15 copies

PCC PYMBLE

YEAR 12 4UNIT MATHEMATICS — TERM II 1986

ADDITIONAL PAPER — Miss Allum

TIME: (3) three hours

- All questions may be attempted. All questions are of equal value
- The questions are not necessarily arranged in order of difficulty. Candidates are advised to read the whole paper carefully at the start of the examination.
- A list of standard integrals is provided at the end of the paper
- Please hand in each question separately.

Question 1

(i) (a) Find $\int \frac{dx}{x \log x}$

(b) Evaluate $\int_1^2 \frac{dx}{\sqrt{x^2+4x+3}}$ correct to three decimal places.

(ii) Using the "integration by parts" technique. or otherwise,

(a) Find $\int e^{2x} \sin 3x \, dx$

(b) Show that, for $n > 1$

$$\int \sin^n x \, dx = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n} \int \sin^{n-1} x \, dx$$

and hence find $\int_0^{\frac{\pi}{2}} \sin^7 x \, dx$ and $\int_0^{\pi} \sin^6 x \, dx$

Question 2

(i) Sketch the following curves, showing all essential features.

(a) $y = e^{-x^2}$

(b) $(x+1)(y-2) = 1$

(c) $y = \max(x, 1-x)$

where $\max(p,q)$ denotes the greater of the two numbers, p and q

(ii) Sketch the curve $9y^2 = x(3-x)^2$ and show that it forms a loop.

Find:

(a) the maximum width of the loop measured parallel to the y-axis

(b) the area enclosed by the loop .

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Question 3

(i) Given $w = 2-3i$

determine

(a) $|w|$ (ie the modulus of w)

(b) $\overline{w}$ (ie the conjugate of w)

(ii) Describe, in geometric terms, the locus (in the Argand plane), represented by

$$2|z| = z+\bar{z}+4$$

(iii) Solve $(4-i)z = 7+2i$ for z in the form a+bi

(iv) Solve $z^2-3z-3-5i = 0$ for z in the form a+bi

(v) Solve $z^5=1-i$ for z in the form $r(\cos\theta + i\sin\theta)$

Question 4

(i) Sketch the conic section: $\frac{x^2}{2} - \frac{y^2}{9} = 1$, giving the co-ordinates of the foci and the equations of the directrices. Find also the eccentricity of the curve.

Find the equation of the normal to $\frac{x^2}{2}-\frac{y^2}{9}=1$ at the point $(2,-3)$

(ii) P is any point on the ellipse whose equation is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, and F is a focus of the ellipse. Prove that the line through F perpendicular to the tangent at P, and OP produced, meet on the directrix corresponding to the focus F.

Question 5

(i) A driver applies the brakes to come to a halt at a set of traffic lights. If the car is initially travelling at 60km/h and covers 30 metres in the time taken to come to a stop, find

(a) the retardation

(b) the time taken to come to a stop

(Assume retardation constant).

(ii) Carbon collected from an ancient campfire contained only one sixth as much C^{14} as carbon from a present day fire. Given that the half-life of C^{14} is 5700 years, find the age of the ancient campfire.

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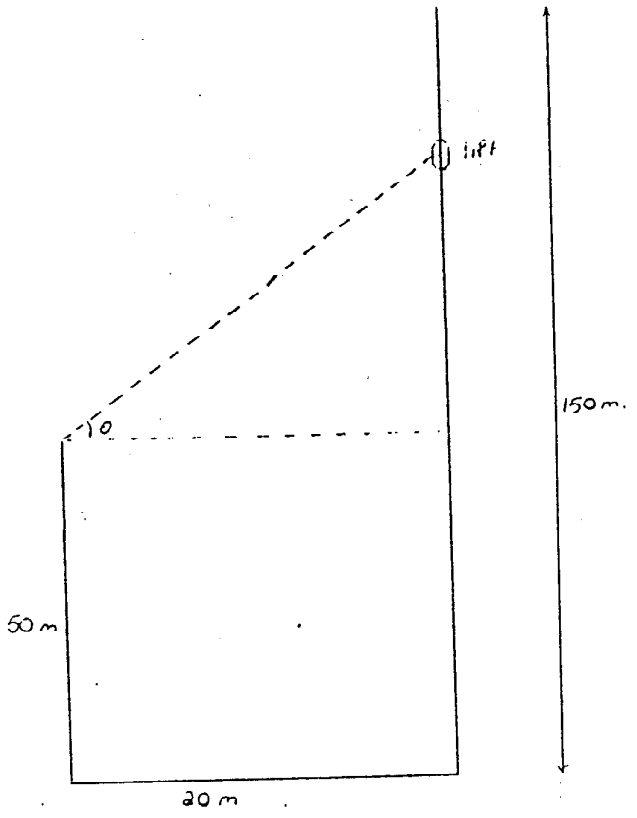
Question 6

(i) A symmectrial concrete road safety bump is made with a parabol cross-section. Its width is 1 metre, its greatest height is 100 mm and its length 6 metres. Calculate the volume of concrete acquired.

(ii) A building 150m high is equipped with an external elevator. This elevator starts at the top at time t=0 and descends at the constant rate of 10m sec $^{-1}$.
You are watching from a window that is 50m above the ground in a building 20m away from the elevator. At what height does the elevator appear to you to be moving fastest (ie when is the rate of change of angle at your eye greatest)?



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Question 7

A function f is called odd if $f(-x) = -f(x)$ for all x.

(i) Prove that every odd function is zero at x=0

(ii) Prove that every odd polynomial P(x) is divisible by x

(iii) The polynomial P(x) is known to be monic, to be an odd function, and to have a root at x=-5. Show that P(x) has degree no less that 3.

(iv) Find a polynomial Q(x) of degree 3 with the properties given in part (iii). State, with reasons, whether there are any other polynomials of degree 3 with these properties.

(v) State the form of the mose general polynomial with the properties given in part (iii) and with degree d in the range $4 \leqslant d \leqslant 6$

Question 8

(i) Find the values of x, $0 \leqslant x \leqslant 2\pi$

for which (a) $\sin 3x = \cos x$

(b) $3 \tan^2 x = 2 \sin x$

(ii) Determine the maximum and minimum values of the function $3x^4-20x^3+36x^2$ and sketch the graph of the function between the values x=-1 and x=4.

Obtain the condition that $3x^4-20x^3+36x^2=k$ should have four distinct roots. Prove that, when this consition is satisfied, the difference between the greatest and least of the four roots is less than $\frac{4}{3}\sqrt{10}$.