SYDNEY GIRLS HIGH SCHOOL

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION 1997

Mathematics 4 Unit

Time Allowed: 3 hours

INSTRUCTIONS:

Answer all questions

All questions are of equal value

Start each question on a new page

Marks may be deducted if all necessary working is not shown

In questions involving gravity, you may use $g = 10ms^{-2}$

Note: This is a Trial Paper and does not necessarily reflect the content or format of the final HSC Examination Paper for this subject.

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Question 1.

a) Given that $z = 3 - 4i$

i) plot on an Argand Diagram A, B, C, D where:

$$A = z \qquad B = \bar{z} \qquad C = \frac{1}{z} \qquad D = z\bar{z}$$

ii) State which of the above complex numbers have

α) Equal arguments $\qquad$ β) Equal moduli

b) Assuming the result to de Moivre's theorem, express

$\left(\frac{i\sqrt{3}}{2} - \frac{1}{2}\right)^8$ in its simplest form

c) Given $-1+i$ is a zero of $x^4 + 2x^3 - x^2 + 2x + 10$, fully factorise the polynomial over the complex field

d) If OABC is a rhombus where A is $2+2i$, C is in the second quadrant and $\angle AOC = 60°$, find the co-ordinates of point B

Question 2: a) Evaluate the following integrals:

i) $\int_0^1 \cos^{-1} x . dx$

ii) $\int_1^2 \frac{dx}{x(1+x)^2}$

iii) $\int_0^4 \frac{x}{\sqrt{x+4}} . dx$

iv) $\int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x + \cos x} . dx$

b) Show that $\int \sec^n x . dx = \frac{1}{n-1}\left(\tan x . \sec^{n-2} x + (n-2)\int \sec^{n-2} x . dx\right)$,

and hence evaluate $\int_0^{\frac{\pi}{4}} \sec^6 x . dx$

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Question 3:

a) If w, w^2 are two of the roots of the equation
$x^3 + px^2 + qx + r = 0$, show that $p = q = r + 1$
given that w is a cube root of unity.

b) i) Find the five fifth roots of unity and plot them on an Argand Diagram .

ii) Factorise $z^5 - 1$ over: α) the complex field
β) the real field

iii) Use the above information to show $\cos\frac{4\pi}{5} + \cos\frac{2\pi}{5} = -\frac{1}{2}$

iv) Hence show that $\cos\frac{2\pi}{5} = \frac{\sqrt{5}-1}{4}$.

Question 4:

a) P($4\sec\theta$, $2\tan\theta$) is a variable point on a hyperbola

i) write down the equation of the hyperbola

ii) write down the co-ordinates of the foci (S & S')

iii) write down the equation of the directrices

iv) write down the equation of the asymptotes

v) sketch the curve

vi) show the equation of the tangent at P is $x\sec\theta - 2y\tan\theta = 4$

vii) express the tangent in terms of its gradient m

viii) write down the two tangents to the above hyperbola that hav gradient equal to 2

b)i) Find the equation of the tangent to the ellipse $\frac{x^2}{27} + \frac{y^2}{9} = 1$

ii) Show that the product of the perpendicular distances from the focii of the ellipse to the tangent is equal to 9.

c) Find the midpoint of the chord $2x + 3y = 12$ to the hyperbola $xy = 2$.

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Question 5:

a) State any asymptotes, show there are no stationary or inflexion points and sketch the curve $y = \dfrac{x(x-4)}{(x-2)^2}$

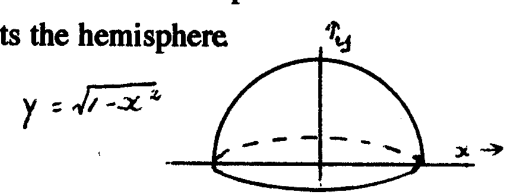
b) Express $\cos^7 x$ in the form $A\cos 7x + B\cos 5x + C\cos 3x + D\cos x$ and hence evaluate $\displaystyle\int_{\frac{\pi}{2}}^{\pi} \cos^7 x \, dx$ (to 2 dec pl)

c) Find the range of values of k such that $x^3 - x^2 - x + k = 0$ has
  i) one real solution
  ii) two real solutions
  iii) three real solutions

Question 6:

a) Prove that $3^{2n+2} - 8n - 9$ is divisible by 64 for all positive integral values of n

b) The semi - circle $y = \sqrt{1-x^2}$ is rotated about the y axis
i) Find the exact value of the volume of the hemisphere so formed.
ii) Show that $y = 2\sin 10°$ bisects the hemisphere



c) The Sydney Swans played a very close match to reach the 1996 Australian Rules Grand Final. When the full time whistle blew at the Preliminary final, Sydney fullback Tony Lockett needed to kick a goal from 60 metres away. He would normally kick a ball at $25ms^{-1}$ about 60cms off the ground. To avoid the defenders, the ball would have to reach the goalposts at a height of at least 3 metres. Use $g = 10ms^{-2}$ and assume projectile motion to determine the angles between which the ball must leave Lockett 's foot to score the goal

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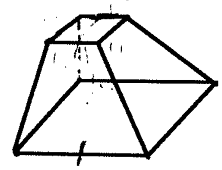
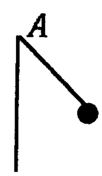
Question 7:

a) An Inca monument is in the shape of the bottom part of a pyramid, the base being a rectangle with sides 20 metres by 16 metres and the top a square with sides 4 metres. If the height is 8 metres, find the volume of the monument.



b) The area enclosed by $y = x^2$, $y = 2x - 1$ and $y = 6x - 9$ is rotated about the Y axis. Find the volume so formed.

c) A 2kg mass revolves in a circular pendulum about a point A at the end of a 2 metre string and angle of $60°$ to the vertical.



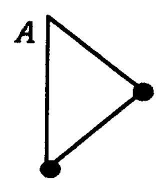
Find i) the tension in the string

ii) the angular speed of the mass.

{Use $g = 10ms^{-2}$ throughout this question}

A second 2 metre string containing a second 2 Kg mass is attached to the first mass so that the second mass is directly under point A.

iii) How fast (in radians per second) must the first mass now be rotating to maintain the system rotating with the same radius.



iv) If when rotating at this new speed, the bottom string is cut; how many centimetres does the first mass rise?

Question 8:

a) Prove that if $\frac{z_2 - z_3}{z_3 - z_1} = \frac{z_3 - z_1}{z_1 - z_2}$, then the points that represent the complex numbers z_1, z_2, z_3 form an equilateral triangle.

b) i) Show that, if $x > 0$

$$\frac{x}{1+x} < \log_e(1+x) < x$$

ii) Hence show that

$$\frac{\pi}{8} - \frac{1}{4}\log_e 2 < \int_0^1 \frac{\log_e(1+x)}{1+x^2}.dx < \frac{1}{2}\log_e 2$$