2008 - SGHS Trial HSC: 4U

Marks

Question 1 (15 marks)

a) Find $\int \cos^2 x \sin x dx$ — 2

b) Find $\int \frac{dx}{\sqrt{4x^2 - 36}}$ — 2

c) Evaluate $\int_0^1 xe^x dx$ — 3

d) Evaluate $\int_0^3 x^2\sqrt{x+1}$ — 4

e) Find real numbers a and b such that

(i) $\frac{4x^2+4x-4}{(x-1)(x+1)^2} = \frac{a}{x-1} + \frac{b}{x+1} + \frac{2}{(x+1)^2}$ — 2

(ii) Hence find $\int \frac{4x^2+4x-4}{(x-1)(x+1)^2}dx$ — 2

Marks

Question 2 (15 marks)

a) Let $z = 2 + 3i$ and $w = 3 - 4i$ Find, in the form $x + iy$,

(i) $\overline{w}$ — 1

(ii) z^2 — 1

(iii) $\frac{z}{w}$ — 1

b) (i) Express $1+\sqrt{3}i$ in modulus-argument form — 2

(ii) Express $\left(1+\sqrt{3}i\right)^8$ in modulus-argument form — 2

(iii) Hence express $\left(1+\sqrt{3}i\right)^8$ in the form $x+iy$ — 1

c) Find, in modulus-argument form, all solutions of $z^3 = 1$ — 2

d) Sketch the region on the Argand Diagram where the inequalities — 3

$|z+\overline{z}| \geq 2$ and $|z-1-i| < 1$ hold simultaneously

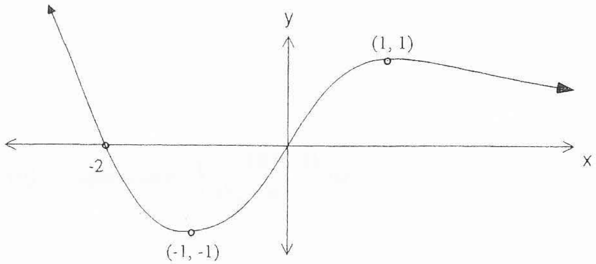
e) Suppose that the complex number z lies on the unit circle, and — 2

$0 \leq \arg(z) \leq \frac{\pi}{2}$. Prove that $2\arg(z-1) = \arg(z) + \pi$

Marks

Question 3 (15 marks)

a)



The diagram shows the graph of $y = f(x)$ The x axis is an asymptote
Draw separate one-third page sketches of the following:

(i) $f(-x)$ 2

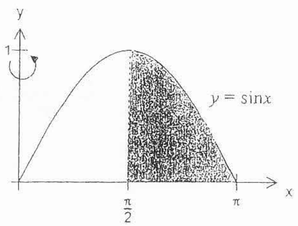
(ii) $f(|x|)$ 2

(iii) $y = \dfrac{1}{f(x)}$ 2

(iv) $y^2 = f(x)$ 2

b) The zeros of $x^3 - 4x^2 + 2x - 1$ are α, β and γ
Find a cubic polynomial with integer coefficients whose zeros are α^2, β^2 and γ^2 3

c) 4



Use the method of cylindrical shells to find the volume of the solid formed when the shaded region bounded by

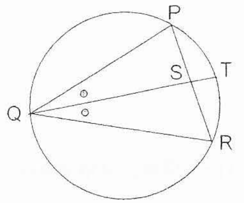
$$y = 0, y = \sin x, x = \frac{\pi}{2}, x = \pi$$

is rotated about the y-axis

Marks

Question 4 (15 marks)

a)



In the diagram, the bisector QT of angle PQR has been extended to intersect the circle PQR at T

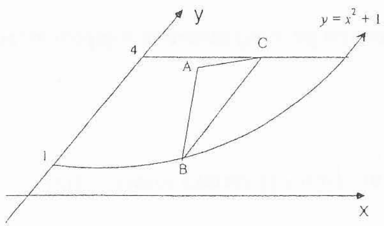
Copy the diagram:

(i) Prove that the triangles QPS and QTR are similar 2

(ii) Show that $QS.QT = QP.QR$ 1

(iii) Prove that $QS^2 = QP.QR - PS.SR$ 3

b) 3



The base of a solid is the region bounded by the curve $y = x^2 + 1$, the y-axis and the lines $y = 1$ and $y = 4$, as shown in the diagram.

Vertical cross-sections taken through this solid in a direction parallel to the y-axis are equilateral triangles. A typical cross-section, ABC is shown.

Find the volume of the solid

c) (i) Suppose that a is a double root of the polynomial equation $P(x) = 0$ 2
Show that $P'(a) = 0$

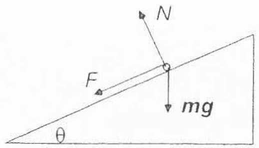
(ii) What feature does the graph of a polynomial have at a root of multiplicity 2? 1

(iii) The polynomial $P(x) = mx^4 - nx^2 + 2$ 3
is divisible by $(x+1)^2$ Find the coefficients m and n

Marks

Question 5 (15 marks)

a)

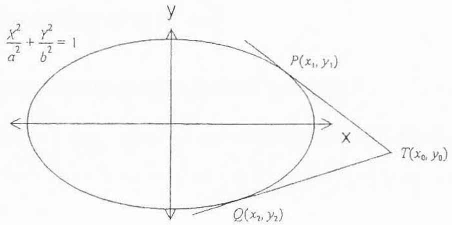


A road contains a bend that is part of a circle of radius r. At the bend, the road is banked at an angle θ to the horizontal. A car travels around the bend at constant speed v. Assume that the car is represented by a point of mass m, and that the forces acting on the car are the gravitational force mg, a sideways friction force F (acting down the road as drawn) and a normal reaction N to the road.

(i) By resolving the horizontal and vertical components of force, find an expression for F 3

(ii) Show that if there is no sideways force $v = \sqrt{gr \tan\theta}$ 2

b)



The points $P(x_1, y_1)$ and $Q(x_2, y_2)$ lie on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

The tangents at P and Q meet at $T(x_0, y_0)$

(i) Show that the equation at P is $\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1$ 2

(ii) Hence show that the chord of contact PQ, has equation $\frac{xx_0}{a^2}+\frac{yy_0}{b^2}=1$ 2

(iii) T lies on the directrix of the ellipse. Prove that the chord PQ passes through the focus $S(ae, o)$ 1

c) (i) Find the equation of the tangent to the curve defined by $x^3+xy-y^2=1$ at the point $(1,1)$ 3

(ii) Show that the curve in (i) has a stationary point if $9x^4+2x^3+1=0$ 2

Marks

Question 6 (15 marks)

a) (i) Prove the identity $\sin(a+b)x+\sin(a-b)x = 2\sin ax\cos bx$ 1

(ii) Hence find $\int \sin 5x \cos 3x\,dx$ 2

b) Consider the following statements about a polynomial $P(x)$

(i) If $P(x)$ is odd, then $P'(x)$ is even 1

(ii) If $P'(x)$ is even, then $P(x)$ is odd 1

Indicate whether each of these statements is true or false. Give reasons for your answers.

c) If $z^6-1=0$

(i) Express all the values of z in modulus argument form 2

(ii) Show that $z^6-1=(z^2-1)(z^2+z+1)(z^2-z+1)$ 1

(iii) Express the roots of $z^4+z^2+1=0$ in the form $x+iy$ 3

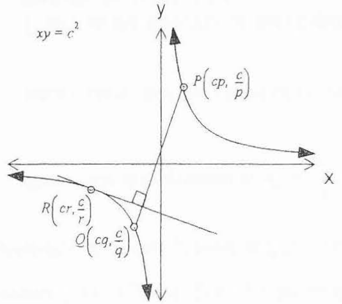
d) (i) Sketch the graph of the function $y=\cos^{-1}\left(\frac{x-1}{2}\right)$ 2

(ii) By adding $y=\sin^{-1}x$ to the graph in (i), solve $\cos^{-1}\left(\frac{x-1}{2}\right)=\sin^{-1}x$ 2

Question 7 (15 marks)

Marks

a)



The points $P(cp, \frac{c}{p}), Q(cq, \frac{c}{q})$ and $R(cr, \frac{c}{r})$ lie on the hyperbola $xy = c^2$

The tangent at R is perpendicular to the line joining P and Q.

Show that

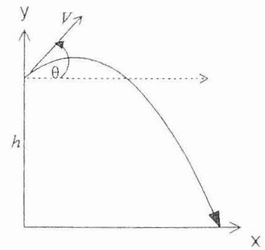
(i) The gradient of the tangent at R is $-\frac{1}{r^2}$ **2**

(ii) $\angle QRP$ is a right angle **3**

(Question 7 continued on next page)

Question 7 Continued

b)



A projectile is launched from the top of a cliff h metres high with an initial velocity of Vms^{-1} at an angle of θ to the horizontal. Given that the horizontal and vertical components of the motion are $\ddot{x} = 0$ and $\ddot{y} = -g$

Show that

(i) $x = Vt\cos\theta$ and $y = Vt\sin\theta - \frac{1}{2}gt^2 + h$ **2**

(ii) The time of flight, T, is given by **2**

$$T = \frac{V\sin\theta + \sqrt{V^2\sin^2\theta + 2hg}}{g}$$

(iii) If $h = \frac{V^2\cos^2\theta}{2g}$ then the range R of the particle is **3**

$$R = \frac{V^2(\sin 2\theta + 2\cos\theta)}{2g}$$

c) $S(n) = log_a x + log_a x^2 + log_a x^3 + \ldots + log_a x^n$

(i) Show that $S(n) = \frac{n(n+1)\log_a x}{2}$ **2**

(ii) Find the value of x if $a = 16$ and $S(100) = 5050$ **1**

Marks

Question 8 (15 marks)

a) Given that $f(x) = ax^3 + bx^2 + cx + d$

Show that if

(i) $f(x)$ has one stationary point then $b^2 = 3ac$ 3

(ii) $f(x)$ has a horizontal point of inflexion then $x = -\frac{c}{b}$ 2

b) Given that $I_n = \int_0^{\pi} x^n \sin x dx$

(i) Show that $I_n = \pi^n - n(n-1)I_{n-2}, n \geq 2$ 3

(ii) Evaluate $\int_0^{\pi} \theta^4 \sin\theta d\theta$ 3

c) (i) Using the fact that $A = \frac{1}{2}ab\sin C$ and $\cos C = \frac{a^2+b^2-c^2}{2ab}$ 2

Show that $A = \frac{1}{4}\sqrt{2a^2b^2 + 2a^2c^2 + 2b^2c^2 - a^4 - b^4 - c^4}$

(ii) Hence or otherwise show that the area A of a triangle with sides a, b and c can be found by using the formula. 2

$A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{1}{2}(a+b+c)$

--- End of Exam ---

Ext 2 Trial 2008

1 a) $\int \cos^2 x \sin x \, dx = -\frac{1}{3}\cos^3 x + C$

b) $\int \frac{dx}{\sqrt{4x^2-36}} = \frac{1}{2}\int \frac{dx}{\sqrt{x^2-9}} = \frac{1}{2}\ln(x+\sqrt{x^2-9}) + C$

c) $\int_0^1 xe^x \, dx$. let $u = x$, $du = dx$; $dv = e^x dx$, $v = e^x$

$\therefore I = xe^x - \int e^x dx$

$= [xe^x - e^x]_0^1$

$= e - e - (0-1)$

$= 1$

d) $\int_0^3 x^2\sqrt{x+1}\, dx$ $\quad u = x+1$, $du = dx$, $(u-1)^2 = x^2$ $\quad x=0, u=1$; $x=3, u=4$

$= \int_1^4 (u-1)^2 \sqrt{u}\, du$

$= \int_1^4 (u^2-2u+1)\sqrt{u}\, du$

$= \int u^{5/2} - 2u^{3/2} + u^{1/2}\, du$

$= \left[\frac{2}{7}u^{7/2} - \frac{4}{5}u^{5/2} + \frac{2}{3}u^{3/2}\right]_1^4$

$= \left(\frac{2}{7}128 - \frac{4}{5}32 + \frac{2}{3}8\right) - \left(\frac{2}{7} - \frac{4}{5} + \frac{2}{3}\right)$

$= \left(\frac{256}{7} - \frac{128}{5} + \frac{16}{3}\right) - \left(\frac{2}{7} - \frac{4}{5} + \frac{2}{3}\right)$

$= 16\left(\frac{16}{7} - \frac{8}{5} + \frac{1}{3}\right) - \left(\frac{2}{7} - \frac{4}{5} + \frac{2}{3}\right)$

$= \frac{1696}{105}$

e) $\frac{4x^2+4x-4}{(x-1)(x+1)^2} = \frac{a}{x-1} + \frac{b}{x+1} + \frac{2}{(x+1)^2}$

$4x^2+4x-4 = a(x+1)^2 + b(x-1)(x+1) + 2(x-1)$

at $x = 1$, $4 = 4a$, $a = 1$

$x = -1$, $-4 = -4$ ✓

Equate x^2: $4 = 1 + b$, $\therefore b = 3$

$\therefore \int \frac{4x^2+4x-4}{(x-1)(x+1)^2} = \int \left(\frac{1}{x-1} + \frac{3}{x+1} + \frac{2}{(x+1)^2}\right) dx = \ln(x-1) + 3\ln(x+1) - \frac{2}{x+1} + C.$

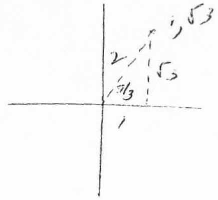
2a) $z = 2+3i$, $w = 3-4i$

i) $\bar{w} = 3+4i$

ii) $z^2 = 4 - 9 + 12i = -5 + 12i$

iii) $\frac{z}{w} = \frac{2+3i}{3-4i}\cdot\frac{3+4i}{3+4i} = \frac{6+8i+9i-12}{9+16} = \frac{-6}{25} + \frac{17i}{25}$

b) i) $1+\sqrt{3}i = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$



ii) $(1+\sqrt{3}i)^8 = 2^8\left(\cos\frac{8\pi}{3} + i\sin 8\pi/3\right)$

$= 256\left(\cos 2\pi/3 + i\sin 2\pi/3\right)$

iii) $= 256\left(-\frac{1}{2} + i\sqrt{3}/2\right)$

$= -128 + 128\sqrt{3}\, i$

c) $z^3 = 1$

let $z = \cos\theta + i\sin\theta$

$\therefore z^3 = \cos 3\theta + i\sin 3\theta = 1$

$\therefore 3\theta = 0, 2\pi, 4\pi$

$\therefore \theta = 0, 2\pi/3, 4\pi/3$

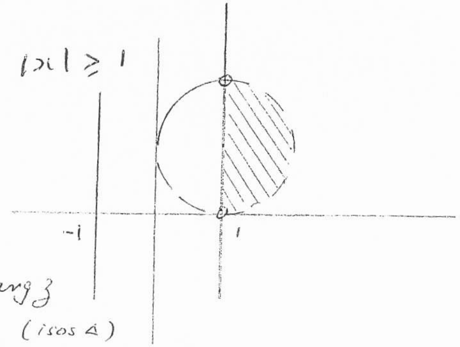
$\therefore z_1 = \cos 0 + i\sin 0 = 1$

$z_2 = \cos 2\pi/3 + i\sin 2\pi/3$

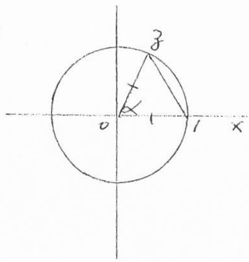
$z_3 = \cos 4\pi/3 + i\sin 4\pi/3$

d) $|z+\bar{z}| \geq 2$, $\therefore |2x| \geq 2$, $|x| \geq 1$

$|z-1-i| < 1$.



e)



let $\angle zOI = \alpha = \arg z$

$\therefore \angle zIO = \pi/2 - \alpha/2$ (isos $\triangle$)

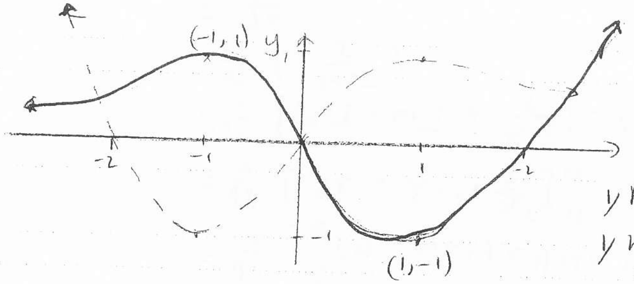
$\therefore \angle zIx = \pi/2 + \alpha/2$ (supp. $\angle$)

i.e. $\arg(z-1) = \pi/2 + \alpha/2$

$\therefore 2\arg(z-1) = \pi + \alpha$

$= \pi + \arg z$.

Question 3.

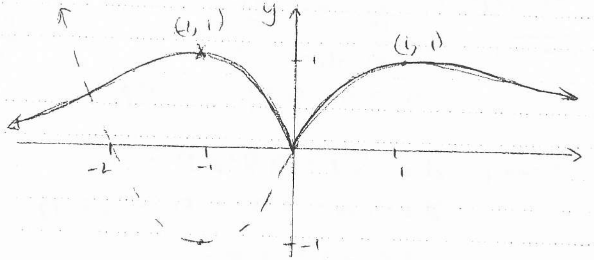


$f(-x)$

(2)

✓ Reflection in y axis

✓ must give co-ords of both T. Pts

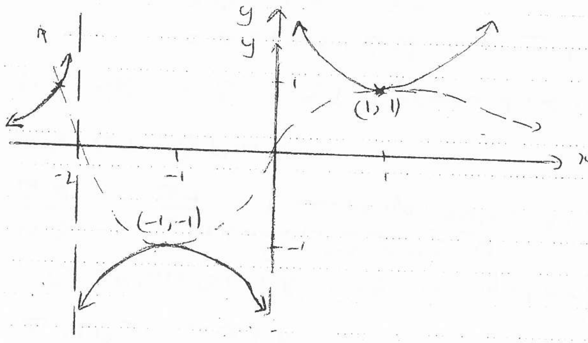


$f|x|$

(2)

✓ Reflection of $f(x) \geq 0$ in y axis

✓ Sharp corner at O

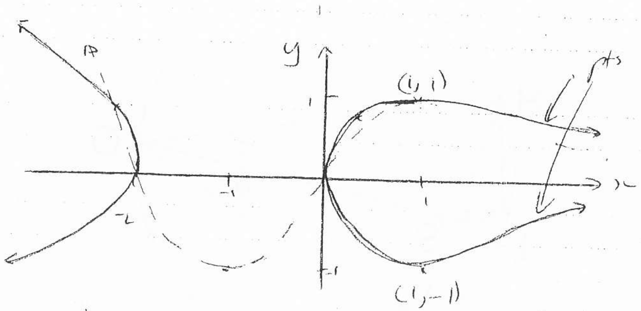


$y = \frac{1}{f(x)}$

✓ 3 Branches

✓ Locate asymptotes

(2)



Pts of inflexion needed for concavity

$y^2 = f(x)$

1 For RHS } Must have correct concavity

1 For LHS

(2)

Q3

b) $x^3 - 4x^2 + 2x - 1 = 0$

let $x = \alpha^2, \beta^2, \gamma^2$

ie $\alpha = \sqrt{x}$ sub in above

$x^{\frac{3}{2}} - 4x + 2x^{\frac{1}{2}} - 1 = 0$

$x^{\frac{1}{2}}(x + 2) = (4x + 1)$

square both sides

$x(x^2 + 4x + 4) = 16x^2 + 8x + 1$

$x^3 + 4x^2 + 4x = 16x^2 + 8x + 1$

$\underline{\underline{x^3 - 12x^2 - 4x - 1 = 0}}$

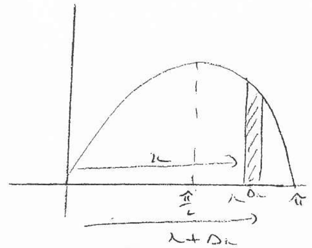
1

2 marks if coefficients are not integers or powers not integers

3

(3)

c)



$h = \sin x$

$V_{shell} = \pi\left[(x + \Delta x)^2 - (x)^2\right] y$ ✓

$= \pi\left[x^2 + 2x\Delta x + (\Delta x)^2 - x^2\right] y$

$= \pi\left[2x\Delta x\right] y \quad (\Delta x)^2$ v. small)

$V_{solid} = \lim_{\delta x \to 0} \sum_{\frac{\pi}{2}}^{\pi} 2\pi x y \delta x$

$= 2\pi \int_{\frac{\pi}{2}}^{\pi} x \sin x \, dx$ ✓

(4)

let $u = x$, $v' = \sin x$, $u' = 1$, $v = -\cos x$ ✓

$V = 2\pi\left\{\left[-x\cos x\right]_{\frac{\pi}{2}}^{\pi} + \int_{\frac{\pi}{2}}^{\pi} \cos x \, dx\right\}$ (if 2π dropped no penalty)

$= 2\pi\left\{(\pi - 0) + \left[\sin x\right]_{\frac{\pi}{2}}^{\pi}\right\}$

$= 2\pi\left\{\pi + (0 - 1)\right\}$

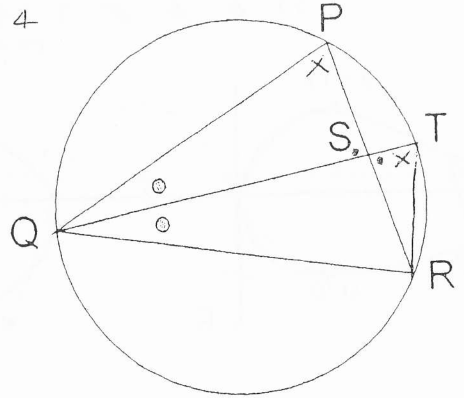
✓ correct evaluation of their $\int$ before simplifying (Must be equivalent value)

$= 2\pi\left[\pi - 1\right]$

$= \underline{2\pi^2 - 2\pi}$ units^3

Question 4

a)



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i) Construct TR

In Δ's QPS, QTR

$\angle PQS = \angle TQR$ (given)

$\angle QPS = \angle QTR$ ($\angle$s in same segment) 1 mark 1 mark (2)

$\therefore \triangle QPS \,|||\, \triangle QTR$ (equiangular)

ii) $\dfrac{QS}{QR} = \dfrac{QP}{QT}$ (or equivalent 1 mark) (corresp sides similar Δ's) (1)

$QS \cdot QT = QP \cdot QR$

iii) $PS \cdot SR = QS \cdot ST$ (product of intersecting chords) 1 mark

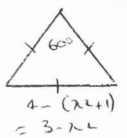
$= QS(QT - QS)$ 1 mark

$= QS \cdot QT - QS^2$ (3)

$QS^2 = QS \cdot QT - PS \cdot SR$

$= QP \cdot QR - PS \cdot SR$ 1 mark (from ii)

b)



$V_{slice} = \frac{1}{2} ab \sin C \, \delta x$

$= \frac{1}{2}(3-x^2)^2 \frac{\sqrt{3}}{2} \delta x$ 1 mark

POI

$4 = x^2 + 1$

$x^2 = 3$

$x = \pm\sqrt{3}$

$V_{solid} = \frac{\sqrt{3}}{4} \lim_{\delta x \to 0} \sum_{0}^{\sqrt{3}} (3-x^2)^2 \, dx$

$= \frac{\sqrt{3}}{4} \int_0^{\sqrt{3}} (9 - 6x^2 + x^4) \, dx$ 1 mark

$= \frac{\sqrt{3}}{4} \left[9x - 2x^3 + \frac{x^5}{5} \right]_0^{\sqrt{3}}$ (3)

$= \frac{\sqrt{3}}{4} \left[(9\sqrt{3} - 6\sqrt{3} + \frac{9\sqrt{3}}{5}) - (0) \right]$ 1 mark

$= \frac{\sqrt{3}}{4} \left(\frac{24\sqrt{3}}{5} \right)$

$= \frac{72}{20}$

$= 3\frac{3}{5}$ units^3

[illegible]

Q4

c) i) let $P(x) = (x-a)^2 Q(x) = 0$

$P'(x) = Q'(x)(x-a)^2 + 2(x-a)Q(x)$ 1 mark

$= (x-a)[Q'(x) + 2(x-a)$ 1 mark (2)

$\therefore P'(a) = 0$

ii) A turning pt. 1 mark (1)

iii) $P(x) = mx^4 - nx^2 + 2$

$P(-1)$ is a root of $P(x)$

$\therefore m - n + 2 = 0$ ① ← 1 mark

$P'(x) = 4mx^3 - 2nx$

Now -1 is a root of above

$\therefore -4m + 2n = 0$ ② ← 1 mark

①×2 $\quad 2m - 2m = -4$ ③ (3)

② + ③ $\quad -2m = -4$

$m = 2$

$n = 4$ } ← 1 mark both correct.

Question Five

a) i) Vertically

$N\cos\theta = F\sin\theta + mg$ ①   1 mark

Horizontally

$N\sin\theta + F\cos\theta = \frac{mv^2}{r}$ 1 mark

From ① $F\sin\theta = N\cos\theta - mg$ $\times\sin\theta$

" ② $F\cos\theta = \frac{mv^2}{r} - N\sin\theta$ $\times\cos\theta$

$F\sin^2\theta = N\cos\theta\sin\theta - mg\sin\theta$ ③   3

$F\cos^2\theta = \frac{mv^2}{r}\cos\theta - N\sin\theta\cos\theta$ ④

③ + ④ $F = \frac{mv^2}{r}\cos\theta - mg\sin\theta$ 1 mark only with working

ii) Put $F = 0$ (1 mark)

$0 = \frac{mv^2}{r}\cos\theta - \cancel{m}g\sin\theta$

$\frac{v^2}{r}\cos\theta = g\sin\theta$ 2

$v^2 = rg\tan\theta$ 1 mark

$v = \sqrt{rg\tan\theta}$

b) i) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\frac{2x}{2a^2} + \frac{2y}{b^2}\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-b^2x}{a^2y}$

at P $m = \frac{-b^2x_1}{a^2y_1}$ 1 mark   2

Eqn of tangent

$y - y_1 = \frac{-b^2x_1}{a^2y_1}(x - x_1)$ 1 mark.

$\frac{yy_1}{b^2} - \frac{y_1^2}{b^2} = -\frac{xx_1}{a^2} + \frac{x_1^2}{a^2}$

$\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = \frac{x_1^2}{a^2} + \frac{y_1^2}{b^2}$

$\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$ (since (x_1,y_1) lies on $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$)

Q5b)ii) The tangent at P in $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ 2

and T lies on it : $\frac{x_1x_0}{a^2} + \frac{y_1y_0}{b^2} = 1$ ①   1 mark

In same manner for QT $\frac{x_2x_0}{a^2} + \frac{y_2y_0}{b^2} = 1$ ②   1 mark

from ① & ② eqn PQ $\frac{xx_0}{a^2} + \frac{yy_0}{b^2} = 1$ ③

iii) $T(\frac{a}{e}, y_0)$ sub in ③ $\frac{\frac{a}{e}x}{a^2} + \frac{yy_0}{b^2} = 1$

1 mark only

$\frac{x}{ae} + \frac{yy_0}{b^2} = 1$ 1 mark   1

at S $y = 0$ i.e. $\frac{x}{ae} = 1 \rightarrow x = ae$

c) i) $3x^2 + x\frac{dy}{dx} + y - 2y\frac{dy}{dx} = 0$ 1 mark

$(x - 2y)\frac{dy}{dx} = -3x^2 - y$

$\frac{dy}{dx} = \frac{-3x^2 - y}{x - 2y}$

at $(1,1)$ $m = \frac{-3-1}{1-2}$ 3

$= 4$ 1 mark

eqn of tangent

$y - 1 = 4(x - 1)$ 1 mark

$y = 4x - 3$

ii) for stny pt $\frac{dy}{dx} = 0$

$\frac{-3x^2 - y}{x - 2y} = 0$ 1 mark

$y = -3x^2$ subst $x^3 + xy - y^2 = 1$ 2

$x^3 + x(-3x^2) - (3x^2)^2 = 1$ 1 mark

$x^3 - 3x^3 - 9x^4 = 1$

$9x^4 + 2x^3 + 1 = 0$

~~1~~ ~~2~~ 3 ~~4~~ 5 ~~6~~

6 a) i) $\sin ax \cos bx + \sin bx \cos ax$

$+ \sin ax \cos bx - \sin bx \cos ax$

$= 2 \sin ax \cos bx$

ii) $\frac{1}{2} \int (\sin 8x + \sin 2x)\, dx$

$= -\frac{1}{16} \cos 8x - \frac{1}{4} \cos 2x + c$

b) i) $P(x) = a_1 x + a_3 x^3 + a_5 x^5 + \ldots$

$P'(x) = a_1 + 3a_3 x^2 + 5a_5 x^4 + \ldots$

$\therefore$ true

ii) $P'(x) = a_0 + a_2 x^2 + a_4 x^4 + \ldots$

$P(x) = a_0 x + \frac{a_2 x^3}{3} + a_4 \frac{x^5}{5} + \ldots + c$

$\therefore$ odd or neither depending on c

$\therefore$ false

c)(i) $\operatorname{cis} 6\theta = 1$

$6\theta = 0°, 360°, 720°, 1080°, 1440°, 1800°$

$\theta = 0°, 60°, 120°, 180°, 240°, 300°$

$z = \operatorname{cis} 0°, \operatorname{cis} 60°, \operatorname{cis} 120°,$

$\operatorname{cis} 180°, \operatorname{cis}(-120°), \operatorname{cis}(-60°)$

ii) $(z^2 - 1)((z^2+1)^2 - z^2)$

$= (z^2-1)(z^4 + 2z^2 + 1 - z^2)$

$= (z^2-1)(z^4 + z^2 + 1)$

$= z^6 + z^4 + z^2 - z^4 - z^2 - 1$

$= z^6 - 1$

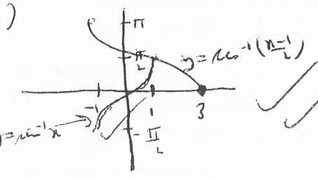
iii) $\operatorname{cis} 60° = \frac{1}{2} + \frac{\sqrt{3}}{2} i$

$\operatorname{cis} 120° = -\frac{1}{2} + \frac{\sqrt{3}}{2} i$

$\operatorname{cis}(-120°) = -\frac{1}{2} - \frac{\sqrt{3}}{2} i$

$\operatorname{cis}(-60°) = \frac{1}{2} - \frac{\sqrt{3}}{2} i$

d)(i)



(ii) $x = 1$

7 a) i) $x \frac{dy}{dx} + y = 0$

$\frac{dy}{dx} = -\frac{y}{x}$

$m_A = -\frac{\frac{c}{r}}{cr}$

$= -\frac{1}{r^2}$

(ii) $m_{PQ} = \frac{\frac{c}{p} - \frac{c}{q}}{cp - cq}$

$= \frac{\frac{q-p}{pq}}{p-q}$

$= -\frac{1}{pq}$

$m_{RQ} = -\frac{1}{rq}$

$m_{PR} = -\frac{1}{rp}$

$-\frac{1}{r^2} \times -\frac{1}{pq} = -1$

$r^2 pq = -1$

$m_{RQ} \times m_{PR}$

$= -\frac{1}{rq} \times -\frac{1}{rp}$

$= \frac{1}{r^2 pq}$

$= \frac{1}{-1}$

$= -1$

$\therefore$ QRP is a right angle

b) i)

$\dot{x} = V \cos\theta$

$x = Vt \cos\theta$

$\dot{y} = -gt + V \sin\theta$

$y = -\frac{gt^2}{2} + Vt \sin\theta + h$

ii) $\frac{gt^2}{2} - Vt \sin\theta - h = 0$

$t = \frac{V \sin\theta + \sqrt{V^2 \sin^2\theta - 4 \times \frac{g}{2} \times -h}}{2 \times \frac{g}{2}}$

$= \frac{V \sin\theta + \sqrt{V^2 \sin^2\theta + 2gh}}{g}$

iii) $R = Vt \cos\theta$

$= V \cos\theta \times$

$\frac{V \sin\theta + \sqrt{V^2 \sin^2\theta + \frac{2gV^2 \cos^2\theta}{2g}}}{g}$

$= \frac{V \sin\theta + \sqrt{V^2 \sin^2\theta + V^2 \cos^2\theta}}{g}$

$\times V \cos\theta$

$= \frac{V \sin\theta + V}{g} \times V \cos\theta$

$= \frac{V^2 \sin\theta \cos\theta + V^2 \cos\theta}{g}$

$= \frac{2V^2 \sin\theta \cos\theta + 2V^2 \cos\theta}{2g}$

$= \frac{V^2 (\sin 2\theta + 2\cos\theta)}{2g}$

c) i) $S(n) = \frac{n}{2}(\log_a x + \log_a x^n)$

$= \frac{n(\log_a x + n \log_a x)}{2}$

$= \frac{n \log_a x \,(1+n)}{2}$

ii) $5050 = \frac{100(101)\log_{16} x}{2}$

$10100 = 10100 \log_{16} x$

$\therefore \log_{16} x = 1$

$x = 16$

8 a) (i) $f'(x) = 0$

$3ax^2 + 2bx + c = 0$

$x = \frac{-2b \pm \sqrt{(2b)^2 - 4 \times 3a \times c}}{2 \times 3a}$

$= \frac{-2b \pm \sqrt{4b^2 - 12ac}}{6a}$

$\therefore 4b^2 - 12ac = 0$

$4b^2 = 12ac$

$b^2 = 3ac$

(ii) $f''(x) = 0$

$6ax + 2b = 0$

$x = -\frac{2b}{6a}$

$= -\frac{b}{3a}$

$= -\frac{b}{\frac{b^2}{c}}$

$= -\frac{c}{b}$

b) (i) $u = x^n \quad v' = \sin x$

$u' = nx^{n-1} \quad v = -\cos x$

$I_n = \left[-x^n \cos x\right]_0^\pi + n\int_0^\pi x^{n-1} \cos x$

$= \pi^n + n \int_0^\pi x^{n-1} \cos x$

$u = x^{n-1} \quad v' = \cos x$

$u' = (n-1)x^{n-2} \quad v = \sin x$

$\int_0^\pi x^{n-1} \cos x = \left[x^{n-1} \sin x\right]_0^\pi - (n-1)\int_0^\pi x^{n-2} \sin x$

$= -(n-1)\int_0^\pi x^{n-2} \sin x$

$\therefore I_n = \pi^n - n(n-1) I_{n-2}$

(ii) $\int_0^\pi \theta^4 \sin\theta \, d\theta$

$= \pi^4 - 12 I_2$

$= \pi^4 - 12(\pi^2 - 2I_0)$

$= \pi^4 - 12\pi^2 + 48$

$I_0 = \int_0^\pi \sin x \, dx$

$= \left[-\cos x\right]_0^\pi$

$= 2$

c) i) $A = \frac{1}{2} ab \sqrt{1 - \left(\frac{a^2+b^2-c^2}{2ab}\right)^2}$

$= \frac{1}{2} ab \sqrt{\frac{4a^2b^2 - a^4 - 2a^2b^2 + b^4 + 2a^2c^2 + 2bc^2}{4a^2b^2}}$

$= \frac{1}{4} \sqrt{2a^2b^2 + 2a^2c^2 + 2b^2c^2 - a^4 - b^4 - c^4}$

(ii) $A = \sqrt{\frac{a+b+c}{2} \times \frac{b+c-a}{2} \times \frac{a+c-b}{2} \times \frac{a+b-c}{2}}$

$= \sqrt{\frac{a+b+c}{2} \times \frac{a+b-c}{2} \times \frac{c+b-a}{2} \times \frac{c-b+a}{2}}$

$= \frac{1}{4}\sqrt{(a^2 + 2ab + b^2 - c^2) \times (c^2 - b^2 + 2ab - a^2)}$

$= \frac{1}{4}\sqrt{-(a^2+b^2-c^2+2ab)(a^2+b^2-c^2-2ab)}$

$= \frac{1}{4}\sqrt{-(a^4 + 2a^2b^2 + b^4 - 2a^2c^2 - 2b^2c^2 + c^4 - 4a^2b^2}$

$= \frac{1}{4}\sqrt{2a^2b^2 + 2a^2c^2 + 2b^2c^2 - a^4 - b^4 - c^4}$