Student Number: ____________________

St George Girls High School

Trial Higher School Certificate Examination

2018

Mathematics Extension 2

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black pen
- Board-approved calculators may be used
- A reference sheet is provided
- In Questions 11 – 16, show relevant mathematical reasoning and/or calculations

Section I	/10
Section II	
Question 11	/15
Question 12	/15
Question 13	/15
Question 14	/15
Question 15	/15
Question 16	/15
Total	**/100**

Total Marks – 100

Section I Pages 2 – 6

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section
- Answer on the multiple choice answer sheet provided at the back of this paper

Section II Pages 7 – 16

90 marks

- Attempt Questions 11 – 16
- Allow about 2 hours and 45 minutes for this section
- Begin each question in a new writing booklet

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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1. Find $\int \frac{1}{x^2+6x+13}\,dx$

   (A) $\sin^{-1}\left(\frac{x+3}{2}\right)+c$

   (B) $\frac{1}{2}\cos^{-1}(x+3)+c$

   (C) $2\tan^{-1}\left(\frac{x+3}{2}\right)+c$

   (D) $\frac{1}{2}\tan^{-1}\left(\frac{x+3}{2}\right)+c$

2. The polynomial $P(x) = x^3 + x - 3$ has roots α, β and γ.

   What is the value of $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$?

   (A) $\frac{1}{3}$

   (B) $-\frac{1}{3}$

   (C) 0

   (D) 3

3. If $= \frac{\sqrt{3}i+1}{i}$, find $\bar{z}$ in modulus-argument form?

   (A) $2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

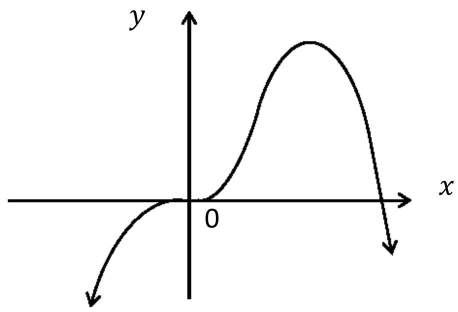
   (B) $2\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$

   (C) $4\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

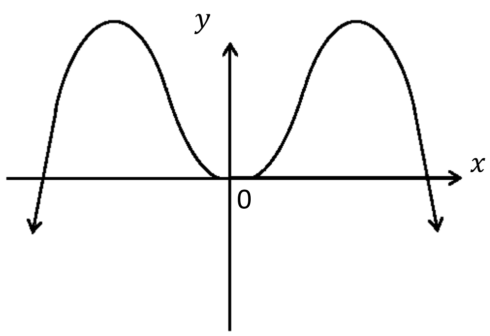
   (D) $4\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)$

Section I (cont'd)

4. The diagram below shows the graph of the function $y = f(x)$.



A second graph is obtained from the function $y = f(x)$.



Which equation best represents the second graph?

(A) $y = [f(x)]^2$

(B) $y = f(|x|)$

(C) $y = |f(x)|$

(D) $y = |f(|x|)|$.

5. It is given that $z = -1 + 2i$ is a root of $z^3 + 4z^2 + 9z + b = 0$, where b is a real number.

What is the value of b?

(A) -10

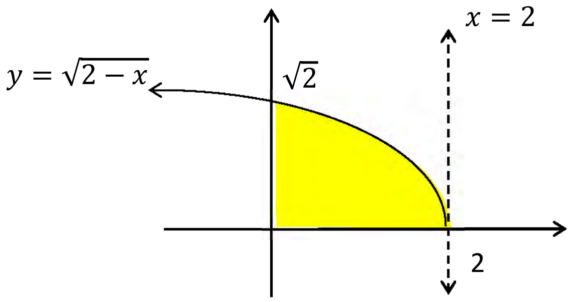
(B) -12

(C) 10

(D) 15

Section I (cont'd)

6. The region bounded by the curve $y = \sqrt{2 - x}$, the x axis and the y axis is rotated about the line x = 2 to form a solid.



Which one of these expressions represents the volume of the solid?

(A) $\pi \int_0^{\sqrt{2}} (2^2 - y^4)\ dy$

(B) $\pi \int_0^{\sqrt{2}} (2^2 - y^2)\ dy$

(C) $\pi \int_0^{\sqrt{2}} (2 - y^2)^2\ dy$

(D) $\pi \int_0^{\sqrt{2}} (2 - y)^2\ dy$

7. Consider the ellipse with the equation $\frac{x^2}{9} + y^2 = 1.$
What are the coordinates of the foci of the ellipse?

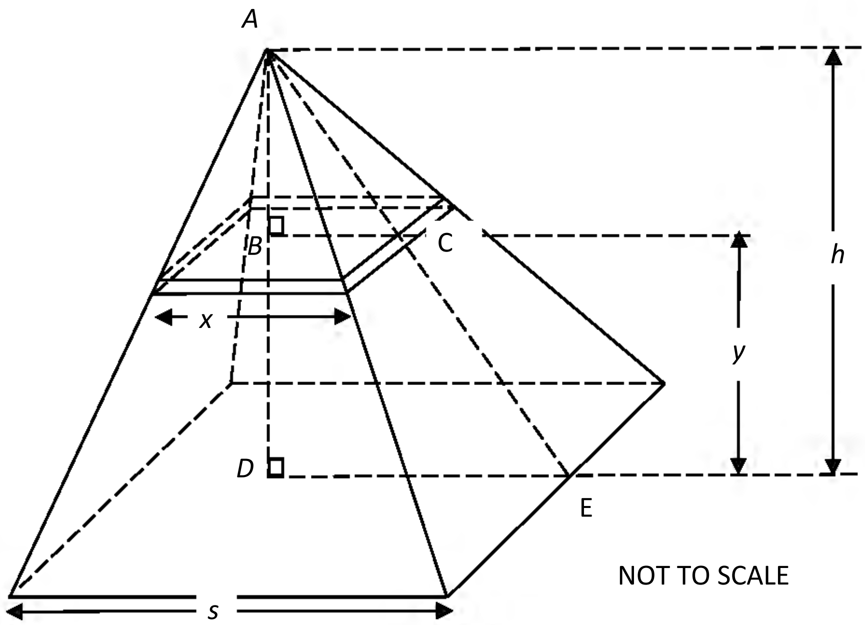
(A) $(\pm 6\sqrt{2}), 0)$

(B) $(0, \pm 6\sqrt{2}))$

(C) $(0, \pm 2\sqrt{2})$

(D) $(\pm 2\sqrt{2}), 0)$

8. Consider the **square** slices in the right **square pyramid** below



Find an expression for x in terms of s, h and y.

(A) $x = \dfrac{s(h+y)}{h}$

(B) $x = \dfrac{s(y-h)}{h}$

(C) $x = \dfrac{s(h-y)}{h}$

(D) $x = \dfrac{h(h-y)}{s}$

Section I (cont'd)

9. A rock of mass m falls vertically from rest at the top of a cliff in a medium whose air resistance is proportional to the velocity of the rock. If the rock falls to ground level under the influence of g, the acceleration due to gravity, which of the following is the correct expression for the velocity of the rock, given that downwards is taken to be the positive direction?

(A) $v = \frac{g}{k}(1 + e^{-kt})$

(B) $v = \frac{g}{k}(1 - e^{-kt})$

(C) $v = \frac{g}{k}(e^{-kt} + 1)$

(D) $v = \frac{g}{k}(e^{-kt} - 1)$

10. A particle is projected with a speed of 20 m/s and passes through a point P whose horizontal distance from the point of projection is 30 m and whose vertical height above the point of projection is 8.75 m. What is the angle of projection? (Take $g = 10\ m/s^2$).

(A) $\tan^{-1}\left(\frac{2}{3}\right)$

(B) $\tan^{-1}\left(\frac{3}{2}\right)$

(C) $\tan^{-1}\left(\frac{3}{4}\right)$

(D) $\tan^{-1}\left(\frac{4}{3}\right)$

End of Section I

Section II
90 marks
Attempt Questions 11 – 16
Allow about 2 hours 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11 – 16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet **Marks**

(a) z is a complex number such that $|z| = 2$ and $\arg z = \frac{\pi}{3}$.

(i) Evaluate z^5 . 1

(ii) Write down z in cartesian form. 1

(iii) Find the value of $\frac{1}{z}$ in cartesian form. 2

(iv) If $\omega = 2 - 3i$, find the value of $\omega^2 z$. 1

(b) Find

(i) $\int \frac{x}{1+x^4}\,dx$. 2

(ii) $\int \tan^3 x\,dx$. 2

(c) By considering the complex number $z = x + iy$ in the Argand plane and on separate Argand diagrams,

(i) sketch the region of the complex plane for which the complex number $z = x + iy$ has a positive real part and $|z + 3i| \leq 2$. 2

(ii) sketch the locus of $\arg \bar{z} = \frac{\pi}{3}$. 1

(d) Find the equation of the normal to the curve $x^2 - xy + y^3 = 1$ at the point $P\ (1, 1)$ to the curve. 3

Question 12 (15 marks) Use a SEPARATE writing booklet **Marks**

(a) Let $Q(x)$ be a polynomial. 3

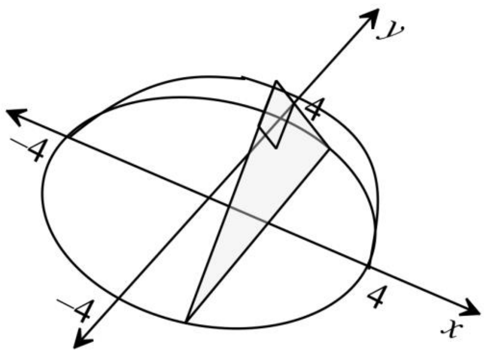
$Q(x) = px^3 + 2x^2 + qx - 4$, where p and q are real numbers. Find the values of p and q given that $(x+1)^2$ is a factor of $Q(x)$.

(b) (i) Find the values of *a*, *b*, and *c* such that:

$$\frac{x+1}{(x+3)(x+2)^2} = \frac{a}{(x+3)} + \frac{b}{(x+2)} + \frac{c}{(x+2)^2}$$ 3

(ii) Hence find $\int \frac{x+1}{(x+3)(x+2)^2}\,dx$. 2

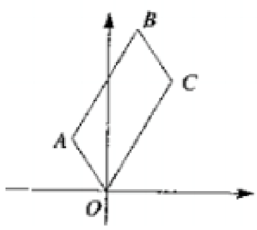
(c) Let S be the solid having its base the region bounded by the curve $x^2 + y^2 = 16$. 4



Every plane of the solid taken perpendicular to the x –axis is an isosceles right-angled triangle with the hypotenuse in the plane of the base.

Find the volume of the solid S.

(d) 3



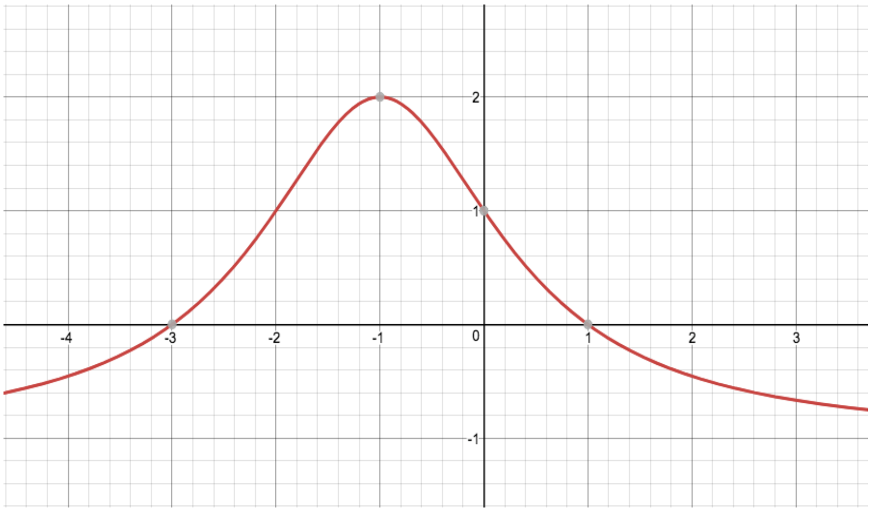
In the diagram above, $OABC$ is a parallelogram with $OA = \frac{1}{2}OC$.

The point A represents the complex number $-\frac{1}{2} + i\frac{\sqrt{3}}{2}$.

If $\angle AOC = \frac{\pi}{3}$, what complex number does C represent?

Question 13 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a)



The graph of $y = f(x)$ is shown above.
Draw separate one-third page sketches of these functions. Indicate clearly any asymptotes and intercepts with the axes.

(i) $y = |f(x)|$ 1

(ii) $y = \{f(x)\}^3$ 2

(iii) $y^2 = f(x)$ 2

(iv) $y = f(1 - x)$ 2

(b) Solve the polynomial equation $x^4 - 6x^3 + 9x^2 + 4x - 12 = 0$, given that the equation has a double root. 3

(c) The region bounded by the parabola $y = 4x(3 - x)$ and the x −axis is rotated about the y −axis to form a solid. 3
Use the method of cylindrical shells to find the volume of the solid.

(d) The equation $x^3 - 5x - 2 = 0$ has roots α, β and γ. 2
Find the equation with integer coefficients that has roots $\alpha + 1$, $\beta + 1$ and $\gamma + 1$.

Question 14 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) The points $P\left(2p, \frac{2}{p}\right), p \neq 0,$ and $Q\left(2q, \frac{2}{q}\right),\ q \neq 0$, are two points on the rectangular hyperbola $xy = 4$.

(i) Show that the equation of the chord PQ is $x + pqy = 2(p + q)$. 1

(ii) Prove that the tangent at P has equation $x + p^2y = 4p$. 2

(iii) The tangents at P and Q intersect at T. Find the coordinates of T. 2

(iv) The line through T, parallel to PQ passes through the point (0, 2). Show that $p + q = 4$. 2

(b) Find $\int \frac{\ln x}{x^2}\, dx$ 2

(c) (i) Prove the identity $\frac{1}{4}\cos 3A = \cos^3 A - \frac{3}{4}\cos A$. 2

(ii) Show that $\cos 3A = \frac{-1}{2\sqrt{2}}$, given that $x = 2\sqrt{2}\ \cos A$ satisfies the cubic equation $x^3 - 6x + 2 = 0$. 2

(iii) What are the three roots of the equation $x^3 - 6x + 2 = 0$? 2

Answer correct to four decimal places.

Question 15 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) By using the substitution $t = \tan\frac{\theta}{2}$, show that $\int_0^{\frac{\pi}{3}} \sec\theta \, d\theta = \ln\ (2 + \sqrt{3})$. 3

(b) $I_n = \int_1^{e} (1 - \ln x)^n \, dx$, n $= 0, 1, 2, 3, \ldots$

(i) Show $I_n = -1 + nI_{n-1}$, $n = 1, 2, 3, \ldots$ 2

(ii) Hence evaluate I_3. 2

(c) The Hyperbola H has equation $9x^2 - 16y^2 = 144$.

(i) Write down the eccentricity for this hyperbola and find the coordinates of its foci S and S'. 2

(ii) If $P(x_1, y_1)$ is an arbitrary point on H, prove that the equation of the tangent T at P is: $9xx_1 - 16yy_1 = 144$. 2

(iii) Hence find the coordinates of the point G at which the tangent T cuts the x – axis. 1

(iv) Hence show that $SP = \frac{5x_1 - 16}{4}$ and that $\frac{SP}{S'P} = \frac{SG}{S'G}$. 3

Question 16 (15 marks) Use a SEPARATE writing booklet. **Marks**

a) (i) A particle of mass m is projected vertically upwards under gravity g, the air resistance to the motion being $\dfrac{mgv^2}{a^2}$ when the speed is v, where a is a constant.

Show that during the upward motion of the particle **2**

$$v\frac{dv}{dx} = -\frac{g}{a^2}(a^2 + v^2)\ .$$

where x is the upward motion of the particle.

(ii) Show that the greatest height reached, given the speed of the projection u, is **3**

$$\frac{a^2}{2g}\ln\left(1 + \frac{u^2}{a^2}\right).$$

b) In the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$, $(a > b)$, B and B' are two points where the ellipse cuts the $y - axis$. The tangents at B and B' to the ellipse intersect the tangent at P in Q and Q' respectively. Let P be the point $(a\ \cos\theta, b\ \sin\theta)$.

Draw a diagram to represent this information.

If the equation of the tangent at P is $bx\cos\theta + ay\sin\theta - ab = 0$,

show that $BQ \times B'Q' = a^2$. **4**

(c) A particle is projected, with an angle of θ, form the origin with initial velocity U to pass through a point (a, b).

(i) Show that the Cartesian equation of the motion of the particle is given by **3**

$$y = \frac{-gx^2}{2U^2}\sec^2\theta + x\tan\theta.$$ You must DERIVE all equations of motion.

(ii) Prove that there are two possible trajectories if: **3**

$$(U^2 - gb)^2 > g^2(a^2 + b^2).$$

End of Examination

MATHEMATICS EXTENSION 2 – QUESTION Multiple Choice Q1 – 10

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
1. $\int \frac{1}{x^2+6x+13}\,dx = \int \frac{1}{x^2+6x+9+13-9}\,dx$		
$= \int \frac{1}{(x+3)^2+4}\,dx$		
$= \frac{1}{2}\tan^{-1}\left(\frac{x+3}{2}\right)+c$		D
2. $P(x) = x^3+x-3$		
$\alpha+\beta+\gamma = 0$		
$\alpha\beta\gamma = 3$		
$\alpha\beta+\beta\gamma+\alpha\gamma = 1$		
$\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma} = \frac{\beta\gamma+\alpha\gamma+\alpha\beta}{\alpha\beta\gamma}$		
$= \frac{1}{3}$		A
3. $z = \frac{\sqrt{3}i+1}{i} \times \frac{i}{i}$		
$= \frac{-\sqrt{3}+i}{-1}$		
$= \sqrt{3}-i$		
$\bar{z} = \sqrt{3}+i$		
$\lvert\bar{z}\rvert = \sqrt{(\sqrt{3})^2+(1)^2}$		
$= \sqrt{4} = 2$		
$\arg\bar{z} = \tan^{-1}\frac{1}{\sqrt{3}}$		
$= \pi/6 \quad \therefore \bar{z} = 2\,\mathrm{cis}\,\pi/6$		A
4. The portion where $x \geq 0$ has been		B
reflected about the y-axis		
So $g(x) = g(-x)$		

MATHEMATICS EXTENSION 2 – QUESTION

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS

5. $z = -1+2i$ is a root of $P(z)$

$\bar{z} = -1-2i$ is also a root.

So $\alpha = -1+2i$, $\beta = -1-2i$ and γ are 3 roots.

Sum of roots: $\alpha + \beta + \gamma = -4$

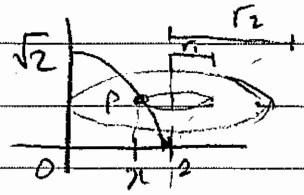
$$-1+2i-1-2i+\gamma = -4$$
$$-2+\gamma = -4$$
$$\gamma = -2$$

Product of roots: $(-1+2i)(-1-2i)-2 = -b$

$$(1+4)\times -2 = -b$$
$$b = 101$$

C

6.



$$r_2 = 2$$
$$r_1 = 2-x$$

$$A_{washer} = \pi(r_2^{\,2} - r_1^{\,2})$$
$$= \pi(2^2 - (2-x)^2)$$
$$= \pi(2^2 - (y^2)^2)$$

$$V = \int_0^{\sqrt{2}} \pi(2^2 - y^4)\,dy$$

A

7. $\dfrac{x^2}{9} + y = 1$ Using $b^2 = a^2(1-e^2)$

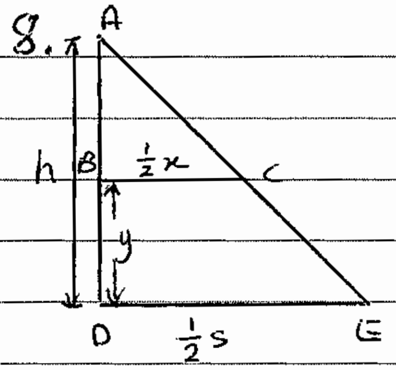
$$1 = 9(1-e^2)$$
$$\frac{1}{9} = 1-e^2$$
$$e^2 = 1 - \tfrac{1}{9} = \tfrac{8}{9}$$
$$e = \frac{2\sqrt{2}}{3}$$

$$\text{Foci} = (\pm ae, 0)$$
$$= \left(\pm 3 \times \frac{2\sqrt{2}}{3}, 0\right)$$
$$= (\pm 2\sqrt{2}, 0)$$

D

MATHEMATICS EXTENSION 2 – QUESTION Multiple Choice		
SUGGESTED SOLUTIONS	**MARKS**	**MARKER'S COMMENTS**

8.



In $\Delta ABC \parallel\!| \Delta ADE$

$$\frac{BC}{DE} = \frac{AB}{AD}$$

$$\frac{\frac{1}{2}x}{\frac{1}{2}s} = \frac{h-y}{h}$$

$$x = \frac{s(h-y)}{h}$$

C

9. $\ddot{x} = g - kv$

$$\frac{dt}{dv} = \frac{1}{g-kv}$$

$$\int_0^t t\,dt = \int_0^v \frac{1}{g-kv}\,dv$$

$$t = -\frac{1}{k}\left[\ln(g-kv) - \ln g\right]$$

$$-kt = \ln(g-kv) - \ln g$$

$$= \ln\left(\frac{g-kv}{g}\right)$$

$$e^{-kt} = \frac{g-kv}{g}$$

$$= 1 - \frac{kv}{g}$$

$$\frac{kv}{g} = 1 - e^{-kt}$$

$$v = \frac{g}{k}\left(1 - e^{-kt}\right)$$

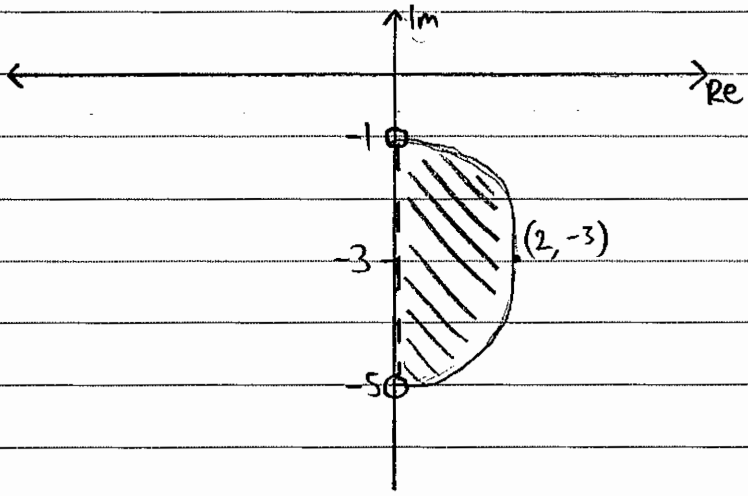
B

MATHEMATICS EXTENSION 2 – QUESTION Multiple Choice		
SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
10. $x = Vt\cos\alpha$, $y = -\frac{1}{2}gt^2 + Vt\sin\alpha$		
$30 = 20t\cos\alpha$		
$t = \frac{3}{2\cos\alpha}$ — ①		
$y = -\frac{gt^2}{2} + Vt\sin\alpha$		
$8{\cdot}75 = -\frac{1}{2}\times 10\times t^2 + 20t\sin\alpha$		
$\times 4$		
$35 = -20t^2 + 80t\sin\alpha$ — ②		
Sub ① in ②		
$35 = -20\left[\frac{3}{2\cos\alpha}\right]^2 + 80\left(\frac{3}{2\cos\alpha}\right)\sin\alpha$		
$= -45\sec^2\alpha + 120\tan\alpha$		
$7 = -9(1+\tan^2\alpha) + 24\tan\alpha$		
$9\tan^2\alpha - 24\tan\alpha + 16 = 0$		
$(3\tan\alpha - 4)^2 = 0$		
$3\tan\alpha = 4$		
$\tan\alpha = \frac{4}{3}$		
$\alpha = \tan^{-1}\frac{4}{3}$		D.

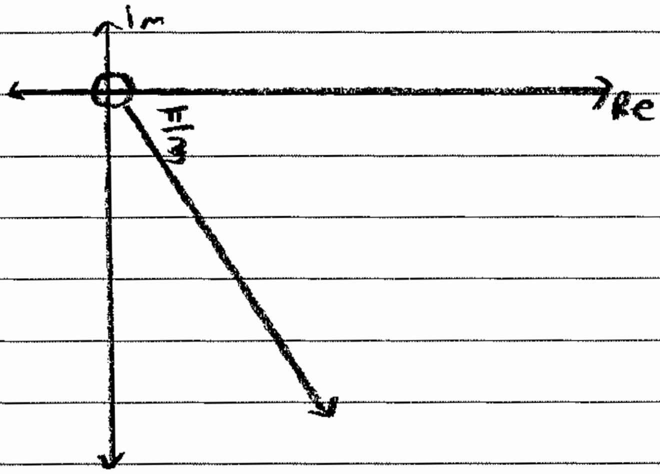
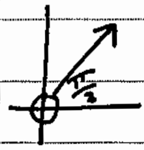
MATHEMATICS EXTENSION 2 – QUESTION 11

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
(a) i $z = 2\text{cis}\frac{\pi}{3}$		
$\therefore z^5 = 2^5 \text{cis}\left(\frac{5\pi}{3}\right)$	1	
$= 32\,\text{cis}\,\frac{5\pi}{3}$		
or $32\,\text{cis}\left(\frac{-\pi}{3}\right)$		
or $16 - 16\sqrt{3}\,i$		
ii $2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right) = 2\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$		
$= 1 + i\sqrt{3}$	1	
iii $\frac{1}{z} = \frac{1}{1+i\sqrt{3}}$		
$= \frac{1}{1+i\sqrt{3}} \times \frac{1-i\sqrt{3}}{1-i\sqrt{3}}$	1	
$= \frac{1-i\sqrt{3}}{1-3i^2}$		
$= \frac{1-i\sqrt{3}}{4}$		
$= \frac{1}{4} - \frac{i\sqrt{3}}{4}$	1	
iv $\omega^2 z = (2-3i)(2-3i)(1+i\sqrt{3})$		
$= (4-12i+9i^2)(1+i\sqrt{3})$		
$= (-5-12i)(1+i\sqrt{3})$		
$= -5 - 12\sqrt{3}\,i^2 - 12i - 5\sqrt{3}\,i$		
$= -5 + 12\sqrt{3} - (5\sqrt{3}+12)\,i$	1	

MATHEMATICS EXTENSION 2 – QUESTION 11

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
b) i let $u = x^2$		
$\frac{du}{dx} = 2x$		
$du = 2x\,dx$		
$\int \frac{x}{1+x^4}\,dx = \frac{1}{2}\int \frac{2x\,dx}{1+(x^2)^2}$		
$= \frac{1}{2}\int \frac{du}{1+u^2}$	1	Everyone remembered to add "+C"! :)
$= \frac{1}{2}\tan^{-1}x^2 + C$	1	
ii $\int \tan^3 x\,dx = \int \tan^2 x \tan x\,dx$		
$= \int (\sec^2 x - 1)\tan x\,dx$	1	
$= \int \sec^2 x \tan x\,dx - \int \tan x\,dx$		
$= \int \sec^2 x \tan x\,dx + \int \frac{-\sin x}{\cos x}\,dx$		
$= \frac{1}{2}\tan^2 x + \ln\lvert\cos x\rvert + C$	1	
c) i 	$1\frac{1}{2}$ $\frac{1}{2}$	Essentially correct region All details (including broken line, open circles, labels on points)

MATHEMATICS EXTENSION 2 – QUESTION 11

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
c) ii 	1	 earned $\frac{1}{2}$ marks
d) $x^2 - xy + y^3 = 1$ Differentiating implicitly, $2x - \left(y + x\frac{dy}{dx}\right) + 3y^2\frac{dy}{dx} = 0$	1	
$3y^2\frac{dy}{dx} - x\frac{dy}{dx} = y - 2x$ $\frac{dy}{dx} = \frac{y-2x}{3y^2-x}$ At $P(1,1)$ $\frac{dy}{dx} = \frac{1-2}{3-1}$ $= -\frac{1}{2}$ $\therefore$ gradient of tangent is $-\frac{1}{2}$	1	
$\therefore$ gradient of normal is 2 Equation of normal $y - 1 = 2(x-1)$ $y = 2x - 2 + 1$ $\therefore y = 2x - 1$	1	

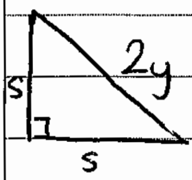
MATHEMATICS EXTENSION 2 – QUESTION 12

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
a) $(x+1)^2$ is a factor of $Q(x)$ means $x=-1$ is a root of multiplicity 2 of $Q(x)$.		
$Q(x) = px^3+2x^2+qx-4$		
$Q'(x) = 3px^2+4x+q$		
Now		
$Q(-1)=0$ and $Q'(-1)=0$		
So $Q(-1) = -p+2-q-4=0$		
$-p-q=2$		
$p+q=-2$ ...①	1	
$Q'(-1) = 3p-4+q=0$		
$3p+q=4$ ...②	1	
② − ①		
$2p = 6$		
$p=3$ sub in ①	½	
$3+q=-2$		
$q=-5$	½	
$\therefore p=3, \quad q=-5$		
		3

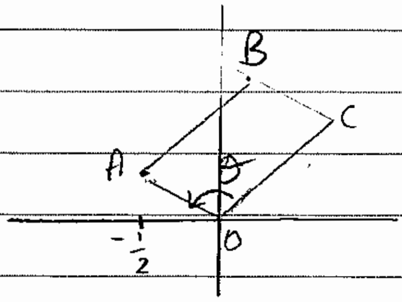
MATHEMATICS EXTENSION 2 – QUESTION 12

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
b) i) $\dfrac{x+1}{(x+3)(x+2)^2} = \dfrac{a}{x+3} + \dfrac{b}{x+2} + \dfrac{c}{(x+2)^2}$		Care needs to be taken when multiplying through by the LCD.
$x+1 = a(x+2)^2 + b(x+2)(x+3) + c(x+3)$		Remember:
when $x=-2$: $-1 = 0+0+c$		$LCD = (x+3)(x+2)^2$
$c = -1$	1	
when $x=-3$: $-2 = a+0+0$		
$a = -2$	1	
Equating coef of x^2:		
$0 = a + b$		
$0 = -2 + b$		
$b = 2$	1	
$\therefore a=-2,\ b=2,\ c=-1$		
ii) $I = \displaystyle\int \frac{x+1}{(x+3)(x+2)^2}\,dx$		This part was done well
$= \displaystyle\int \frac{-2}{x+3}\,dx + \int \frac{2}{x+2}\,dx - \int \frac{dx}{(x+2)^2}$	1	
$= -2\ln\lvert x+3\rvert + 2\ln\lvert x+2\rvert - \displaystyle\int (x+2)^{-2}\,dx$		
$= 2\left[\ln\lvert x+2\rvert - \ln\lvert x+3\rvert\right] - \dfrac{(x+2)^{-1}}{-1} + c$	1	
$= 2\ln\left\lvert\dfrac{x+2}{x+3}\right\rvert + \dfrac{1}{x+2} + c$		
		2

MATHEMATICS EXTENSION 2 – QUESTION 12

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
c) Area of right-angled isosceles triangle:		
 $A = \frac{1}{2}bh$		
$A = \frac{1}{2}s^2 \quad --- (1)$		Care needs to be taken when finding the area of the right angled isosceles triangle w.r.t x.
Using Pythagoras Theorem:		
$s^2 + s^2 = (2y)^2$		
$2s^2 = 4y^2$		
$\frac{s^2}{2} = y^2 \quad -- (2)$	1	
but $x^2 + y^2 = 16$		
$y^2 = 16 - x^2 \quad ---- (3)$		
sub (2) in (3)		
$\frac{s^2}{2} = 16 - x^2$		
$s^2 = 2(16 - x^2)$ sub in (1)		
$A = \frac{1}{2}\left[2(16 - x^2)\right]$		
$= 16 - x^2$	1	
$\delta V = (16 - x^2)\,\delta x$		
$V = \lim_{\delta x \to 0} \sum_{x=-4}^{4} (16 - x^2)\,\delta x$		
$V = \int_{-4}^{4} 16 - x^2 \, dx$		
$= 2 \times \int_{0}^{4} 16 - x^2 \, dx$	1	
$= 2\left[16x - \frac{x^3}{3}\right]_0^4$		
$= 2\left(\left[16 \times 4 - \frac{(4)^3}{3}\right] - 0\right)$		
$= \frac{256}{3} u^3$	1	(4)

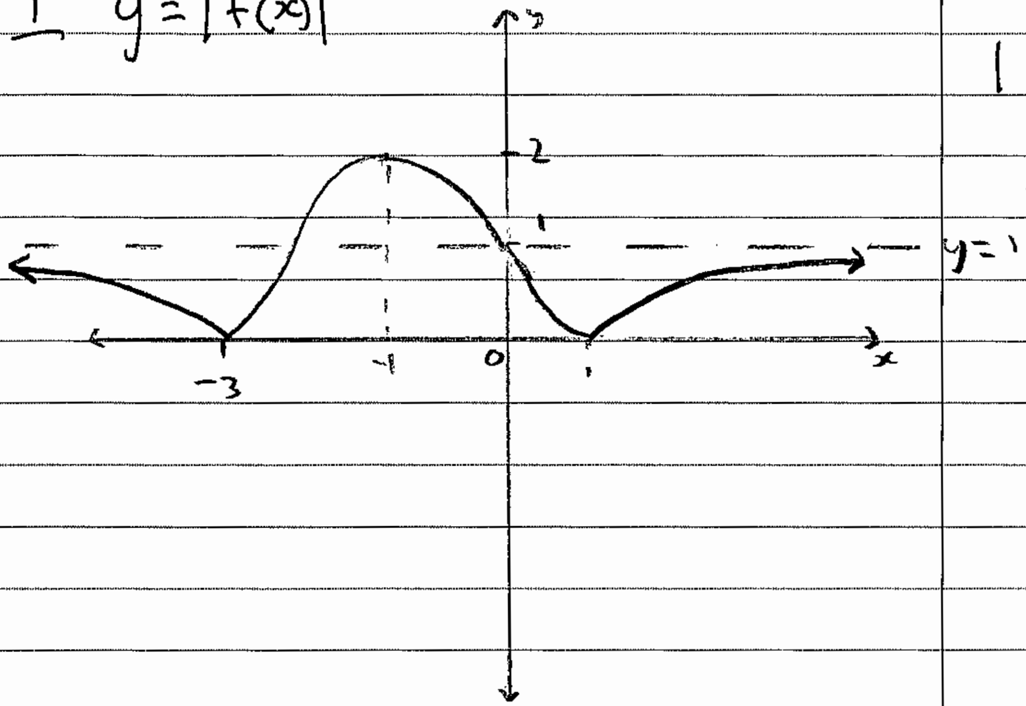
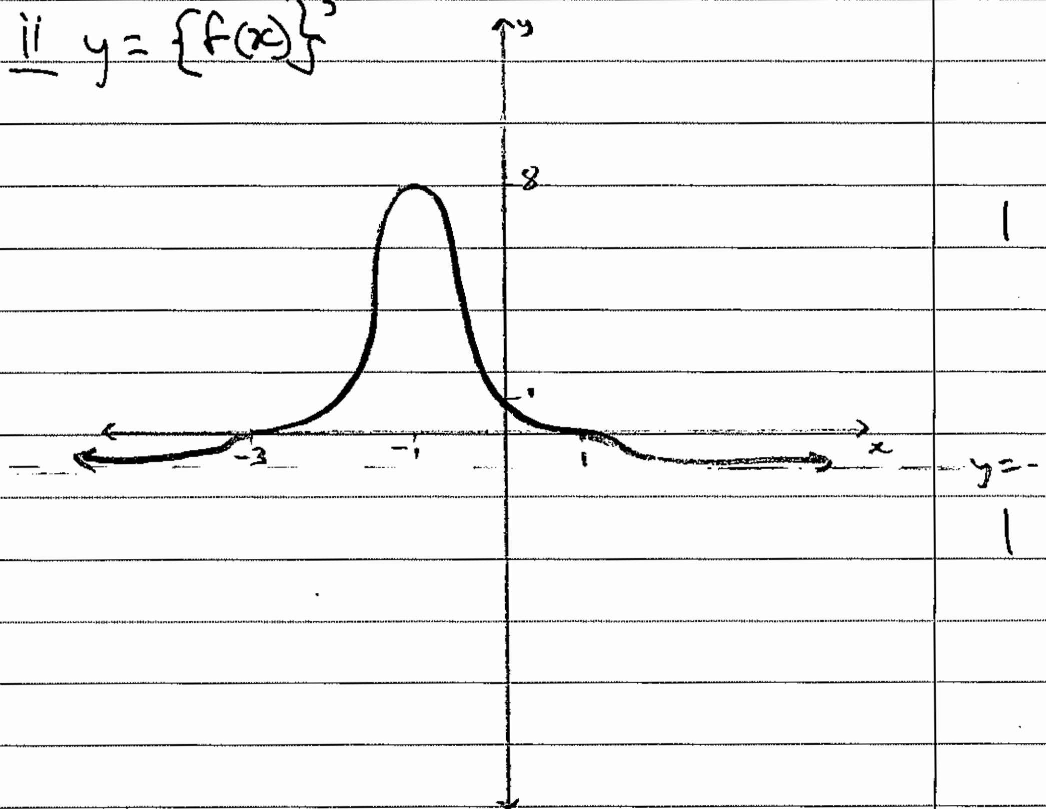
MATHEMATICS EXTENSION 2 – QUESTION 12

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
d) $\vec{OA} = \frac{1}{2}\vec{OC}$		
So		
$\lvert OA\rvert = \frac{1}{2}\lvert OC\rvert$ 		Please refer to alternative method overleaf.
i.e $\lvert OC\rvert = 2\lvert OA\rvert$		
but		
$\lvert OA\rvert = \sqrt{\left(\frac{-1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$		
$= \sqrt{\frac{1}{4} + \frac{3}{4}}$		
$= \sqrt{\frac{4}{4}}$		
$= 1$		
$\therefore \lvert OC\rvert = 2 \times 1$		
$= 2$	1	
and		
$\arg OA = \tan^{-1} \frac{\sqrt{3}/2}{-1/2}$		
$= \tan^{-1} -\sqrt{3}$		
$= 2\pi/3$		
but $\angle AOC = \pi/3$		
$\therefore \angle COX = \frac{2\pi}{3} - \frac{\pi}{3}$		
$= \pi/3$	1	
So $C = 2 \text{ cis } \pi/3$		
$= 2(\cos \pi/3 + i \sin \pi/3)$	1	
$= 2\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$		
$= 1 + \sqrt{3}\, i$		3

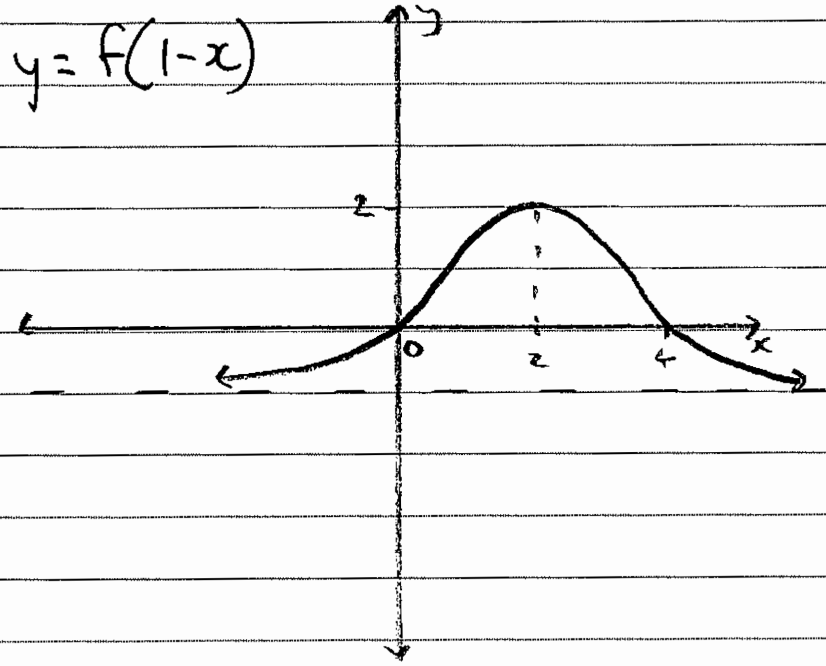
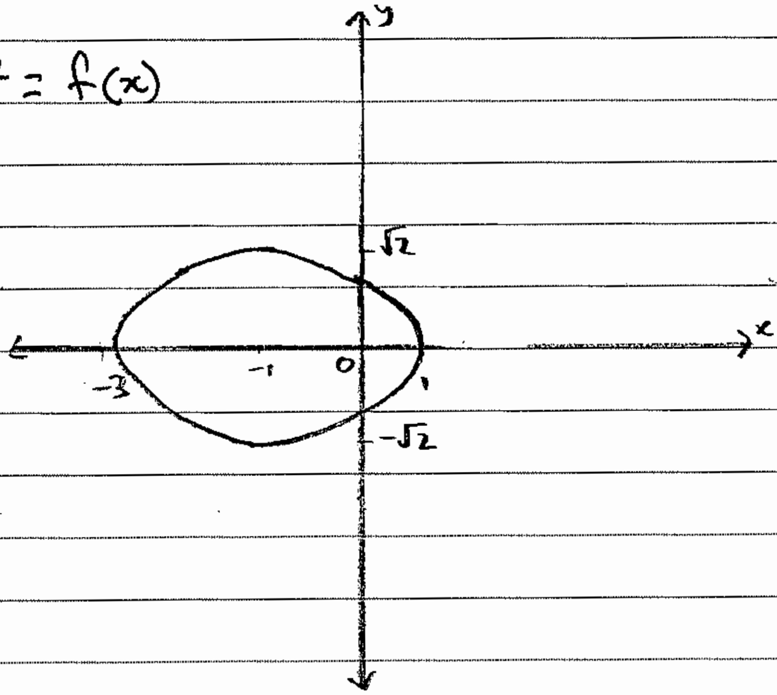
MATHEMATICS EXTENSION 2 – QUESTION 12

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
Alternative solution 12 (d)		NOTE:
Also $\vec{OA} = \operatorname{cis}\frac{2\pi}{3}$		Multiplying $C(z)$ through by $\operatorname{cis}\theta$ in an anticlockwise direction gives $A(z)$
and $\vec{OC} = 2 \times \vec{OA} \times \operatorname{cis}\left(-\frac{\pi}{3}\right)$		i.e
$= 2 \times \operatorname{cis}\frac{2\pi}{3} \times \operatorname{cis}\left(\frac{-\pi}{3}\right)$		$A(z) = C(z)\operatorname{cis}\theta$ So to find C
$= 2\operatorname{cis}\left(\frac{2\pi}{3} + \frac{-\pi}{3}\right)$		we use
$= 2\operatorname{cis}\pi/3$		$C(z) = A(z)\operatorname{cis}(-\theta)$

MATHEMATICS EXTENSION 2 – QUESTION 13

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
a)		
i $y = \lvert f(x) \rvert$ 	1	Crucial features: • sharp point at x-intercepts • asymptote at $y=1$ (though no marks deducted this time)
Parts ii – iv were essentially:	1	concept
	1	execution
ii $y = \{f(x)\}^3$ 	1	Crucial features • Horizontal inflexions at $x=-3$ and $x=1$ • Maximum at $y=8$ • Asymptote at $y=-1$
	1	• Smooth, symmetric curve

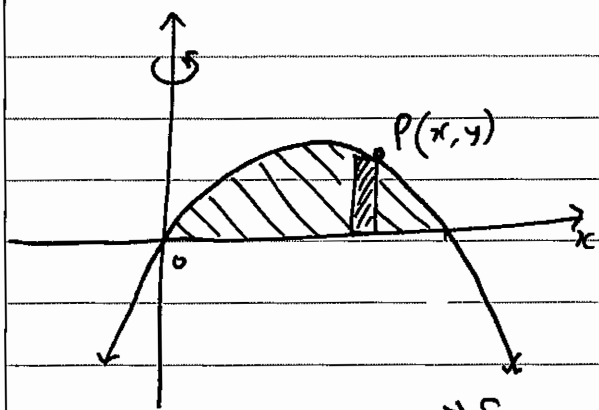
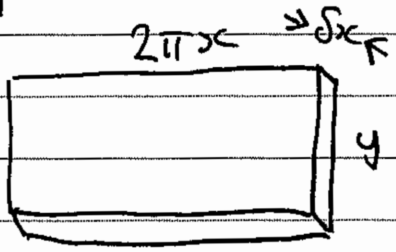
MATHEMATICS EXTENSION 2 – QUESTION 13

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
iv $y = f(1-x)$ 	1 1	Crucial features: • Intercepts at $x=0$ and $x=4$ • Asymptote at $y=-1$ • Maximum at $(2,2)$
Note: part marks could be earned by building up your answer with a series of simpler transformations eg. sketching $y=f(-x)$ was helpful.		
iii $y^2 = f(x)$ 	1 1	Crucial features: • Vertical at x-intercepts • Maximum at $y=\sqrt{2}$ • Smooth, symmetric curve

MATHEMATICS EXTENSION 2 – QUESTION 13

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
b) Let $P(x) = x^4 - 6x^3 + 9x^2 + 4x - 12$		
$P'(x) = 4x^3 - 18x^2 + 18x + 4$		
$P(2) = 16 - 48 + 36 + 8 - 12$		
$= 0$		
$P'(2) = 32 - 72 + 36 + 4$		
$= 0$		
$\therefore x = 2$ is the double root		
$\therefore (x-2)^2$ is a factor	1	
$x^2 - 2x - 3$		
$x^2 - 4x + 4 \overline{) x^4 - 6x^3 + 9x^2 + 4x - 12}$		
$x^4 - 4x^3 + 4x^2$		
$-2x^3 + 5x^2 + 4x$		Note that this question asked for a solution. Factorising was useful, but not enough to earn full marks.
$-2x^3 + 8x^2 - 8x$		
$-3x^2 + 12x - 12$		
$-3x^2 + 12x - 12$		
0		
$\therefore x^4 - 6x^3 + 9x^2 + 4x - 12 = (x-2)^2(x^2 - 2x - 3)$	1	
$= (x-2)^2(x-3)(x+1)$		
$\therefore$ the solutions of $x^4 - 6x^3 + 9x^2 + 4x - 12 = 0$		
are $x = -1,\ x = 2,\ x = 3$	1	
OR $x = 2$ is the double root (as above)	1	
Let the other roots be α and β		
$\alpha + \beta + 2 + 2 = 6$ (sum of roots)		
$\alpha + \beta = 2$		
$\alpha \times \beta \times 2 \times 2 = -12$ (product)		
$\alpha\beta = -3$	1	
$\therefore$ the roots are $-1, 3, 2$ and 2		
$\therefore x = -1,\ x = 2,\ x = 3$	1	

MATHEMATICS EXTENSION 2 – QUESTION 13

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
c)		
 		
		
$\therefore \delta V = 2\pi x \cdot y \cdot \delta x$	1	
$= 2\pi x \cdot 4x(3-x) \cdot \delta x$		
$\therefore V = \int_0^3 2\pi x \cdot 4x(3-x)\, dx$		
$= \int_0^3 24\pi x^2 - 8\pi x^3\, dx$	1	
$= \left[8\pi x^3 - 2\pi x^4 \right]_0^3$		
$= (8\pi \cdot 27 - 2\pi \cdot 81) - (0 - 0)$		
$= 54\pi$ units^3	1	

MATHEMATICS EXTENSION 2 – QUESTION 13

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
d) If $x^3 - 5x - 2 = 0$ ①		
Let $X = y + 1$		
$\therefore y = X - 1$		
sub $x-1$ into ①		
$(x-1)^3 - 5(x-1) - 2 = 0$	1	
$x^3 - 3x^2 + 3x - 1 - 5x + 5 - 2 = 0$		
$x^3 - 3x^2 - 2x + 2 = 0$	1	
Note that this question requested an equation. Giving a function (e.g. $P(x) = x^3 - 3x^2 - 2x + 2$) as your answer does not answer the question.		

MATHEMATICS EXTENSION 2 – QUESTION 14

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
a) i) $P\left(2p, \frac{2}{p}\right)$ $Q\left(2q, \frac{2}{q}\right)$		Part (a) i) ii) and iii) was well done.
Gradient of PQ: $m_{PQ} = \dfrac{\frac{2}{p} - \frac{2}{q}}{2p - 2q}$		
$= \dfrac{\frac{2q-2p}{pq}}{2p-2q}$		
$= \dfrac{2q-2p}{pq} \times \dfrac{1}{2(p-q)}$		
$= -\dfrac{\cancel{2}\cancel{(p-q)}}{pq} \times \dfrac{1}{\cancel{2}\cancel{(p-q)}}$		
$= \dfrac{-1}{pq}$	$\frac{1}{2}$	
Equation of chord PQ		
$y - \dfrac{2}{p} = \dfrac{-1}{pq}(x - 2p)$	$\frac{1}{2}$	
$pqy - 2q = -x + 2p$		
$\therefore x + pqy = 2(p+q)$		1
ii) For $xy = 4$.		
Differentiating implicitly wrt x		
$y + x.\dfrac{dy}{dx} = 0$		
$\dfrac{dy}{dx} = \dfrac{-y}{x}$		
$= \dfrac{-2p}{-2/p}$		
$= \dfrac{-2}{p} \times \dfrac{1}{2p}$		
$= -\dfrac{1}{p^2}$		

MATHEMATICS EXTENSION 2 – QUESTION 14

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
a) Cont'd		
Equation of tangent:		
$y - \frac{2}{p} = \frac{-1}{p^2}(x - 2p)$	1	
$p^2 y - 2p = -x + 2p$		2
$x + p^2 y = 4p \quad \text{---} \ (1)$		
iii) Similarly tangent at Q:		
$x + q^2 y = 4q \quad \text{---} \ (2)$		
$(1) - (2) \quad (p^2 - q^2)y = 4(p-q)$		
$\cancel{(p-q)}(p+q)y = 4\cancel{(p-q)}$		
$y = \frac{4}{p+q}$ sub in (1)	1	
$x + p^2\left(\frac{4}{p+q}\right) = 4p$		
$x = 4p - \frac{4p^2}{p+q}$		
$= \frac{4p(p+q) - 4p^2}{p+q}$		
$= \frac{\cancel{4p^2} + 4pq - \cancel{4p^2}}{p+q}$		
$= \frac{4pq}{p+q}$	1	
$\therefore$ T is $\left(\frac{4pq}{p+q}, \frac{4}{p+q}\right)$		2

MATHEMATICS EXTENSION 2 – QUESTION 14

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
a) iv) Using T and Gradient of PQ, the line through T, parallel to PQ is:		Some students did not substitute the point T and (0,2) correctly in the equation of a line, otherwise the question was done quite well.
$y - \frac{4}{p+q} = -\frac{1}{pq}\left(x - \frac{4pq}{p+q}\right)$	1	
Sub (0,2) in this equation		
$2 - \frac{4}{p+q} = -\frac{1}{\cancel{pq}}\left(\frac{-4\cancel{pq}}{p+q}\right)$	1	
$2(p+q) - 4 = 4$		
$2(p+q) = 8$		
$p+q = 4$		2
b) Using Integration by parts		
$\int \frac{\ln x}{x^2}\,dx = uv - \int vu'\,dx$ $u = \ln x$, $v' = \frac{1}{x^2} = x^{-2}$, $u' = \frac{1}{x}$, $v = \frac{x^{-1}}{-1} = \frac{-1}{x}$		
$= \ln x \cdot \frac{-1}{x} - \int -\frac{1}{x} \cdot \frac{1}{x}\,dx$	1	
$= \frac{-\ln x}{x} + \int \frac{1}{x^2}\,dx$		
$= \frac{-\ln x}{x} - \frac{1}{x} + c$	1	
		2

MATHEMATICS EXTENSION 2 – QUESTION 14

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
c) i) $\cos 3A = \cos(2A + A)$		
$= \cos 2A \cos A - \sin 2A \sin A$		
$= (2\cos^2 A - 1)\cos A - 2\sin A \cos A \sin A$	1	
$= 2\cos^3 A - \cos A - 2\cos A \sin^2 A$		
$= 2\cos^3 A - \cos A - 2\cos A(1 - \cos^2 A)$	1	
$= 2\cos^3 A - \cos A - 2\cos A + 2\cos^3 A$		2
$= 4\cos^3 A - 3\cos A$		
$\div 4$		
$\frac{1}{4}\cos 3A = \cos^3 A - \frac{3}{4}\cos A$ ....(1)		
ii) For $x^3 - 6x + 2 = 0$ ....(2)		
Sub $x = 2\sqrt{2}\cos A$ in (2)		
$(2\sqrt{2}\cos A)^3 - 6(2\sqrt{2}\cos A) + 2 = 0$	1	
$16\sqrt{2}\cos^3 A - 12\sqrt{2}\cos A + 2 = 0$		
$16\sqrt{2}\cos^3 A - 12\sqrt{2}\cos A = -2$		
$\cos^3 A - \frac{3}{4}\cos A = \frac{-2}{16\sqrt{2}}$	1	
from (1)		
$\frac{1}{4}\cos 3A = \frac{-2}{16\sqrt{2}}$		
$\cos 3A = \frac{-1}{8\sqrt{2}} \times 4$		
$= \frac{-1}{2\sqrt{2}}$		2
iii) Three roots of $x^3 - 6x + 2 = 0$ are 3 roots of		
$\cos 3A = -\frac{1}{2\sqrt{2}}$		
$3A = 2n\pi + \cos^{-1}\left(\frac{-1}{2\sqrt{2}}\right)$		
$A = \frac{2n\pi}{3} + \frac{\cos^{-1}\left(\frac{-1}{2\sqrt{2}}\right)}{3}$		1 mark to get 1 solution and ½ mark each to get the other 2.
Take:		
$n = 0$, $A_1 \doteqdot \frac{\cos^{-1}\left(\frac{-1}{2\sqrt{2}}\right)}{3}$ $\therefore x = 2\sqrt{2}\cos A_1 \doteqdot 2.2618$		
$n = 1$, $A_2 = \frac{2\pi}{3} + \frac{\cos^{-1}\left(\frac{-1}{2\sqrt{2}}\right)}{3}$, $x = 2\sqrt{2}\cos A_2 \doteqdot -2.26017$		
$n = 2$, $A_3 = \frac{4\pi}{3} + \frac{\cos^{-1}\left(\frac{-1}{2\sqrt{2}}\right)}{3}$, $x = 2\sqrt{2}\cos A_3 \doteqdot 0.3398$		2
$\therefore x = 2.2618$, $x = -2.6017$, $x = 0.3399$		

MATHEMATICS EXTENSION 2 – QUESTION 14

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS

c) Alternative solution to c(i)

Using De Moivre's Theorem

For $(\cos A + i\sin A)^3 = \cos 3A + i\sin 3A$

$$\text{LHS} = \cos^3 A + 3\cos^2 A\, i\sin A + 3\cos A\, i^2 \sin^2 A + i^3 \sin^3 A$$

$$= \cos^3 A + 3\cos A \sin A\, i - 3\cos A \sin^2 A - i\sin^3 A$$

Equating Real Parts

$$\cos 3A = \cos^3 A - 3\cos A \sin^2 A$$

$$= \cos^3 A - 3\cos A(1 - \cos^2 A)$$

$$= \cos^3 A - 3\cos A + 3\cos^2 A$$

$$= 4\cos^3 A - 3\cos A$$

$\div$

$$\frac{1}{4}\cos 3A = \cos^3 A - \frac{3}{4}\cos A$$

MATHEMATICS EXTENSION 2 – QUESTION 15

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
a) Let $t = \tan\frac{\theta}{2}$ $\frac{dt}{d\theta} = \frac{1}{2}\sec^2\frac{\theta}{2}$ $= \frac{1+\tan^2\frac{\theta}{2}}{2}$ (since $\sec^2\theta = 1+\tan^2\theta$) $= \frac{1+t^2}{2}$ $\therefore \frac{d\theta}{dt} = \frac{2}{1+t^2}$	1	This is a "show" question – don't take shortcuts. Show how you obtain every result.
$d\theta = \frac{2\,dt}{1+t^2}$		
OR $t = \tan\frac{\theta}{2}$ $\tan^{-1}t = \theta/2$ $\theta = 2\tan^{-1}t$ $\frac{d\theta}{dt} = \frac{2}{1+t^2}$	1	When doing integration by substitution, never mix variables. Your integral should progress from one wholly in terms of θ, to one wholly in terms of t.
$d\theta = \frac{2\,dt}{1+t^2}$ when $\theta = 0$, $t = \tan\frac{0}{2}$ $= 0$ when $\theta = \frac{\pi}{3}$, $t = \tan\frac{\pi/3}{2}$ $= \tan\frac{\pi}{6}$ $= \frac{1}{\sqrt{3}}$ $\therefore \int_0^{\frac{\pi}{3}} \sec\theta\,d\theta = \int_0^{\frac{\pi}{\sqrt{3}}} \frac{1}{\cos\theta}\,d\theta$		
$= \int_0^{\frac{1}{\sqrt{3}}} \frac{1+t^2}{1-t^2} \cdot \frac{2\,dt}{1+t^2}$	1	

MATHEMATICS EXTENSION 2 – QUESTION 15

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
$= \int_0^{\frac{1}{\sqrt{3}}} \frac{2}{1-t^2}\,dt$		
Let $\frac{2}{(1-t)(1+t)} \equiv \frac{a}{1-t} + \frac{b}{1+t}$		
$\therefore 2 = a(1+t) + b(1-t)$		
when $t=1$, $\cancel{2=} = 2$		
$\therefore a = 1$		
when $t=-1$, $2 = 2b$		
$b = 1$		
$= \int_0^{\frac{1}{\sqrt{3}}} \frac{1}{1+t} + \frac{1}{1-t}\,dt$		
$= \Big[\ln\lvert 1+t\rvert - \ln\lvert 1-t\rvert\Big]_0^{\frac{1}{\sqrt{3}}}$		
$= \left[\ln\frac{\lvert 1+t\rvert}{\lvert 1-t\rvert}\right]_0^{\frac{1}{\sqrt{3}}}$		
$= \ln\left(\frac{1+\frac{1}{\sqrt{3}}}{1-\frac{1}{\sqrt{3}}}\right)$		
$= \ln\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)$		
$= \ln\frac{(\sqrt{3}+1)(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$		
$= \ln\left(\frac{4+2\sqrt{3}}{3-1}\right)$	1	for full solution
$= \ln(2+\sqrt{3})$		

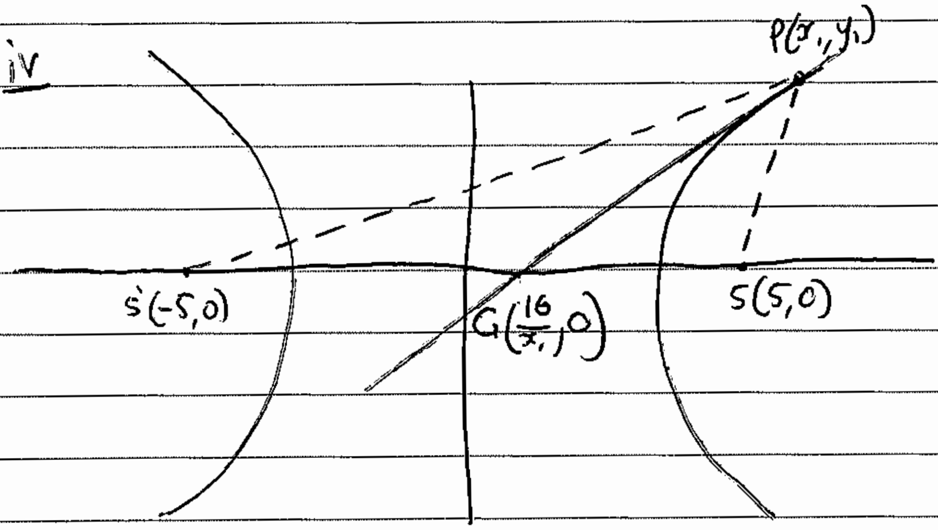
MATHEMATICS EXTENSION 2 – QUESTION 15

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
b)		
i $I_n = \int_1^e (1-\ln x)^n \, dx, \quad n = 0,1,2\ldots$		
let $u = (1-\ln x)^n$		
$\therefore u' = -\frac{1}{x}\cdot n(1-\ln x)^{n-1}$		
let $v' = 1$		
$\therefore v = x$		
$\therefore I_n = \left[x(1-\ln x)^n\right]_1^e - \int_1^e x\cdot -\frac{1}{x}\cdot n(1-\ln x)^{n-1}\,dx$	2	
$= \left[e(1-1) - 1(1-0)\right] + n\int (1-\ln x)^{n-1}\,dx$		
$= -1 + nI_{n-1}$		
ii $I_3 = -1 + 3I_2$		
$= -1 + 3(-1 + 2I_1)$		
$= -1 + 3(-1 + 2(-1 + I_0))$		
$= -1 + 3(-1 + 2(-1 + (e-1)))$		
$= -1 + 3(-1 + 2(-1 + e - 1))$		
$= -1 + 3(-1 + 2(e-2))$		
$= -1 + 3(-1 + 2e - 4)$		
$= -1 + 3(2e - 5)$		
$= -1 + 6e - 15$		
$= 6e - 16$		
$\left[I_0 = \int_1^e (1-\ln x)^0 = \int_1^e 1 = [x]_1^e = e - 1 \right]$		
2 marks for full solution		
1 mark for significant progress towards solution		

MATHEMATICS EXTENSION 2 – QUESTION 15

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
c) i $9x^2 - 16y^2 = 144$		
$\therefore \frac{x^2}{16} - \frac{y^2}{9} = 1$ $\quad a^2 = 16 \quad b^2 = 9$ $\quad a = 4 \quad b = 3$		
$b^2 = a^2(e^2 - 1)$		
$9 = 16(e^2 - 1)$		
$\frac{9}{16} = e^2 - 1$		
$e^2 = \frac{25}{16}$		
$e = \frac{5}{4} \quad (e > 0)$	1	
Foci: $(\pm ae, 0) \quad (\pm 4 \times \frac{5}{4}, 0)$		
$\therefore S(5, 0), \quad S'(-5, 0)$	1	
ii $\frac{x^2}{16} - \frac{y^2}{9} = 1$		
$\frac{2x}{16} - \frac{2y}{9} \cdot \frac{dy}{dx} = 0$		
$\frac{dy}{dx} = \frac{2x}{16} \times \frac{9}{2y}$		
$= \frac{9x}{16y}$		
at $P(x_1, y_1)$, $m_T = \frac{9x_1}{16y_1}$	1	
$\therefore$ equation of tangent to H at P		
$y - y_1 = \frac{9x_1}{16y_1}(x - x_1)$		
$16y_1 y - 16y_1^2 = 9x_1 x - 9x_1^2$		

MATHEMATICS EXTENSION 2 – QUESTION 15

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
$\therefore\ 9x_1x - 16y_1y = 9x_1^2 - 16y_1^2$	1	
$= 144$ (since $P(x_1, y_1)$ lies on H)		
$\therefore\ 9x_1x - 16y_1y = 144$		
iii when $y = 0$		
$9x_1x - 16y_1(0) = 144$		
$x = \dfrac{144}{9x_1}$		
$= \dfrac{16}{x_1}$		
$\therefore\ G\left(\dfrac{16}{x_1}, 0\right)$	1	
iv 		$9x^2 - 16y^2 = 144$ $\Rightarrow y^2 = \dfrac{9x^2 - 144}{16}$
$SP^2 = (x_1 - 5)^2 + (y_1 - 0)^2$		
$= x_1^2 - 10x_1 + 25 + y_1^2$		
$= x_1^2 - 10x_1 + 25 + \dfrac{9x_1^2 - 144}{16}$ (since $P(x_1, y_1)$ lies on H)		
$= \dfrac{16x_1^2 - 160x_1 + 400 + 9x_1^2 - 144}{16}$		
$= \dfrac{25x_1^2 - 160x - 256}{16}$		

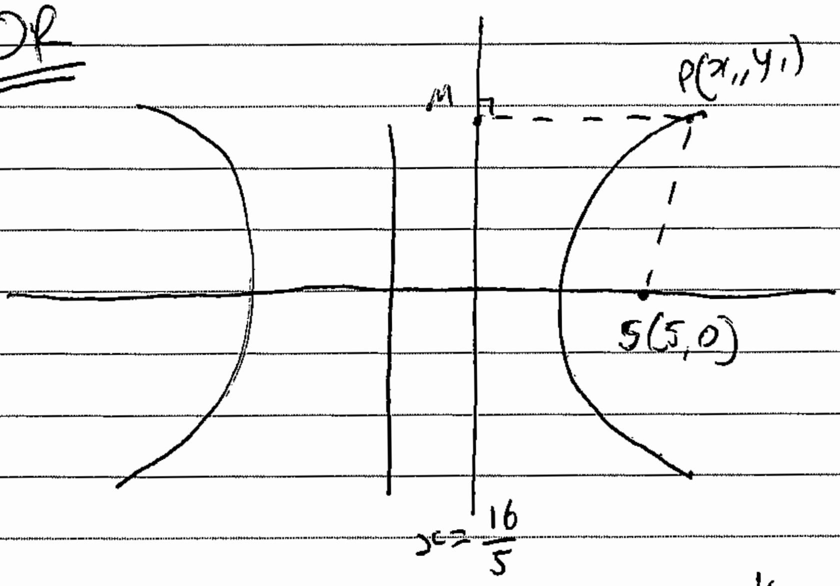
MATHEMATICS EXTENSION 2 – QUESTION 15

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
$= \dfrac{(5x_1-16)^2}{4^2}$		
$\therefore SP = \dfrac{5x_1-16}{4}$	1	
$S'P^2 = (x_1 - -5)^2 + (y_1 - 0)^2$		
$= x_1^2 + 10x_1 + 25 + y_1^2$		
$= x_1^2 + 10x_1 + 25 + \dfrac{9x_1^2 - 144}{16}$		
$= \dfrac{16x_1^2 + 160x + 400 + 9x_1^2 - 144}{16}$		
$= \dfrac{25x_1^2 + 160x - 256}{16}$		
$= \dfrac{(5x_1+16)^2}{4^2}$		
$\therefore S'P = \dfrac{5x_1+16}{4}$	1	for any other distance (i.e. $S'P$, SG or $S'G$)
$\therefore \dfrac{SP}{S'P} = \dfrac{(5x_1-16)/4}{(5x_1+16)/4}$		
$= \dfrac{(5x_1-16)}{(5x_1+16)}$		
$SG = 5 - \dfrac{16}{x_1}$		
$= \dfrac{5x_1-16}{x_1}$		
$S'G = 5 + \dfrac{16}{x_1}$		
$= \dfrac{5x_1+16}{x_1}$		
$\therefore \dfrac{SG}{S'G} = \dfrac{(5x_1-16)/x_1}{(5x_1+16)/x_1}$	1	full solution
$= \dfrac{5x_1-16}{5x_1+16}$		
$= \dfrac{SP}{S'P}$		

MATHEMATICS EXTENSION 2 – QUESTION 15

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS

OR



directrix has equation $x = \frac{a}{e} \Rightarrow \frac{4}{5/4}$

$\therefore x = \frac{16}{5}$

Let M be the the foot of the perpendicular from the directrix to P

$\therefore M\left(\frac{16}{5}, y_1\right)$

From the definition of a hyperbola:

$$SP = ePM$$

$$= \frac{5}{4}\left(x_1 - \frac{16}{5}\right)$$

$$= \frac{5}{4}\left(\frac{5x_1 - 16}{5}\right)$$

$$= \frac{5x_1 - 16}{4}$$ — 1

Similarly, $S'P = ePM'$

$$= \frac{5}{4}\left(x_1 - -\frac{16}{5}\right)$$

$$= \frac{5}{4}\left(x_1 + \frac{16}{5}\right)$$

$$= \frac{5}{4}\left(\frac{5x_1 + 16}{5}\right)$$

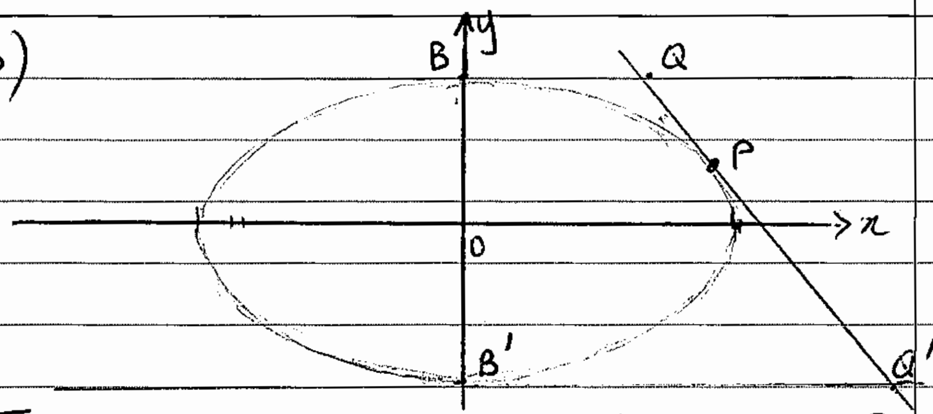
$$= \frac{5x_1 + 16}{4}$$ — 1

Then as above.

MATHEMATICS EXTENSION 2 – QUESTION 16

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
a) i)		
$m\ddot{x} = -mg - R$		
$= -mg - \dfrac{mgv^2}{a^2}$		
$\ddot{x} = -g - \dfrac{gv^2}{a^2}$	1	
but $\ddot{x} = v.\dfrac{dv}{dx}$		
$v.\dfrac{dv}{dx} = -g\left(1 + \dfrac{v^2}{a^2}\right)$		
$= -g\left(\dfrac{a^2+v^2}{a^2}\right)$	1	
$= \dfrac{-g}{a^2}(a^2+v^2)$		
ii) $\dfrac{dv}{dx} = \dfrac{-g}{a^2}\left(\dfrac{a^2+v^2}{v}\right)$		
$\dfrac{dx}{dv} = \dfrac{-a^2}{g}\left(\dfrac{v}{a^2+v^2}\right)$	1/2	
$x = \dfrac{-a^2}{g}\int \dfrac{v}{a^2+v^2}\,dv$		
$= \dfrac{-a^2}{2g}\ln\lvert a^2+v^2) + c$	1/2	
when $t=0$, $x=0$, $v=U$		
$0 = \dfrac{-a^2}{2g}\ln(a^2+U^2) + c$		
$c = \dfrac{a^2}{2g}\ln(a^2+u^2)$		
$x = \dfrac{-a^2}{2g}\ln(a^2+v^2) + \dfrac{a^2}{2g}\ln(a^2+u^2)$	1	
$= \dfrac{a^2}{2g}\ln\left[\dfrac{a^2+u^2}{a^2+v^2}\right]$ For greatest height $v=0$		
$x_{MAX} = \dfrac{a^2}{2g}\ln\left[\dfrac{a^2+u^2}{a^2}\right]$	1	
$= \dfrac{a^2}{2g}\ln\left[1 + \dfrac{u^2}{a^2}\right]$		3

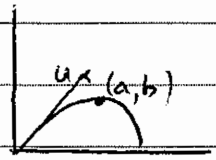
MATHEMATICS EXTENSION 2 – QUESTION 16

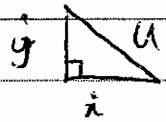
SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
b) 		
The equation of the tangent at P		
$bx\cos\theta + ay\sin\theta - ab = 0 \quad \ldots (1)$		
Sub $y = b$ in (1) to obtain x coord of Q.		
$bx\cos\theta + ab\sin\theta - ab = 0$		
$bx\cos\theta = ab - ab\sin\theta$		
$x = \dfrac{a\cancel{b}(1-\sin\theta)}{\cancel{b}\cos\theta}$		
$= \dfrac{a(1-\sin\theta)}{\cos\theta}$	½	
$\therefore BQ = \dfrac{a(1-\sin\theta)}{\cos\theta}$	½	
Sub $y = -b$ in (1) to obtain x-coord of Q'		
$bx\cos\theta - ab\sin\theta - ab = 0$		
$x = \dfrac{a\cancel{b}(1+\sin\theta)}{\cancel{b}\cos\theta}$		
$x = \dfrac{a(1+\sin\theta)}{\cos\theta}$	½	
$\therefore BQ' = \dfrac{a(1+\sin\theta)}{\cos\theta}$	½	
Now $BQ \times BQ' = \dfrac{a(1-\sin\theta)}{\cos\theta} \times \dfrac{a(1+\sin\theta)}{\cos\theta}$	1	
$= \dfrac{a^2(1-\sin^2\theta)}{\cos^2\theta}$	1	
$= \dfrac{a^2\cos^2\theta}{\cos^2\theta}$		
$= a^2$		4

MATHEMATICS EXTENSION 2 – QUESTION 16

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS

c)





Initial Velocities

$\dot{x} = u\cos\theta$

$\dot{y} = u\sin\theta$

Vertical Component

$\ddot{y} = -g$

$\dot{y} = -gt + c$

when $t=0$, $\dot{y} = u\sin\theta$

$u\sin\theta = 0 + c$

$\therefore \dot{y} = -gt + u\sin\theta$

$y = -\dfrac{gt^2}{2} + ut\sin\theta + c_2$ --- (1)

when $t=0$, $y=0$,

$0 = 0 + 0 + c_2 \quad \therefore c_2 = 0$

$y = -\frac{1}{2}gt^2 + ut\sin\theta$

Horizontal Component

$\ddot{x} = 0$

$\dot{x} = c_3$

when $t=0$, $\dot{x} = u\cos\theta$

$\dot{x} = u\cos\theta$

$x = ut\cos\theta + c_4$

when $x=0$, $t=0$,

$0 = 0 + c_4$

$c_4 = 0$

$x = ut\cos\theta$ --- (2)

$t = \dfrac{x}{u\cos\theta}$ --- (3) **2**

Sub (3) in (1)

$$y = -\frac{1}{2}g\left(\frac{x}{u\cos\theta}\right)^2 + u\left(\frac{x}{u\cos\theta}\right)\sin\theta$$ **1**

$$= -\frac{1}{2}g\left(\frac{x^2}{u^2\cos^2\theta}\right) + \frac{x\sin\theta}{\cos\theta}$$

$$y = -\frac{1}{2}g\frac{x^2}{u^2}\sec^2\theta + x\tan\theta \quad \text{--- (4)}$$

Marker's comments:

2 marks for fully deriving the vertical and horizontal equations of motion

1 mark for only stating them.

3

MATHEMATICS EXTENSION 2 – QUESTION 16

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
c) ii)		
From (4)		
$y = \frac{-g}{2u^2} x^2(1+\tan^2\theta) + x\tan\theta$ ...(5)		
when $x=a$, $y=b$ sub in (5)		
$b = \frac{-g}{2u^2} \cdot a^2(1+\tan^2\theta) + a\tan\theta$		
$b = \frac{-ga^2}{2u^2} - \frac{ga^2}{2u^2}\tan^2\theta + a\tan\theta$	1	
$\frac{ga^2}{2u^2}\tan^2\theta - a\tan\theta + \frac{ga^2}{2u^2} + b = 0$		
$ga^2\tan^2\theta - 2u^2a\tan\theta + ga^2 + 2u^2b = 0$		
Quad Equation in $\tan\theta$		
For 2 distinct points, $\Delta > 0$		
$b^2 - 4ac > 0$		
$(-2u^2a)^2 - 4(ga^2)(ga^2+2u^2b) > 0$	1	
$4u^4a^2 - 4(g^2a^4 + 2ga^2bu^2) > 0$		
$4u^4a^2 - 8ga^2bu^2 - 4g^2a^4 > 0$		
$\div 4a^2$ $u^4 - 2gbu^2 - g^2a^2 > 0$		
$u^4 - 2gbu^2 > g^2a^2$		
By Completing the square on LHS		
$u^4 - 2gbu^2 + g^2b^2 > g^2a^2 + g^2b^2$	1	
$(u^2 - gb)^2 > g^2(a^2+b^2)$		
		3