Sydney Girls High School

4 unit mathematics

Trial HSC Examination 1986

1. **(a)** Find the following integrals:
(i) $\int \frac{3+x}{3-x}\ dx$ **(ii)** $\int \sin^{-1} x\ dx$
(b) Evaluate the following:
(i) $\int_0^{\frac{\pi}{8}} \sin 5\theta \cos 3\theta\ d\theta$ **(ii)** $\int_0^2 \frac{dx}{(4+x^2)^{3/2}}$ **(iii)** $\int_0^1 \frac{x\ dx}{(x+1)(x+3)^2}$

2. **(a)** Prove De Moivres Theorem, $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$ for positive and negative integral powers on n. Hence or otherwise
(i) Express $(\frac{1}{2} + \frac{i}{2})^{-6}$ in the form $a + ib$
(ii) Solve $z^5 + 32 = 0$ over the complex field and sketch your answers on an Argand diagram.
(b) **(i)** If $|z| = 1$, what is the maximum value of $\arg(z + 2)$
(ii) Express $\frac{(1+\sqrt{3}i)(2+2i)}{(3-3i)}$ in the form $a + ib$.
(iii) Shade the region defined by $\{|\arg z| \leq \frac{\pi}{3}\} \cap \{z + \overline{z} \leq 4\} \cap \{|z| \geq 2\}$ **and** find the area of this shaded region.

3. **(a)** A point P lies on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, that has the points S and S' as its foci.
(i) Show that SP and $S'P$ make equal angles with the tangent at P.
(ii) Show that the sum of the lengths of SP and $S'P$ is equal to the length of the major axis.
(b) Find the equation of the tangent to $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ in terms of the gradient of the tangent. Hence or otherwise find the equations of the tangents to $\frac{x^2}{9} - \frac{y^2}{4} = 1$ that have a gradient of 2.

4. **(a)** For what values of K will $xe^{-x} = K$ have two values of x.
(b) Find the relation between the height and radius of a cylinder, if the volume of the cylinder is fixed but the surface area is to be a minimum.

5. **(a)** Two cuts are made on a circular log, one perpendicular to its axis and the other inclined at $60°$ to the axis. If the cuts meet at the edge of the log, and the diameter of the log cuts meet at the edge of the log, and the diameter of the log is 30 centimetres, find the volume of the resulting solid.
(b) The area enclosed by $y = \sin x$, $0 \leq x \leq \pi$ and the x axis is rotated about the line $x = -\frac{\pi}{2}$. Find the volume of the solid.

(c) Find a reduction formula for $\int \sin^n x\ dx$ in terms of $\int \sin^{n-2} x\ dx$ and hence evaluate $\int_0^{\frac{\pi}{2}} \sin^9 x\ dx$

6. **(a)** Solve for x if $x^2 - 2bx + c = 0$ and hence or otherwise factorise $x^4 - 2bx^3 + (c-1)x^2 + 2bx - c$ over the field of real numbers.
(b) Find a and b if $x = 2$ is a double root of $x^4 + ax^3 + 9x^2 + bx + 4 = 0$. Hence find all the roots of the equation over the complex field.
(c) If $x^3 - 4x + 1 = 0$ has roots α, β, γ find **(i)** $\sum \alpha^2$ **(ii)** $\sum \alpha^3$ **(iii)** $\sum \alpha^2\beta^2$
(d) A polynomial is known to be odd, monic and of degree nine. If it is further known that this polynomial has a triple root at 2 and single root at 4, sketch the polynomial and write down its equation.

7. **(a)** Solve $2\sqrt{x+1} - \sqrt{4x-1} = 1$ for x.
(b) Solve $\sin 5x = \cos 5x$ for x and for $0 \leq x \leq \pi$.
(c) Find the exact value of $\sin\{\cos^{-1}(-\frac{4}{5}) + \tan^{-1}(\frac{4}{3})\}$.
(d) Twenty years before retirement an employee places $40,000 in a Water Board loan that pays 13% interest per annum over the twenty years. The interest is paid six monthly and transferred directly into a bank account that pays 10% per annum interest, the interest also being paid six monthly but allowed to accumulate in that account. At the end of the twenty years what is the maximum amount of interest that the employee could have earnt?

8. **(a)** Research has shown that 90% of 4 Unit students, 70% of 3 Unit students and 40% of 2 Unit students pass a particluar first year course at University. If the students studying this course are evenly distributed from each of the levels of Mathematics, find the probability that two students chosen at random will pass this course.
(b) The motion of a pendulum moved according to: $\ddot{x} = -\frac{g}{L}x$, where
$\ddot{x}$ is the accelereation
x is the horizontal distance from the equilibrium position
g is the force due to gravity
L is the length of the pendulum
On a good day, the old clock in the school is run by the motion of a pendulum whereby each time the pendulum stops, the time is advanced by one second. If the acceleration due to gravity can be approximated to 9.8 ms^{-2}, find -
(i) how long the length of the pendulum should be (to the nearest centimetre)
(ii) the speed at which the pendulum passes through the equilibrium position if the maximum horizontal distance from that position is 12 centimetres.