Student Number: ____________________

St George Girls High School

Trial Higher School Certificate Examination

2019

Mathematics Extension 2

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black pen
- Board-approved calculators may be used
- A reference sheet is provided
- In Questions 11 – 16, show relevant mathematical reasoning and/or calculations

Section I	/10
Section II	
Question 11	/15
Question 12	/15
Question 13	/15
Question 14	/15
Question 15	/15
Question 16	/15
Total	**/100**

Total Marks – 100

Section I Pages 2 – 5

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section
- Answer on the multiple choice answer sheet provided at the back of this paper

Section II Pages 6 – 11

90 marks

- Attempt Questions 11 – 16
- Allow about 2 hours and 45 minutes for this section
- Begin each question in a new writing booklet

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

1. If $z = \frac{3+4i}{1+2i}$, the imaginary part of z is:

   (A) -2

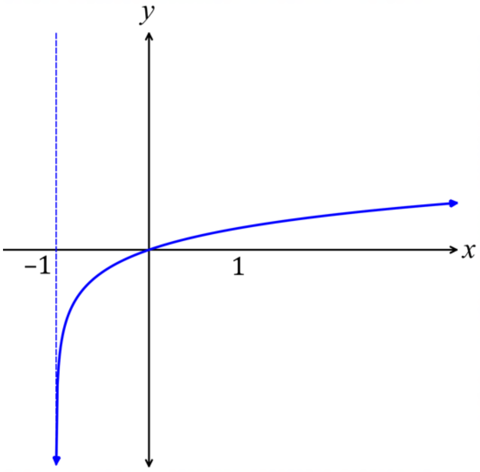
   (B) $-\frac{2}{5}$

   (C) $-\frac{2}{5}i$

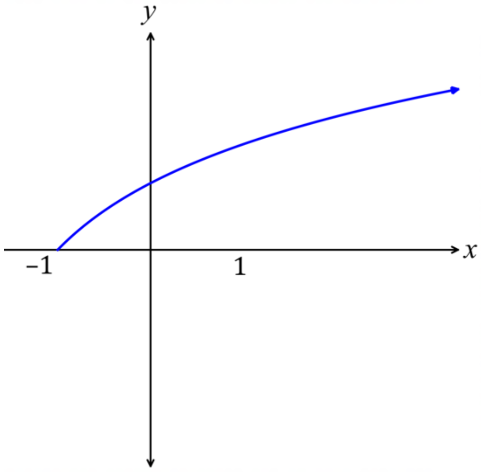
   (D) $-2i$

2. Which graph best represents $y = |\log_e(x+1)|$?

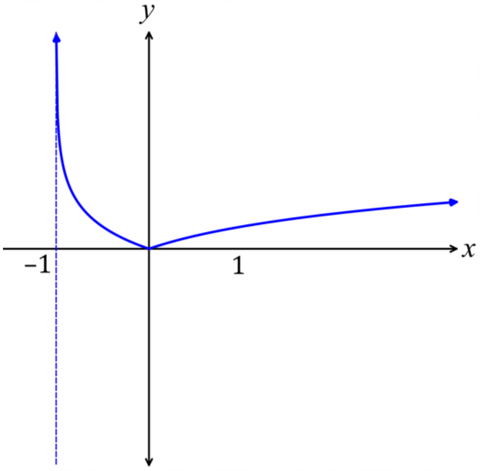
(A)



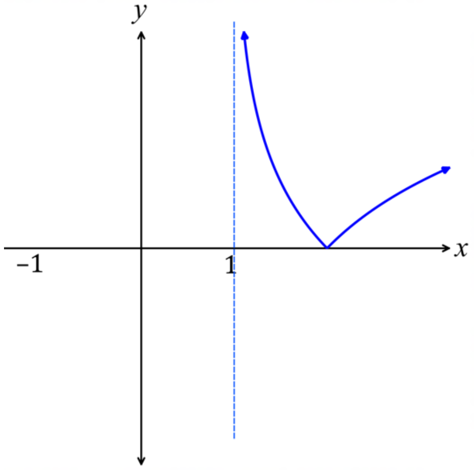
(B)



(C)



(D)



Section I (cont'd)

3. The equation $2x^3 - 4x^2 - 8x - 1 = 0$ has roots α, β and γ.
What is the value of $\alpha^{-3}\beta^{-3}\gamma^{-3}$?

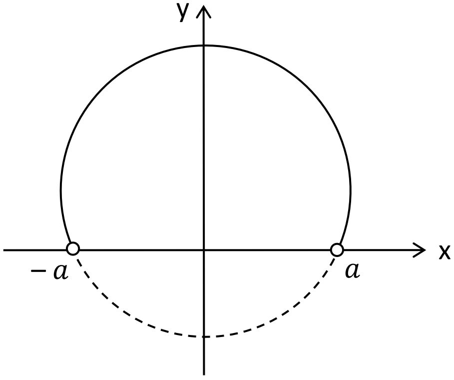
(A) –8

(B) $-\frac{1}{8}$

(C) $\frac{1}{8}$

(D) 8

4. The diagram shows the solution of an equation.



Which of these could be the equation, if z is a point on the circumference of the circle?

(A) $\text{Arg}\,(z - a) = \text{Arg}\,(z + a)$

(B) $\text{Arg}\,(z - a) - \text{Arg}\,(z + a) = \frac{\pi}{2}$

(C) $\text{Arg}\,(z + a) - \text{Arg}\,(z - a) = \frac{\pi}{3}$

(D) $\text{Arg}\,(z - a) - \text{Arg}\,(z + a) = \frac{\pi}{3}$

Section I (cont'd)

5. Find $\int \frac{dx}{x^2 + 4x + 9}$.

(A) $\frac{1}{\sqrt{5}} \tan^{-1}\left(\frac{x + 2}{\sqrt{5}}\right) + c$

(B) $\frac{1}{5} \tan^{-1}\left(\frac{x + 2}{\sqrt{5}}\right) + c$

(C) $\frac{1}{\sqrt{5}} \tan^{-1}\left(\frac{x}{\sqrt{5}}\right) + c$

(D) $\frac{1}{5} \tan^{-1}\left(\frac{x}{\sqrt{5}}\right) + c$

6. The polynomial $P(x) = x^3 - 6x^2 + 9x + c$ has a double zero.
What are the values of c?

(A) $c = -4$ or $c = -1$

(B) $c = -4$ or $c = 0$

(C) $c = 4$ or $c = 7$

(D) $c = 4$ or $c = 0$

7. Which of the following is an expression for $\int xe^{-x}\ dx$?

(A) $-xe^{-x} - e^{-x} + c$

(B) $-xe^{-x} + e^{-x} + c$

(C) $xe^{-x} - e^{-x} + c$

(D) $xe^{-x} + e^{-x} + c$

Section I (cont'd)

8. What is the eccentricity of the ellipse $4x^2 + 9y^2 = 16$?

(A) $\dfrac{13}{3}$

(B) $\dfrac{\sqrt{5}}{9}$

(C) $\dfrac{\sqrt{13}}{3}$

(D) $\dfrac{\sqrt{5}}{3}$

9. The circle $x^2 + y^2 = 9$ is rotated about the line $x = 4$ to form a solid.
What is an expression for the volume of the solid using the method of cylindrical shells?

(A) $2\pi \int_0^3 (4 - x)\sqrt{9 - x^2}dx$

(B) $4\pi \int_0^3 (4 - x)\sqrt{9 - x^2}dx$

(C) $2\pi \int_{-3}^3 (4 - x)\sqrt{9 - x^2}dx$

(D) $4\pi \int_{-3}^3 (4 - x)\sqrt{9 - x^2}dx$

10. A body of mass m kg moves in a straight line with initial speed U ms^{-1} subject to a resistance of magnitude $m(1 + v)$ Newtons when its speed is v ms^{-1} . What is the time taken in seconds by the body to come to rest?

(A) $\dfrac{1}{1 + U}$

(B) $\sqrt{1 + U}$

(C) $\ln(1 + U)$

(D) e^{1+U}

End of Section I

Section II
90 marks
Attempt Questions 11 – 16
Allow about 2 hours 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11 – 16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet **Marks**

a) For $z = 3 + i$ and $w = 2 + 2i$, find the values of:

(i) $\arg(w - z)$ 1

(ii) $w + \overline{2\imath z}$ 2

b) Use a suitable trigonometric substitution to evaluate the following integral. 3

$$\int_0^1 \sqrt{1 - x^2}dx$$

c) (i) Express $\dfrac{x}{(x+1)(x^2+1)}$ in the form $\dfrac{A}{x+1} + \dfrac{Bx + C}{x^2+1}$. 2

(ii) Hence find $\displaystyle\int \frac{x}{(x+1)(x^2+1)}dx$. 2

d) (i) Find real numbers a and b such that $(a + ib)^2 = -3 + 4i$. 3

(ii) Hence solve the equation $z^2 - 3z + (3 - i) = 0$. 2

Question 12 (15 marks) Use a SEPARATE writing booklet **Marks**

a) A rectangular hyperbola A has the equation $xy = 18$.

(i) Show that the equation of the normal N_1 to A at the point $P\ (2, 9)$ is given by $2x - 9y + 77 = 0$. 2

(ii) Find, in general form, an equation of the normal N_2 to A at the point $Q\ (9, 2)$. 1

(iii) Find the coordinates of point B where N_1 and N_2 intersect. 1

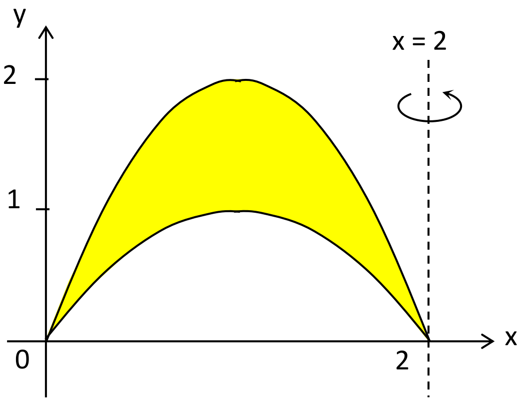
(iv) Show that the quadrilateral $OPBQ$ is a rhombus. 2

b) Find all the roots of the equation: 3

$$18x^3 + 3x^2 - 28x + 12 = 0$$

given that two of the roots are equal.

c) The shaded region bounded by the parabolas $y = 2x - x^2$ and $y = 4x - 2x^2$ between $x = 0$ and $x = 2$ is as shown in the diagram. 3



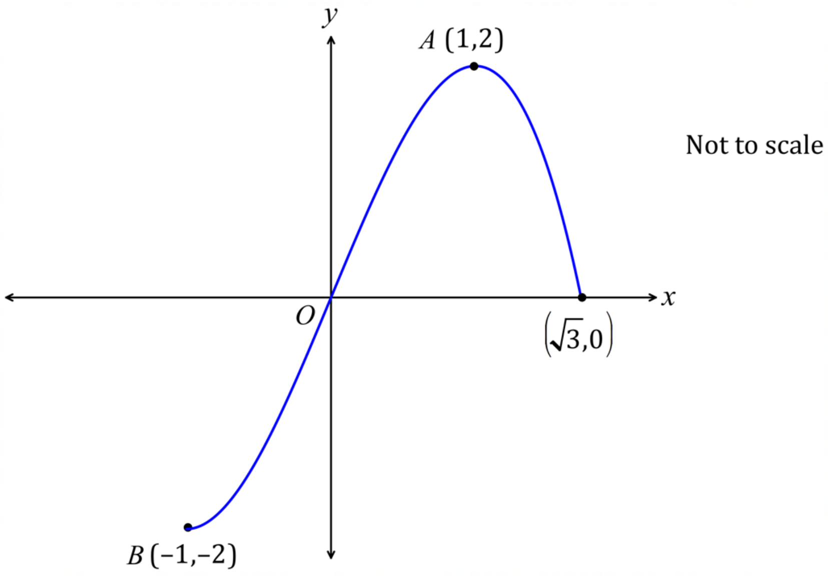
Use the cylindrical shells method to find the volume of the solid formed when the shaded area is rotated about the line $x = 2$.

d) On an Argand diagrams, shade in the regions containing all points representing complex numbers z such that: 3

$$|z| \leq |z - 2| \text{ and } -\frac{\pi}{4} \leq \arg z \leq \frac{\pi}{4}$$

Question 13 (15 marks) Use a SEPARATE writing booklet. **Marks**

a)



The graph of $y = f(x)$ is shown above. Draw separate one-third page sketches of these functions. Indicate clearly any asymptotes and intercepts with the axes.

(i) $y = f(|x|)$ 2

(ii) $y = \sqrt{f(x)}$ 2

(iii) $y = [f(x)]^2$ 2

(iv) $y = f^{-1}(x)$ 2

b) A solid is formed by rotating about the x –axis, the region bounded by the parabola $y^2 = 4ax$, the x –axis and ordinate $x = a$. Find the volume of this solid using the method of slicing. 3

c) $I_n = \int_1^e (1 - \log_e x)^n \, dx$ n = 0,1,2, …

(i) Show that $I_n = -1 + nI_{n-1}$ n = 1,2,3, … 2

(ii) Hence evaluate $\int_1^e (1 - \log_e x)^3 \, dx$. 2

Question 14 (15 marks) Use a SEPARATE writing booklet. **Marks**

a) A particle of mass m falls from rest in a medium where the resistance to motion has magnitude mkv for some positive constant k when the speed is v, and the terminal velocity of the particle is V. The particle falls a distance x in time t, and the acceleration due to gravity is g.

(i) Show that $\ddot{x} = \frac{g}{V}(V - v)$. 2p

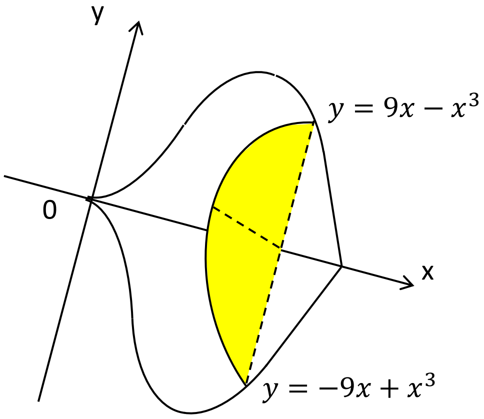
(ii) If the particle attains half its terminal velocity in time T and falls a distance X in this time, show that $VT - X = \frac{V^2}{2g}$. 4

b) Sketch the graph of $y = x^2 + \frac{1}{x^2}$. 3

c) Find the equation of the tangent to the curve $x^2y + xy^2 - 6 = 0$ at the point (1, 2). 3

d) The base of a solid is the region bounded by the curves $y = 9x - x^3$, $y = -9x + x^3$ between $x = 0$ and $x = 3$. 3

Each cross section perpendicular to the x −axis is a semi circle as shown in the diagram.



Find the volume of this solid.

Question 15 (15 marks) Use a SEPARATE writing booklet. **Marks**

a) $P(a\cos\theta, b\sin\theta)$ lies on the ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1 \quad a>b>0.$

The tangent and the normal at P cut the y −axis at A and B respectively, and $S(ae, 0)$ is a focus of the ellipse.

(i) Show that the tangent to the ellipse at P is the following equation. 2

$$\frac{x\cos\theta}{a}+\frac{y\sin\theta}{b}=1$$

(ii) Show that the normal to the ellipse at P is the following equation. 2

$$\frac{ax}{\cos\theta}-\frac{by}{\sin\theta}=a^2-b^2$$

(iii) Why is $\angle ASB = 90°$? 3

(iv) Deduce that A, P, S and B are concyclic points. 1

b) A particle is initially at $x = 1$ with a velocity of 2 m/s.
The acceleration of the particle is given by $a=\frac{1}{2}\left(1-\frac{1}{x^2}\right)$, where x is the displacement of the particle from 0.

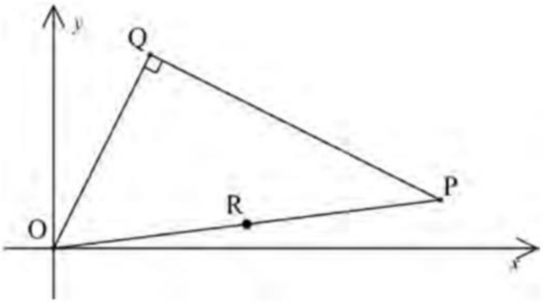
(i) Prove that $\dfrac{dx}{dt}=\dfrac{1+x}{\sqrt{x}}$. 3

(ii) Find the time taken for the particle to reach $x = 3$. 4

Question 16 (15 marks) Use a SEPARATE writing booklet. **Marks**

a) In the Argand diagram below, OPQ is a triangle, which is right-angled at Q.

The point R is the midpoint of OP.



(i) If $\overrightarrow{OP} = z$ and $\overrightarrow{OQ} = w$ show that $\overrightarrow{OR} = \frac{1}{2}(1 - ki)w$, where k is a constant, $k > 0$. 3

(ii) Express $\overrightarrow{RQ}$ in terms of w and hence show $|\overrightarrow{OR}| = |\overrightarrow{RQ}|$. 2

b) If $x > 1$, show that $\tan^{-1}\left(\frac{x+1}{x-1}\right) - \tan^{-1}\left(\frac{1}{x}\right) = \frac{\pi}{4}$. 3

c) A particle of mass m moves along the x −axis, beginning at $x = 0$.
It experiences a resistive force R given by $R = kv$, where k is a constant and v is the velocity of the particle.

(i) Show that its speed v, is given by $v = v_0 e^{-\frac{kt}{m}}$, where v_0 is the initial speed. 3

(ii) Show that the displacement (x) of the particle after t seconds is given by 3

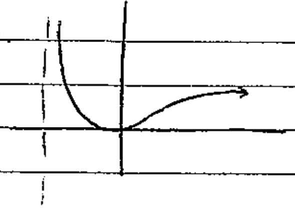
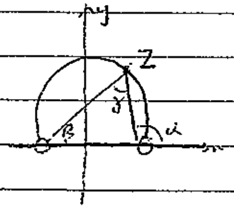
$$x = \frac{mv_0}{k}\left[1 - e^{-\frac{kt}{m}}\right].$$

(iii) Show that its limiting position x_L is given by $x_L = \frac{mv_0}{k}$. 1

End of Examination

Year 12 Ext 2 HSC Trial HSC Exam

MATHEMATICS EXTENSION 2 – QUESTION MC

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
1. $z = \frac{3+4i}{1+2i} \cdot \frac{1-2i}{1-2i}$ $= \frac{3+4i-6i+8}{5}$ $= \frac{11-2i}{5}$ $= \frac{11}{5} - \frac{2}{5}i$	B	
2. 	C	
3. $\alpha\beta\gamma = \frac{1}{2}$ $\alpha^{-3}\beta^{-3}\gamma^{-3} = (\alpha\beta\gamma)^{-3}$ $= \left(\frac{1}{2}\right)^{-3}$ $= 8$	D	
4.  $\arg(z-a) = \alpha$ and $\arg(z+a) = \beta$ then $\alpha = \beta + \gamma$ (ext ∠'s property) i.e. $\arg(z-a) = \arg(z+a) + \gamma$ So $\arg(z-a) - \arg(z+a) = \gamma$ But ∠ at centre is less than π	D	

∴ ∠ at z must be less than $\pi/2$

$\therefore \arg(z-a) - \arg(z+a) = \pi/2$

MATHEMATICS EXTENSION 2 – QUESTION MC

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
5. $\int \frac{dx}{x^2+4x+9} = \int \frac{dx}{x^2+4x+4+5}$ $= \int \frac{dx}{(x+2)^2+5}$ $= \frac{1}{\sqrt{5}} \tan^{-1}\left(\frac{x+2}{\sqrt{5}}\right) + c$	A	
6. $P(x) = x^3 - 6x^2 + 9x + c$ $P'(x) = 3x^2 - 12x + 9$ $= 3(x^2 - 4x + 3)$ $= 3(x-3)(x-1)$ $P'(x) = 0$ and $P(x) = 0$ for a double root. For $x = 3$, $P(3) = 0$ $P(3) = 3^3 - 6(3)^2 + 9(3) + c$ $0 = 27 - 54 + 27 + c$ $c = 0$ $P(1) = 0$ $0 = 1 - 6 \times 1 + 9 \times 1 + c$ $c = -4$	B	
7. $\int x e^{-x} dx$ Integrating by Parts $I = -x \cdot e^{-x} - \int -e^{-x} dx$ $= -xe^{-x} + \int e^{-x} + c$ $= -xe^{-x} - e^{-x} + c$	A	

MATHEMATICS EXTENSION 2 – QUESTION MC

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS

8. $4x^2+9y^2=16$

$\frac{x^2}{4}+\frac{y^2}{16}=1$

$a^2=4 \qquad b^2=16/9$

$b^2=a^2(1-e^2)$

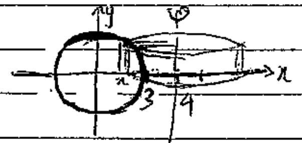
$\frac{16}{9}=4(1-e^2)$

$\frac{16}{36}=1-e^2$

$e^2=\frac{5}{9}$

$e=\frac{\sqrt{5}}{3}$ D

9. By Cylindrical Shells



Outer radius $= 4-x+\delta x$

Inner radius $= 4-x$

$\delta V = 2\pi(R^2-r^2).y$

$= 2\pi\left[(4-x+\delta x)^2-(4-x)^2\right]\sqrt{9-x^2}$

$= 2\pi\left[(4-x)^2+2(4-x)\delta x+(\delta x)^2-(4-x)^2\right] \times \sqrt{9-x^2}$

but $(\delta x)^2 \to 0$

$= 2\pi\left[2(4-x)\,\delta x\right]\sqrt{9-x^2}$

$= 4\pi(4-x)\sqrt{9-x^2}\,\delta x$

$V = \lim_{\delta x\to 0}\sum_{x=-3}^{3} 4\pi(4-x)\sqrt{9-x^2}\,\delta x$

$= 4\pi\int_{-3}^{3}(4-x)\sqrt{9-x^2}\,dx$ D

MATHEMATICS EXTENSION 2 – QUESTION MC

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS

10.

$\frac{dv}{dt}=-(1+v)$

$\frac{dt}{dv}=\frac{-1}{1+v}$

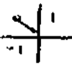
$\int_0^t dt=\int_u^0 \frac{-1}{1+v}\,dv$

$t=-\left[\ln(1+v)\right]_u^0$

$=-(0-\ln(1+u))$

$t=\ln(1+u)$ C

MATHEMATICS EXTENSION 2 – QUESTION 11

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
a) i) $\arg(w-z) = \arg(2+2i-(3+i))$		
$= \arg(2+2i-3-i)$		
		Many students
$= \arg(-1+i)$	½	gave the answer as
$= \tan^{-1}(-1)$		$-\frac{\pi}{4}$ not realising
$= \frac{3\pi}{4}$		that $(-1+i)$
	½	is in the 2nd
		Quad  ①
ii) $2+2i+2i(3+i) = 2+2i+6i-2$		
$= 2+2i-2-6i$	1	A few student
$= -4i$	1	took the conjugate
		of $\overline{6i-2}$ as
		$6i+2$ which
		is incorrect.
		②
b) Use the substitution $x = \sin\theta$		
$\frac{dx}{d\theta} = \cos\theta$		
$dx = \cos\theta \, d\theta$		
when $x = 1$, $\theta = \pi/2$		
when $x = 0$, $\theta = 0$		
$\int_0^1 \sqrt{1-x^2}\,dx = \int_0^{\pi/2} \sqrt{1-\sin^2\theta}\cdot\cos\theta\,d\theta$		Some errors
$= \int_0^{\pi/2} \sqrt{\cos^2\theta}\,\cos\theta\,d\theta$	1	were made with the substitution
$= \int_0^{\pi/2} \cos^2\theta\,d\theta$		otherwise this question was
		well answered.
$= \int_0^{\pi/2} \frac{1}{2} + \frac{1}{2}\cos 2\theta\,d\theta$	1	
$= \left[\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta\right]_0^{\pi/2}$		
$= \frac{\pi}{4}$	1	③

MATHEMATICS EXTENSION 2 – QUESTION 11

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
c) i)		
$\frac{x}{(x-1)(x^2+1)} = \frac{A}{x+1} + \frac{Bx+1}{x^2+1}$		Well answered.
$x = A(x^2+1) + (Bx+c)(x+1)$		
when $x = -1$		
$-1 = 2A$		
$A = -\frac{1}{2}$	½	
when $x = 0$		
$0 = -\frac{1}{2}(1) + (0+c)(0+1)$		
$0 = -\frac{1}{2} + c$		
$c = \frac{1}{2}$	½	
when $x = 1$		
$1 = -\frac{1}{2}(1+1) + (B+\frac{1}{2})(1+1)$		
$1 = -1 + 2B + 1$		
$2B = 1$		
$B = \frac{1}{2}$	½	
$\therefore$		②
$\frac{x}{(x+1)(x^2+1)} = \frac{-1}{2(x+1)} + \frac{x+1}{2(x^2+1)}$	½	For expression written with A, B and c.
ii) $\int \frac{x\,dx}{(x+1)(x^2+1)} = \frac{-1}{2}\int \frac{dx}{x+1} + \frac{1}{2}\int \frac{x+1}{x^2+1}\,dx$		
		Overall quite
$= \frac{-1}{2}\int \frac{dx}{x+1} + \frac{1}{2}\int \frac{x}{x^2+1}\,dx + \frac{1}{2}\int \frac{dx}{x^2+1}$		well done.
$= \left[-\frac{1}{2}\ln\lvert x+1\rvert + \frac{1}{4}\ln\lvert x^2+1\rvert + \frac{1}{2}\tan^{-1}x\right] + c$		
		②

MATHEMATICS EXTENSION 2 – QUESTION 11

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
d) $(a+ib)^2 = -3+4i$		
$a^2+2iba-b^2=-3+4i$		
Equating part.		
$a^2-b^2=-3$ ---- ①		
$2ab = 4$ --- ②		
From ② $a = \frac{2}{b}$ sub in ①	1	
$\frac{4}{b^2}-b^2=-3$		
$4-b^4=-3b^2$		
$b^4-3b^2-4=0$		
$(b^2-4)(b^2+1)=0$		1 mark was
$b^2=4$ or $b^2=-1$ no solutio		lost if students
as b is real		factorised incorrectly
$\therefore b=\pm 2$	1	
when $b=2$, $a=1$		Some students
when $b=-2$, $a=-1$	1	did not realise
	③	that since $ab>0$
ii) $z=\frac{3\pm\sqrt{9-4(3-i)}}{2}$		the $a>0$ & $b>0$
		(or $a<0$ & $b<0$)
$=\frac{3\pm\sqrt{9-12+4i}}{2}$		½ mark was
		taken off if
$=\frac{3\pm\sqrt{-3+4i}}{2}$ from	½	student did not
		clearly show this
$=\frac{3\pm(1+2i)}{2}$ from (i)	½	
$\therefore z=\frac{3+1+2i}{2}$ or $z=\frac{3-1-2i}{2}$		
$=\frac{4+2i}{2}$ or $=\frac{2-2i}{2}$		
$\therefore z= 2+i$ or $z=1-i$	1	②

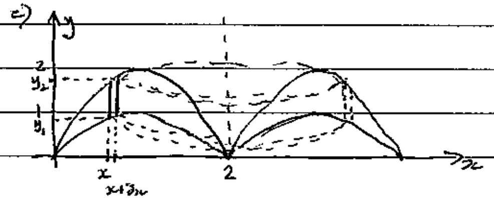
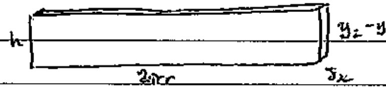
MATHEMATICS EXTENSION 2 – QUESTION 12

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
a) (i) $y=18x^{-1}$		
$\frac{dy}{dx}=-18x^{-2}$		
$\therefore$ gradient of tangent at $P(2,9)$ is $\frac{-18}{2^2}=\frac{-9}{2}$		
$\therefore$ gradient of normal at P is $\frac{2}{9}$	1	
Equation of N_1 is		
$y-9=\frac{2}{9}(x-2)$	1	
$9y-81=2x-4$		
$2x-9y+77=0$ —— ①		
(ii) gradient of normal at $Q(9,2)$ is $\frac{9}{2}$		
Equation of N_2 is		
$y-2=\frac{9}{2}(x-9)$		
$2y-4=9x-81$		
$9x-2y-77=0$ —— ②	1	
(iii) Multiply ① by 2		
$4x-18y+154=0$ —— (1A)		
② × 9		
$81x-18y-693=0$ —— (2A)		
(2A) − (1A)		
$77x \quad -847=0$		
$x=11$		
sub. into ①		
$22-9y+77=0$		
$9y=99$		
$y=11$		
$\therefore$ Coordinates of B $(11,11)$	1	
(iv) $O(0,0)$, $P(2,9)$, $Q(9,2)$ and $B(11,11)$		
$m_{OP}=\frac{9}{2}$ $m_{BP}=\frac{2}{9}$ $m_{OQ}=\frac{2}{9}$ $m_{BQ}=\frac{9}{2}$		

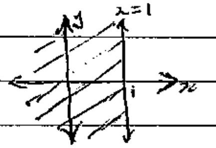
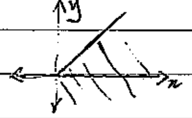
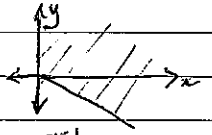
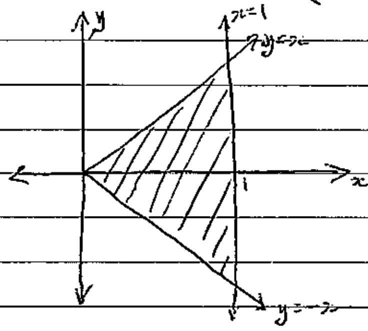
MATHEMATICS EXTENSION 2 – QUESTION 12

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
a) (iv) continued		
$\therefore$ $OP \parallel BQ$ and $OQ \parallel BP$		
$\therefore$ OPQB is a parallelogram	1	
$OP = \sqrt{2^2+9^2}$ $\quad OQ = \sqrt{9^2+2^2}$		
$= \sqrt{85}$ $\quad = \sqrt{85}$		
$\therefore$ adjacent sides equal		
$\therefore$ rhombus	1	
OR	OR	
$OP = \sqrt{85}$ $\quad OQ = \sqrt{85}$ (from above)	1	
$PB = \sqrt{(11-2)^2+(11-9)^2}$ $\quad QB = \sqrt{(11-9)^2+(11-2)^2}$		
$= \sqrt{9^2+2^2}$ $\quad = \sqrt{9^2+2^2}$		
$= \sqrt{85}$ $\quad = \sqrt{85}$	1	
$\therefore$ all four sides equal		
$\therefore$ rhombus		
OR	OR	
midpoint $OB = \left(\frac{11}{2}, \frac{11}{2}\right)$		
midpoint $PQ = \left(\frac{2+9}{2}, \frac{9+2}{2}\right)$		
$= \left(\frac{11}{2}, \frac{11}{2}\right)$		
$\therefore$ OB and PQ bisect each other	1	
$m_{OB} = \frac{11}{11}$ $\quad m_{PQ} = \frac{9-2}{2-9}$		
$= 1$ $\quad = -1$		
$m_{OB} \times m_{PQ} = -1$		
$\therefore OB \perp PQ$	1	
$\therefore$ diagonals bisect at 90°		
$\therefore$ rhombus		
b) Let $P(x) = 18x^3 + 3x^2 - 28x + 12$		
$P'(x) = 54x^2 + 6x - 28$	1	1 mark for differentiating
$= 2(27x^2 + 3x - 14)$		to find possible
$= 2(9x+7)(3x-2)$		double roots
Possible double roots when $P'(x) = 0$		

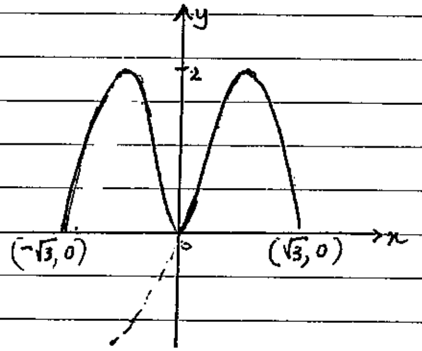
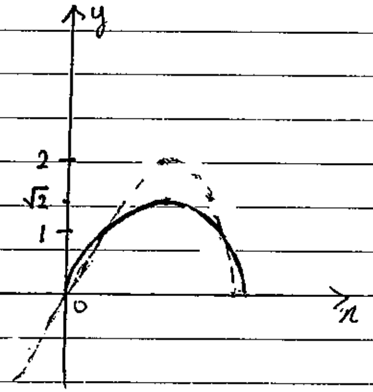
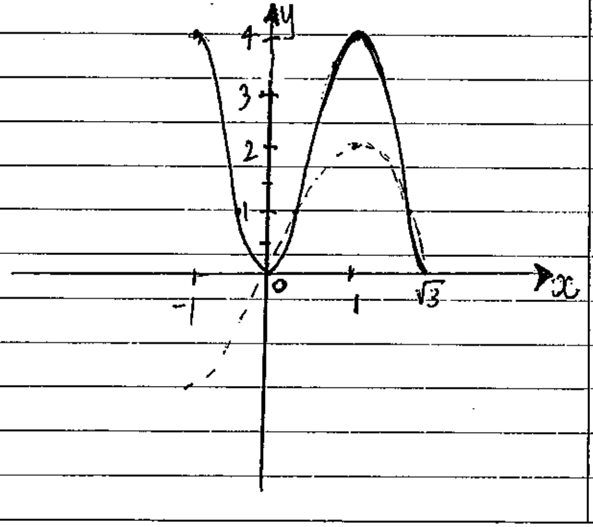
MATHEMATICS EXTENSION 2 – QUESTION 12

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
b) continued		
$9x + 7 = 0$ $\quad 3x - 2 = 0$		
$x = \frac{-7}{9}$ $\quad x = \frac{2}{3}$		
$P\left(\frac{-7}{9}\right) = 18\left(\frac{-7}{9}\right)^3 + 3\left(\frac{-7}{9}\right)^2 - 28\left(\frac{-7}{9}\right) + 12$		
$= \frac{2187}{81}$ $\quad P\left(\frac{2}{3}\right) = 18\left(\frac{2}{3}\right)^3 + 3\left(\frac{2}{3}\right)^2 - 28\left(\frac{2}{3}\right) + 12$		
$\neq 0$ $\quad = 0$		
$\therefore x = \frac{2}{3}$ is a double root	1	1 mark for finding
$x = \frac{-7}{9}$ is not a root		the double root
If $\alpha = \frac{2}{3}$ is the double root let β be the		
other root		
$\alpha + \alpha + \beta = \frac{-3}{18}$ (sum of the roots)		
$2 \times \frac{2}{3} + \beta = \frac{-3}{18}$		
$\beta = \frac{-3}{18} - \frac{4}{3}$		
$= \frac{-3}{2}$		1 mark for finding
$\therefore$ the roots are $\frac{2}{3}, \frac{2}{3}, \frac{-3}{2}$	1	the third root and
		listing all 3 roots
c) 		
		
$\delta V = 2\pi r h \delta x$ $\quad r = 2-x,\ h = y_2 - y_1$		
$= 2\pi(2-x)(y_2-y_1)\delta x$	1	1 mark for setting
$= 2\pi(2-x)(4x-2x^2-(2x-x^2))\delta x$		up the equation of
$= 2\pi(2-x)(2x-x^2)\delta x$		the volume of the shell
$= 2\pi(4x-4x^2+x^3)\delta x$		
$V = \lim_{\delta x \to 0} \sum_{x=0}^{2} 2\pi(4x-4x^2+x^3)\delta x$	1	1 mark for setting up
$= \int_0^2 2\pi(4x-4x^2+x^3)\,dx$		the integral

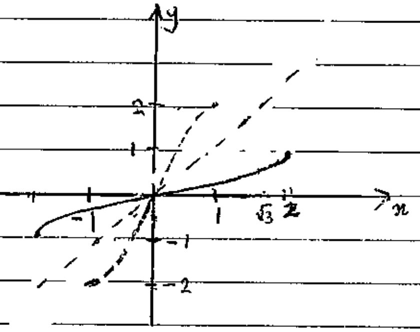
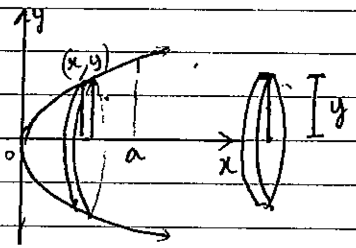
MATHEMATICS EXTENSION 2 – QUESTION 12

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
c) continued		
$V = 2\pi\int_0^2 (4x - 4x^2 + x^3)\,dx$		
$= 2\pi\left[2x^2 - \frac{4}{3}x^3 + \frac{x^4}{4}\right]_0^2$		
$= 2\pi\left(8 - \frac{32}{3} + 4 - 0\right)$		1 mark for integration
$= \frac{8\pi}{3}\ u^3$	1	and correct calculation
d) Let $z = x + iy$ $\quad |z| = |x+iy|$		
$= \sqrt{x^2+y^2}$		
$|z-2| = |x+iy-2|$		
$= |x-2+iy|$		
$= \sqrt{(x-2)^2+y^2}$		
$|z| \leq |z-2|$		
$\sqrt{x^2+y^2} \leq \sqrt{(x-2)^2+y^2}$		
$x^2+y^2 \leq (x-2)^2+y^2$		
$\leq x^2-4x+4+y^2$		
$0 \leq -4x+4$		
$4x \leq 4$		
$x \leq 1$ 		
$\arg z \leq \frac{\pi}{4}$ 		
$\arg z \geq -\frac{\pi}{4}$ 		
	1 1 1	for region $x \leq 1$ for less than $y=x$ for greater than $y=-x$

MATHEMATICS EXTENSION 2 – QUESTION 13

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
13a) i) 		1 mark – point $(-1,2)$ & $(-\sqrt{3},0)$ 1 mark – shape ②
		1 mark – shape $\frac{1}{2}$ mark – behaviour between 0 & 1 (i.e $\sqrt{f(x)} > f(x)$) $\frac{1}{2}$ mark – point $(1,\sqrt{2})$ ②
		$\frac{1}{2}$ mark – shape $\frac{1}{2}$ concavity change at $x=0$ & $x=\sqrt{3}$ 1 mark for 2 points $(1,4)$, $(-1,4)$ ②

MATHEMATICS EXTENSION 2 – QUESTION 13

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
iv) 		This part was not well done.
Restricted domain: $-1 \leq x \leq 1$		
Many students did not restrict the domain and sketched the inverse relation. $-\frac{1}{2}$ marks		②
13b) 		Some students took the slice parallel to the axis of rotation. Volumes by slicing requires the slice to be perpendicular to the axis of rotation.
$\delta V = \pi y^2 \delta x$ $= \pi (4ax)\delta x$ $V = \lim_{\delta x \to 0} \sum_{n=0}^{a} \pi 4ax\delta x$	1	
$= 4a\pi \int_0^a x\,dx$ $= 4a\pi \left[\frac{x^2}{2}\right]_0^a$	1	
$V = 2a^3\pi u^3$	1	
		③

MATHEMATICS EXTENSION 2 – QUESTION 13

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
13c)		
$I_n = \int_1^e (1-\log_e x)^n . 1\, dx$		
$u = (1-\log_e x)^n$ $u' = n(1-\log_e x)^{n-1} . \frac{-1}{x}$ $v' = 1$ $v = x$		
$\therefore I_n = \left[x.(1-\log_e x)^n\right]_1^e - \int_1^e x . n(1-\log_e x)^{n-1} . \frac{-1}{x}\, dx$		1 mark
$= \left[(e(1-\log_e e)^n) - 1(1-\log_e 1)^n\right] + \int_1^e n(1-\log_e x)^{n-1}\, dx$ $= -1 + \int_1^e n(1-\log_e x)^{n-1}\, dx$		1 mark ②
$= -1 + n I_{n-1}$, $n = 1, 2, 3,$		
ii) $I_3 = -1 + 3 I_2$ $= -1 + 3(-1 + 2 I_1)$ $= -1 - 3 + 6 I_1$ $= -4 + 6 I_1$ $= -4 + 6(-1 + I_0)$ $= -4 - 6 + 6 I_0$		
$= -10 + 6 I_0$ but $I_0 = \int_1^e 1\, dx$ $= x\Big]_1^e$ $= e - 1$		1 mark
$\therefore I_3 = -10 + 6(e-1)$ $= -10 + 6e - 6$ $= -16 + 6e$		
$\therefore \int_1^e (1-\log_e x)^3 = -16 + 6e$		1 mark ②

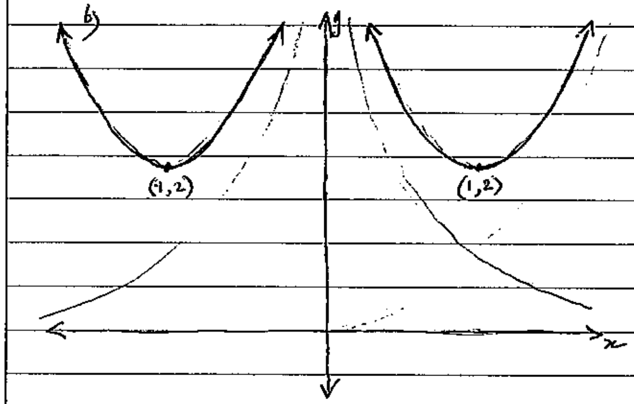
MATHEMATICS EXTENSION 2 – QUESTION 14

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
a)(i) Forces acting on the particle 		
Net force $m\ddot{x} = mg - mkv$		
$\ddot{x} = g - kv$		
As $\ddot{x} \to 0$ $v \to \frac{g}{k}$		
For terminal velocity V $\ddot{x} = 0$, $v = V$	1	1 mark for stating that terminal velocity occurs when $\ddot{x} = 0$
$\therefore V = \frac{g}{k}$		
$k = \frac{g}{V}$		
$\therefore \ddot{x} = g - \frac{g}{V}v$	1	1 mark for finding and using $k = \frac{g}{V}$
$= \frac{gV - gv}{V}$		
$= \frac{g}{V}(V - v)$		
(ii) Initial conditions $t=0$, $x=0$, $v=0$		
when $t = T$, $x = X$, $v = \frac{V}{2}$		
$\ddot{x} = \frac{g}{V}(V - v)$		
$v\frac{dv}{dx} = \frac{g}{V}(V - v)$		
$\frac{dv}{dx} = \frac{g(V-v)}{Vv}$		
$\frac{dx}{dv} = \frac{Vv}{g(V-v)}$		
$\int_0^X dx = \int_0^{\frac{V}{2}} \frac{Vv}{g(V-v)}dv$	1	1 mark for setting up this integral
$X = \frac{V}{g}\int_0^{\frac{V}{2}} \frac{v}{V-v}dv$		
$= -\frac{V}{g}\int_0^{\frac{V}{2}} \frac{-v}{V-v}dv$		
$= -\frac{V}{g}\int_0^{\frac{V}{2}} \frac{V-v-V}{V-v}dv$		
$= -\frac{V}{g}\int_0^{\frac{V}{2}} \left(1 - \frac{V}{V-v}\right)dv$		

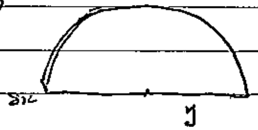
MATHEMATICS EXTENSION 2 – QUESTION 14

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
a)(ii) continued		
$X = -\frac{V}{g}\left(\left[v\right]_0^{\frac{V}{2}} + V\int_0^{\frac{V}{2}} \frac{-dv}{V-v}\right)$		
$= -\frac{V}{g}\left(\frac{V}{2} + V\left[\ln(V-v)\right]_0^{\frac{V}{2}}\right)$		
$= \frac{-V^2}{2g} - \frac{V^2}{g}\left(\ln(V - \frac{V}{2}) - \ln V\right)$		
$= \frac{-V^2}{2g} - \frac{V^2}{g}\ln\frac{V}{2V}$		
$= \frac{-V^2}{2g} - \frac{V^2}{g}\ln\frac{1}{2}$		
$= \frac{-V^2}{2g} + \frac{V^2}{g}\ln 2$ ①	1	1 mark for finding this integral
$\ddot{x} = \frac{g}{V}(V - v)$		
$\frac{dv}{dt} = \frac{g}{V}(V - v)$		
$\frac{dt}{dv} = \frac{V}{g(V-v)}$		
$\int_0^T dt = \int_0^{\frac{V}{2}} \frac{V}{g(V-v)}dv$		
$T = -\frac{V}{g}\int_0^{\frac{V}{2}} \frac{-dv}{V-v}$		
$= -\frac{V}{g}\left[\ln(V-v)\right]_0^{\frac{V}{2}}$		
$= -\frac{V}{g}\left(\ln(V - \frac{V}{2}) - \ln V\right)$		
$= -\frac{V}{g}\ln\frac{V}{2V}$		
$= -\frac{V}{g}\ln\frac{1}{2}$		
$= \frac{V}{g}\ln 2$	1	1 mark for setting up and finding this integral

MATHEMATICS EXTENSION 2 – QUESTION 14

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
a) (ii) continued		
$VT = \frac{V^2}{g}\ln 2$		
from ①		
$VT - X = \frac{V^2}{g}\ln 2 + \frac{V^2}{2g} - \frac{V^2}{g}\ln 2$	1	1 mark for combining the two integrals
$VT - X = \frac{V^2}{2g}$		
b) 	3	1 mark for each arm and 1 mark for vertical asymptote at $x = 0$
c) $2xy + x^2\frac{dy}{dx} + y^2 + 2y\frac{dy}{dx} = 0$	1	1 mark for implicit differentiation
$x^2\frac{dy}{dx} + 2y\frac{dy}{dx} = -2xy - y^2$		
$(x^2+2y)\frac{dy}{dx} = -(2xy+y^2)$		
$\frac{dy}{dx} = \frac{-(2xy+y^2)}{x^2+2y}$		
at $(1,2)$ $\frac{dy}{dx} = \frac{-(2\times1\times2+2^2)}{1^2+2\times2}$		
$= \frac{-8}{5}$	1	1 mark for gradient of tangent
Equation of the tangent		
$y - 2 = \frac{-8}{5}(x-1)$		

MATHEMATICS EXTENSION 2 – QUESTION 14

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
c) continued		
$5y - 10 = {}^-8x + 8$		
$8x + 5y - 18 = 0$	1	1 mark for equation of tangent
d) 		
$\delta V = \frac{1}{2}\pi r^2\,\delta x$ $= \frac{1}{2}\pi y^2\,\delta x$ $= \frac{1}{2}\pi(9x-x^3)^2\,\delta x$	1	1 mark for volume of slice
$V = \lim_{\delta x\to 0}\sum_{x=0}^{3}\frac{1}{2}\pi(9x-x^3)^2\,\delta x$		
$= \int_0^3 \frac{1}{2}\pi(9x-x^3)^2\,dx$		
$= \frac{\pi}{2}\int_0^3 (81x^2 - 18x^4 + x^6)\,dx$	1	1 mark for setting up the integral
$= \frac{\pi}{2}\left[27x^3 - \frac{18}{5}x^5 + \frac{x^7}{7}\right]_0^3$		
$= \frac{\pi}{2}\left(27^2 - \frac{18}{5}(3^5) + \frac{(3)^7}{7} - 0\right)$		
$= \frac{2916}{35}\pi \quad u^3$	1	1 mark for integrating and finding the volume.

MATHEMATICS EXTENSION 2 – QUESTION 15

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
15 a) i)		
$x = a\cos\theta$ $\quad y = b\sin\theta$		
$\frac{dx}{d\theta} = -a\sin\theta$ $\quad \frac{dy}{d\theta} = b\cos\theta$		
$\therefore \frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx}$		
$= b\cos\theta \times \frac{-1}{a\sin\theta}$		
$= -\frac{b\cos\theta}{a\sin\theta}$		1 mark for gradient
Equation of tangent at P		
$y - b\sin\theta = \frac{-b\cos\theta}{a\sin\theta}(x - a\cos\theta)$		1 mark for Equation of tangent.
$ya\sin\theta - ab\sin^2\theta = -b\cos\theta x + ab\cos^2\theta$		
$xb\cos\theta + ya\sin\theta = ab\sin^2\theta + ab\cos^2\theta$		
$= ab(\sin^2\theta + \cos^2\theta)$		
$\therefore xb\cos\theta + ya\sin\theta = ab$		
$\div ab$		
$\frac{x\cos\theta}{a} + \frac{y\sin\theta}{b} = 1$		②
ii) Gradient of normal is $\frac{a\sin\theta}{b\cos\theta}$	½	
Equation of normal is:		
$y - b\sin\theta = \frac{a\sin\theta}{b\cos\theta}(x - a\cos\theta)$	½	
$by\cos\theta - b^2\sin\theta\cos\theta = ax\sin\theta - a^2\sin\theta\cos\theta$	½	
$ax\sin\theta - by\cos\theta = a^2\sin\theta\cos\theta - b^2\sin\theta\cos\theta$		
$= \sin\theta\cos\theta(a^2 - b^2)$	½	
$\therefore \frac{ax}{\cos\theta} - \frac{by}{\sin\theta} = a^2 - b^2$		②

MATHEMATICS EXTENSION 2 – QUESTION 15

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
a) iii) To find the coordinates of A,		
sub $x=0$ into equation of tangent		
$\frac{y\sin\theta}{b} = 1$		
$y = \frac{b}{\sin\theta}$		
$\therefore$ A is $(0, \frac{b}{\sin\theta})$	1	
To find the coordinates of B		
sub $x=0$ into equation of normal		
$\frac{-by}{\sin\theta} = a^2 - b^2$		
$\frac{by}{\sin\theta} = b^2 - a^2$		
$y = \frac{(b^2 - a^2)\sin\theta}{b}$		
$\therefore$ B is $(0, \frac{(b^2 - a^2)\sin\theta}{b})$	1	
Gradient of AS : $m_{AS} = \frac{\frac{b}{\sin\theta} - 0}{a - ae}$		
$= \frac{-b}{ae\sin\theta}$		
Gradient of BS : $m_{BS} = \frac{(b^2 - a^2)\sin\theta}{-aeb}$		
$= \frac{(a^2 - b^2)\sin\theta}{aeb}$		From $b^2 = a^2(1-e^2)$
		$= a^2 - a^2e^2$
$\therefore m_{BS} = \frac{a^{\cancel{2}}e^2\sin\theta}{\cancel{a}eb}$		So $a^2 - b^2 = a^2e^2$.
$= \frac{ae\sin\theta}{b}$	½	

MATHEMATICS EXTENSION 2 – QUESTION 15

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
15a) iii)		
Now		
$m_{AS} \times m_{BS} = \frac{-\cancel{b}}{a\cancel{e}\sin\cancel{\theta}} \times \frac{a\cancel{e}\sin\cancel{\theta}}{\cancel{b}}$	$\frac{1}{2}$	
$= -1$		
∴ AS is perpendicular to BS.		3
iv) $\angle APB = 90°$ (tangents and normals at right angles to each other)		
$\angle ASB = 90°$ (proven above)		
∴ $\angle APB = \angle ASB$	1	Many students gave the reason "the angles in the same segment are equal". This reason applies only if the points are already concyclic. Care needs to be taken.
Since the interval AB subtends equal angles at two points, at P and S, on the same side of it, ∴ A & B and S + P form a cyclic quadrilateral ∴ are concyclic points.		
		1

MATHEMATICS EXTENSION 2 – QUESTION 15

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
15(b)		
i) $\frac{d(\frac{1}{2}v^2)}{dx} = \frac{1}{2}\left(1-\frac{1}{x^2}\right)$		
$\frac{1}{2}v^2 = \frac{1}{2}\int 1-\frac{1}{x^2}\,dx$		
$= \frac{1}{2}\left[x+\frac{1}{x}\right]+c$		
$v^2 = x + \frac{1}{x} + c_1$	1	
when $x=1$, $v=2$		
$4 = 1+1+c_1$		
$c_1 = 2$	1	
∴ $v^2 = x + \frac{1}{x} + 2$ $= \frac{x^2+2x+1}{x}$ $= \frac{(x+1)^2}{x}$		
$v = \pm\frac{x+1}{\sqrt{x}} \Rightarrow \frac{dx}{dt} = \pm\frac{1+x}{\sqrt{x}}$		
but when $x=1$, $v=2>0$ ∴ $\frac{dx}{dt} = \frac{1+x}{\sqrt{x}}$	1	Many students did not state that $v>0$ since $x=1$ ∴ lost ½ marks
		3

MATHEMATICS EXTENSION 2 – QUESTION 15

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
b) ii) $\frac{dt}{dx} = \frac{\sqrt{x}}{1+x}$		
$\int_0^t dt = \int_1^3 \frac{\sqrt{x}}{1+x}\,dx$		
$t = \int_1^3 \frac{\sqrt{x}}{1+x}\,dx$	$\frac{1}{2}$	
Let $u = \sqrt{x}$ i.e. $x = u^2$	$\frac{1}{2}$	
$\frac{dx}{du} = 2u$		
when $x=3,\ u=\sqrt{3}$		
$x=1 \quad u=1$		
$\therefore t = \int_1^{\sqrt{3}} \frac{u . 2u\,du}{1+u^2}$	$\frac{1}{2}$	
$= \int_1^{\sqrt{3}} \frac{2u^2}{1+u^2}\,du$		
$= \int_1^{\sqrt{3}} \frac{2u^2+2-2}{1+u^2}\,du$		
$= \int_1^{\sqrt{3}} \left(2 - \frac{2}{1+u^2}\right) du$	$\frac{1}{2}$	
$= \left[2u - 2\tan^{-1}u\right]_1^{\sqrt{3}}$	1	
$= 2\left[\left(\sqrt{3} - 2\tan^{-1}\sqrt{3}\right) - \left(1 - \tan^{-1}1\right)\right]$		
$= 2\left[\sqrt{3} - 2\tan^{-1}\sqrt{3} - 1 + \frac{\pi}{4}\right]$		
$= 2\sqrt{3} - 4\tan^{-1}\sqrt{3} - 2 + \frac{\pi}{2}$		
$= 2\sqrt{3} - 4\pi/6 - 2 + \pi/2$		④
$= 2\sqrt{3} - 2 - \pi/6$	1	

$\doteqdot 0{\cdot}94$ sec

MATHEMATICS EXTENSION 2 – QUESTION 16

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
a) (i) $\vec{OR} = \frac{1}{2}\vec{OP}$ ①		
$\vec{QP}$ is a 90° clockwise rotation of $\vec{OQ}$		
multiplied by a factor of k, $k>0$		
$\vec{QP} = -i\vec{OQ} \times k$		
$= -ki\vec{OQ}$		
$= -kiw$	1	1 mark for finding
		$\vec{QP}$ in terms of w
$\vec{OP} = \vec{OQ} + \vec{QP}$		and k
$= w + (-kiw)$	1	1 mark for finding
		$\vec{OP}$ in terms of w
From ① $\vec{OR} = \frac{1}{2}\vec{OP}$	1	1 mark for relating
$= \frac{1}{2}(w - kiw)$		$\vec{OR}$ to $\vec{OP}$
$= \frac{1}{2}(1-ki)w$		
(ii) $\vec{RQ} = \vec{OQ} - \vec{OR}$		
$= w - \frac{1}{2}(1-ki)w$		
$= w - \frac{w}{2} + \frac{kiw}{2}$		
$= \frac{w}{2} + \frac{kiw}{2}$		
$= \frac{w}{2}(1+ki)$	1	
$\lvert\vec{OR}\rvert = \left\lvert\frac{1}{2}(1-ki)w\right\rvert$		
$= \lvert\tfrac{1}{2}\rvert\,\lvert 1-ki\rvert\,\lvert w\rvert$		
$= \frac{1}{2}\sqrt{1+(-k)^2}\,\lvert w\rvert$		
$= \frac{1}{2}\sqrt{1+k^2}\,\lvert w\rvert$		
$\lvert\vec{RQ}\rvert = \left\lvert\frac{1}{2}(1+ki)w\right\rvert$		
$= \lvert\tfrac{1}{2}\rvert\,\lvert 1+ki\rvert\,\lvert w\rvert$		
$= \frac{1}{2}\sqrt{1+k^2}\,\lvert w\rvert$		
$= \lvert\vec{OR}\rvert$	1	

MATHEMATICS EXTENSION 2 – QUESTION 16

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
b) Let $\alpha = \tan^{-1}\left(\frac{x+1}{x-1}\right)$ and $\beta = \tan^{-1}\left(\frac{1}{x}\right)$		
$\tan\alpha = \frac{x+1}{x-1}$ $\quad \tan\beta = \frac{1}{x}$ $\quad x > 1$	1	1 mark for substitution
$\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta}$		
$= \frac{\frac{x+1}{x-1} - \frac{1}{x}}{1+\left(\frac{x+1}{x-1}\right)\left(\frac{1}{x}\right)}$	1	1 mark for finding $\tan(\alpha-\beta)$
$= \frac{x(x+1)-(x-1)}{x(x-1)} \div \frac{x(x-1)+x+1}{x(x-1)}$		
$= \frac{x^2+1}{x(x-1)} \times \frac{x(x-1)}{x^2+1}$		
$= 1$		
$\alpha - \beta = \tan^{-1}(1)$		
$= \frac{\pi}{4}$	1	1 mark for $\alpha-\beta = \frac{\pi}{4}$
$\therefore \tan^{-1}\frac{x+1}{x-1} - \tan^{-1}\frac{1}{x} = \frac{\pi}{4}$		
c) (i) when $t=0$, $x=0$, $v=v_0$		
$m\ddot{x} = -kv$		
$\ddot{x} = -\frac{kv}{m}$		
$\ddot{x} = \frac{dv}{dt}$		
$\frac{dv}{dt} = -\frac{kv}{m}$		
$\frac{dt}{dv} = -\frac{m}{kv}$		
$\int_0^t dt = \int_{v_0}^{v} \frac{-m}{kv}\,dv$	1	1 mark for setting up the integral
$t = -\frac{m}{k}\int_{v_0}^{v} \frac{1}{v}\,dv$		
$= -\frac{m}{k}\left[\ln\lvert v\rvert\right]_{v_0}^{v}$		
$= -\frac{m}{k}(\ln v - \ln v_0)$	1	1 mark for finding the integral

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MATHEMATICS EXTENSION 2 – QUESTION

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
c)(i) continued		
$-\frac{kt}{m} = \ln\frac{v}{v_0}$		
$e^{-\frac{kt}{m}} = \frac{v}{v_0}$	1	1 mark for taking each side to e
$v = v_0 e^{-\frac{kt}{m}}$		
(ii) $\frac{dx}{dt} = v_0 e^{-\frac{kt}{m}}$		
$\int_0^x dx = \int_0^t v_0 e^{-\frac{kt}{m}}\,dt$	1	1 mark for setting up the integral
$x = v_0\int_0^t e^{-\frac{kt}{m}}\,dt$		
$= v_0\left[-\frac{m}{k}e^{-\frac{kt}{m}}\right]_0^t$	1	1 mark for finding the integral
$= v_0\left(-\frac{m}{k}e^{-\frac{kt}{m}} + \frac{m}{k}e^0\right)$		
$= \frac{mv_0}{k}\left(-e^{-\frac{kt}{m}} + 1\right)$	1	1 mark for simplificatio
$= \frac{mv_0}{k}\left(1 - e^{-\frac{kt}{m}}\right)$		
(iii) as $t \to \infty$ $\quad e^{-\frac{kt}{m}} \to 0$	1	
$\therefore x \to \frac{mv_0}{k}$		
$\therefore x_L = \frac{mv_0}{k}$		