STUDENT NUMBER

CLASS: 12M1 12M2

NORMANHURST BOYS HIGH SCHOOL
NEW SOUTH WALES

2015

YEAR 12

YEARLY EXAMINATION

Mathematics Extension 2

Outcomes

YO1	Uses a variety of methods to: determine the important features of the graphs of a wide variety of functions; and solve problems involving polynomials.
YO2	Applies a number of techniques to integration, including partial fractions and integration by parts.
YO3	Makes use of appropriate techniques to solve problems involving complex numbers.
YO4	Determines volumes through the use of techniques involving slices and shells and applies techniques involving conics.
YO5	Solves problems in mechanics involving resolution of forces and resisted motion, and makes use of appropriate techniques from the Extension1.

General Instructions

- Reading time - 5 minutes
- Working time - 3 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- Show relevant mathematical reasoning and/or calculations in Questions 11-16

Total marks - 100

Section I

10 marks

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II

90 marks

- Attempt Questions 11-16
- Allow about 2 hour 45 minutes for this section

Outcomes Tally Sheet

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NORMANHURST BOYS HIGH SCHOOL
NEW SOUTH WALES

2015
HIGHER SCHOOL CERTIFICATE
TRIAL EXAMINATION

Mathematics Extension 2

OUTCOMES ASSESSED: A student

YO1	Uses a variety of methods to: determine the important features of the graphs of a wide variety of functions; and solve problems involving polynomials.
YO2	Applies a number of techniques to integration, including partial fractions and integration by parts.
YO3	Makes use of appropriate techniques to solve problems involving complex numbers.
YO4	Determines volumes through the use of techniques involving slices and shells and applies techniques involving conics.
YO5	Solves problems in mechanics involving resolution of forces and resisted motion, and makes use of appropriate techniques from the Extension1.

OUTCOME QUESTION	YO1 GRAPHS & POLYNOMIAS	YO2 INTEGRALS	YO3 COMPLEX NOS	YO4 CONICS & VOLUMES	YO5 MECHANICS & EXTENSION 1	TOTALS
Q 1 – 10	6 8 9 /3	5 /1	2 7 /2	3 4 10 /3	1 /1	/10
Q 11		b c d /10	a e /5			/15
Q 12	e /3		a b /5	c d /7		/15
Q 13	b /4			a /6	c /5	/15
Q 14		b /5	a /3	c /4	d /3	/15
Q 15	a b /7				c /8	/15
Q 16		c /3	ai ii /4		aiii b /8	/15
TOTALS	/17	/19	/19	/20	/25	/100

Section I

Attempt Questions 1 – 10.
1 mark each. Total: **10 marks**
Allow about **15 minutes** for this section.

Use the **multiple-choice answer sheet** for Questions 1-10.

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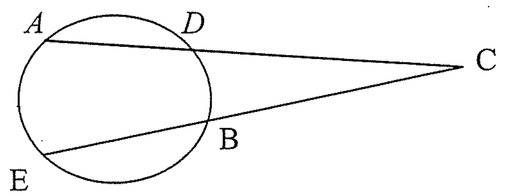
1 In the diagram below $DA = x,\ DC = y,\ BE = s$ and $BC = t$.



The correct relationship between x, y, s and t is

(A) $xy - st = 0$ (B) $xt - sy = 0$

(C) $st - xy = (y - t)(y+t)$ (D) $st - xy = (x - s)(x+s)$

2 The locus of the graph of $\arg\left(\dfrac{z-2}{z+2i}\right) = \dfrac{\pi}{2}$ is

(A) a semicircle passing through the origin
(B) a circle with centre at the origin
(C) an ellipse with a focus at the origin
(D) a hyperbola not passing through the origin

3 What is the volume of the solid formed when the region bounded by the curves $y = 2x^3$ and $y = 2\sqrt{x}$ is rotated about the x-axis?

(A) $\dfrac{5\pi}{14}$ cubic units

(B) $\dfrac{10\pi}{14}$ cubic units

(C) $\dfrac{5\pi}{7}$ cubic units

(D) $\dfrac{10\pi}{7}$ cubic units

4 The normal to the point $P(cp, \frac{c}{p})$ on the rectangular hyperbola $xy = c^2$ has the equation

$p^3x - py + c - cp^4 = 0$. The normal cuts the hyperbola at another point $Q(cq, \frac{c}{q})$.

What is the relationship between p and q?

(A) $pq = -1$

(B) $p^2q = -1$

(C) $p^3q = -1$

(D) $p^4q = -1$

5 Which of the following is an expression for $\int \frac{1}{x^2 - 6x + 13} dx$?

(A) $\frac{1}{2}\tan^{-1}\frac{(x-3)}{4} + C$

(B) $\frac{1}{2}\tan^{-1}\frac{(x-3)}{2} + C$

(C) $\frac{1}{4}\tan^{-1}\frac{(x-3)}{4} + C$

(D) $\frac{1}{4}\tan^{-1}\frac{(x-3)}{2} + C$

6 What is the number of asymptotes on the graph of $f(x) = \frac{x^2}{x^2 - 1}$?

(A) 1

(B) 2

(C) 3

(D) 4

7 What is $(-1+i)^n$ expressed in modulus-argument form? (n is a positive integer)

(A) $\left(\cos\frac{n\pi}{4} + i\sin\frac{n\pi}{4}\right)$

(B) $(\sqrt{2})^n\left(\cos\frac{n\pi}{4} + i\sin\frac{n\pi}{4}\right)$

(C) $\left(\cos\frac{3n\pi}{4} + i\sin\frac{3n\pi}{4}\right)$

(D) $(\sqrt{2})^n\left(\cos\frac{3n\pi}{4} + i\sin\frac{3n\pi}{4}\right)$

8 Let α, β and γ be the roots of the equation $x^3+2x^2+5=0$.

Which of the following polynomial equations have the roots α^2, β^2 and γ^2?

(A) $x^3-4x^2-20x-25=0$

(B) $x^3-4x^2-10x-25=0$

(C) $x^3-4x^2-20x-5=0$

(D) $x^3-4x^2-10x-5=0$

9 When $x^y=e$ is implicitly differentiated the result for $\frac{dy}{dx}$ is

(A) $\frac{-y}{x\log_e x}$

(B) $\frac{y}{x\log_e x}$

(C) $\frac{-x\log_e x}{y}$

(D) $\frac{x\log_e x}{y}$

10 Points $P(a\cos\theta,\ b\sin\theta)$ and $Q(a\cos\phi,\ b\sin\phi)$ lie on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

The chord PQ subtends a right angle at $(0,0)$.

Which of the following is the correct expression?

(A) $\tan\theta\tan\phi=-\frac{b^2}{a^2}$

(B) $\tan\theta\tan\phi=-\frac{a^2}{b^2}$

(C) $\tan\theta\tan\phi=\frac{b^2}{a^2}$

(D) $\tan\theta\tan\phi=\frac{a^2}{b^2}$

Section II

Attempt Questions 11 – 16.
Allow about **2 hours and 45 minutes** for this section.
All questions are worth **15 marks each.** Total: **90 marks**
Answer each question in a new writing booklet.

Include relevant mathematical reasoning and/or calculations in your responses.

Question11 (15 marks) **Begin a new writing booklet.** **Marks**

(a) (i) On the Argand diagram sketch the graph of $\left|z-\left(\sqrt{2}+\sqrt{2}i\right)\right|=1$. **2**

(ii) Find the largest possible value of both $|z|$ and $\arg z$ if z satisfies $\left|z-\left(\sqrt{2}+\sqrt{2}i\right)\right|=1$. **2**

(b) (i) Find real numbers *a, b* and *c* such that **2**

$$\frac{3x^2-3x+2}{(2x-1)\left(x^2+1\right)}=\frac{a}{2x-1}+\frac{bx+c}{x^2+1}$$

(ii) Hence evaluate $\int \frac{3x^2-3x+2}{(2x-1)\left(x^2+1\right)}dx$ in simplest form. **2**

(c) Use the substitution $x=u^2$ ($u>0$) to evaluate $\int \frac{1}{x(1+\sqrt{x})}dx$. **3**

(d) Find the exact value of $\int_0^{\frac{\pi}{6}} \sec 4x \tan 4x dx$. **3**

(e) Let $z_1=\cos\theta_1+i\sin\theta_1$ and $z_2=\cos\theta_2+i\sin\theta_2$ for real θ_1 and θ_2.

Show that $z_1z_2=\cos(\theta_1+\theta_2)+i\sin(\theta_1+\theta_2)$. **1**

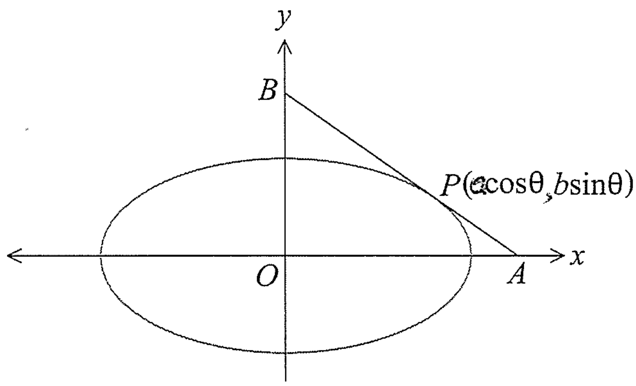
Question 12 (15 marks) **Begin a new writing booklet.** **Marks**

(a) Let $z_1 = \dfrac{a}{1+i}$ and $z_2 = \dfrac{b}{1+2i}$ where a and b are real numbers. **3**
What is the value of a and b, if $z_1 + z_2 = 1$?

(b) Let $z = 1+i$ be a root of the polynomial $z^2 - biz + c = 0$ where b and c are real numbers. Find the value of b and c. **2**

(c) The parabola $y = 4 - x^2$ is rotated about the line $y = 4$ for $\{x : 0 \le x \le 2\}$ to form a solid. Use the method of slicing to find the volume of the solid. **3**

(d) The point P lies on the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ where $a > b > 0$.



(i) Use the parametric representation of an ellipse to show that the equation of the tangent is $\dfrac{x}{a}\cos\theta + \dfrac{y}{b}\sin\theta = 1$. **2**

(ii) The tangent at P cuts the x-axis at A, y-axis at B and C is the foot of the perpendicular from P to the y-axis. Show that $OC \times OB = b^2$. **2**

(e) (i) Consider the function $f(x) = x^4 - 4x^3$. Sketch the graph of $y = f(x)$. **2**

(ii) Hence or otherwise find the number of real roots of the equation $x^4 - 4x^3 = kx$, where k is a positive real number. **1**

Question 13 (15 marks) **Begin a new writing booklet.** **Marks**

(a) Hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ has focus S on the positive x-axis and the corresponding directrix cuts the asymptotes to the hyperbola at points P and Q in the first and fourth quadrants respectively.

(i) Show that PS is perpendicular to the asymptote through P. **2**

(ii) Show that $PS = b$. **1**

(iii) A circle with centre S touches the asymptotes of the hyperbola. Deduce that the point of contact are the points P and Q. **1**

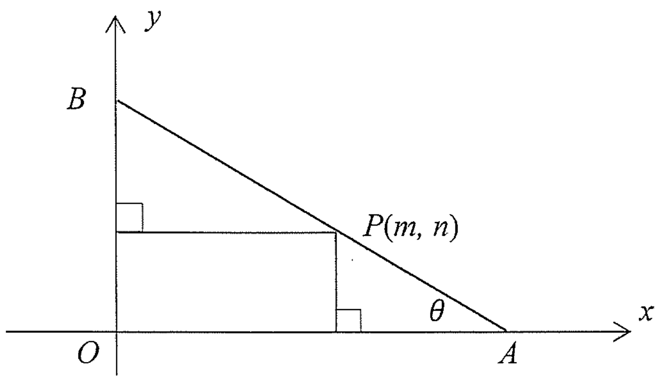
(iv) The circle with centre S touches the asymptotes of the hyperbola and cuts the hyperbola at the points R and T. Show that RT is a diameter of the circle if $a = b$. **2**

(b) By first sketching $f(x) = \frac{x^2}{x^2 - 1}$, draw separate one-third page sketches of: **1**

(i) $y = |f(x)|$ **1**

(ii) $y = log_2\,[f(x)]$ **2**

(c) The line AB drawn through a fixed point $P(m, n)$ cuts the x-axis at A and the y-axis at B. Angle $BAO = \theta$.



(i) Show that $L(\theta)$, the length of AB, is given by $L(\theta) = m\,sec\theta + n\,cosec\theta$. **2**

(ii) Prove that $tan^3\theta = \frac{n}{m}$ when $L'(\theta) = 0$. **3**

Show that this generates a length of $\sqrt{(m^{\frac{2}{3}} + n^{\frac{2}{3}})}$ for AB.

Marks

Question 14 (15 marks) **Begin a new writing booklet.**

(a) (i) Show that $z\overline{z} = |z|^2$ for any complex number z. **1**

(ii) A sequence of complex numbers z_n is given by the rule $z_1 = w$ and $z_n = v\overline{z}_{n-1}$ where w is a given complex number and v is a complex number with modulus 1. Show that $z_3 = w$. **2**

(b) Let $I_n = \int_0^{\frac{\pi}{2}} \sin^n x dx$, where n is positive integer.

(i) Show that $I_n = (n-1)\int_0^{\frac{\pi}{2}} \sin^{n-2} x \cos^2 x dx$ when $n \geq 2$. **2**

(ii) Prove that $I_n = \dfrac{(n-1)}{n} I_{n-2}$ when $n \geq 2$. **2**

(iii) What is the value of I_4? **1**

(c) A solid is formed by rotating about the y-axis the region bounded by the curve $y = \log_e x$ and the x-axis between $1 \leq x \leq e$. **4**

Find the volume of this solid using the method of cylindrical shells.

(d) Show, by mathematical induction for even integer values of n, that **3**

$$2n^2 \geq n^2 + n + 2 \quad \text{for } n > 1.$$

Question 15 (15 marks) **Begin a new writing booklet.** **Marks**

(a) (i) The polynomial $P(x)$ has a double root at $x = \alpha$. **1**
Prove that $P'(x)$ has a root at $x = \alpha$.

(ii) The polynomial $P(x) = x^3 - ax^2 + b$ has a double root at $x = \alpha$. **2**
Show that $4a^3 - 27b = 0$.

(b) A and B are on the curve $y = x^4 + 4x^3$ at $x = \alpha$ and $x = \beta$ respectively.
The line $y = mx + b$ is a tangent to the curve at both points A and B.

(i) The zeros of the equation $x^4 + 4x^3 - mx - b = 0$ are α, α, β and β. **1**
Explain this result.

(ii) Use relationships between the coefficients and the roots to find the values for m and b. **3**

(c) A rock is dropped under gravity g, from rest, at the top of a cliff.
The vertical distance travelled is represented by x in time t.
Air resistance is proportional to the velocity v of the rock.

(i) Explain why $\dfrac{dv}{dt} = g - kv$. **1**

(ii) Show that $v = \dfrac{g}{k}(1 - e^{-kt})$ when $t \geq 0$. **3**

(iii) Show that $x = -\dfrac{1}{k}v + \dfrac{g}{k^2}\log_e\left(\dfrac{g}{g - kv}\right)$. **3**

(iv) Verify that $kv < g$. **1**

Question 16 (15 marks) **Begin a new writing booklet.** **Marks**

(a) Let $z = r(\cos\theta + i\sin\theta)$ where $z \neq 0$ and $n \geq 1$ for integer n.

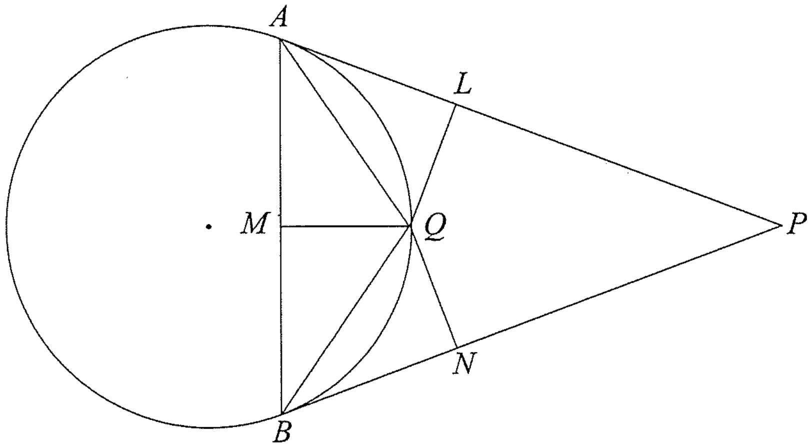
(i) Use De Moivre's theorem to show $z^n - \frac{1}{z^n} = 2i\sin n\theta$ for $n \geq 1$. **2**

(ii) You may assume $\left(z - \frac{1}{z}\right)^5 = (z^5 - \frac{1}{z^5}) - 5(z^3 - \frac{1}{z^3}) + 10(z - \frac{1}{z})$.

Show that $\sin^5\theta = \frac{1}{16}(\sin 5\theta - 5\sin 3\theta + 10\sin\theta)$. **2**

(iii) Hence solve $\sin 5\theta = 5\sin 3\theta$ to the nearest radian for $0 \leq \theta \leq \pi$. **3**

(b) Tangents PA and PB are drawn to a circle. Point Q is on the minor arc AB. Perpendiculars QL, QM and QN are drawn from Q to PA, AB and PB respectively.



(i) Show that $\Delta BNQ \parallel\!| \Delta AMQ$ and $\Delta ALQ \parallel\!| \Delta BMQ$. **3**

(ii) Hence show that QN, QM and QL form a geometric sequence. **2**

(c) Find a primitive function of $\frac{1}{1+\sin x}$. **3**

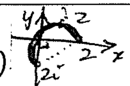
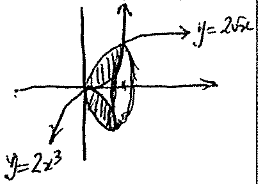
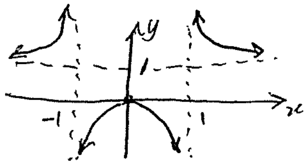
End of paper

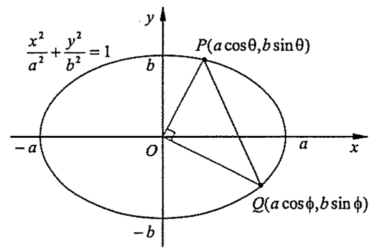
ACE Examination 2015

HSC Mathematics Extension 2 Yearly Examination

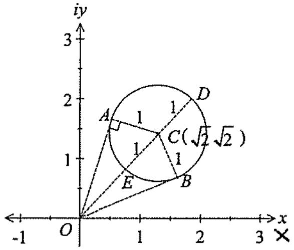
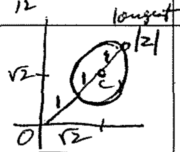
Worked solutions and marking guidelines

SOLUTIONS

Section I		
		Criteria
1	$y\times(x+y)=t\times(s+t)$ ∴ $ts-xy=y^2-t^2$ $xy+y^2=ts+t^2$ $\quad ts-xy=(y-t)(y+t)$	1 Mark: C
2	$\arg\left(\frac{z-2}{z-2i}\right)=\frac{\pi}{2}$ $\quad \frac{\pi}{2}=\arg(z-2)-\arg(z-2i)$ Semi circle $\quad z=0 \quad \arg\left(\frac{-2}{-2i}\right)=\frac{\pi}{2}$  angle in a semicircle is 90°.	1 Mark: A
3	Slices are taken perpendicular to the axis of rotation (x-axis). The base is an annulus. $A=\pi(r_2^2-r_1^2)=\pi((2\sqrt{x})^2-(2x^3)^2)$ $=\pi(4x-4x^6)=4\pi(x-x^6)$ $V=\lim_{\delta x\to 0}\sum_{x=0}^{1}4\pi(x-x^6)\delta x$ $=\int_0^1 4\pi(x-x^6)dx=4\pi\int_0^1(x-x^6)dx$ $=4\pi\left[\frac{x^2}{2}-\frac{x^7}{7}\right]_0^1=4\pi\left[(\frac{1}{2}-\frac{1}{7})-0\right]=\frac{10\pi}{7}$ 	1 Mark: D
4	$Q(cq,\frac{c}{q})$ is on the normal and satisfies the equation. $p^3cq-p\frac{c}{q}+c-cp^4=0$ $p^3q^2-p+q-qp^4=0$ $p^3q(q-p)=-(q-p)$ or $p^3q=-1$	1 Mark: C
5	$\int\frac{1}{x^2-6x+13}dx=\int\frac{dx}{(x-3)^2+2^2}=\frac{1}{2}\tan^{-1}\frac{(x-3)}{2}+C$	1 Mark: B
6	 $\lim_{x\to\infty}\frac{x^2}{x^2-1}$ 3 Asy $=\lim_{x\to\infty}\frac{1}{1-\frac{1}{x^2}}$ $=\frac{1}{1-0}=1$ 3 ASYMPTOTES	1 Mark: C

7	$\tan\theta=\frac{1}{-1}$ or $\theta=\frac{3\pi}{4}$ $\quad r^2=x^2+y^2=1^2+1^2$ or $r=\sqrt{2}$ $-1+i=\sqrt{2}(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4})$ $(-1+i)^n=(\sqrt{2})^n\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)^n$ $=(\sqrt{2})^n\left(\cos\frac{3n\pi}{4}+i\sin\frac{3n\pi}{4}\right)$	1 Mark: D
8	If α, β and γ are zeros of $x^3+2x^2+5=0$ then the polynomial equation with roots α^2, β^2 and γ^2 is: $(\sqrt{x})^3+2(\sqrt{x})^2+5=0$ $(\sqrt{x})^3=-(2x+5)$ $x^3=4x^2+20x+25$ $x^3-4x^2-20x-25=0$	1 Mark: A
9	$xy=e$ $\ln x^y=\ln e$ $y\ln x=1$ ∴ $y.\frac{1}{x}+\ln x.\frac{dy}{dx}=0$ $\ln x\frac{dy}{dx}=-\frac{y}{x}$ so $\frac{dy}{dx}=\frac{-y}{x\log_e x}$	1 Mark: A
10	 POQ is a right-angled triangle. Therefore $OP^2+OQ^2=PQ^2$. $a^2\cos^2\theta+b^2\sin^2\theta+a^2\cos^2\varphi+b^2\sin^2\varphi$ $=a^2(\cos\theta-\cos\varphi)^2+b^2(\sin\theta-\sin\varphi)^2$ $a^2(\cos^2\theta+\cos^2\varphi)+b^2(\sin^2\theta+\sin^2\varphi)$ $=a^2(\cos\theta-\cos\varphi)^2+b^2(\sin\theta-\sin\varphi)^2$ $0=-2a^2\cos\theta\cos\varphi-2b^2\sin\theta\sin\varphi$ $2b^2\sin\theta\sin\varphi=-2a^2\cos\theta\cos\varphi$ Hence $\frac{\sin\theta\sin\varphi}{\cos\theta\cos\varphi}=\frac{-2a^2}{2b^2}$ or $\tan\theta\tan\phi=-\frac{a^2}{b^2}$ OR slope $OP=\frac{b\sin\theta}{a\cos\theta}$ slope $OQ=\frac{b\sin\phi}{a\cos\phi}$ $\angle POQ=90°$ ∴ slope $OP\times$ slope $OQ=-1$ $\frac{b\sin\theta}{a\cos\theta}\times\frac{b\sin\phi}{a\cos\phi}=-1$ $\frac{b^2}{a^2}\tan\theta.\tan\phi=-1$ So $\tan\theta.\tan\phi=\frac{-a^2}{b^2}$	1 Mark: B

Section II

	Solution	Criteria
11(a) (i)	$\left\|z-(\sqrt{2}+\sqrt{2}i)\right\|=1$ Represents a circle with centre $(\sqrt{2},\sqrt{2})$ and radius of 1 unit.  $\angle DOX = 45^\circ$ $\angle DOB = 30^\circ$ $\therefore \angle BOX = 15^\circ$ So $\angle AOX = 15+30+30$ largest $= 75^\circ$ $\arg z = \frac{5\pi}{12}$	2 Marks: Correct answer. 1 Mark: Draws a circle or states the radius or centre.
11(a) (ii)	$OC=\sqrt{(\sqrt{2})^2+(\sqrt{2})^2}=2$ $\therefore OE=1$ and $OD=3$ ~~and therefore $1\le\|z\|\le 3$~~ so longest $\|z\|=3$ Arg $OC=\frac{\pi}{4}$ $\sin\angle AOC=\frac{1}{2}$, $\angle AOC=\frac{\pi}{6}$ $\quad\sin\angle BOC=\frac{1}{2}$, $\angle BOC=\frac{\pi}{6}$ $\frac{\pi}{4}-\frac{\pi}{6}\le\arg z\le\frac{\pi}{4}+\frac{\pi}{6}$ or $\frac{\pi}{12}\le\arg z\le\frac{5\pi}{12}$  $\sqrt{2^2+\sqrt{2}^2}=2=OC$  largest arg z $\arg z = 75^\circ = \frac{5\pi}{12}$	2 Marks: Correct answer. 1 Mark: Finds $\|z\|$ or arg z or shows some understanding. $\|z\|=3$ $\arg z=\frac{5\pi}{12}$
11(b) (i)	$\frac{3x^2-3x+2}{(2x-1)(x^2+1)}=\frac{a}{2x-1}+\frac{bx+c}{x^2+1}$ $3x^2-3x+2=a(x^2+1)+(bx+c)(2x-1)$ Let $x=\frac{1}{2}$ $\quad$ and $x=0$ $\frac{5}{4}=a\times\frac{5}{4}$ or $a=1$ $\quad 2=a+c(-1)$ $c=-1$ Equating the coefficients of x^2 $\quad 3=a+2b$ or $b=1$ $\therefore a=1,\ b=1$ and $c=-1$	2 Marks: Correct answer. 1 Mark: Makes some progress in finding a, b or c.
11(b) (ii)	$\int\frac{3x^2-3x+2}{(2x-1)(x^2+1)}dx=\int\left(\frac{1}{2x-1}+\frac{x-1}{x^2+1}\right)dx$ $=\int\left(\frac{1}{2x-1}+\frac{x}{x^2+1}-\frac{1}{x^2+1}\right)dx$ $=\frac{1}{2}\log_e\|2x-1\|+\frac{1}{2}\log_e\|x^2+1\|-\tan^{-1}x+C$ $=\frac{1}{2}\log_e\left[\|2x-1\|(x^2+1)\right]-\tan^{-1}x+C$	2 Marks: Correct answer. 1 Mark: Correctly finds one of the integrals.

11(c)	Let $x=u^2$ then $\frac{dx}{du}=2u$ or $dx=2udu$ $\int\frac{1}{x(1+\sqrt{x})}dx=\int\frac{1}{u^2(1+u)}2udu$ $=2\int\frac{1}{u(1+u)}du$ $=2\int\left(\frac{1}{u}-\frac{1}{1+u}\right)dx$ $=2\left[\log_e u-\log_e(1+u)\right]$ $=2\log_e\left\|\frac{u}{1+u}\right\|+C$ $=2\log_e\frac{\sqrt{x}}{1+\sqrt{x}}+C$	3 Marks: Correct answer. 2 Marks: Finds the primitive function. 1 Mark: Sets up the integral in terms of u
11(d)	$\int_0^{\frac{\pi}{6}}\sec 4x\cdot\tan 4x\,dx$ $=\int_0^{\frac{\pi}{6}}\frac{1}{\cos 4x}\cdot\frac{\sin 4x}{\cos 4x}dx$ $=\int_0^{\frac{\pi}{6}}\sin 4x\cdot(\cos 4x)^{-2}dx$ $=-\frac{1}{4}\int_0^{\frac{\pi}{6}}-4\sin 4x(\cos 4x)^{-2}dx$ $=\left[-\frac{1}{4}\frac{(\cos 4x)^{-1}}{-1}\right]_0^{\frac{\pi}{6}}$ $=\frac{1}{4}\left(\sec\frac{2\pi}{3}-\sec 0\right)$ $=\frac{1}{4}\left(\frac{1}{\cos 120^\circ}-\frac{1}{\cos 0^\circ}\right)$ $=\frac{1}{4}\times(-2-1)$ $=-\frac{3}{4}$	3 Mark: Correct answer. 2 Mark: 1 error only 1 Mark: ~~Correct~~ partially answer.
11(e)	$z_1z_2=(\cos\theta_1+i\sin\theta_1)(\cos\theta_2+i\sin\theta_2)$ $=\cos\theta_1\cos\theta_2+i^2\sin\theta_1\sin\theta_2+i\sin\theta_1\cos\theta_2+i\sin\theta_2\cos\theta_1$ $=\cos\theta_1\cos\theta_2-1\sin\theta_1\sin\theta_2+i(\sin\theta_1\cos\theta_2+\sin\theta_2\cos\theta_1)$ $=\cos(\theta_1+\theta_2)+i\sin(\theta_1+\theta_2)$.	2 Marks: Correct answer. 1 Mark: Substitutes into $z_1+z_2=1$ and uses the conjugate.

ALTERNATE SOLUTIONS TO Q16(c)

Q16(c)

$\int\frac{1}{\sin x+1}dx\times\frac{1-\sin x}{1-\sin x}$

$=\int\frac{1-\sin x}{1-\sin^2 x}dx=\int\frac{1-\sin x}{\cos^2 x}dx$

$=\int\frac{1}{\cos^2 x}-\frac{\sin x}{\cos x}\cdot\frac{1}{\cos x}dx$

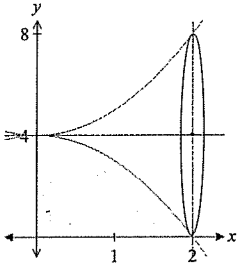
$=\int\sec^2 x-\sec x\tan x\,dx$

$=\boxed{\tan x-\sec x+C}$

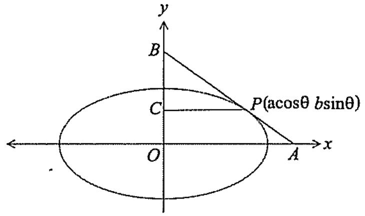
$=\frac{\sin x-1}{\cos x}\times\frac{\cos x}{\cos x}$

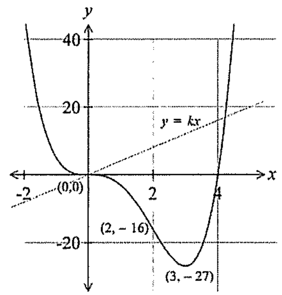
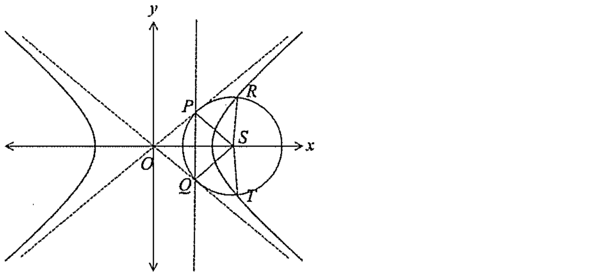
$=\frac{(\sin x-1)\times\cos x}{(1-\sin x)(1+\sin x)}$

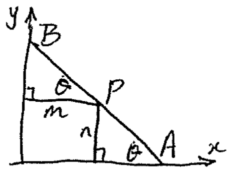
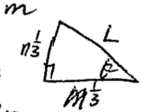
$=\frac{-\cos x}{\ldots}$

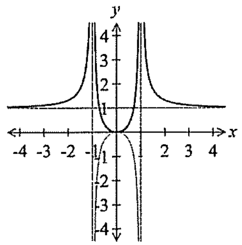
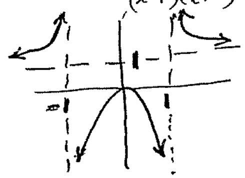
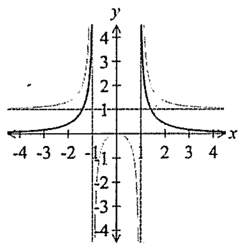
12(a)	$\frac{a}{1+i}+\frac{b}{1+2i}=1$ $a(1+2i)+b(1+i)=(1+i)(1+2i)$ $a+b+i(2a+b)=1+2i+i+2i^2$ $=-1+3i$ $a+b=-1$ $2a+b=3$ − $-a=-4$ $\therefore a=4$ So $a+b=-1$ $4+b=-1$ $b=-5$.	3 Marks: Correct answer. 1 Mark: Shows some understanding of the problem 2 Mark: Shows good understanding
12(b)	$z=1+i$ satisfies the polynomial $z^2-biz+c=0$ $(1+i)^2-bi(1+i)+c=0$ $1+2i-1-bi+b+c=0$ $(b+c)+(2-b)i=0$ Equating real and imaginary parts Therefore $b=2$ and $c=-2$	2 Marks: Correct answer. 1 Mark: Uses the factor theorem.
12(c)	 Same volume as $y=x^2$ rotated about the x-axis. Area of the slice is a circle radius is y and height x $A=\pi y^2$ $=\pi x^4$ $\delta V=\delta A.\delta x$ $V=\lim_{\delta x\to 0}\sum_{x=0}^{2}\pi x^4\delta x$ $=\int_0^2 \pi x^4 dx$ $=\pi\left[\frac{1}{5}x^5\right]_0^2$ $=\frac{\pi}{5}\times 2^5=\frac{32\pi}{5}$ cubic units	3 Marks: Correct answer. 2 Marks: Correct integral for the volume of the solid. 1 Marks: Sets up the area of the slice

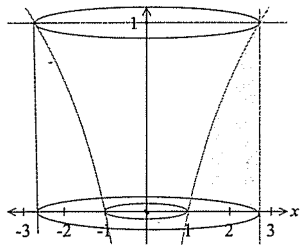
HSC Mathematics Extension 2

12(d) (i)	To find the equation of tangent through P $y=b\sin\theta$ $\frac{dy}{d\theta}=b\cos\theta$ $x=a\cos\theta$ $\frac{dx}{d\theta}=-a\sin\theta$ $\frac{dy}{dx}=\frac{dy}{d\theta}\times\frac{d\theta}{dx}$ $=b\cos\theta\times\frac{1}{-a\sin\theta}=\frac{-b\cos\theta}{a\sin\theta}$ Equation of the tangent $y-y_1=m(x-x_1)$ $y-b\sin\theta=\frac{-b\cos\theta}{a\sin\theta}(x-a\cos\theta)$ $ay\sin\theta-ab\sin^2\theta=-bx\cos\theta+ab\cos^2\theta$ $bx\cos\theta+ay\sin\theta=ab(\sin^2\theta+\cos^2\theta)$ $\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1$	2 Marks: Correct answer 1 Mark: Correctly calculates the gradient
12(d) (ii)	 At B $x=0$ and $\frac{0}{a}\cos\theta+\frac{y}{b}\sin\theta=1$ or $y=b\text{cosec}\,\theta$ Point B is $(0,b\text{cosec}\theta)$ and Point C is $(0,b\sin\theta)$ $OC\times OB=b\sin\theta\times b\text{cosec}=b^2$	2 Marks: Correct answer 1 Mark: Finds the coordinates or B or C.
12(e) (i) Not necessary	$f(x)=x^4-4x^3, f'(x)=4x^3-12x^2, f''(x)=12x^2-24x$ Stationary points $f'(x)=0$ $4x^3-12x^2=0$ or $4x^2(x-3)=0$ or $x=0$ or 3 $f''(0)=0$ possible point of inflection. $f''(3)=36>0$ (3, −27) is a Minima Points of inflection $f''(x)=0$ $12x^2-24x=0$ or $12x(x-2)$ or $x=0$ or $x=2$ $f''(0^-)>0$ and $f''(0^+)<0$ Hence (0,0) is a point of inflection $f''(2^-)<0$ and $f''(2^+)>0$ Hence (2,–16) is a point of inflection EASIEST CURVE SKETCHING. $f(x)=x^3(x-4)$ $f(x)=0$ $0=x^3(x-4)$ $x=0$ $x=4$ As $x=0$ is from a cubic and $x=4$ is from power of 1, both are odd ∴ curve cuts through.	2 Marks: Correct sketch 1 Marks: Turning point and cutting at $x=4$. 1 Mark: Point of inflexion at $x=0$ (like $y=x^3$)

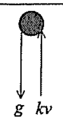
		
12(e) (ii)	The real solution of $x^4 - 4x^3 = kx$ is given by the x values where $y = x^4 - 4x^3$ and $y = kx$ intersect. If $k > 0$ then from the graph there are 2 real roots.	1 Mark: Correct answer.
13(a) (i)	 Focus $S(ae, 0)$ and at P (directrix $x = \frac{a}{e}$ and asymptote $y = \frac{b}{a}x$) At P $x = \frac{a}{e}$ and $y = \frac{b}{a} \times \frac{a}{e} = \frac{b}{e}$ $\quad \therefore P\left(\frac{a}{e}, \frac{b}{e}\right)$ Gradient $PS = \frac{\frac{b}{e}}{\frac{a}{e} - ae} = \frac{b}{a(1-e^2)}$ Gradient $OP = \frac{b}{a}$ $\therefore m_1 m_2 = \frac{b}{a(1-e^2)} \times \frac{b}{a} = \frac{b^2}{a^2(1-e^2)} = \frac{b^2}{-b^2} = -1$ Hence PS is perpendicular to OP.	2 Marks: Correct answer. 1 Mark: Finds the coordinates of P or shows some understanding of the problem.
13(a) (ii)	$PS^2 = \left(\frac{a}{e} - ae\right)^2 + \left(\frac{b}{e}\right)^2$ $= \frac{1}{e^2}\left[a^2(1-e^2)^2 + b^2\right]$ $= \frac{1}{e^2}\left[-b^2(1-e^2) + b^2\right]$ $= \frac{1}{e^2}\left[b^2 e^2\right]$ $PS = b$	1 Mark: Correct answer.

13(a) (iii)	Perpendicular distance from S to P is b (from parts (i) and (ii)). Tangent to a circle is perpendicular to the radius through the point of contact. Therefore P is the point of contact of a circle with centre S and radius b. Similarly, by symmetry Q is the point of contact of a circle with centre S and radius b.	1 Mark: Correct answer.
13(a) (iv)	If $a = b$ then $b^2 = a^2(e^2 - 1)$ $b^2 = b^2(e^2 - 1)$ $e^2 = 2$ or $e = \sqrt{2}$ Hence $S(a\sqrt{2}, 0)$ Using the locus definition of a hyperbola with $SR = ST = b$ $\frac{b}{x - \frac{a}{e}} = e$ $b = e(x - \frac{a}{e})$ $x = \frac{a+b}{e} = \frac{a+a}{\sqrt{2}} = a\sqrt{2}$ Therefore, if $a = b$, R and T have the same x coordinate ($a\sqrt{2}$) as S. Hence R, S and T are collinear and RT is the diameter of the circle with centre S.	2 Marks: Correct answer. 1 Mark: Finds the eccentricity or the x-coordinate of R or T in terms of a, b and e.
13(c) (i)	13(c)(i)  $\frac{n}{PA} = \sin\theta$ $\frac{n}{\sin\theta} = PA$ $n \operatorname{cosec}\theta = PA$ ✓ $\cos\theta = \frac{m}{BP}$ $\therefore BP = \frac{m}{\cos\theta}$ $= m\sec\theta$ ✓ So $BA = L(\theta) = m\sec\theta + n\operatorname{cosec}\theta$ (i)	Marks: 2 Correct answer. Marks: 1 Makes significant progress towards the solution.
(ii)	(ii) $L(\theta) = m(\cos\theta)^{-1} + n(\sin\theta)^{-1}$ $\frac{dL}{d\theta} = -m(\cos\theta)^{-2} \cdot -\sin\theta - n(\sin\theta)^{-2}\cos\theta$ $= m\frac{\sin\theta}{\cos^2\theta} - \frac{n\cos\theta}{\sin^2\theta}$ ✓ $\frac{dL}{d\theta} = 0 \quad 0 = \frac{m\sin\theta}{\cos^2\theta} - n\frac{\cos\theta}{\sin^2\theta}$ So $m\sin^3\theta = n\cos^3\theta$ ✓ $\tan^3\theta = \frac{n}{m}$ Now $\tan\theta = \frac{n^{\frac{1}{3}}}{m^{\frac{1}{3}}}$  ✓ $\therefore L^2 = (n^{\frac{1}{3}})^2 + (m^{\frac{1}{3}})^2$ So $L = \sqrt{n^{\frac{2}{3}} + m^{\frac{2}{3}}}$ (ii)	2 Marks: As indicated 1 Mark: As indicated. 3 Marks: All correct.

13(b) (i)	$y=\left\|\frac{x^2}{x^2-1}\right\|=\left\|\frac{x^2}{(x+1)(x-1)}\right\|$ (asymptote at $x=\pm1$) $y=\left\|\frac{x^2}{x^2-1}\right\|=\left\|\frac{1}{1-\frac{1}{x^2}}\right\|$ $x\to\pm\infty,\ y\to1$ (asymptote at $y=1$) $y=f(x)=\frac{x^2}{x^2-1}=\frac{x^2}{(x-1)(x+1)}$ 	2 Marks: Correct answer. 1 Mark: $y=\frac{x^2}{x^2-1}$ Determines the asymptotes or shows some understanding. 1 Mark for $y=\left\|\frac{x^2}{x^2-1}\right\|$
13(b) (ii)		2 Marks: Correct answer. 1 Mark: Determines the asymptotes or shows some understanding. (no negative logs)
14(a) (i)	Let $z=a+ib$ where a and b are real. $z\bar{z}=(a+ib)(a-ib)$ $=a^2-i^2b^2$ $=a^2+b^2=\|z\|^2$	1 Mark: Correct answer.
14(a) (ii)	Now $z_1=w$, $z_2=v\bar{z}_1=v\bar{w}$ and $\|v\|=1$ $z_3=v\bar{z}_2$ $=v\times\overline{v\bar{w}}$ $=v\bar{v}w$ $=\|v\|^2 w$ from (i) $=w$	2 Marks: Correct answer. 1 Mark: Uses the formula to obtain an expression for z_3.

14(b) (i)	Integration by parts $I_n=\int_0^{\frac{\pi}{2}}\sin^{n-1}x\sin x\,dx$ $=-\left[\sin^{n-1}x\cos x\right]_0^{\frac{\pi}{2}}+(n-1)\int_0^{\frac{\pi}{2}}\sin^{n-2}x\cos^2 x\,dx$ $=(n-1)\int_0^{\frac{\pi}{2}}\sin^{n-2}x\cos^2 x\,dx$	2 Marks: Correct answer. 1 Mark: Sets up the integration and shows some understanding.
14(b) (ii)	$I_n=(n-1)\int_0^{\frac{\pi}{2}}\sin^{n-2}x\cos^2 x\,dx$ $=(n-1)\int_0^{\frac{\pi}{2}}\sin^{n-2}x(1-\sin^2 x)dx$ $=(n-1)\int_0^{\frac{\pi}{2}}(\sin^{n-2}x-\sin^n x)dx$ $=(n-1)\left[I_{n-2}-I_n\right]=(n-1)I_{n-2}-nI_n+I_n$ $nI_n=(n-1)I_{n-2}$ $I_n=\frac{(n-1)}{n}I_{n-2}$	2 Marks: Correct answer. 1 Mark: Makes some progress towards the solution.
14(b) (iii)	$I_4=\frac{(4-1)}{4}I_2$ $=\frac{3}{4}\times\frac{(2-1)}{2}I_0$ $=\frac{3}{8}\times\int_0^{\frac{\pi}{2}}1dx=\frac{3\pi}{16}$	1 Mark: Correct answer.
14(c)	 Cylindrical shell – inner radius x, outer radius $x+\delta x$, height y. $\delta V=\pi\left[(x+\delta x)^2-x^2\right]y$ $=\pi\left[2x\delta x+\delta x^2\right]y=\pi(2x+\delta x)(\log_e x)\delta x$ $V=2\times\lim_{\delta x\to0}\sum_{x=1}^{e}\pi(2x+\delta x)\log_e x\delta x=2\pi\int_1^e(x\log_e x)dx$ $=2\pi\left(\left[\log_e x\times\frac{1}{2}x^2\right]_1^e-\int_1^e\left(\frac{1}{2}x^2\times\frac{1}{x}\right)dx\right)$ $=2\pi\left[\frac{1}{2}e^2-\frac{1}{2}\int_1^e x\,dx\right]=\pi\left(e^2-\left[\frac{x^2}{2}\right]_1^e\right)=\frac{\pi}{2}(e^2+1)$	4 Marks: Correct answer. 3 Marks: Correct integral for the volume of the solid. 2 Marks: Correct expression for δV. 1 Mark: Determines the radius or height of the cylindrical shell.

14(d)	14(d) Step 1 Show true for $n=2$ (EVEN!!!) $LHS = 2n^2 = 8 \quad RHS = 2^2+2+2 = 8$ TRUE $8 \geq 8$ Step 2 Assume true for $n=2k$ $2(2k)^2 \geq (2k)^2+(2k)+2$ $4k^2 \geq 2k^2+k+1$ ——— (*) Step 3 Prove true for $n=2k+2$ To prove: $2(2k+2)^2 \geq (2k+2)^2+(2k+2)+2$ That is $4k^2+8k+4 \geq 2(k^2+2k+1)+(k+1)+1$ $4k^2+8k+4 \geq 2k^2+5k+4$ LHS $4k^2+8k+4 \geq (2k^2+k+1)+8k+4$ from (*) $\geq 2k^2+9k+5$ $\geq 2k^2+5k+4$ as required $\therefore 4k^2+8k+4 \geq 2k^2+5k+4$ Step 4 The statement is true for $n=2$. Since it is true for $n=2k+2$ it is true for $n=2+2=4$, $n=4+2=8$ and so on for all even values of n.	3 Marks: Correct answer. 2 Marks: Proves the result true for $n=2$ and attempts to use the result of $n=k$ to prove the result for $n=2k+2$. 1 Mark: Proves the result true for $n=2$
15(a) (i)	$P(x)=(x-\alpha)^2Q(x)$ $P'(x)=(x-\alpha)^2Q'(x)+2(x-\alpha)Q(x)$ $=(x-\alpha)[(x-\alpha)Q'(x)+2Q(x)]$ Therefore $P'(\alpha)=0$ and $x=\alpha$ is a root of $P'(x)$.	1 Marks: Correct answer.
15(a) (ii)	$P(x)=x^3-ax^2+b$ has a root $x=\alpha$ $P(\alpha)=\alpha^3-a\alpha^2+b=0$ (1) $P'(x)=3x^2-2ax$ $P'(\alpha)=3\alpha^2-2a\alpha=0$ Take $\alpha=\frac{2a}{3}$ ($\alpha=0$ is a null result) $=\alpha(3\alpha-2a)=0$ Substituting $\alpha=\frac{2a}{3}$ into equation (1) $\left(\frac{2a}{3}\right)^3-a\left(\frac{2a}{3}\right)^2+b=0$ $\frac{8a^3}{27}-\frac{4a^3}{9}+b=0$ $-\frac{4a^3}{27}+b=0 \quad \therefore -4a^3+27b=0$ So $4a^3-27b=0$	2 Marks: Correct answer. 1 Mark: Shows some understanding of the problem. $P'(\alpha)=0$ and $\alpha=\frac{2a}{3}$ or $a=\frac{3\alpha}{2}$
15(b) (i)	Solving the equations $y=x^4+4x^3$ and $y=mx+b$ simultaneously has two solutions ($x=\alpha$ and $x=\beta$). $\therefore x^4+4x^3-mx-b=0$ is degree 4 with multiple roots at $x=\alpha$ and $x=\beta$. Zeros are α,α,β and β.	1 Mark: Correct answer.

15(b) (ii)	$\alpha+\alpha+\beta+\beta=-\frac{b}{a}$ $2(\alpha+\beta)=-4$ or $\alpha+\beta=-2$ $\alpha\alpha+\alpha\beta+\alpha\beta+\alpha\beta+\alpha\beta+\beta\beta=\frac{c}{a}$ $\alpha^2+\beta^2+4\alpha\beta=0$ $(\alpha+\beta)^2+2\alpha\beta=0$ $\alpha\beta=-2$ $\alpha\alpha\beta+\alpha\alpha\beta+\alpha\beta\beta+\alpha\beta\beta=-\frac{d}{a}$ and $\alpha\alpha\beta\beta=\frac{e}{a}$ $2\alpha\beta(\alpha+\beta)=m \qquad (\alpha\beta)^2=-b$ $m=8 \qquad b=-4$	3 Marks: Correct answer. 2 Marks: Makes significant progress towards the solution. 1 Mark: Uses the relationships between the roots and coefficients.
15(c) (i)	Newton's second law: $\ddot{x}=g-kv$ $\frac{dv}{dt}=g-kv$ 	1 Mark: Correct answer.
15(c) (ii)	$\frac{dv}{dt}=g-kv$ $\frac{dt}{dv}=\frac{1}{g-kv}$ $t=-\frac{1}{k}\log_e(g-kv)+C$ Initial conditions $t=0$ and $v=0$ $0=-\frac{1}{k}\log_e(g)+C$ $C=\frac{1}{k}\log_e g$ $t=-\frac{1}{k}\log_e(g-kv)+\frac{1}{k}\log_e g$ $=\frac{1}{k}\log_e\left(\frac{g}{g-kv}\right)$ $kt=\log_e\left(\frac{g}{g-kv}\right)$ $e^{kt}=\frac{g}{g-kv}$ $ge^{kt}-kve^{kt}=g$ $kve^{kt}=ge^{kt}-g$ $v=\frac{g}{k}(1-e^{-kt})$	3 Marks: Correct answer. 2 Marks: Correctly substitutes the initial conditions into the expression for t 1 Mark: Finds the correction expression for t.

Question	Solution	Marks
15(c) (iii)	$\frac{dv}{dt}=v\frac{dv}{dx}$ $v\frac{dv}{dx}=g-kv$ $\frac{dv}{dx}=\frac{g-kv}{v}$ $\frac{dx}{dv}=\frac{v}{g-kv}$ $x=\int\frac{-\frac{1}{k}(g-kv)+\frac{g}{k}}{g-kv}dv$ $=-\frac{1}{k}v-\frac{g}{k^2}\log_e(g-kv)+C$ When $x=0$ and $v=0$ $0=-\frac{1}{k}\times 0-\frac{g}{k^2}\log_e(g-k\times 0)+C$ $C=\frac{g}{k^2}\log_e g$ $x=-\frac{1}{k}v-\frac{g}{k^2}\log_e(g-kv)+\frac{g}{k^2}\log_e g$ $=-\frac{1}{k}v+\frac{g}{k^2}\log_e\left(\frac{g}{g-kv}\right)$ (iv) $\lim_{t\to\infty} v=\lim_{t\to\infty}\frac{g}{k}(1-e^{-kt})$ $=\frac{g}{k}$ So V approaches $\frac{g}{k}$, $v<\frac{g}{k}$ ∴ $kv<g$ 1 MARK OR $g-kv>0$ from the log $g>kv$ $\frac{g}{k}>v$ ∴ $g>kv$	3 Marks: Correct answer. 2 Marks: Makes significant progress towards the solution. 1 Mark: Uses results for part (i) to determine an expression for $\frac{dx}{dv}$
16(a) (i)	$z^n=[\cos\theta+i\sin\theta]^n$ $=\cos n\theta+i\sin n\theta$ $\frac{1}{z^n}=[\cos\theta+i\sin\theta]^{-n}$ $=\cos n\theta-i\sin n\theta$ $z^n-\frac{1}{z^n}=\cos n\theta+i\sin n\theta-\cos n\theta+i\sin n\theta$ $=2i\sin n\theta$	2 Marks: Correct answer. 1 Mark: Uses De Moivre's theorem
16(a) (ii)	$\left(z-\frac{1}{z}\right)^5=z^5+5z^4\left(-\frac{1}{z}\right)+10z^3\left(-\frac{1}{z}\right)^2+10z^2\left(-\frac{1}{z}\right)^3$ $+5z\left(-\frac{1}{z}\right)^4+\left(-\frac{1}{z}\right)^5$ $=\left(z^5-\frac{1}{z^5}\right)-5\left(z^3-\frac{1}{z^3}\right)+10\left(z-\frac{1}{z}\right)$ $(2i\sin\theta)^5=2i\sin 5\theta-10i\sin 3\theta+20i\sin\theta$ $\sin^5\theta=\frac{1}{16}(\sin 5\theta-5\sin 3\theta+10\sin\theta)$	2 Marks: Correct answer. 1 Marks: Makes significant progress.

Question	Solution	Marks
16(a) (iii)	$16\sin^5\theta=\sin 5\theta-5\sin 3\theta+10\sin\theta$ $16\sin^5\theta-10\sin\theta=\sin 5\theta-5\sin 3\theta=0$ So solve $16\sin^5\theta-10\sin\theta=0$ $(8\sin^4\theta-5)\sin\theta=0$ ∴ $\sin\theta=0$ or $\sin\theta=\pm\sqrt[4]{\frac{5}{8}}$ in $0\leq\theta\leq\pi$ $\theta=0,\pi$ $\theta=1.09546^c$, $\pi-1.09546$ $=1.09576^c$ 2.04613^c To nearest radian $\theta=0,3,1,2$	3 Marks: 0,1,2,3 c Correct answer. 2 Marks: Half solution 1 Mark: Makes some progress to $16\sin^5\theta=10\sin\theta$
16(b) (i)	Consider ΔBNQ and ΔAMQ. $\angle QBN=\angle QAM$ (angle between a tangent and a chord equals the angle in the alternate segment) $\angle QNB=\angle QMA=90^\circ$ (perpendiculars from Q) $\therefore \Delta BNQ \parallel\vert \Delta AMQ$ (Two angles of one triangle are respectively equal to two angles of another triangle) $\Delta ALQ \parallel\vert \Delta BMQ$ is a similar proof. Consider ΔALQ and ΔBMQ. $\angle QAL=\angle QBM$ (angle between a tangent and a chord equals the angle in the alternate segment) $\angle QLA=\angle QMB=90^\circ$ (perpendiculars from Q) $\therefore \Delta ALQ \parallel\vert \Delta BMQ$ (Two angles of one triangle are respectively equal to two angles of another triangle)	3 Marks: Correct answer. 2 Marks: Makes significant progress towards the solution. 1 Mark: Applies a relevant circle theorem.
16(b) (ii)	$\frac{QN}{QM}=\frac{QB}{QA}$ (matching sides in similar triangles $\Delta BNQ \parallel\vert \Delta AMQ$) $\frac{QM}{QL}=\frac{QB}{QA}$ (matching sides in similar triangles $\Delta ALQ \parallel\vert \Delta BMQ$) $\therefore\frac{QN}{QM}=\frac{QM}{QL}$ This represents a geometric sequence QN, QM, QL,…	2 Marks: Correct answer. 1 Mark: Matches the sides in the similar triangles.
16(c)	$\int\frac{dx}{1+\sin x}$ t result substitution $dx=\frac{2dt}{1+t^2}$ $t=\tan\frac{x}{2}$ so $\sin x=\frac{2t}{1+t^2}$ $=\int\frac{\frac{2dt}{1+t^2}}{1+\frac{2t}{1+t^2}}=\int\frac{2dt}{1+t^2+2t}$ $=\int\frac{2dt}{(t+1)^2}=\int 2(t+1)^{-2}dt$ $=\frac{2(t+1)^{-1}}{-1}+c$ $=\frac{-2}{t+1}+c$ PRIMITIVE $=\frac{-2}{\tan\frac{x}{2}+1}+c$ OTHERS COULD BE $\frac{-\cos x}{1+\sin x}+c$ or $\tan x-\sec x+c$	3 Marks: Correct answer. 2 Marks: Makes significant progress towards the solution. 1 Mark: Sets up $f(x)$ and uses calculus.