{15} + \operatorname{cosec}\frac{8\pi}{15} + \operatorname{cosec}\frac{16\pi}{15} + \operatorname{cosec}\frac{32\pi}{15} = 0$$ **3**



**Question 8** (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) (i) Sketch the graph of $y = \sec x$ for $0 \le x \le \frac{\pi}{2}$ indicating any important features. **2**

On the same set of axes, sketch the graph of $y = \sec^{-1} x$, again indicating any important features.

(ii) If $x = \sec y$, find $\frac{dx}{dy}$ and hence if $y = \sec^{-1} x$, find $\frac{dy}{dx}$. **2**

(b) The points $P(a\cos\theta, b\sin\theta), Q(-a\sin\theta, b\cos\theta)$ lie on the ellipse $E$, given by $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

(i) Show that if $O$ is the centre of $E$, then $OP^2 + OQ^2 = a^2 + b^2$. **2**

(ii) The equations of the tangents at $P$ and $Q$ are : **3**

$$\frac{x\cos\theta}{a} + \frac{y\sin\theta}{b} = 1$$

$$\frac{-x\sin\theta}{a} + \frac{y\cos\theta}{b} = 1$$

Show that the point of intersection $T$ of the two tangents at $P$ and $Q$ is given by $T(a(\cos\theta - \sin\theta), b(\sin\theta + \cos\theta))$.

(iv) Show that the locus of $T$ is given by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$ **2**

(v) If $\alpha$ is the angle between the tangents at $P$ and $Q$, show that $\tan\alpha = 2\frac{\sqrt{1-e^2}}{e^2\sin 2\theta}$ **4**

**End of paper**