ABBOTSLEIGH

2009

YEAR 12
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

General Mathematics

General Instructions

- Reading time – 5 minutes.
- Working time – 2 ½ hours.
- Write using blue or black pen.
- Calculators may be used.
- A Formulae sheet is provided with this paper.

Total marks - 100

Section I
22 Marks
- Attempt Questions 1-22
- Allow about 30 minutes for this section.
- Give your answers on the multiple choice answer sheet.

Section II
78 Marks
- Attempt Questions 23 - 28
- Allow about 2 hours for this section.
- Use a separate writing booklet for each question.

Outcomes Assessed

Preliminary course

- **P1** develops a positive attitude to mathematics and appreciates its capacity to provide enjoyment and recreation
- **P2** applies mathematical knowledge and skills to solving problems within familiar contexts
- **P3** develops rules to represent patterns arising from numerical and other sources
- **P4** represents information in symbolic, graphical and tabular forms
- **P5** represents the relationships between changing quantities in algebraic and graphical form
- **P6** performs calculations in relation to two-dimensional and three-dimensional figures
- **P7** determines the degree of accuracy of measurements and calculations
- **P8** models financial situations using appropriate tools
- **P9** determines an appropriate form of organisation and representation of collected data
- **P10** performs simple calculations in relation to the likelihood of familiar events
- **P11** justifies his/her response to a given problem using appropriate mathematical terminology

HSC course

- **H1** appreciates the importance of mathematics and its usefulness in contributing to society
- **H2** integrates mathematical knowledge and skills from different content areas in exploring new situations
- **H3** develops and tests a general mathematical relationship from observed patterns
- **H4** analyses representations of data in order to make inferences, predictions and conclusions
- **H5** makes predictions about the behaviour of situations based on simple models
- **H6** analyses two-dimensional and three-dimensional models to solve practical and mathematical problems
- **H7** interprets the results of measurements and calculations and makes judgements about reasonableness
- **H8** makes informed decisions about financial situations
- **H9** develops and carries out statistical processes to answer questions which she/he and others have posed
- **H10** solves problems involving uncertainty using basic principles of probability
- **H11** uses mathematical argument and reasoning to evaluate conclusions drawn from other sources, communicating his/her position clearly to others

SECTION I

22 Marks
Attempt Question 1 – 22
Allow about 30 minutes for this section.

Use the multiple-choice answer sheet.

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ⬭ B ● C ⬭ D ⬭

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ⬭ D ⬭

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and drawing an arrow as follows:

correct ↙

A ⊗ B ⊗ C ⬭ D ⬭

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1 The Merino Sheep Company records the amount of rain in the rain gauge each morning at 8.00am. Which of the following best describes the type of data collected?

(A) Discrete (B) Stratified (C) Categorical (D) Continuous

2 Expand and simplify $3(m-3)-2(4m+7)$

(A) $-5m-23$ (B) $4-3m$ (C) $5-5m$ (D) $-5m-18$

3 The table of values below gives four points which lie on a straight line.

x	5	7	8	11
y	3	7	9	15

The equation of this straight line is

(A) $y=7-2x$ (B) $y=5x-3$ (C) $y=2x-7$ (D) $y=5x+2$

4 Considering error in measurement, calculate the largest possible area of a rectangle which measures 11.9 m by 7.4 m.

(A) 87.0975 m^2 (B) 87.8385 m^2 (C) 88.2825 m^2 (D) 89.0275 m^2

5 Joanna has a ten-pin bowling average of 152 with a standard deviation of 6. Her lowest score has been 130 and her highest score 190. Between which two numbers will 68% of her scores lie?

(A) 130 and 190 (B) 130 and 152 (C) 146 and 158 (D) 140 and 164

6 Chloe invests $\$2500$ in an investment account which pays interest at 8% pa compounded every six months. How much interest has she earned after 6 years?

(A) $\$1467.19$ (B) $\$3967.19$ (C) $\$1502.58$ (D) $\$4002.58$

7 The ratio of blue balls to green balls in a hat is $3:4$.

The number of balls in the hat could be:

(A) 9 (B) 16 (C) 34 (D) 49

8 Sven throws 3 darts at a dartboard 15 times and records his scores in a stem and leaf plot. After a visit to the spa Sven repeats the experiment.

Before Spa				After Spa			
	9	1	20				
9	8	2	30	5			
8	7	1	40	7			
8	6	0	50	3	8		
6	4	1	60	1	2	6	7
		2	70	1	3	4	5
			80	1	1	2	
			90				

Which of the statements below is true?

(A) The mean has increased and the range has increased

(B) The range has risen but the mean has fallen

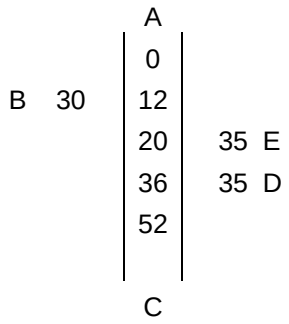
(C) Both the mean and the range have decreased

(D) The mean has increased and the range has decreased

9 Jane buys a new viola for $\$9500$. Each year, the viola depreciates by $\$600$. Calculate the salvage value of the viola after 3 years, correct to the nearest dollar.

(A) $\$7700$ (B) $\$7815$ (C) $\$11300$ (D) $\$5900$

10 The following is a field book entry of an olive orchard in Mudgee. All measurements are given in metres.



The area of AEDC, correct to the nearest square metre is:

(A) 780 (B) 910 (C) 1190 (D) 1970

11 The solution of $\frac{x-2}{3}+4=\frac{x}{5}$ is:

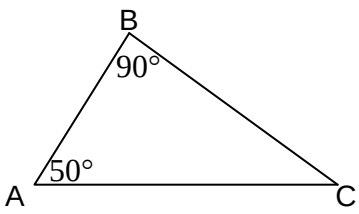
(A) -25 (B) $-4\frac{1}{6}$ (C) -1 (D) 3

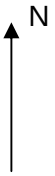
12 On Lucy's credit card daily interest is charged at 0.04% on outstanding balances.
Lucy buys two gifts at $54 each on the 8th August.

How much interest does Lucy pay if she pays off her credit card on 10th September?

(A) $\$0.71$ (B) $\$1.43$ (C) $\$71.28$ (D) $\$142.56$

13 Three towns A, B and C are situated as shown in the diagram. C is due east of town A.



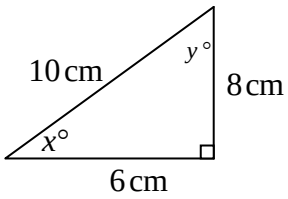


Not to scale

The bearing of C from B is

(A) $040°$ (B) $130°$ (C) $140°$ (D) $310°$

14



Not to scale

In the triangle above, which of the following ratios is the largest?

(A) $\sin x$ (B) $\sin y$ (C) $\tan x$ (D) $\tan y$

15 The lift at Centre Point Tower travels 298 metres in 42 seconds. Its average speed to the nearest kilometre per hour is:

(A) 2 km/h (B) 7 km/h (C) 26 km/h (D) 118 km/h

16 In NSW postcodes consist of four digits, with the first number always a '2'. The number of distinct possible postcodes in NSW is given by:

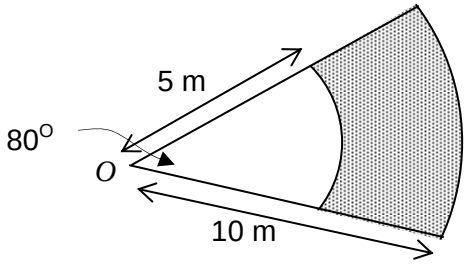
(A) 2^4 (B) $2\times10\times9\times8$ (C) 10^3 (D) 10^4

17 A hospitality class was given a short topic test. The mean mark in the test was 71 and the standard deviation was 12. A z-score of -2 for this test would represent a mark of:

(A) 47 (B) 69 (C) 73 (D) 95

18 The area of the shaded region is approximately:

Not to scale



(A) 236 m^2 (B) 53 m^2 (C) 52 m^2 (D) 70 m^2

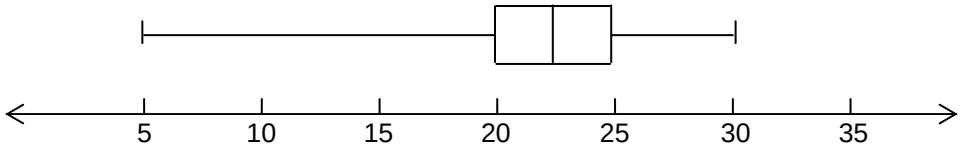
19 The speed of a ski lift (*s*) moving up an incline is inversely proportional to the weight (*w*) the lift carries.

A ski life carrying a total weight of 600 kg can move up the incline at a speed of 25 km/h.

If the ski lift moves at 35 km/h, total weight it is carrying is approximately:

(A) 430 kg (B) 590 kg (C) 610 kg (D) 840 kg

20 Which of the following is **not** true of the data represented in the box and whisker plot below:



(A) $\frac{1}{4}$ of the scores are between 5 and 20

(B) $\frac{1}{2}$ if the scores are between 20 and 25

(C) the lowest $\frac{1}{4}$ of the data is spread over a wide range

(D) most of the data is contained between the scores of 5 and 20

21 Look at the table of monthly loan repayments per $1000 shown below.

	Interest Rate (pa)			
Term	9%	10%	11%	12%
10	$12.67	$13.22	$13.78	$14.35
15	$10.14	$10.75	$11.37	$12.00
20	$9.00	$9.65	$10.32	$11.01
25	$8.39	$9.09	$9.80	$10.53

Daniel has an $80 000 mortgage at 10%pa over 10 years. After interest rates rise to 12% Daniel extends the term of his loan to 15 years.

What is the change in Daniel's monthly repayments?

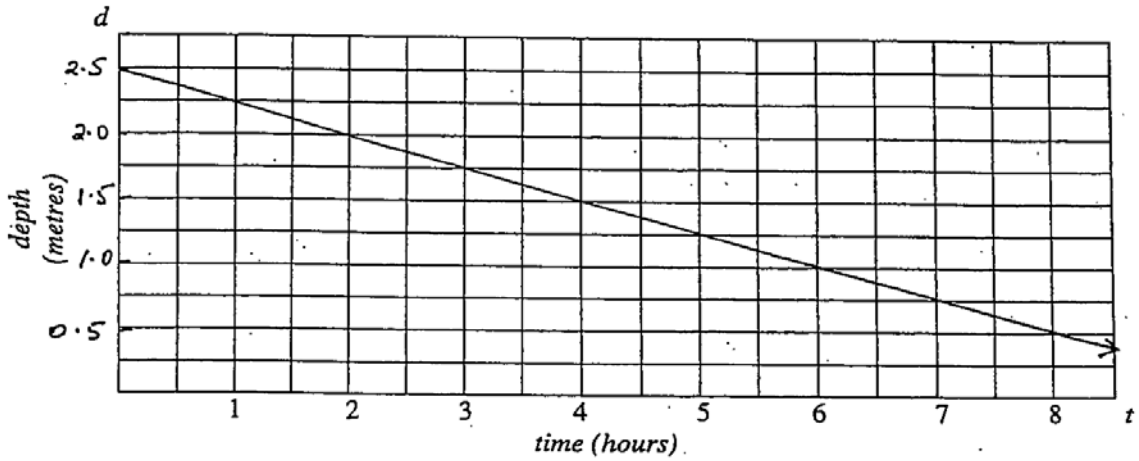
(A) They increased by $1.13 per month

(B) They decreased by $1.22 per month

(C) They increased by $90.40 per month

(D) They decreased by $97.60 per month

22 A swimming pool was emptied and the depth of water recorded as it fell. The graph below shows the change in depth.



What is the equation of the depth of water over time?

(A) $d = 2.5 - 0.25t$

(B) $d = t + 0.25$

(C) $d = 2.5$

(D) $d = 0.25t + 2.5$

SECTION II
78 Marks
Attempt questions 23-28.
Allow approximately 2 hours.

Question 23 (13 marks)

(a) A car is advertised for sale as follows:

Cash: $25 070
Terms: $5 070 deposit and $1033.33 per calendar month for 2 years.

(i) What is the total amount paid for the car on these terms? **1**

(ii) Calculate the annual simple interest rate charged by the car dealer when the car is purchased on these terms. **2**

(b) The frequency distribution below shows the results of a student survey about the number of movies watched in the last week.

Number of Movies	Number of students
1	5
2	7
3	10
4	13
5	5

What is the relative frequency of watching four movies? **1**

(c) Georgia bought a new machine costing $121 000 for her confectionary factory.

(i) What would be the value of the machine after 3 years if the depreciation is calculated using the declining balance method and the annual depreciation rate is 9%? **1**

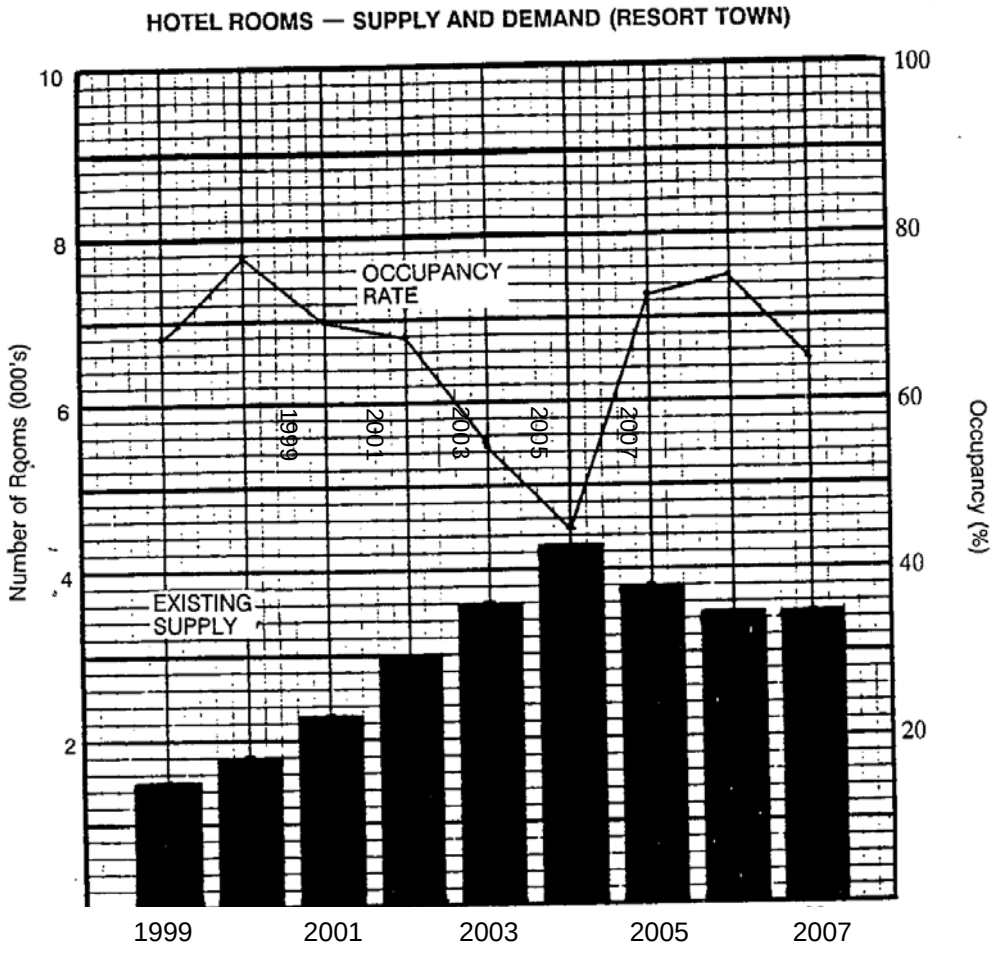
(ii) Use the 'guess, check and refine' method to calculate how long it would take for the machine to halve in value using the declining balance method. **2**

Question 23 Continued on Page 10

Question 23 continued

(d) The height of a Year 12 student was measured to be 172 cm correct to the nearest centimetre. What is the percentage error of this measurement correct to 2 decimal places? **2**

(e) The information below refers to the supply and demand for hotel



(i) How many rooms were available in 2003? **1**

(ii) How many rooms were unoccupied in 2000? **1**

(iii) When did the greatest increase in occupancy rate occur? Justify your answer. **2**

Question 24 (13 marks)

(a) Solve (i) $2x^2 + 7 = 25$ **2**

(ii) $\frac{x-3}{2} - \frac{1-x}{3} = 5$ **3**

(b) A gardening expert recommends 15 litres of water per week for each 1 metre height of gum tree. A garden hose fills a 10 litre bucket in 30 seconds.

Calculate the time that should be spent each week watering a 9 metre high gum tree with a hose. **2**

(c) The Victoria Parks Corporation conducted a survey on the population of koalas in the Otway National Park. In January 2008, in a particular region of the park, Simone caught and tagged 56 koalas. In January this year, the same region of the park, Simon caught 42 koalas and discovered that 13 of them had been tagged in the previous year. Estimate the total population of koalas in the National Park. **2**

(d) When people make a toast to celebrate an event they often 'clink' glasses. Each person in a group 'clinks' with everyone else.

A rule to establish the number of clinks could be:

$C = \frac{n(n-1)}{2}$ where 'C' is the number of clinks and 'n' is the number of people.

(i) Copy and complete the table below.

No of people 'n'	2	3	4	5	6	
No of clinks 'C'	1	3				28

2

(ii) How many people would be required to produce 500 clinks? Explain how you found your answer. **2**

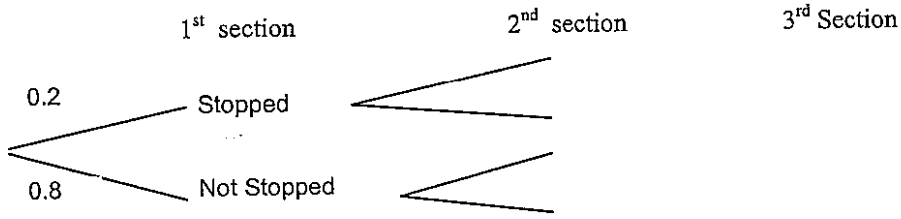
Question 25 (13 marks)

(a) The average temperature in June for the past *30* years in a city has been $18.4°C$. This year the new June temperature average was $22.1°C$.

What is the new 31 year average? *(Answer correct to one decimal place)* **2**

(b) On his way to work Leonardo passes three lots of roadwork. At the first section of roadwork the traffic is regularly stopped for 20% of the time and at the second and third sections it is regularly stopped for 10% of the time.

(i) Copy and complete the probability tree diagram below to show the possible outcomes for Leonardo passing through the three sections of roadwork. **2**



(ii) What is the probability that he travels to work without being stopped by roadwork? **1**

(iii) What is the probability that he is stopped at least once for roadwork? **1**

(iv) What is the probability that he is stopped exactly once for roadwork? **2**

(c) Standard time for Sydney is based on the $150°E$ meridian and time for New York is based on the $75°W$ meridian. At 4.00am on Monday in Sydney, Sef decides to call a friend in New York. What time and day is it in New York? **2**

(d) Mr Peabody bought 500 Restac $2 shares at $1.50 each

Six months later the shares paid a dividend of 8% and the market price was $1.65 per share.

(i) How much dividend did Mr Peabody receive? **1**

(ii) Calculate the percentage yield of the shares. **2**

Question 26 (13 marks)

(a) A football commentator makes the following statement about two football teams. “The Dogs are a much bigger team, man for man than the Tigers.” The mass of each player in the two teams is measured and five figure summaries of both sets of results are shown below.

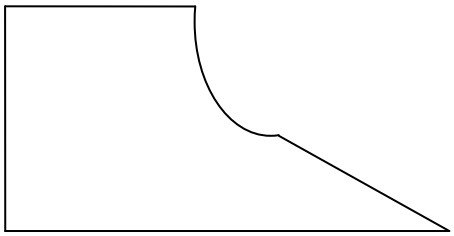
Statistical Measure	Dogs	Tigers
Lowest Score	65	60
Lower Quartile	67	70
Median	85	90
Upper Quartile	110	110
Highest Score	130	120

(i) From the information on the table, draw a box and whisker plot for each team, using the same scale, to compare the two sets of data. **3**

(ii) Do either team’s results appear to be skewed, and if so, what is the type of skew? **1**

(iii) Does the data support the commentator’s statement? Give reasons. **1**

(b)



Drawn to scale 1 **:** 1000

The shape drawn above is to be a paved area:

(i) Use your ruler to determine the values of $h, d_f, d_m \text{ and } d_l$ required to calculate the area using Simpson’s Rule. Express each length correct to the nearest 5mm **2**

(ii) Use one application of Simpson’s Rule to calculate the approximate area of this shape. **2**

(iii) A constant depth of 10cm of sand will be used as a base for the pavers. What volume of sand will be used? **2**

Question 26 Continued on Page 14

Question 26 continued

(c) Helga has a combination lock with three dials. Each dial is numbered from 1 to 16.

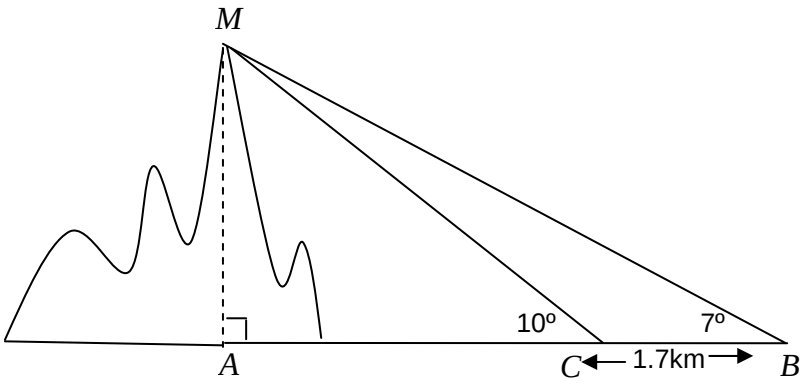
- She remembers that the number for the first dial is less than 10
- She remembers that the number for the second dial is even
- She can't remember anything about the third dial

Based on what Helga can remember, how many different lock combinations are possible? **2**

Question 27 (13 marks)

(a) During a trek in Nepal, Angelina recorded the angle of elevation from various points to the top of a mountain from a point B, the angle was $7°$, from another point C, on the same level as B, the angle was $10°$. The point C was $1.7km$ closer to the base of the mountain than was B.

The diagram below represents this information. In the diagram AM is the mountain height.



Not to scale

(i) Find the size of $\angle BMC$ **1**

(ii) Calculate the length of CM to 2 decimal places **2**

(iii) Find the height of the mountain **2**

(b) The distance between Capetown ($34°S, 17°E$) and Sydney ($34°S, 151°E$) is 12 500km.

(i) Show that the radius of the parallel of latitude on which both Sydney and Capetown both lie is $5300\,\text{km}$ (correct to 2 significant figures) **2**

(ii) Hence calculate the shortest distance around the globe between Sydney and Santiago ($34°S, 71°W$). Answer in kilometres correct to 3 significant figures. **2**

Question 27 Continued on Page 15

(c) Norah would like to determine if there is a correlation between the mean January temperature and the number of callouts to domestic disputes by the Police in January. The data below was collected.

Mean January Temperature.	*Number of Domestic Callouts.*
29	18
30	20
31	22
32	22
33	23
35	25
36	26
37	26
38	28
39	30

This data is graphed on page 19. Use it to help answer this question.

(i) Describe the correlation between the temperature and the number of callouts. **1**

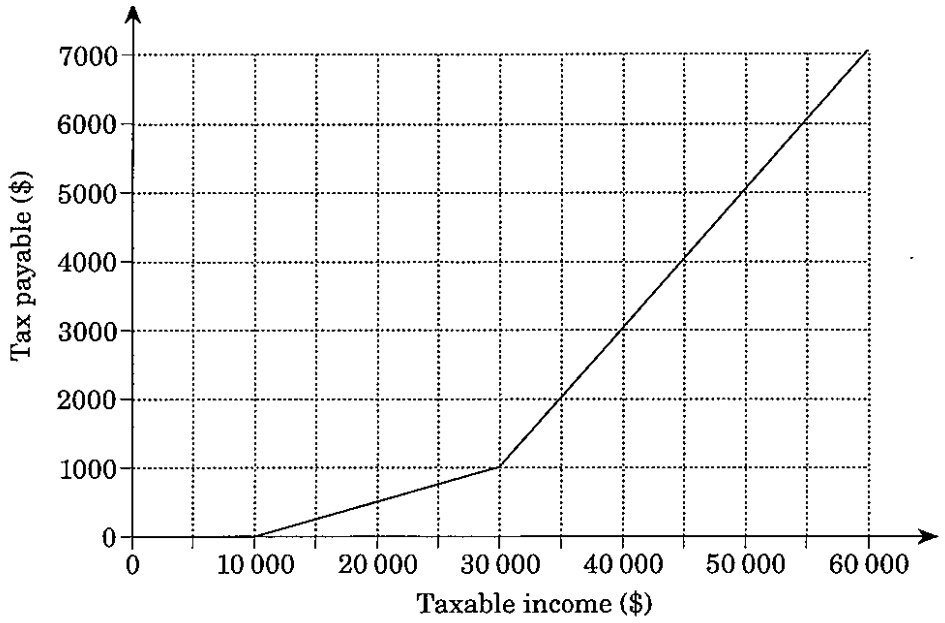
(ii) Estimate the correlation coefficient for the data **1**

(iii) On the graph on page 19 draw in the **median regression line** for the data. **2**

DETACH THE GRAPH ON PAGE 19 AND ATTACH TO YOUR ANSWER BOOKLET FOR THIS QUESTION.

Question 28 (13 marks)

(a) The graph shows the tax payable for taxable incomes up to $60 000 in a proposed tax system.



Calculate the tax rate as a percentage for incomes of $30 000 and above. **2**

(b) Hassan borrowed $75 000 to buy a home unit which he is going to use as a rental investment. This table shows some of the figures involved in the repayment of his loan.

	Home Loan Table			
	Amount =	$75 000	This table assumes the same number of days in each month. Interest = rate/12 × principal	
	Annual interest rate	10%		
	Monthly repayment R =	$900		
N	*Principal (P)*	*Interest (I)*	*P + I*	*P + I − R*
1	$75 000.00	$625.00	$75 625.00	$74 725.00
2	$74 725.00	$622.71	$75 347.71	$74 447.71
3	$74 447.71	$620.40	$75 068.11	$74 168.11
4	$74 168.11	*A*	*B*	*C*
185	$1219.77	$10.16	$1229.93	$329.93
186	$329.93			

(i) What fraction of Hassan's first repayment of $900 will be interest? **1**

(ii) After Hassan's 186th repayment, the loan will be repaid. Calculate the value of his last repayment. **2**

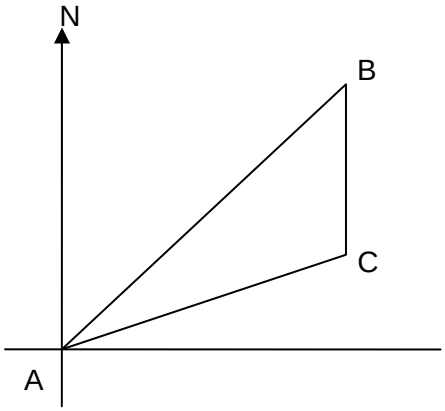
(iii) Calculate the total amount of interest Hassan will pay on this loan. **2**

Question 28 Continued on Page 17

Question 28 continued

(c) Robert Rowlands commutes $30km$ by train from home A into the city B, on a bearing of $040°T$. He then changes trains and travels due south for $16km$ to reach work C.

(i) Copy the diagram below and mark in all the given information. **2**



(ii) Calculate the length of the shortest distance between home and work to the nearest kilometre. **2**

(iii) Find the angle BAC to the nearest degree **2**

(iv) On what bearing would he have to travel to take the direct route from A to C? **1**

END OF EXAMINATION

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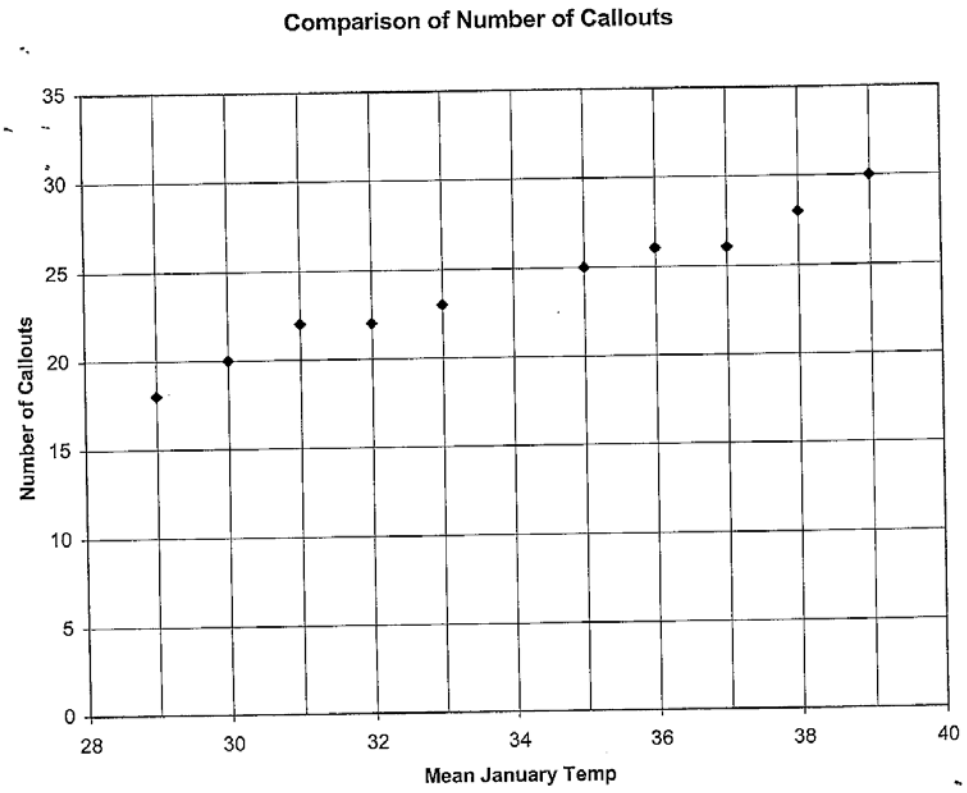
STUDENT NUMBER ……

GRAPH FOR QUESTION 27 (c) DETATCH AND HAND IN WITH YOUR ANSWERS



DETACH THIS PAGE AND HAND IT IN WITH YOUR ANSWER PAGE FOR QUESTION 27(c)(iii).

EXTRA QUESTIONS

1.

3. When a parachutist jumps from a plane, the distance he has fallen in metres (D) varies directly with the square of the time (t) he has been falling, in seconds. After 3 seconds the parachutist has fallen 44.1 metres.

(i) Write a formula for D in terms of t and find the constant of variation. **2**

(ii) Find the distance the parachutist has fallen after 4.5 seconds, correct to the nearest metre. **1**

(iii) How long will it take the parachutist to fall 500 metres? **1**

2009 Yr 12 Trial HSC

Section I

(1) Rain collected 8:00 am each morning

Data collected (D) Continuous

(2) $3(m-3) - 2(4m+7)$

$= 3m - 9 - 8m - 14$

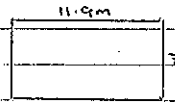
$= -5m - 23$ (A)

(3) A: $y = 7 - 2x$ B: $y = 5x - 3$ C: $y = 2x - 7$

$= 7 - 2(5)$ $= 5(5) - 3$ $= 2(5) - 7$

$= 7 - 10$ $= 25 - 3$ $= 10 - 7$

$= -3$ ✗ $= 22$ ✗ $= 3$ ✓ (C)

(4) 

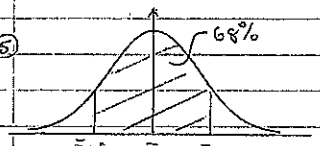
$l = 11.9$m $w = 7.4$m

$l \approx 11.95$m $w \approx 7.45$m

$A = l \times w$

$= 11.95 \times 7.45$

$= 89.0275\text{m}^2$ (D)

(5) 

average = 152

standard deviation = 6

lowest score = 130

highest score = 190

68% of scores = mean ± 1 standard deviation

$= 152 \pm 6$

ie 146 and 158 (C)

(6) $2500 invested

8% p.a. compounded 6 monthly $= \frac{8\%}{2} = 4\% = 0.04$

6 years $= 6 \times 2 = 12$ lots of 6 months

$A = P(1+r)^n$

$= 2500(1+0.04)^{12}$

$= \$4002.580546$

$\approx \$4002.58$

→ Interest = $4002.58 − $2500

= $1502.58

(C)

(7) blue : green

3 : 4

ie 7 balls, and multiples there of

ie 7, 14, 21, 28, 35, 42, 49, ... (D)

(8) Before: 201, 209, 302, 308, 309, 401, 407, 408, 500, 506, 508, 601, 604, 606, 702

After: 305, 407, 503, 508, 601, 602, 606, 607, 701, 703, 704, 705, 801, 801, 802

Before: mean $= \frac{6572}{15} = 438.1\dot{3}$ range = 702 − 201 = 501

After: mean $= \frac{9356}{15} = 623.7\dot{3}$ range = 802 − 305 = 497

Mean has increased and the range has decreased (D)

(9) $9500

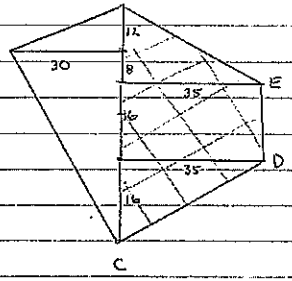
Depreciates by $600 each year, for 3 years

$S = V_0 - Dn$

$= \$9500 - \$600 \times 3$

$= 9500 - 1800$

$= \$7700$ (A)

(10) 

Area of AEDC

Trapezium: Area $= \frac{(a+b)}{2} \times h$

$= \left(\frac{16+52}{2}\right) \times 35$

$= 34 \times 35$

$= 1190\text{m}^2$ (C)

(11) $\frac{x-2}{3} + 4 = \frac{x}{5}$ (× 15 to cancel out all fractions)

$15\left(\frac{x-2}{3}\right) + 15(4) = 15^{3}\left(\frac{x}{5}\right)$

$5(x-2) + 15 \times 4 = 3x$

$5x - 10 + 60 = 3x$

$2x = -50$

$x = -25$ (A)

(12) 0.04% daily interest rate

August 8th : 2 × $54 = $108 for 2 gifts

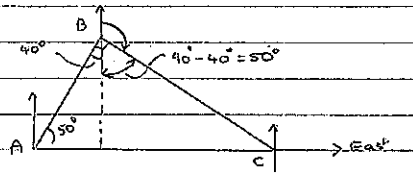
Sept 10th : Pays off card

∴ 33 days.

$$\text{Amount} = \frac{0.04}{100} \times \$108 \times 33 = \$1.4256$$

$$\approx \$1.43 \quad \text{(B)}$$

(13)

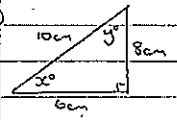


Bearing of C from B

$= 180° - 50°$

$= 130°$ (B)

(14)



$\sin x° = \frac{opp}{hyp} = \frac{8}{10} = 0.8$

$\sin y° = \frac{opp}{hyp} = \frac{6}{10} = 0.6$

$\tan x° = \frac{opp}{adj} = \frac{8}{6} = (1.3)$

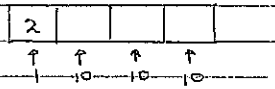
$\tan y° = \frac{opp}{adj} = \frac{6}{8} = 0.75$ (C)

(15) $$\text{Speed} = \frac{298\text{m}}{42\text{secs}} \times \frac{60\times60}{1000} = 25.54285714$$

$$\approx 26 \frac{\text{km}}{\text{h}} \quad \text{(C)}$$

$(\approx 7.01 \text{m/s})$

(16)



Number of district postcodes

$= 1 \times 10 \times 10 \times 10$

$= 10^3$ (C)

(17) mean = 71

standard deviation = 12

z-score of −2

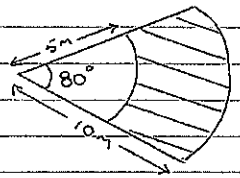
$$z = \frac{x - \bar{x}}{s}$$

$$-2 = \frac{x - 71}{12}$$

$-24 = x - 71$

$x = 47$ (A)

(18)



$$A_L = \frac{\text{Large } \theta}{360°} \times \pi r^2 \qquad A_S = \frac{\text{Small } \theta}{360°} \times \pi r^2$$

$$= \frac{80°}{360°} \times \pi \times 10^2 \qquad = \frac{80°}{360°} \times \pi \times 5^2$$

$= 69.81317008$ $= 17.45329252$

Shaded Area $= 69.81317008 - 17.45329252$

$= 52.35987756 \approx 52\text{m}^2$ (C)

(19) Speed $\propto \frac{1}{\text{weight}}$

i.e. $S \propto \frac{1}{w} \Rightarrow S = \frac{k}{w}$

$25 = \frac{k}{600}$ $\Leftrightarrow S = 25$ and $w = 600$

$k = 25 \times 600$

$= 15000$

$\therefore S = \frac{15000}{w}$

Given $S = 35$ km/h

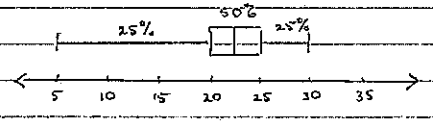
$35 = \frac{15000}{w}$

$w = \frac{15000}{35}$

$= 428.57142857$

≈ 430 kg (A)

(20)



D : most of the data is contained between the scores of 5 and 20

– since only 25%, not "most" (D)

(21) $80000 (÷1000 = 80 "units" for table)

10% p.a.

10 years Repayment = $13.22 × 80 = $1057.60

$80000

12% pa

15 years Repayment = $12.00 × 80 = $960

∴ $1057.60 − $960 = $97.60 decrease per month (D)

(22) $\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{-1}{4} = -0.25$

y-intercept = 2.5

Eqn: $d = -0.25t + 2.5$

$d = 2.5 - 0.25t$ (A)

(1) D	(4) D	(7) D	(10) C	(13) B	(16) C	(19) A	(22) A
(2) A	(5) C	(8) D	(11) A	(14) C	(17) A	(20) D	
(3) C	(6) C	(9) A	(12) B	(15) C	(18) C	(21) D	

Section II

(23) (a) Cash: $25070

Terms: $5070 deposit + $1033.33 per calendar month for 2 years

(i) Terms: $5070 + $1033.33 × 12 × 2

= $5070 + $24799.92

= $29869.92 ✓

(ii) S.I. ⇒ Interest = $29869.92 − $25070

= $4799.92

$I = PRT$

$4799.92 = 20000 \times R \times 2$ $\quad P = \$25070 - \$5070$

$\therefore R = \frac{4799.92}{20000 \times 2}$ $\quad = \$20000$

$= 0.119998 \quad (\times 100\%)$

$= 11.9998\%$

$R \approx 12\%$ p.a. ✓

(b) Relative frequency of watching four movies $= \frac{13}{40} \times \frac{100}{1} = 32.5\%$

(c) $121000 cost for confectionary machine

(i) Declining balance method of depreciation

Depreciation rate of 9% (= 0.09)

3 years.

$S = V_0(1-r)^n$

$= 121000(1-0.09)^3$

$= 91182.091$

$= \$91182.09$ ✓

(ii) Guess, check, refine to get half the value

$60500 = 121000(1-0.09)^n$

$0.5 = 0.91^n$

$n = 7: \quad 0.91^7 = 0.516761019$

$n = 8: \quad 0.91^8 = 0.470252527$

(continued)

(23) (c) (ii) continued

Using logs

$\log 0.5 = \log 0.91^n$

$= n \log 0.91$

$n = \frac{\log 0.5}{\log 0.91}$

$= 7.349614956.$

ie b/w 7 and 8 years.

∴ Will take 8 years

(d) correct to nearest cm

172 cm ie $171.5 \leq x < 172.5$

% error of this measurement.

error = 0.5 cm

$\frac{0.5}{172} \times \frac{100}{1}\% = 0.2906976744.$

$\approx 0.29\%$ (2 dec pls)

(e) Hotel Rooms

(i) Rooms available in 2003 $\quad 3.6 \times 1000$

$= 3600$ rooms ✓

(ii) Rooms unoccupied in 2000

N° of rooms $\quad 1.800 \times 1000$

$= 1800$

Occupancy rate 78%.

∴ Unoccupied = 22%

$\frac{22}{100} \times 1800 = 396$ ✓

(iii) Greatest increase in occupancy rate

rate = slope of line graph. ie % occupancy / time

greatest slope in 2004 − 2005 ✓

(24) (a) (i) $2x^2 + 7 = 25$

$2x^2 = 18$

$x^2 = 9$

$x = \pm\sqrt{9}$

$x = \pm 3$ ✓

(ii) $\frac{x-3}{2} - \frac{1-x}{3} = 5$

(x 6 for all terms)

$\frac{6^3(x-3)}{2} - \frac{6^2(1-x)}{3} = 6(5)$

$3(x-3) - 2(1-x) = 30$

$3x - 9 - 2 + 2x = 30$

$5x - 11 = 30$

$5x = 41$

$x = 8.2$ ✓

(b) 15 L/week for each 1m in height

∴ 9m high tree = 9 x 15

= 135 litres in a week

10 litre bucket ⇒ $\frac{135}{10} = 13.5$ bucketfuls of H_2O

Time ⇒ 13.5 buckets x 30 seconds

= 405 seconds (6.75 minutes = 6 min 45s) ✓

(c) 56 koalas tagged Jan 2008

13 tagged koalas out of 42 at a later date

$\frac{56}{x} = \frac{13}{42}$

$13x = 42 \times 56$

$x = 180.9230769$

$x \approx 181$ koalas in total population ✓

(d) $C = \frac{n(n-1)}{2}$ where C = n° of clinks

n = n° of people

(continued)

(24) (d) (Continued)

(i) $n = 4$: $C = \frac{4(4-1)}{2} = \frac{4(3)}{2} = 6$

$n = 5$: $C = \frac{5(5-1)}{2} = \frac{5(4)}{2} = 10$

$n = 6$: $C = \frac{6(6-1)}{2} = \frac{6(5)}{2} = 15$

$C = 28$: $28 = \frac{n(n-1)}{2}$

$56 = n(n-1)$ ∴ $n = 8$ and $n-1 = 7$

N° of people "n"	2	3	4	5	6	(8)
N° of clinks "C"	1	3	(6)	(10)	(15)	28

✓

(ii) N° of people to produce a minimum of 500 clinks

$500 = \frac{n(n-1)}{2}$

$1000 = n(n-1)$

Guess and check

$\sqrt{1000} = 31.6227766$

∴ answers close to 31 or 32

$32 \times 31 = 992$ → 496 clinks

$33 \times 32 = 1056$ → 528 clinks

∴ Need a minimum of 33 people to get 500 clinks ✓

(you get 28 extra)

Quadratic Formula

$n(n-1) = 1000$

$n^2 - n - 1000 = 0$

$n = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1000)}}{2(1)}$

$= \frac{1 \pm \sqrt{4001}}{2}$

$= 32.1267292$ or -31.1267292 not possible

then $32 \times 33 = 1056$

(25) (a) Average temp = 18.4°C for 30 years

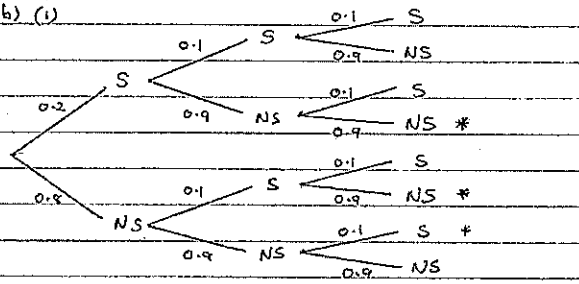
New average? ⇒ 22.1°C for the next year

$$31\text{year Average} = \frac{(30 \times 18.4) + (1 \times 22.1)}{31} = \frac{574.1}{31}$$

$$= 18.51935484$$

$$\approx 18.5°C \quad (1 \text{ dec pl.})$$

(b) (i)



(ii) Probability not stopped (NS, NS, NS)

$$= 0.8 \times 0.9 \times 0.9$$

$$= 0.648$$

(iii) Probability stopped at least once

= 1 − Probability not stopped

$$= 1 - 0.648$$

$$= 0.352$$

(iv) Probability stopped exactly once

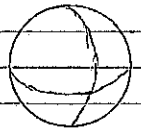
= 3 results (*)

$$= (0.2 \times 0.9 \times 0.9) + (0.8 \times 0.1 \times 0.9) + (0.8 \times 0.9 \times 0.1)$$

$$= 0.162 + 0.072 + 0.072$$

$$= 0.306$$

(c)



Sydney 150°E
New York 75°W
} Difference in longitude = 225°

∴ time diff = $\frac{225}{15}$ = 15 hrs

Sydney: 4:00 am Mon

NY: 1:00 pm Sunday

(25) (d) 500 Reshac $2 shares bought at $1.50 each

Dividend of 8% and market price was $1.65 per share

(i) Dividend = $500 \times \frac{8}{100} \times \$2.00 = \$80$

(ii) Percentage yield = $\frac{\text{amount of dividend}}{\text{market price}} \times 100\%$

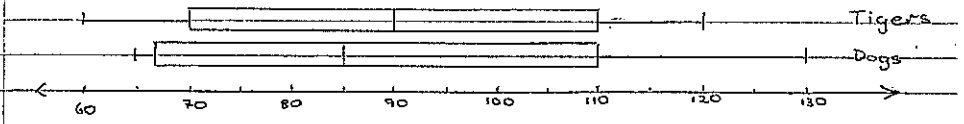
$$= \frac{80}{500 \times 1.65} \times \frac{100}{1}\%$$

$$= \frac{80}{825} \times \frac{100}{1}\%$$

$$= 9.6969\%$$

$$\approx 9.7\%$$

(26) (a) (i) Box-and-whisker plot



(ii) Dogs results are skewed: tail out to right

∴ positively skewed

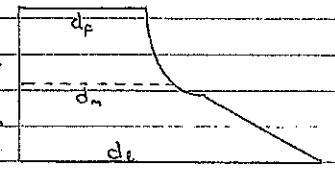
(iii) "Dogs are a much bigger team, man for man, than the Tigers" not true

Dogs IQR 67–110 (50% of team) median 85

Tigers IQR 70–110 (50% of team) median 90

- more of the Tigers are heavier than the Dogs (for the middle 50%)
- median of Dogs is lower than the middle of Tigers so comparing these players, "Dog" is not heavier than "Tiger"

(b) (i)



h = 15 mm Scale 1:1000

d_f = 25 mm

d_m = 30 mm

d_l = 60 mm

(continued)

(26) (b) (ii) Simpson's rule

$$A = \frac{h}{3}(d_f + 4d_m + d_l)$$

$$= \frac{15}{3}(25 + 4(30) + 60)$$

$$= 1025 \text{ mm}^2 \text{ on diagram}$$

$\therefore$ $A = 1025 \text{ m}^2$ for paved area

(iii) Volume = A × depth

$$= 1025 \text{ m}^2 \times \frac{10}{100}$$

$$= 102.5 \text{ m}^3$$

(c)

9	8	16

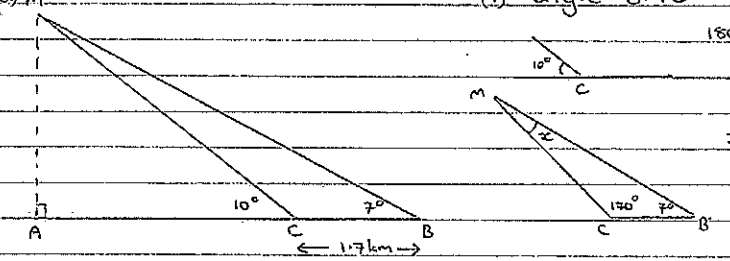
$9 \times 8 \times 16$

$= 1152$ ways ✓

Dials can be numbered 1 to 16

- 1st dial less than 10 ie 9 ways
- 2nd dial even ie 8 ways
- 3rd dial any value ie 16 ways

(27) (a)



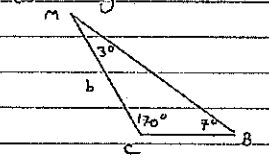
(i) angle BMC

$180° - 10° = 170°$

$x = 180° - (170° + 7°)$

$x° = 3°$ ✓

(ii) length of cm to 2 dec. places



$$\frac{m}{\sin M} = \frac{b}{\sin B}$$

$$\frac{1.7}{\sin 3°} = \frac{b}{\sin 7°}$$

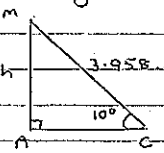
$$b = \frac{1.7}{\sin 3°} \times \sin 7°$$

$$= 3.958614663$$

$$= 3.96 \text{ km} \checkmark \text{ (2 dec pls.)}$$

(continued)

(27) (a) (iii) height of the mountain



$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$

$$\sin 10° = \frac{h}{3.958...}$$

$h = 3.958614663 \times \sin 10°$

$= 0.687406222 \text{ km}$

$= 687.406222 \text{ m}$

$\approx$ 687 m ✓ (nearest m)

(b) $D \propto t^2$ D = distance in metres

t = time in seconds

(i) $D = kt^2$

subst. $t = 3$, $D = 44.1$

$44.1 = k(3)^2$

$k = \frac{44.1}{9}$

$k = 4.9$

$\therefore$ $D = 4.9t^2$ ✓

(ii) distance fallen after 4.5 seconds

$D = 4.9t^2$

$= 4.9 \times (4.5)^2$

$= 99.225 \text{ m}$

$= 99 \text{ m}$ ✓ (to nearest metre)

(iii) time to fall 500m.

$D = 4.9t^2$

$500 = 4.9t^2$

$t^2 = \frac{500}{4.9}$

$= 102.04081633$

$t = \pm\sqrt{102.04081633}$ but consider only +ve value

$= 10.101525445$

$\approx$ 10.1 seconds ✓ (to 1 dec. pl.)

(continued)

(27) (c) (i) correlation : strong positive linear

(ii) correlation coefficient ≈ 0·8 (approx) ✓

(iii) median regression line

lower summary pt. (30, 20)

upper summary pt (38, 28)

middle summary pt (34, 24) → 22, 23, 25, 26 (median = 24)

line through lower and upper summary points passes through the middle summary point, so this is the median regression line

(28) (a) Tax rate as a percentage of incomes of $30000 and above

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{6000}{30000} = 0.2 \quad (\times 100\%)$$

$\therefore$ rate = 20% ✓

(b) (i) $900 − $625 = $275 is not interest

$$\text{Interest fraction} = \frac{625}{900} = \frac{25}{36}$$ ✓

(ii) Last repayment

$329.93 + I

$$I = \frac{10\%}{12} \times \$329.93$$

$$= 2.749416667$$

≈ $2·75

$\therefore$ Repayment = P + I

= $329.93 + $2.75

= $332.68

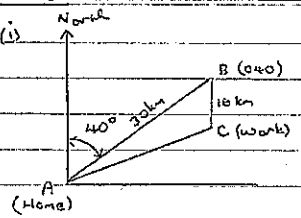
(iii) Total amount of interest

Amt paid = $900 × 185 + $332.68 = $166832·68

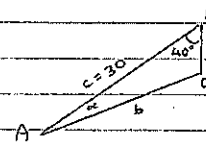
$\therefore$ Interest = $166832·68 − $75000

= $91832·68

(25) (c) (i)



(ii) Shortest length between home and work.



$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$= 16^2 + 30^2 - 2(16)(30)\cos 40°$$

$$= 420.5973346$$

$$b = \sqrt{420.5973346}$$

$$= 20.50546983$$

≈ 21 km (to nearest km)

(iii) Sine Rule to find ∠BAC

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{16}{\sin\alpha} = \frac{20.50846983}{\sin 40°}$$

$$= 31.90551517$$

$$\sin\alpha = \frac{16}{31.90551517}$$

$$= 0.501480697$$

$$\alpha = \sin^{-1}(0.501480697)$$

$$= 30.0980106°$$

= 30° (to nearest degree)

(iv) Bearing

40° + 30° = 70°

$\therefore$ bearing 070°T