Merewether High School
Year 12 Trial HSC Examination 2001

MATHEMATICS
Extension 2 paper

Time Allowed: 3 hours plus 5 minutes reading time

Instructions:

- All questions may be attempted
- Start each question on a new page
- In every question all necessary working should be shown – full marks may not be awarded for answers without suitable working
- Approved silent calculators may be used
- A Table of Standard Integrals is provided
- Hand in the paper in TWO bundles, Questions 1, 2, 3, 4 and Questions 5, 6, 7, 8.

Question 1 **15 Marks**

(a) Evaluate $\int_0^2 \sqrt{4-x^2}\,dx$ 2

(b) Find $\int \frac{2x-3}{x^2-4x+5}\,dx$ 3

(c) Find $\int \sin^{-1} x\,dx$ 2

(d) Evaluate $\int_0^{\frac{\pi}{4}} \sin^2 x \cos^2 x\,dx$ 2

(e) (i) Given that $I_n = \int_0^{\frac{\pi}{2}} \cos^n x\,dx$, prove that $I_n = \left(\frac{n-1}{n}\right) I_{n-2}$, where n is an integer and $n \geq 2$. 6

(ii) Hence evaluate $\int_0^{\frac{\pi}{2}} \cos^5 x\,dx$.

Question 2 **15 marks**

(a) If w is the complex number $2\sqrt{3}i - 2$, 5

(i) find $|w|$;

(ii) find $\arg w$;

(iii) write w in modulus argument form;

(iv) show that $w^2 = 4\overline{w}$.

(b) Sketch the locus described by $\arg(z+1) = \frac{3\pi}{4}$. 2

(c) Sketch the locus described by $|z-1| = |z+i|$ 2

(d) Evaluate $\frac{1}{\left(-1+i\sqrt{3}\right)^6}$. 2

(e) Find the locus of z if $w = \frac{z-2}{z}$, given that w is purely imaginary. 4

Question 3 **15 marks**

(a) By means of the substitution $x = a - u$, or otherwise, prove that 8

$$\int_0^a f(x)dx = \int_0^a f(a-x)dx.$$

Hence prove that $\int_0^\pi \frac{x \sin x\,dx}{1+\cos^2 x} = \int_0^\pi \frac{(\pi - x)\sin x\,dx}{1+\cos^2 x} = \frac{\pi^2}{4}$

(b) Sketch the following curves for $-2\pi \leq x \leq 2\pi$.
Do each sketch on a separate diagram.

(i) $y = |\sin x|$ 2

(ii) $y = \sin|x|$ 2

(iii) $y^2 = \sin x$ 3

Question 4 **15 marks**

(a) (i) Show that $(1+i)$ is a zero of the polynomial $P(x) = x^3 + x^2 - 4x + 6$. 5

(ii) Using (i), resolve $P(x)$ into irreducible factors over the field of:

(α) Complex Numbers;

(β) Real Numbers.

(b) If $x^4 + 2x^3 - 12x^2 + 14x - 5 = 0$ has a root of multiplicity 3, find all the roots of the equation. 4

(c) (i) Assuming a, b, c, d are real, show that if the roots of the equation $\left(a^2+b^2\right)x^2 + 2(ac+bd)x + c^2 + d^2 = 0$ are real then they are equal. 6

(ii) Show that the double root in (i) is $x = \frac{-c}{a}$.

Question 5 **15 marks**

(a) The ellipse E has Cartesian equation $\frac{x^2}{25}+\frac{y^2}{16}=1$.

(i) Sketch the curve and write down the: **4**
(α) eccentricity;
(β) coordinates of the foci S and S';
(γ) equations of the directerices.

(ii) (α) Show that the point P on E can be represented by the coordinates $(5\cos\theta, 4\sin\theta)$. **5**

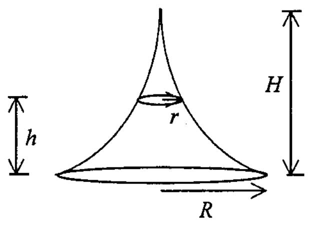
(β) Prove that $PS+PS'$ is independent of the position of P on the curve.

(b) Show that the tangents at the points $P\left(cp,\frac{c}{p}\right)$ and $Q\left(cq,\frac{c}{q}\right)$ on the rectangular hyperbola $xy=c^2$ meet at the point $\left(\frac{2c\,pq}{p+q},\frac{2c}{p+q}\right)$. **6**

Question 6 **15 marks**

5

(a) A spire is constructed as shown in the diagram. Each cross-section parallel to the base is a circle whose radius r is given by $r=\sqrt{R^2-\frac{R^2h^2}{H^2}}$ where R is The radius of the base, H is the height of the spire and h is the distance from the base to the circular cross-section.



Prove that the volume of the spire is $V=\frac{2}{3}\pi R^2 H$.

(b) The area enclosed by the curve $y=(x-4)^2$ and the line $y=16$ is rotated about the y axis. Using cylindrical shells find the volume of the solid generated. **5**

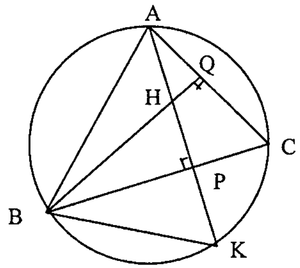
(c) Using the slice technique, find the volume of the solid obtained by revolving $y=\cos x$ about the line $y=-1$ over the interval $-\pi\le x\le\pi$. **5**

Question 7 **15 Marks**

(a) Find $\int\frac{(x^2+x)\,dx}{(x-1)(x^2+1)}$ **3**

4

(b) The altitudes AP and BQ of an acute angled triangle meet at H.
AP produced cuts the circle through A, B and C at K.
Prove that HP=PK.



(c) (i) The equation $x^3+3hx+g=0$ has two equal roots. Show that this equal root α has the value $\alpha=\sqrt[3]{\frac{g}{2}}$, hence prove that $g^2+4h^3=0$. **8**

(ii) Using part (i) or otherwise, solve the equation $x^3-12x+16=0$ given that it has two equal roots.

(iii) The tangent to the curve $y=x^3$ at $P(2,8)$ intersects the curve again at Q. Find the coordinates of Q.

Question 8 **15 Marks**

(a) A farmer using explosives to blow a stump out of the ground uses enough explosive to hurl debris in all directions with a velocity of 15ms^{-1}. **7**
If the explosion occurs on level ground, show that any person at ground level $10\sqrt{5}$ metres from the explosion could be struck by debris at two instants $\sqrt{5}$ seconds and 2 seconds after the blast, respectively.
(Take the acceleration due to gravity as g = 10 ms^{-2} and air resistance to be zero)

(b) A curve is defined by the parametric equations $x=\cos^3\theta,\quad y=\sin^3\theta$ for $0<\theta<\frac{\pi}{4}$. **8**

(i) Show that the equation of the normal to the curve at the point $P(\cos^3\phi,\sin^3\phi)$ is $x\cos\phi-y\sin\phi=\cos 2\phi$.

(ii) The normal at P cuts the x axis at A and the y axis at B.
Show that $AB=2\cot 2\phi$.

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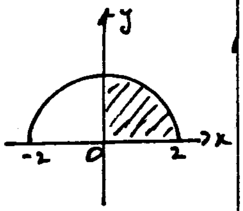
Q1 (a)

$\int_0^2 \sqrt{4-x^2}\cdot dx$



$= \frac{1}{4}\cdot\pi\cdot 2^2$

$= \pi$

(b) $\int \frac{2x-3}{x^2-4x+5}\cdot dx$

$= \int \frac{2x-4}{x^2-4x+5}\cdot dx + \int \frac{1\,dx}{(x-2)^2+1}$

$= \log|x^2-4x+5| + \tan^{-1}(x-2) + C$

(c) $\int \sin^{-1}x\cdot dx = x\sin^{-1}x - \int \frac{x\cdot dx}{\sqrt{1-x^2}}$

$= x\sin^{-1}x + \sqrt{1-x^2} + C$

(d) $\int_0^{\pi/4} \sin^2 x\cos^2 x\cdot dx$

$= \int_0^{\pi/4}\left(\frac{1}{2}\sin 2x\right)^2\cdot dx$ $\quad \boxed{\cos 4x = 1-2\sin^2 2x}$

$= \frac{1}{4}\int_0^{\pi/4}\frac{1}{2}(1-\cos 4x)\cdot dx$

$= \frac{1}{8}\left[x - \frac{1}{4}\sin 4x\right]_0^{\pi/4}$

$= \frac{1}{8}\left[\frac{\pi}{4}\right]$

$= \frac{\pi}{32}$

(e) $I_n = \int_0^{\pi/2}\cos^n x\cdot dx$

$= \int_0^{\pi/2}\cos x\cdot\cos^{n-1}x\cdot dx$

$= \left[\sin x\cos^{n-1}x\right]_0^{\pi/2} - \int \sin x(n-1)\cos^{n-2}x\cdot -\sin x\cdot dx$

$= (n-1)\int_0^{\pi/2}\sin^2 x\cos^{n-2}x\cdot dx$

$= (n-1)\int(1-\cos^2 x)\cos^{n-2}x\cdot dx$

$\therefore I_n = (n-1)I_{n-2} - (n-1)I_n$

$nI_n = (n-1)I_{n-2}$

$I_n = \frac{n-1}{n}I_{n-2}$

(ii) $I_5 = \int_0^{\pi/2}\cos^5 x\cdot dx$

i.e. $I_5 = \frac{4}{5}I_3$

$= \frac{4}{5}\cdot\frac{2}{3}I_1$

$= \frac{8}{15}$

where $I_1 = \int_0^{\pi/2}\cos x\cdot dx = 1$

Q2 (a) $\omega = 2\sqrt{3}i - 2$

$|\omega| = \sqrt{(2\sqrt{3})^2+2^2} = 4$

$\arg\omega = \tan^{-1}(-\sqrt{3})$

$= \pi - \tan^{-1}(\sqrt{3})$

$= \pi - \frac{\pi}{3}$

$= \frac{2\pi}{3}$

(iii) $\omega = 4\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

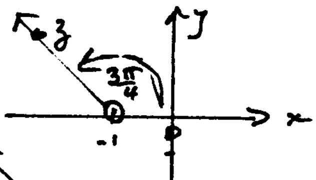
(iv) $\omega^2 = (2\sqrt{3}i - 2)^2$

$= -8(1+\sqrt{3}i)$

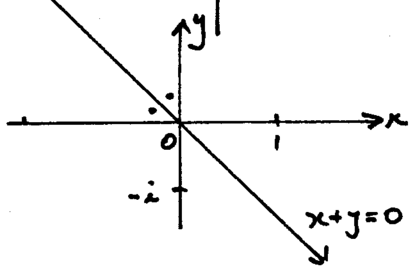
$= 4(-2-2\sqrt{3}i)$

i.e. $\omega^2 = 4\overline{\omega}$.

(b)



(c)



(d) $\frac{1}{(-1+i\sqrt{3})^6} = (-1+i\sqrt{3})^{-6}$

$= \left(2\operatorname{cis}\frac{2\pi}{3}\right)^{-6}$

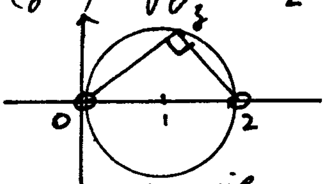
$= 2^{-6}\operatorname{cis}(-4\pi)$

$= \frac{1}{2^6}$

$= \frac{1}{64}$

(e) if ω is purely imaginary $\arg\omega = \pm\frac{\pi}{2}$

i.e. $\arg(z-2) - \arg z = \pm\frac{\pi}{2}$



Locus of z is the circle $(x-1)^2+y^2=1$ or $|z-1|=1$ excluding $(0,0)$ & $(2,0)$

Q3 (a) if $x = a-u$

$dx = -du$

$x=0,\ u=a$

$x=a,\ u=0$

$\therefore \int_0^a f(x)dx = \int_a^0 f(a-u)\cdot -du$

$= \int_0^a f(a-u)\cdot du$

$= \int_0^a f(a-x)\cdot dx$

$\int_0^\pi \frac{x\sin x\cdot dx}{1+\cos^2 x} = \int\frac{(\pi-x)\sin(\pi-x)}{1+\cos^2(\pi-x)}\cdot dx$

$= \int\frac{(\pi-x)\sin x}{1+\cos^2 x}\cdot dx$

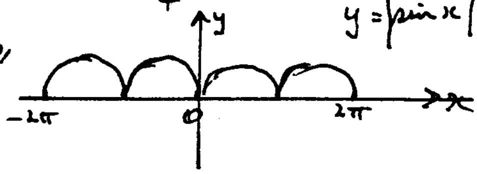
$= \int\frac{\pi\sin x\cdot dx}{1+\cos^2 x} - \int\frac{x\sin x}{1+\cos^2 x}\cdot dx$

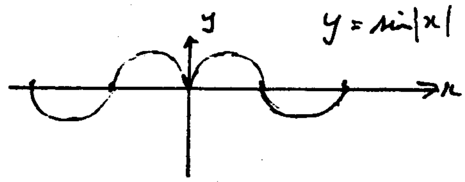
$\therefore 2I = \int\frac{\pi\sin x}{1+\cos^2 x}\cdot dx$

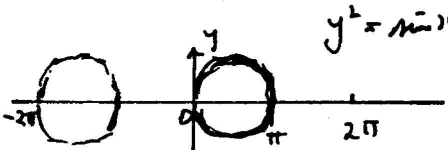
$I = -\frac{\pi}{2}\left[\tan^{-1}\cos x\right]_0^\pi$

$= \frac{\pi^2}{4}$

(b)







Q4 (a) $P(x) = x^3+x^2-4x+6$

$P(1+i) = (1+i)^3+(1+i)^2-4(1+i)+6$

$= 2i(1+i)+2i-4-4i+6$

$= 0$

because coeffs. are real $(1-i)$ is also a root

i.e. x^2-2x+2 is a factor

(ii) $P(x) = (x-(1+i))(x-(1-i))(x+3)$

(iii) $P(x) = (x^2-2x+2)(x+3)$

(b) $P(x) = x^4+2x^3-12x^2+14x-5$

$P'(x) = 4x^3+6x^2-24x+14$

$P''(x) = 12x^2+12x-24$

$= 12(x-1)(x+2)$

if $P(x)$ has a triple root it must be $x=1$

$\therefore P(x) = (x-1)^3(x+5)$

$\therefore$ Roots are $1, 1, 1, -5$

(c) $\Delta = 4(ac+bd)^2 - 4(a^2+b^2)(c^2+d^2)$

$= -4(ad-bc)^2$

if roots are real, they must be equal i.e. $ad = bc$

(ii) if roots are equal say α

$2\alpha = -\frac{2(ac+bd)}{a^2+b^2}$

i.e. $\alpha = -\frac{(ac+bd)}{a^2+b^2}$

$= -\frac{\left(ac+\frac{b^2c}{a}\right)}{a^2+b^2} = -\frac{c}{a}$.

Q5 (a) $\frac{x^2}{25}+\frac{y^2}{16}=1 \rightarrow a=5,\ b=4$

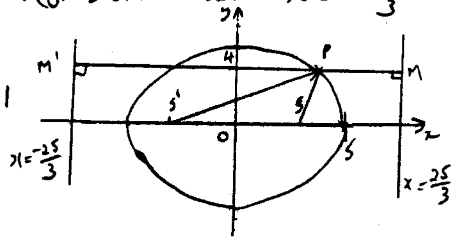
(i) (α) $b^2=a^2(1-e^2)$ 1

$\frac{16}{25}=1-e^2$, $e^2=\frac{9}{25}$

$e=\frac{3}{5}$

(β) $S(3,0)$, $S'(-3,0)$ 1 $ae=3$, $\frac{a}{e}=\frac{5}{3/5}=\frac{25}{3}$

(γ) Directrices $x=\pm\frac{25}{3}$ 1



1

(ii) ② $P(5\cos\theta, 4\sin\theta)$: LHS $=\frac{x^2}{25}+\frac{y^2}{16}$

No LHS $=\rightarrow 1$
Deriv of Para 1

1 $=\frac{25\cos^2\theta}{25}+\frac{16\sin^2\theta}{16}$

1 $=\cos^2\theta+\sin^2\theta=1=$ RHS

$\therefore P$ lies on E

(β) ③ $PS=e\,PM$

$PS+PS'=e\,PM+e\,PM'$ 1

$=e(PM+PM')$

$=e\,MM'$ 1

$=e\times 2\times\frac{a}{e}$ 1

$=2a$

which is independent of the Position of P on the curve

(b) ⑥ $xy=c^2$ $P(cp,\frac{c}{p})$

$y=\frac{c^2}{x}$ $Q(cq,\frac{c}{q})$

$\frac{dy}{dx}=\frac{-c^2}{x^2}$ 1

Tangent at P: $y-\frac{c}{p}=\frac{-c^2}{c^2p^2}(x-cp)$ 1

$p^2y-cp=-x+cp$

$x+p^2y=2cp$ (i)

Tan at Q: $x+q^2y=2cq$ (ii) 1

(i) − (ii) $(p^2-q^2)y=2c(p-q)$ 1

$y=\frac{2c\cancel{(p-q)}}{\cancel{(p-q)}(p+q)}$ 1

$x=2cp-\frac{2cp^2}{p+q}$

$=\frac{2cp(p+q-p)}{p+q}$

$=\frac{2cpq}{p+q}$ 1

$\therefore$ Tangents at P and Q meet at $\left(\frac{2cpq}{p+q},\frac{2c}{p+q}\right)$

Q6

(a) ⑤

$r=\sqrt{R^2-\frac{R^2h^2}{H^2}}$

$A=\pi r^2$

$=\pi\left(R^2-\frac{R^2h^2}{H^2}\right)$ 1

δh $\delta V=\pi\left(R^2-\frac{R^2h^2}{H^2}\right)\delta h$

$V=\int_0^H \pi\left(R^2-\frac{R^2h^2}{H^2}\right)dh$ 1

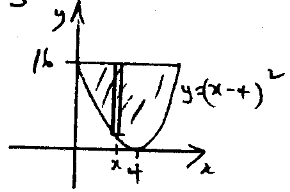
$=\pi\left[R^2h-\frac{R^2h^3}{3H^2}\right]_0^H$ 1

$=\pi\left[R^2H-\frac{R^2H^3}{3H^2}-0\right]$ 1

$=\pi R^2\left(H-\frac{H}{3}\right)$

$=\frac{2\pi R^2H}{3}$ 1

(b) ⑤



1

$\delta A=(16-y)\delta x$

$\delta V=2\pi x(16-y)\delta x$ 1

$V=2\pi\int_0^8 x(16-y)\,dx$ 1

$=2\pi\int_0^8 x(16-(x-4)^2)\,dx$

$=2\pi\int_0^8 x(8x-x^2)\,dx$

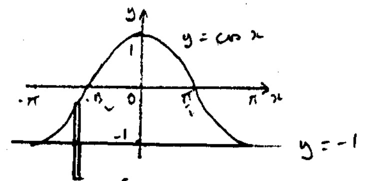
$=2\pi\int_0^8 (8x^2-x^3)\,dx$

$=2\pi\left[\frac{8x^3}{3}-\frac{x^4}{4}\right]_0^8$ 1

$=2\pi\left[\frac{8^4}{3}-\frac{8^4}{4}-0\right]$ 1

$=2\pi\times 8^4\times\frac{1}{12}=\frac{2048\pi}{3}$ $(682\tfrac{2}{3}\pi)$

(c) ⑤



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$A=\pi(y+1)^2$ 1

$\delta V=\pi(y+1)^2\cdot\delta x$

$V=\int_{-\pi}^{\pi}\pi(y+1)^2\,dx$ 1

$=\pi\int_{-\pi}^{\pi}(\cos x+1)^2\,dx$

$=\pi\int_{-\pi}^{\pi}(1+2\cos x+\cos^2x)\,dx$

$=\pi\int_{-\pi}^{\pi}\left(1+2\cos x+\frac{1+\cos 2x}{2}\right)dx$ 1

$=\pi\left[\frac{3x}{2}+2\sin x+\frac{\sin 2x}{4}\right]_{-\pi}^{\pi}$

$=\pi\left[\frac{3\pi}{2}+0+0-\left(\frac{-3\pi}{2}+0+0\right)\right]$

$=3\pi^2$ 1

π^2 with incorrect working − pay 3

Q7

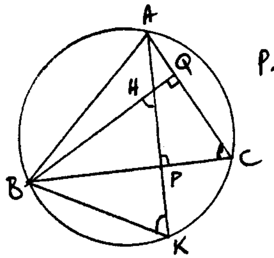
(a) ③

$$\int \frac{(x^2+x)\,dx}{(x-1)(x^2+1)}$$

$$1 = \int \left(\frac{1}{x-1} + \frac{1}{x^2+1}\right) dx$$

1, 1 $\quad = \ln|x-1| + \tan^{-1}x + c$

(b) ④



Prove $HP = PK$

$H\hat{Q}C = H\hat{P}C = 90^\circ$ (BQ + AP are altitudes)

$\therefore$ HQCP is a cyclic quad. (opp ∠s supplementary)

$\therefore B\hat{H}P = Q\hat{C}P$ (ext ∠ cyclic quad equals int opp ∠)

and $A\hat{C}B = A\hat{K}B$ (angles in the same segment)

But $Q\hat{C}P$ and $A\hat{C}B$ are the same angle

$\therefore B\hat{H}K = H\hat{K}B$

$\therefore \triangle BHK$ is isosceles (base angles equal)

$BP \perp HK$ (given) is an altitude of $\triangle BHK$

$\therefore BP$ bisects HK (altitude bisects base of isos $\triangle$)

$\therefore HP = PK$

c) i) ④ $x^3 + 3hx + g = 0$

roots α, α, β.

$\therefore 2\alpha + \beta = 0 \Rightarrow \beta = -2\alpha$

$\alpha^2\beta = -g$

$\therefore \alpha^2 \times (-2\alpha) = -g$

$\alpha^3 = \frac{g}{2}$

2 $\quad \alpha = \sqrt[3]{g/2}$

$f(x) = x^3 + 3hx + g$
$f'(x) = 3x^2 + 3h$
α = double root.
$\therefore \alpha^3 + 3h\alpha + g = 0$
$+ \quad 3\alpha^2 + 3h = 0$
$h = -\alpha^2$
$\therefore \alpha^3 + 3\alpha.(-\alpha^2) + g = 0$
$2\alpha^3 = g$
$\alpha = \sqrt[3]{\frac{g}{2}}$

and $\left(\sqrt[3]{\frac{g}{2}}\right)^3 + 3h.\sqrt[3]{\frac{g}{2}} + g = 0$

$\frac{g}{2} + 3h.\sqrt[3]{\frac{g}{2}} + g = 0$

$\frac{\cancel{3}g}{2} = -\cancel{3}h \times \sqrt[3]{g/2}$

$\frac{g^3}{8} = -h^3 \times \frac{g}{2}$

$\frac{g^2}{4} = -h^3$

2 $\quad g^2 + 4h^3 = 0$

ii) ② $x^3 - 12x + 16 = 0 \qquad \therefore h = -4$, $g = 16$

α, α, β

$\therefore \alpha = \sqrt[3]{\frac{16}{2}} = 2$

$\beta = -2 \times 2 = -4$

$\therefore$ Roots are $2, 2, -4$ $\quad$ 2

i.e. $x = 2, -4$

iii) ② $y = x^3 \qquad P(2, 8)$

$\frac{dy}{dx} = 3x^2 \qquad$ at P, $\frac{dy}{dx} = 12$

Tangent at P: $\quad y - 8 = 12(x-2)$

$y = 12x - 16$

For Q, $\quad x^3 = 12x - 16$

$x^3 - 12x + 16 = 0$ $\quad$ 2

From (ii) $\quad x = -4$ for Q

$y = -64$

$\therefore Q(-4, -64)$

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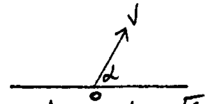
Q8 (a) ⑦



$V = 15\,ms^{-1}$

$g = 10$

$t = 0, x = 0, y = 0$

$\ddot{y} = -g$

$\left.\begin{matrix} x = 10\sqrt{5} \\ y = 0 \end{matrix}\right\}$, show $t = \sqrt{5}$ or 2:

$\ddot{x} = 0 \qquad \ddot{y} = -g$

$\dot{x} = V\cos\alpha \qquad \dot{y} = -gt + C$

$= 15\cos\alpha \qquad$ 1 $\quad \dot{y} = 15\sin\alpha - 10t$

1 $\quad x = 15t\cos\alpha \qquad$ 1 $\quad y = 15t\sin\alpha - 5t^2$

$x = 10\sqrt{5}$, $\frac{10\sqrt{5}}{15} = t\cos\alpha \qquad y = 0$, $\sin\alpha = \frac{5t^2}{15t}$

1 $\quad \cos\alpha = \frac{2\sqrt{5}}{3t} \qquad$ 1 $\quad = \frac{t}{3}$

But $\sin^2\alpha + \cos^2\alpha = 1$

1 $\quad \frac{20}{9t^2} + \frac{t^2}{9} = 1$

$20 + t^4 = 9t^2$

$t^4 - 9t^2 + 20 = 0$

$(t^2 - 4)(t^2 - 5) = 0$

$(t-2)(t+2)(t-\sqrt{5})(t+\sqrt{5}) = 0$

$\therefore t = 2, \sqrt{5}$ or $-2, -\sqrt{5}$.

1 $\quad t > 0$

$\therefore$ Person $10\sqrt{5}$m from explosion could be struck after $\sqrt{5}$ or 2 seconds

(b) ⑧ $\quad x = \cos^3\theta \qquad 0 < \theta < \frac{\pi}{4}$

$y = \sin^3\theta$

(i) 4 $\quad P(\cos^3\phi, \sin^3\phi)$

$\frac{dx}{d\theta} = 3\cos^2\theta.(-\sin\theta)$

$\frac{dy}{d\theta} = 3\sin^2\theta.\cos\theta$ $\quad$ } 1 frame

$\frac{dy}{dx} = \frac{3\sin^2\theta\cos\theta}{-3\sin\theta.\cos^2\theta}$ $\quad$ 1

$= -\tan\theta$

at P, $\theta = \phi \qquad \therefore \frac{dy}{dx} = -\tan\phi$

grad of normal $= \cot\phi$

Eqn: $\quad y - \sin^3\phi = \frac{\cos\phi}{\sin\phi}(x - \cos^3\phi)$ $\quad$ 1

$y\sin\phi - \sin^4\phi = x\cos\phi - \cos^4\phi$

$x\cos\phi - y\sin\phi = \cos^4\phi - \sin^4\phi$

$x\cos\phi - y\sin\phi = (\cos^2\phi + \sin^2\phi)(\cos^2\phi - \sin^2\phi)$

$x\cos\phi - y\sin\phi = 1 \times \cos 2\phi$

$x\cos\phi - y\sin\phi = \cos 2\phi$ $\quad$ 1

(ii) 4 $\quad A(?, 0) \qquad B(0, ?) \qquad AB = ?$

$y = 0, \quad x\cos\phi = \cos 2\phi$

$x = \frac{\cos 2\phi}{\cos\phi}$ $\quad$ 1

$A\left(\frac{\cos 2\phi}{\cos\phi}, 0\right)$

$x = 0, \quad -y\sin\phi = \cos 2\phi$

$y = \frac{-\cos 2\phi}{\sin\phi}$ $\quad$ 1

~~B(cos~~ $\quad B\left(0, \frac{-\cos 2\phi}{\sin\phi}\right)$ $\quad$ 1

$AB^2 = \frac{\cos^2 2\phi}{\cos^2\phi} + \frac{\cos^2 2\phi}{\sin^2\phi}$

$= \frac{\cos^2 2\phi}{\cos^2\phi.\sin^2\phi}(\sin^2\phi + \cos^2\phi)$

$= \frac{\cos^2 2\phi}{\sin^2\phi\cos^2\phi}$ $\quad$ 1

$AB = \frac{\cos 2\phi}{\sin\phi\cos\phi}$

$= \frac{2\cos 2\phi}{\sin 2\phi}$

$= 2\cot 2\phi$ $\quad$ 1

(a) $\rightarrow \sin 2\theta = \frac{4\sqrt{5}}{9}$

$\therefore \cos 2\theta = \frac{1}{9}$

$\cos^2\theta = \frac{1 + \cos 2\theta}{2} = \frac{5}{9} \quad \therefore \cos\theta = \frac{\sqrt{5}}{3}$

$\sin^2\theta = \frac{1 - \cos 2\theta}{2} = \frac{4}{9} \quad \therefore \sin\theta = \frac{2}{3}$

etc.