SYDNEY BOYS HIGH
MOORE PARK, SURRY HILLS

2002
HIGHER SCHOOL CERTIFICATE
TRIAL EXAMINATION

Mathematics Extension 2

General Instructions
- Reading time — 5 minutes
- Working time — 3 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total marks — 120
- Attempt questions 1–8
- All questions are of equal value, the mark value is shown beside each part.

Examiner: D.M.Hespe

Note: This is an assessment task only and does not necessarily reflect the content or format of the Higher School Certificate.

Total marks – 120
Attempt Questions 1–8
All questions are of equal value

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Marks

Question 1 (15 marks) Use a SEPARATE writing booklet.

(a) Find

(i) $\int \sin^{-1} x \, dx$ **2**

(ii) $\int \frac{x}{1+x^4} \, dx$ **2**

(iii) $\int \tan^3 x \, dx$ **2**

(b) Evaluate $\int_0^{\frac{\pi}{2}} \frac{dx}{1+\cos x}$ using the substitution $t = \tan \frac{x}{2}$. **3**

(c) Given that $I_n = \int_1^e (\ln x)^n \, dx, \quad n = 0, 1, 2, \ldots,$ **3**

show that $I_n = e - nI_{n-1}$.

(d) If $x = \frac{\pi}{4} - u$,

(i) Show that $\tan x = \frac{1-\tan u}{1+\tan u}$. **1**

(ii) Hence or otherwise, show that $\int_0^{\frac{\pi}{4}} \ln(1+\tan x) \, dx = \frac{\pi}{8} \ln 2$. **2**

Marks

Question 2 (15 marks) Use a SEPARATE writing booklet.

(a) Explain the flaw in this "proof" that $i = -i$. **2**

$$i = i$$
$$\sqrt{-1} = \sqrt{-1}$$
$$\sqrt{\frac{-1}{1}} = \sqrt{\frac{1}{-1}}$$
$$\frac{\sqrt{-1}}{\sqrt{1}} = \frac{\sqrt{1}}{\sqrt{-1}}$$
$$\frac{i}{1} = \frac{1}{i} = \frac{-(-1)}{i} = \frac{-i^2}{i} = -i$$
$$\therefore i = -i$$

(b) $u = -3 - 4i$ and $v = 1 - i$ are two complex numbers. Express in the form $x + iy$, where x and y are real:

(i) $\bar{u} - v$ **1**

(ii) $\dfrac{2u}{v}$ **2**

(iii) $\sqrt{u}$ **2**

(c) On an Argand diagram sketch the region defined by **2**

$$\{z: -\tfrac{\pi}{6} \le \arg z \le \tfrac{\pi}{6} \cap |z| \le 1\}.$$

(d) (i) If a, b are the complex numbers represented by the points A and B on an Argand diagram, what geometrical properties correspond to the modulus and argument of $\dfrac{b}{a}$? **2**

(ii) Show that, if the four points representing the complex numbers z_1, z_2, z_3, and z_4 are concyclic, the fraction $\dfrac{(z_1 - z_2)(z_3 - z_4)}{(z_3 - z_2)(z_1 - z_4)}$ must be real. **4**

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Marks

Question 3 (15 marks) Use a SEPARATE writing booklet.

(a) Reduce to irreducible factors over the complex field: $x^3 - 4x^2 + 7x - 6$. **3**

(b) Find the polynomial $f(x)$ of the fourth degree such that $f(0) = f(1) = 1$, $f(2) = 13$, $f(3) = 73$ and $f'(0) = 0$. **4**

(c) (i) Prove that if $P(x)$ has a root of multiplicity m, then $P'(x)$ has a root of multiplicity $m - 1$. **2**

(ii) Find the value of c if the polynomial $5x^5 - 3x^3 + c$ has a positive repeated root. **3**

(d) Let α, β, γ be the roots of the equation $x^3 + px + q = 0$, where $q \neq 0$. Find, in terms of p and q, the coefficients of the cubic equation whose roots are α^{-1}, β^{-1}, and γ^{-1}. **3**

Marks

Question 4 (15 marks) Use a SEPARATE writing booklet.

(a) Solve the simultaneous equations: **4**

$$x^2+xy+y^2 = 7,$$

$$2x^2-xy+y^2 = 28.$$

(b) Show that if $b^2 < 4ac$, the value of the function ax^2+bx+c will have the same sign as a for all real values of x. **2**

(c) (i) By considering the expression $x^2-2xy+5y^2+2x-14y+k$ as a quadratic function of x, show that it is positive for all real values of x and y if $k>10$. **4**

(ii) Show that if $k=10$, the expression may be written in the form $(x+py+q)^2+(ry+s)^2$, and hence find the simultaneous values of x and y for which the expression is zero. **4**

(iii) Deduce the minimum value of the expression for a general value of k. **1**

Marks

Question 5 (15 marks) Use a SEPARATE writing booklet.

(a) A particle P of mass m starts from rest at a point O and falls under gravity in a medium where the resistance to its motion has magnitude mkv, v being the speed of the particle and k is a constant.

(i) Draw a diagram to show the *forces* acting on the particle during this downward path, and hence write down the equation of motion. **2**

(ii) Show that the expression for its velocity v at any time t is given by **2**

$$v = \frac{g}{k}\left(1 - e^{-kt}\right).$$

(iii) Explain what is meant by the *terminal velocity* and find an expression for the terminal velocity V_T. **3**

(b) A second particle Q, also of mass m, is fired vertically upwards from O with initial speed u, so that P and Q leave O simultaneously.

(i) Draw a diagram to show the *forces* acting on the particle during this ~~downward~~ upward path, and hence write down the equation of motion. **2**

(ii) Find an expression for the time t when Q comes to rest. **3**

(c) Show that, at the instant Q comes to rest, the velocity of P is given by: **3**

$$v = \frac{V_T\, u}{V_T + u}.$$

Marks

Question 6 (15 marks) Use a SEPARATE writing booklet.

(a) (i) Show that if x is real, the expression $\dfrac{(x-2)^2}{x-1}$ cannot take any value between -4 and 0. **2**

(ii) Sketch the graph of the expression. **3**

(iii) Show that the equation $\dfrac{(x-2)^2}{x-1} = \dfrac{k}{x}$ has three real roots if k is positive, but only one real root if k is negative. **2**

(b) For $z = r(\cos\theta + i\sin\theta)$, find r and the smallest positive value of θ which satisfy the equation $2z^3 = 9 + 3\sqrt{3}i$. **2**

(c) Using the method of shells find the volume of the solid formed when the region bounded by the curve $y = x^2 + 1$ and the x-axis between $x = 0$ and $x = 2$ is rotated about the y-axis. **3**

(d) Explain why, if $\lim\limits_{n\to\infty}\left(\sqrt{n^2+n} - n\right) = \lim\limits_{n\to\infty}\left(n \times \sqrt{1+\frac{1}{n}} - n\right)$, then the limit is not zero, but a half. **3**

Marks

Question 7 (15 marks) Use a SEPARATE writing booklet.

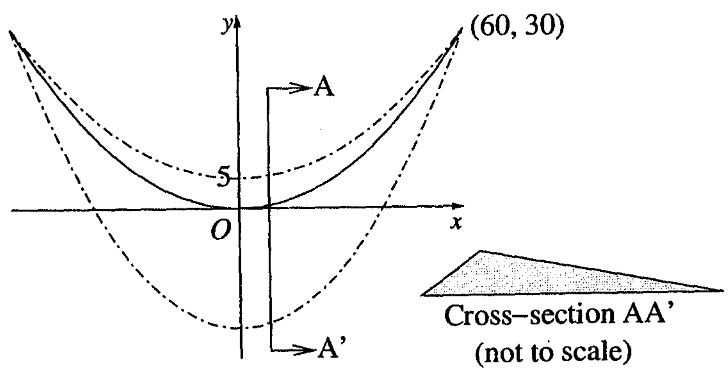
(a) Find the rational roots of $x^4+2x^3-17x^2-18x+32=0$ using the substitution $y=x^2+x$, or otherwise. **2**

(b) (i) Prove that the medians of a triangle are concurrent at a point which is a point of trisection of each median. [A *median* of a triangle is a line from a vertex to the mid point of the opposite side.] **3**

(ii) If the medians of triangle ABC meet at G, and AG is produced to K so that $AG = GK$, prove that the triangle BGK is similar to the triangle whose sides are equal in length to the three medians. **3**

(iii) Also show that the area of the triangle whose sides are equal in length to the medians is ¾ of the area of triangle ABC. **2**

(c) **5**



Barcan sand dunes are parabolic in plan view and are triangular in cross section with the inner face having an angle of repose of $\tan^{-1}\frac{3}{4}$ to the horizontal and the outer face at $\tan^{-1}\frac{1}{6}$ to the horizontal. The figure above shows one such dune (dimensions are in metres). Calculate the volume of sand.

Marks

Question 8 (15 marks) Use a SEPARATE writing booklet.

(a) (i) A circular disc, centre A, of radius a, rolls without slipping along the axis of x. Initially the point P on the edge of the disc is at the origin of coordinates. Prove that, when the radius AP has turned through an angle θ, the coordinates of P are: $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$. **3**

(ii) The length, ℓ, of a curve, $y = f(x)$, is given by **3**

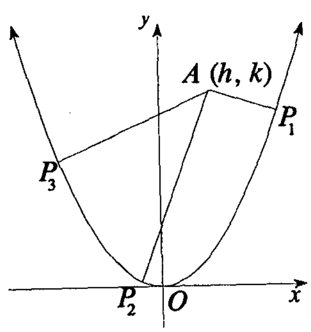
$$\ell = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx.$$

When P is again in contact with the axis of x, prove that the length of its path is $8a$.

(b) Sum the series, n being a positive integer, **4**

$$1 + x\cos x + x^2\cos 2x + x^3\cos 3x + \cdots + x^n\cos nx.$$

(c) (i)



Prove that, in general, three normals can be drawn from any point to a parabola. **3**

(ii) Also show that if P_1, P_2, and P_3 have coordinates (x_1, y_1), (x_2, y_2), and (x_3, y_3) respectively, then $x_1 + x_2 + x_3 = 0$. **2**

End of paper

(a) (i) $\int \sin^{-1}x\,dx = \int \sin^{-1}x \cdot \frac{d(x)}{dx}\,dx$

$= x\sin^{-1}x - \int x \cdot \frac{1}{\sqrt{1-x^2}}\,dx$ 1

$= x\sin^{-1}x + \sqrt{(1-x^2)} + C$ 1

(ii) $I = \int \frac{x}{1+x^4}\,dx$

let $u = x^2 \Rightarrow du = 2x\,dx$

$\therefore I = \frac{1}{2}\int \frac{du}{1+u^2}$ 1

$= \frac{1}{2}\tan^{-1}u + c$

$= \frac{1}{2}\tan^{-1}(x^2) + c$ 1

(iii) $I = \int \tan^3 x\,dx = \int \tan x \cdot \tan^2 x\,dx$

ie $I = \int \tan x(\sec^2 x - 1)\,dx$

$= \int \tan x \sec^2 x\,dx - \int \tan x\,dx$ 1

$= \frac{1}{2}\tan^2 x - \log_e(\cos x)^{-1}$ 1

$= \frac{1}{2}\tan^2 x + \log_e(\cos x)$

$\int_0 \frac{dx}{1+\cos x}$

$t = \tan\frac{x}{2}$

$\frac{dt}{dx} = \frac{1}{2}\sec^2\frac{x}{2}$

$\frac{dx}{dt} = \frac{2}{\sec^2\frac{x}{2}}$

$\frac{dx}{dt} = \frac{2}{1+t^2}$

When $x=0, t=$

When $x=\frac{\pi}{2}, t$

$= \int_0^1 \frac{2\,dt}{\left(1+\frac{1-t^2}{1+t^2}\right)(1+t^2)}$

$= 2\int_0^1 \frac{dt}{2}$ 1

$= [t]_0^1$

$= 1$ ½

(c) $I_n = \int_1^e (\ln x)^n\,dx$

$= \int_1^e (\ln x)^n \cdot \frac{d(x)}{dx}\,dx$ (½)

$= [x\ln x]_1^e - \int_1^e x \cdot n(\ln x)^{n-1} \cdot \frac{1}{x}\,dx$

$= e - n\int_1^e (\ln x)^{n-1}\,dx$

$\therefore I_n = e - nI_{n-1}$

(d) (i) $\tan\left(\frac{\pi}{4} - u\right) = \frac{\tan\frac{\pi}{4} - \tan u}{1 + \tan\frac{\pi}{4}\tan u}$

$= \frac{1-\tan u}{1+\tan u}$

(ii)

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Question 2

(a) The real number law

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

does not necessarily hold for complex numbers (other than real ones).

$\therefore$ Line four is incorrect. (2)

(b) $u = -3-4i$; $v = 1-i$

(i) $\bar{u} - v = -3+4i-(1-i)$

$= -4+5i$ (1)

(ii) $\frac{2u}{v} = 1-7i$ (calculator)

OR $\frac{2u}{v} = \frac{2(-3-4i)}{1-i} \times \frac{1+i}{1+i}$

$= \frac{2(-3-4i)(1+i)}{2}$

$= -3-3i-4i+4$

$= 1-7i$ (2)

(iii) $\sqrt{u} = 1-2i$ (calculator)

OR Let $\sqrt{u} = a+ib$

$\therefore (a+ib)^2 = -3-4i$

$a^2-b^2+2abi = -3-4i$

$\therefore a^2-b^2 = -3$; $2ab = -4$

$b = -\frac{2}{a}$

$\therefore a^2 - \left(\frac{-2}{a}\right)^2 = -3$

$a^4+3a^2-4=0$

$(a^2+4)(a^2-1)=0$

$\therefore a = \pm 2i$, or ± 1

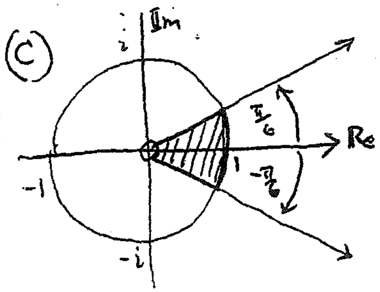
but a is real

$\therefore a = \pm 1$

$b = \mp 2$

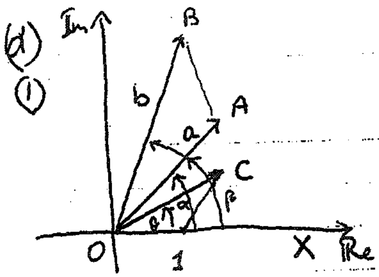
Hence $\sqrt{u} = 1-2i$ (Princi... Ro...) (2)

(c)



(2)

(d) (i)



Let $|a| = OA$, $|b| = OB$

Now $\left|\frac{b}{a}\right| = \frac{|b|}{|a|}$ (2)

and $\arg\left(\frac{b}{a}\right) = \arg b - \arg a$

i.e. $\angle COX = \angle BOX - \angle AOX$

(b)

Let $z = x\,cis\,\theta$, for real x

$$1 + z + z^2 + z^3 + \ldots + z^n = \frac{z^{n+1} - 1}{z - 1}$$

$$\text{Re}(1 + z + z^2 + z^3 + \ldots + z^n) = 1 + x\cos\theta + x^2\cos 2\theta + \ldots + x^n \cos n\theta$$

(De Moivre's Theorem)

$$\therefore 1 + x\cos\theta + x^2\cos 2\theta + \ldots + x^n\cos n\theta = \text{Re}\left(\frac{z^{n+1} - 1}{z - 1}\right) \checkmark$$

So

$$\frac{z^{n+1} - 1}{z - 1} = \frac{x^{n+1} cis(n+1)\theta - 1}{x\,cis\,\theta - 1} = \frac{x^{n+1}\cos(n+1)\theta - 1 + ix^{n+1}\sin(n+1)\theta}{x\cos\theta - 1 + ix\sin\theta} \checkmark$$

$$= \frac{x^{n+1}\cos(n+1)\theta - 1 + ix^{n+1}\sin(n+1)\theta}{x\cos\theta - 1 + ix\sin\theta} \times \frac{x\cos\theta - 1 - ix\sin\theta}{x\cos\theta - 1 - ix\sin\theta}$$

$$= \frac{a + ib}{(x\cos\theta - 1)^2 + x^2\sin^2\theta} = \frac{a + ib}{x^2\cos^2\theta - 2x\cos\theta + 1 + x^2\sin^2\theta}$$

$$= \frac{a + ib}{x^2 - 2x\cos\theta + 1}$$

$$\text{So Re}\left(\frac{z^{n+1} - 1}{z - 1}\right) = \text{Re}\left(\frac{a + ib}{x^2 - 2x\cos\theta + 1}\right) = \frac{a}{x^2 - 2x\cos\theta + 1}$$

From above

$$\frac{z^{n+1} - 1}{z - 1} = \frac{a + ib}{x^2 - 2x\cos\theta + 1}$$

$$\text{So } a = (x\cos\theta - 1)\left[x^{n+1}\cos(n+1)\theta - 1\right] + x^{n+1}\sin(n+1)\theta(x\sin\theta)$$

$$= x^{n+2}\cos\theta\cos(n+1)\theta - x^{n+1}\cos(n+1)\theta - (x\cos\theta - 1) + x^{n+2}\sin(n+1)\theta\sin\theta$$

$$= 1 - x\cos\theta + x^{n+2}\cos\theta\cos(n+1)\theta + x^{n+2}\sin(n+1)\theta\sin\theta - x^{n+1}\cos(n+1)\theta$$

$$= 1 - x\cos\theta + x^{n+2}\cos n\theta - x^{n+1}\cos(n+1)\theta \checkmark$$

$$\therefore 1 + x\cos\theta + x^2\cos 2\theta + \ldots + x^n\cos n\theta = \frac{1 - x\cos\theta + x^{n+2}\cos n\theta - x^{n+1}\cos(n+1)\theta}{1 - 2x\cos\theta + x^2} \checkmark$$

) $P(x) = x^3 - 4x^2 + 7x - 6$.

now $P(2) = 8 - 16 + 14 - 6$
$= 0$.

$\therefore$ $x - 2$ is a factor.

	1	−4	7	−6
2		2	−4	6
	1	−2	3	0

$\therefore P(x) = (x-2)(x^2 - 2x + 3)$
$= (x-2)((x-1)^2 - (\sqrt{2}i)^2)$
$= (x-2)(x-1-\sqrt{2}i)(x-1+\sqrt{2}i)$ (3)

b). Let $f(x) = ax^4 + bx^3 + cx^2 + dx + e$
now $f'(x) = 4ax^3 + 3bx^2 + 2cx + d$.

now $f'(0) = 0$
$\therefore$ $d = 0$

$f(0) = 1$
$\therefore$ $e = 1$

$f(1) = 1$
$\therefore$ $a + b + c = 0$ — A

$f(2) = 13$
$\therefore$ $16a + 8b + 4c = 12$ — B

$f(3) = 73$
$\therefore$ $81a + 27b + 9c = 72$ — C

Solving (A), (B) & (C)
$a = 1,\ b = 0,\ c = -1,\ d = 0,\ e = 1$

$\therefore$ $f(x) = x^4 - x^2 + 1$. (7)

(c) (i) $P(x) = (x - \alpha)^m \cdot Q(x)$.
$P'(x) = m(x-\alpha)^{m-1} Q(x) + (x-\alpha)^m \cdot Q'(x)$
$= (x-\alpha)^{m-1}[m\,Q(x) + (x-\alpha) \cdot Q'(x)]$
$= (x-\alpha)^{m-1} \cdot R(x)$. Q.E.D.

(ii) Let $P(x) = 5x^5 - 3x^3 + c = 0$
have pos. repeated root.
$P'(x) = 25x^4 - 9x^2 = 0$.
$\therefore x = 0$ or $x = \pm\frac{3}{5}$
Clearly $x = \frac{3}{5}$.

$\therefore 5\left(\frac{3}{5}\right)^5 - 3\left(\frac{3}{5}\right)^3 + c = 0$
$c = \frac{81}{125} - \frac{2?}{6?}$
$c = \frac{162}{625}$

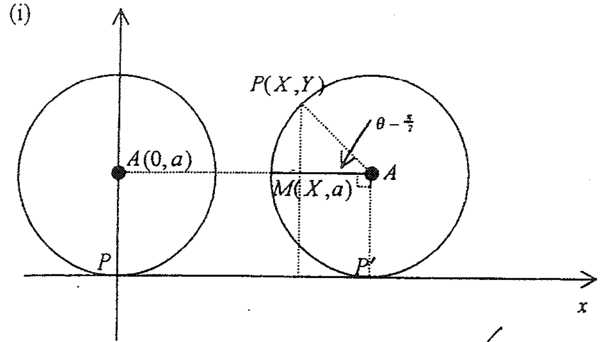
(d) Given $x^3 + px + q = 0$ — (1)
let $y = \frac{1}{x} \Rightarrow x = \frac{1}{y}$ in (1)

$\left(\frac{1}{y}\right)^3 + p \cdot \frac{1}{y} + q = 0$
$1 + py^2 + qy^3 = 0$

OR $qx^3 + px^2 + 1 = 0$

$\therefore$ co-efficients are $q, p, 0, 1$.

(i)



On the x-axis $PP' = a\theta$ since there is no slipping ✓

Let $\angle PAP' = \theta \Rightarrow \angle PAM = \theta - \frac{\pi}{2}$

$Y = a + a\sin(\theta - \frac{\pi}{2}) = a - a\sin(\frac{\pi}{2} - \theta) = a(1-\cos\theta)$ ✓

$X = a\theta - a\cos(\frac{\pi}{2} - \theta) = a\theta - a\cos(\theta - \frac{\pi}{2}) = a(\theta - \sin\theta)$ ✓

$\sin(90^\circ - A) = \cos A$
$\cos(90^\circ - A) = \sin A$
$\sin(-A) = -\sin A$
$\cos(-A) = \cos A$

$x = a(\theta - \sin\theta)$ $\quad$ $y = a(1-\cos\theta)$

$\frac{dx}{d\theta} = a(1-\cos\theta)$ $\quad$ $\frac{dy}{d\theta} = a\sin\theta$

$$\frac{dy}{dx} = \frac{dy}{d\theta}\Big/\frac{dx}{d\theta} = a\sin\theta / a(1-\cos\theta) = \frac{\sin\theta}{1-\cos\theta}$$

$$\sqrt{1+\left(\frac{dy}{dx}\right)^2} = \sqrt{1+\left(\frac{\sin\theta}{1-\cos\theta}\right)^2} = \sqrt{\frac{(1-\cos\theta)^2+\sin^2\theta}{(1-\cos\theta)^2}}$$

$$= \sqrt{\frac{1-2\cos\theta+\cos^2\theta+\sin^2\theta}{(1-\cos\theta)^2}}$$

$$= \sqrt{\frac{2-2\cos\theta}{(1-\cos\theta)^2}} = \sqrt{\frac{2(1-\cos\theta)}{(1-\cos\theta)^2}} = \frac{\sqrt{2}}{\sqrt{1-\cos\theta}}$$ ✓

We need to use the following substitution

$x = a(\theta - \sin\theta)$

$dx = a(1-\cos\theta)d\theta$

$0 \le \theta \le 2\pi$

Q5

i) $m\ddot{x} = mg - mkv$

$\ddot{x} = g - kv$

ii) $\frac{dv}{dt} = g - kv$

$\frac{dt}{dv} = \frac{1}{g-kv}$

$t = -\frac{1}{k}\ln(g-kv) + c$

$v=0, t=0$ $\quad$ $c = \frac{1}{k}\ln g$

$-kt = \ln\left[\frac{g-kv}{g}\right]$

$e^{-kt} = \frac{g-kv}{g}$

$g - kv = ge^{-kt}$

$v = \frac{g}{k}(1-e^{-kt})$

iii) terminal velocity - net accel^n zero. max velocity as $t \to \infty$

$V_t = \frac{g}{k}$

) $\quad$ $a = -mg - mkv$

$\frac{dv}{dt} = -g - kv$

$dt = \frac{-1}{g+kv}\,dv$

$t = -\frac{1}{k}\ln(g+kv) + c$

$v = u, t = 0$ $\quad$ $c = \frac{1}{k}\ln(g+ku)$

$t = \frac{1}{k}\ln\left[\frac{g+ku}{g+kv}\right]$

@ comes to rest $v = 0$

$t = -\frac{1}{k}\ln\left(\frac{g}{g+ku}\right)$

or $\frac{1}{k}\ln\left(\frac{g+ku}{g}\right)$

(c) $V = \frac{g}{k}(1-e^{-kt})$

$t = \frac{1}{k}\ln\left(\frac{g+ku}{g}\right)$

$= \frac{g}{k}\left(1-e^{\ln\frac{g}{g+ku}}\right)$

$= \frac{g}{k}\left(1-\frac{g}{g+ku}\right)$

$= \frac{g}{k}\left(\frac{ku}{g+ku}\right)$

$V = \frac{gu}{g+ku}$

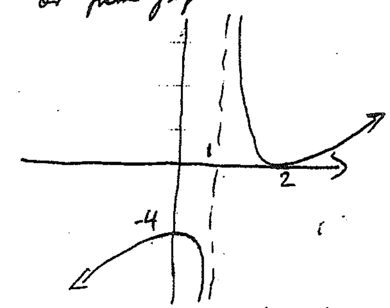
$V = \frac{\frac{g}{k}u}{\frac{g}{k}+u}$

$V = \frac{V_t u}{V_t + u}$

Q6 $\quad$ let $y = \frac{(x-2)^2}{x-1}$

Solve as a quadratic in x for Δ.

or from graph



above y does not exist between 0 & 4

(ii) $\frac{(x-2)^2}{(x-1)} = \frac{k}{x}$.

Draw $y = \frac{k}{x}$ and $y = -\frac{k}{x}$ on above graph $y = \pm\frac{k}{x}$ will

cut at the place ...

$y = -\frac{k}{x}$ will cut ...

b) $2z^3$

$= 2r^3(\cos 3\theta + ...$

$= 2r^3\cos 3\theta = ...$

$2r^3\sin 3\theta = ...$

$\tan 3\theta = ...$

$= \frac{\sqrt{3}}{3}$

$3\theta = \frac{\pi}{6}$

$\theta = \frac{\pi}{18}$

$2r^3\sin\frac{\pi}{6} = ...$

$r^3 = ...$

$r = \sqrt[3]{3...}$

$r = (3...$

$r = 3^{\frac{...}{2}}$

c) $V = \pi\int_0^2 x(...$

$= 12\pi$

d) $\sqrt{n^2+n} - n \times ...$

$= \frac{n^2+n-...}{\sqrt{n^2+n}...}$

$= \frac{n}{\sqrt{n^2+n}...}$

$= \frac{1}{\frac{1}{n}\sqrt{n^2+...}}$

$= \frac{1}{\frac{n}{n}\sqrt{1+...}}$

$= \frac{1}{\sqrt{1+\frac{1}{n}}}$

as $n \to \infty$

$\lim = \frac{1}{1+...}$