DANIEL HUYNH

Ext 2

NORTH SYDNEY BOYS HIGH SCHOOL

2005
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics
Extension 2

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write on one side of the paper (with lines) in the booklet provided
- Write using blue or black pen
- Board approved calculators may be used
- All necessary working should be shown in every question
- Each new question is to be started on a **new page.**

- Attempt all questions

Class Teacher:
(Please tick or highlight)
- Mr Ee
- ✓ Ms Silverman
- Mr WEiss

Student Number: 9270

(To be used by the exam markers only.)

Question No	1	2	3	4	5	6	7	8	Total	Total
Mark	13/15	14/15	15/15	12/15	14/15	12/15	14/15	9+4/15	103/120	/100

Question 1 Marks

(a) Express $z = \dfrac{7+4i}{3-2i}$ in the form $a + ib$ where a and b are real. 2

(b) (i) Express $z = -\sqrt{3} + i$ in modulus-argument form. 2

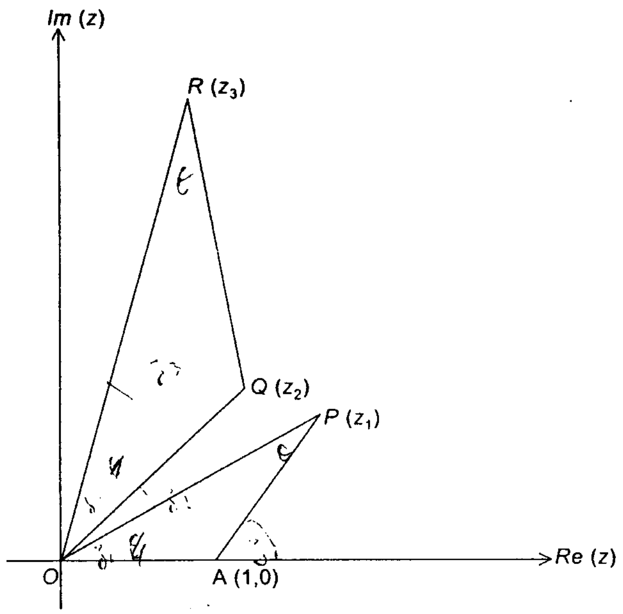
(ii) Hence show that $z^7 + 64z = 0$ 2

(c) On an Argand diagram sketch the locus of z satisfying :

(i) $\arg(z-1) = \dfrac{2\pi}{3}$ 2

(ii) $Re\, z = |z|$ 2

(d)



In the Argand diagram above, ΔOQR is constructed similar to ΔOAP.

Show that

(i) $|z_3| = |z_1||z_2|$ 2

(ii) $\arg z_3 = \arg z_1 + \arg z_2$ 2

(iii) What is the significance of these results? 1

Question 2 Marks

(a) Find :

(i) $\int \frac{e^x}{e^{2x}+1}\,dx$ 2

(ii) $\int x e^{2x}\,dx$ 2

(iii) $\int \frac{dx}{\sqrt{2x-x^2}}$ 2

(b) Evaluate $\int_0^{\frac{\pi}{2}} \frac{1}{5+3\cos\theta}\,d\theta$ 4

(c) (i) Show that $\int_0^a f(x)dx = \int_0^a f(a-x)\,dx$ 2

(ii) Hence evaluate $\int_0^{\frac{\pi}{2}} \frac{\cos x - \sin x}{1+\sin x\cos x}\,dx$ 3

Question 3

(a) (i) Prove that if the polynomial $P(x)$ has a root of multiplicity m then $P'(x)$ has a root of multiplicity $(m-1)$. 3

(ii) Find the value of k so that $5x^5 - 3x^3 + k = 0$ has two equal roots, both positive. 4

(b) If α, β, γ are the roots of $x^3 + px + q = 0$, find in terms of p and q

(i) $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}$ 2

(ii) $\alpha^3+\beta^3+\gamma^3$ 2

(c) Use the method of cylindrical shells to find the volume obtained when the region bounded by the curve $y=\sqrt{x}$ and the x-axis, between $x=0$ and $x=1$, is rotated about the line $x=1$ 4

Question 4 **Marks**

(a) (i) Show that the sum of the distances from any point P on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ to the two foci is equal to $2a$. 3

(ii) If $A=-2+3i$ and $B=8+3i$ verify that $|z-A|+|z-B|=20$ is an ellipse on the Argand diagram and find its eccentricity. 3

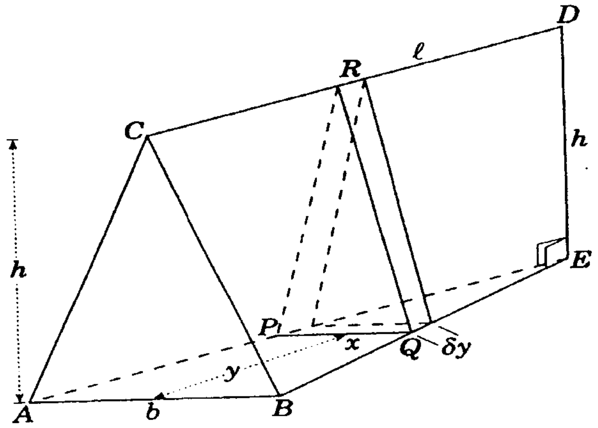
(b) Consider the function $y=\sin^{-1}(\sin x)$

(i) What is the range? 1

(ii) What is the period? 1

(iii) Sketch the function for $-2\pi \le x \le 2\pi$ 2

(c)



ABC is an isosceles triangle with $AC=BC$ and $AB=b$. $ABCDE$ is a wedge shape with height $DE=h$ and length $CD=\ell$. Triangle ABC and line DE are perpendicular to the plane of ABE as shown in the diagram.

Consider a slice of the wedge of height h and depth δy as in the diagram. The slice is parallel to the plane ABC at PQR.

(i) Calculate the area of the triangle PQR as a function of y. 2

(ii) Hence calculate the volume of the wedge. 3

Question 5

(a) The tangent to the hyperbola $xy=c^2$ at the point $T\left(ct,\frac{c}{t}\right)$ meets the x and y axes at F and G respectively and the normal at T meets the line $y=x$ at H.

(i) Show that the tangent at T is $x+t^2y=2ct$. 3

(ii) Show that the normal at T is $t^3x-ty=c(t^4-1)$. 2

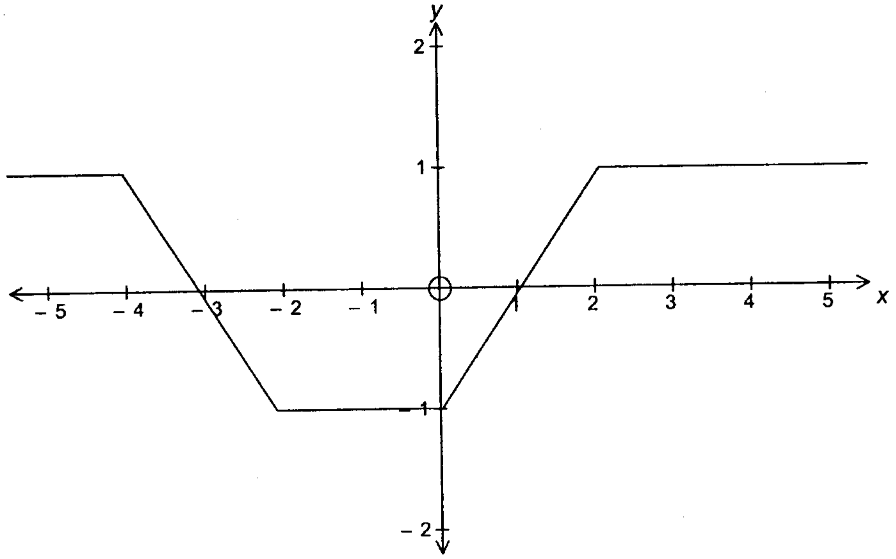
(iii) Prove that $FH \perp HG$. 6

(b) Given that the roots of the equation $x^3+ax^2+bx+c=0$ form a geometric sequence, determine the relationship between a, b and c. 4

Question 6

Marks

(a)



The diagram is a sketch of the function $y = f(x)$

On separate diagrams sketch:

(i) $y = |f(x)|$ 1

(ii) $y = f(|x|)$ 2

(iii) $y = \dfrac{1}{f(x)}$ 2

(iv) $\cos^{-1}(f(x))$ 2

(b) Let $z = \cos\theta + i\sin\theta$.

(i) Show that $z^n - z^{-n} = 2i\sin n\theta$ 2

(ii) Expand $(z - z^{-1})^3$ 2

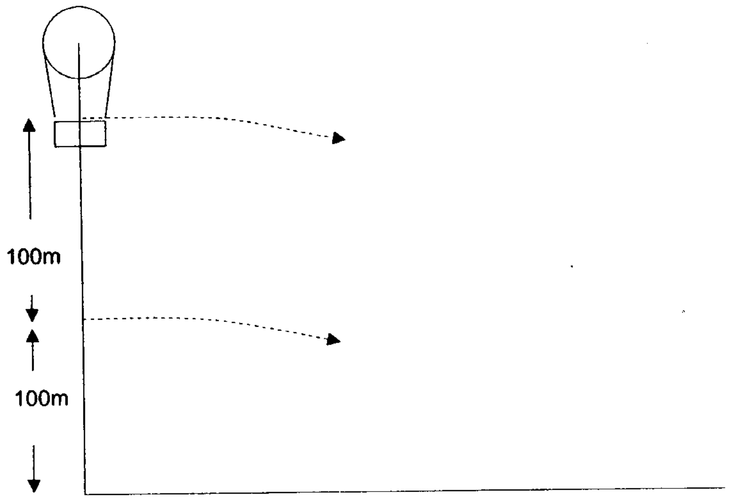
(iii) Hence show that $\sin^3\theta = \dfrac{1}{4}(3\sin\theta - \sin 3\theta)$ 2

(iv) Evaluate $\displaystyle\int_0^{\pi/2} \sin^3\theta \, d\theta$ 2

Question 7 **Marks**

(a) A particle moving with simple harmonic motion has a speed of 32m/s and 24m/s when its distances from the centre of motion are respectively 3 m and 4 m. Find the periodic time of the motion. 3

(b) A balloon rises vertically from level ground. Two projectiles are fired horizontally in the same direction from the balloon at a velocity of 80ms^{-1}. The first is fired at a point 100 m from the ground and the second when it has risen a further 100 m from the ground. How far apart will the projectiles hit the ground? (Use $g = 10ms^{-2}$) 4



(c) If $y = \frac{1}{2}\left(e^{x} - e^{-x}\right)$

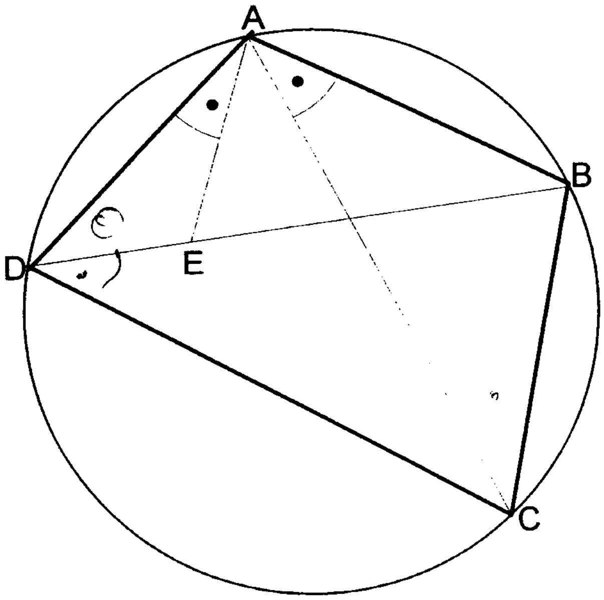
(i) Show that $x = \log_e \left| y + \sqrt{y^2 + 1} \right|$ 3

(ii) Show that $\left(\frac{dy}{dx}\right)^2 - y^2 = 1$ 2

(iii) Hence deduce that $\int \frac{dy}{\sqrt{y^2 + 1}} = \log_e \left| y + \sqrt{y^2 + 1} \right|$ 3

Question 8 **Marks**

(a)



ABCD is a cyclic quadrilateral.
E is a point on diagonal BD such that $\angle DAE = \angle BAC$.
Prove that :

(i) $AB \times CD = AC \times BE$ 3

(ii) $BC \times DA = AC \times DE$ 2

(iii) $AB \times CD + BC \times DA = AC \times BD$ 2

(b) If $I_n = \int \sec^n x \, dx$

(i) Prove that $I_n = \dfrac{\sec^{n-2} x \tan x}{n-1} + \dfrac{n-2}{n-1} I_{n-2}$ 4

(ii) Hence evaluate $\displaystyle\int_0^{\pi/4} \sec^6 x \, dx$ 4