Student number: ....

Ascham School

General Mathematics Trial HSC Examination July 2011

General Instructions

- Reading time – 5 minutes.
- Working time – 2 ½ hours.
- Write using blue or black pen.
- Calculators may be used.
- A Formulae sheet is provided with this paper.

Total marks - 100

Section I
22 Marks
- Attempt Questions 1-22
- Allow about 30 minutes for this section.
- Give your answers on the multiple choice answer sheet.

Section II
78 Marks
- Attempt Questions 23 - 28
- Allow about 2 hours for this section.
- Use a separate writing booklet for each question.

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Section I

22 marks
Attempt Questions 1–22
Allow about 30 minutes for this section

Use the multiple choice answer sheet for questions 1 - 22

1   Expand and simplify: $7 - 3(5x + 2)$

    (A) $1 - 15x$ B $20x + 2$
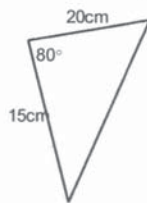
    (C) $9 - 15x$ (D) $20x + 8$

2   If $P = 5.6 \times 10^{-2}$ and $Q = 8.745 \times 10^{2}$ then $\dfrac{P}{Q}$ is closest to

    (A) 6.40×10^{-5} (B) 6.4037

    (C) 0.000006404 (D) 640366

3   Five girls try out for two places on a doubles rowing team. The number of teams that can be chosen is:

    (A)   5   (B)   10

    (C)   20   (D)   25

4

The length of the remaining side in this triangle is closest to:

    (A)   521 cm   (B)   34 cm

    (C)   23 cm   (D)   2 cm

5   The daily cost of renting a car is $50 plus 25 cents per kilometre. The total daily cost in dollars (D) can be expressed in terms of the number of kilometres (k) driven. Which equation best represents this information?

    (A) $D = 50 + 25k$ (B) $D = 50 + \dfrac{k}{4}$

    (C) $D = 75k$ (D) $D = 50 + 0.25$

6   When a normal die is rolled 120 times, how many times would you expect a number less than 3 to be rolled?

    (A)   80   (B)   20

    (C)   40   (D)   60

7   The force F on a particular moving object, moving with a velocity of v is defined by the equation $F = \dfrac{3v^2}{4}$. Which of the following could be the value of v if the value of F is 12?

    (A)   16   (B)   4

    (B)   3   (D)   9

8   Polly measures the width of a piece of paper with a ruler calibrated in millimetres. She measures it to be 16.3 cm. What is the percentage error of her measurement?

    (A)   31%   (B)   16.3%

    (C)   6.1%)   (D)   0.31%

9   Two towns are 150 km apart but on a map they are 6 cm apart. What is the scale of the map?

   (A)   1:2 500 000   (B)   1:250 000

    (C)   1:25 000   (D)   1:40 000 000

10 In a business venture there is a 40% chance of making a profit of $50 000, a 25% chance of making neither a profit nor a loss, and a 35% chance that the business will incur a loss of $40 000.
What is the financial expectation of this business venture?

(A) - $6 000

(B) - $4 000

(C) $2 000

(D) $6 000

11 The surface area of a cube is 96cm^2. What is the volume of the cube?

(A) 48cm^3

(B) 24cm^3

(C) 52cm^3

(D) 64cm^3

12 Three scores have a mean of 70. When another score is added the mean becomes 71. What is the fourth score?

(A) 70

(B) 141

(C) 71

(D) 74

13 At the end of every month Polly deposits $250 in an account that has an interest rate of 6% per annum compounded monthly. How much will she have after 10 years correct to the nearest dollar?

(A) $455

(B) $18 120

(C) $40 970

(D) $50 000

14
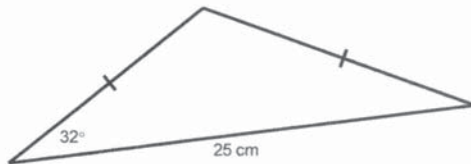
What is the length of the equal sides in the diagram?

(A) 11.9 cm

(B) 14.7 cm

(C) 15.6 cm

(D) 22.5 cm

15 Which of the following equations is equivalent to $y = bx + 2ax$ with the subject changed to a.

(A) $a = y - bx - 2x$

(B) $a = \frac{b-y}{2x}$

(C) $a = \frac{y-bx}{2x}$

(D) $a = \frac{bx-y}{2x}$

16 A laboratory tests the fat content of a sample of frozen meals. Manufacturers have labelled the meals as "no fat" or "containing fat". The results of the tests are shown in the following two-way table.

	No fat	Containing fat	Total
Tested as no fat	84	5	89
Tested as containing fat	4	27	31
Total	88	32	120

What percentage of frozen meals were incorrectly labelled

(A) 3.3

(B) 7.5

(C) 5.6

(D) 26.7

17 In right angled triangle PQR, the right angle occurs at Q.
If $\text{Tan } P = \frac{p}{r}$, which of the following expressions is correct?

(A) $\text{Tan } Q = \frac{\sqrt{p^2+r^2}}{p}$

(B) $\text{Sin } P = \frac{\sqrt{p^2+r^2}}{p}$

(C) $\text{Cos } P = \frac{r}{p}$

(D) $\text{Cos } R = \frac{p}{\sqrt{p^2+r^2}}$

18 The height of a plant is recorded over a four week period and the results plotted on a graph as shown.

During which week did the plant have the greatest percentage growth?

(A) week 1

(B) week 2

(C) week 3

(D) week 4

19 The results of a test are recorded in the stem and leaf plot below

Stem	Leaf
5	4 5 7
6	2 4 6 6
7	7 9 9
8	1

Polly sat the test late and her mark of 78 has to be added to the display above. Which of the following statistics will be changed by the addition of Polly's mark?

(A) Mean

(B) Median

(C) Mode

(D) Range

20 At a childcare centre the number of staff needed for the number of children attending is shown in the following graph.

If the number of children drops from 45 to 20 how many staff would no longer be required?

(A) 1

(B) 2

(C) 3

(D) 4

21 The table below shows the monthly repayments per $1000 on a bank home loan.

Term of Loan (years)	6.00%	6.25%	6.50%	6.75%	7.00%	7.25%	7.50%
5	$19.33	$19.45	$19.57	$19.68	$19.80	$19.92	$20.04
10	$11.10	$11.23	$11.35	$11.48	$11.61	$11.74	$11.87
15	$8.44	$8.57	$8.71	$8.85	$8.99	$9.13	$9.27
20	$7.16	$7.31	$7.46	$7.60	$7.75	$7.90	$8.06
25	$6.44	$6.60	$6.75	$6.91	$7.07	$7.23	$7.39

Determine the monthly repayment for a loan of $120 000 at 6·5% p.a. interest rate over 20 years.

(A) $7.46

(B) $89.52

(C) $895.20

(D) $7460

22 The depreciation on my new laptop computer worth $4000 can be found using the declining balance method. The table below shows the value of the laptop after a number of years.

Years	Salvage Value
0	$4000
1	$3200
2	$2560
3	$2048
4	$1638.40
5	$1310.72
6	$1048.58
7	$838·86
8	$671.09
9	$536.87
10	$429.50

In which year will the laptop first be worth half of its original value?

(A) 2nd

(B) 3rd·

(C) 4th

(D) 5th

End of Section 1

Section 2

78 marks

Attempt questions 23-28

Allow about 2 hours for this section

Answer each question in a separate booklet – extra booklets available.

All necessary working must be shown.

Question 23 (13 marks)   Use a separate answer booklet

(a) The major axis of an ellipse measures 24cm and the minor axis 18cm. Find its area, correct to 2 significant figures.   **2**

(b) As part of a project to beautify the school grounds, the construction of a fish pond is being considered. A sketch is shown below.
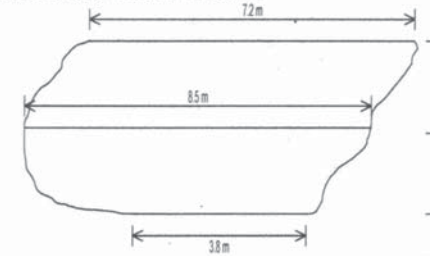
(i) Use Simpson's Rule to calculate the area of the pond's surface.   **2**

(ii) The pond has an even depth of 1.5 metres. Each fish introduced into the pond requires 1.75 m^3 of water.   **2**

What is the maximum number of fish that can successfully be introduced into the pond?

(c) A rectangular sheet of steel 1m by 1.25m is rolled into an open cylindrical drum as shown in the diagram below.


1.25m

1m
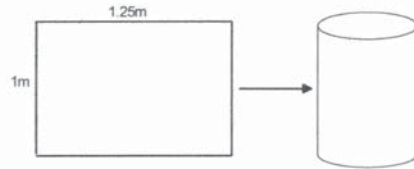
(i)   Show that the drum has a radius of approximately 20 cm   **2**

(ii)   Calculate the volume of the drum in cubic metres correct to 3 significant figures.   **2**

(iii)   What is the capacity of the drum to the nearest litre?   **1**

(d) The area chart below shows the percentage of glass, plastic and paper recycled by the end of each year over a four year period

Materials Recycling
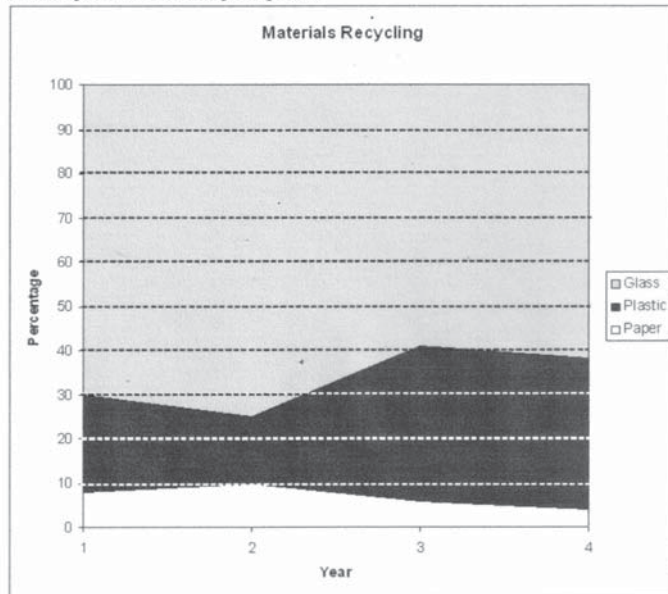
(i)   What percentage of recycled material was plastic in the second year?   **1**

(ii)   Describe what happened to recycled glass as a percentage of recycled material from the end of year 2 to the end of year 3.   **1**

Question 24   Take a new booklet

(a) 2000 teenagers were questioned about their mobile phone use over a month. The box and whisker plot shows the number of text messages received.
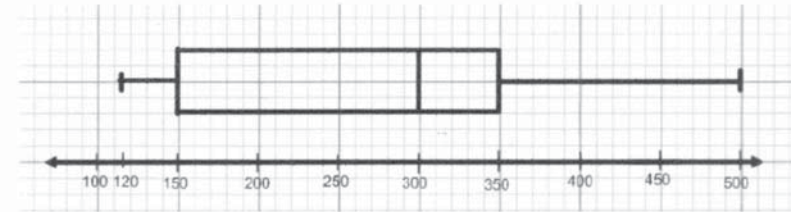
(i)   Find the inter-quartile range.   **1**

(ii)   How many teenagers received between 300 and 350 text messages that month?   **1**

(iii)   Is this data symmetrical, negatively skewed or positively skewed?   **1**

(b) The maths test results of two students, Polly and Molly, are being compared.
Molly, who studies well for HSC assessments, has percentage marks over the year of
78, 29, 65, 94, 88, 57.

Polly has a mean of 68 and a standard deviation of 16.

(i)   Calculate the mean and standard deviation for Molly's six tests.   **2**

(ii)   Polly and Molly are both contenders for the Maths Prize which is awarded for   **1**
excellence and consistency over the year. Which student do you think should be
awarded the prize based on your answers to (i)? Refer to the statistics in your
explanation.

(c) A stratified random sample survey is to be conducted to determine whether hats should be part of the school uniform. The table shows how many students are in each year group.

| Year 7	100
Year 8	120
Year 9	150
Year 10	140
Year 11	110
Year 12	100

If 50 students are to be interviewed, how many of these should be from Year 12?   **2**

Question 24 continued

(d) The probability that Polly can solve suduko puzzles of various levels of difficulty is given in the table.

Moderate	0.8
Hard	0.6
Diabolical	0.2

(i)   What is the probability that she fails to solve a moderate puzzle?   **1**

(ii)   On Sunday Polly does three suduko puzzles, one moderate, one hard and one **2** diabolical. Copy and complete the probability tree diagram below


S= solves
F = fails to solve
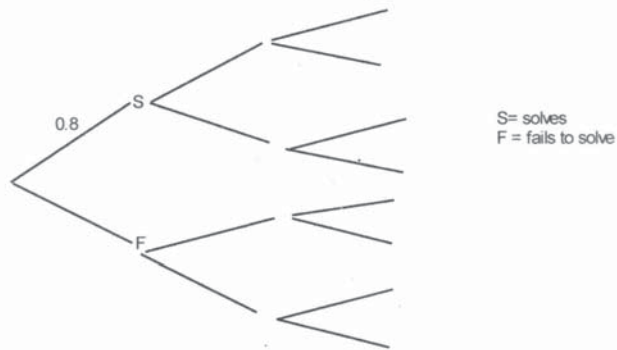
(iii)   What is the probability that Polly solves all three puzzles?   **1**

(iv)   What is the probability that she solves at least one puzzle?   **2**

Question 25 Take a new booklet

(a) Polly needs to borrow money to buy a house. She has arranged a bank loan at 9.25% p.a. to be repaid over 20 years at $640 a month.

Polly gets a pay rise and can afford to increase her repayments by $100 per month to $740 per month. The graphs below show how her increased repayment shortens the time it takes her to repay the loan.
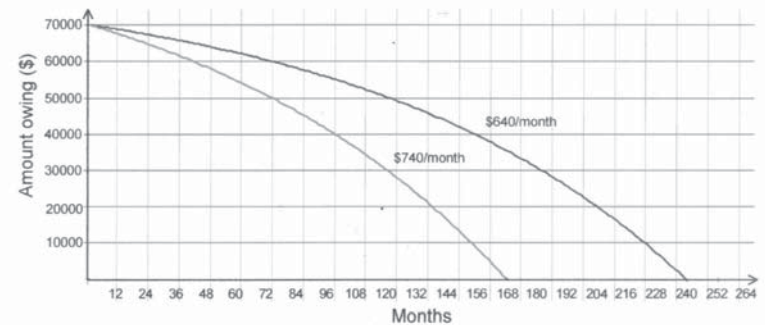
(i)   How much did Polly borrow?   **1**

(ii)   How many years sooner did Polly repay the loan by increasing her monthly **1** repayment to $740?

(iii)   How much money did she save by repaying $740 per month rather than $640 per **2** month?

(b) The relationship between the rate of increase (R) in the temperature of water being heated by a coil as a current (I) passes through it is given by the equation
$$R = kI^2$$
Where R is measured in °C per minute, I is measured in amps and is positive, and k is a constant.

(i)   It is found that the water temperature rises by 3.5°per minute when the current is **2** 1.5 amps. Find k correct to 3 significant figures.

(ii)   What current would raise the temperature of the water by 2° per minute? Answer **2** correct to 2 significant figures.

Question 25 (c) is on page 19

Question 26 Take a new booklet

(a) The sun rises in Christmas Island 3 hours 24 minutes later than in Sydney.

    (i)   What is the difference in longitude between Christmas Island and Sydney?   **2**

    (ii)   Sydney has longitude 152°E. What is the longitude of Christmas Island?   **1**

(b) Polly travels from Perth (31°S, 115°E) to Beijing (39°N, 115°E).

    (i)   Show that the distance from Perth to Beijing is approximately 4200 nautical miles.   **2**

    (ii)   If Polly travels at 240 knots, how long will the journey take?   **1**

    (iii)   If it is 2pm on Saturday when Polly leaves Perth, what day and time is it on her arrival in Beijing?   **1**

    (iv)   (α) If it is 10.25am in Beijing (+8 on GMT) what time is it in Frankfurt (+1 on GMT)?   **1**

        (β) Polly now travels by plane to Frankfurt (39°N, 10°W), leaving Beijing at 10.25am on Monday. The journey takes 18 hours 45 minutes. What will be the day and local time on her arrival in Frankfurt?   **1**

(c) Molly begins at the top of the Chairlift at an altitude of 900m from the base of the mountain. The angle of depression from the top of the chairlift to the bottom is 30° while the angle of depression from the top of the chairlift to the train is 5°.

Not to scale
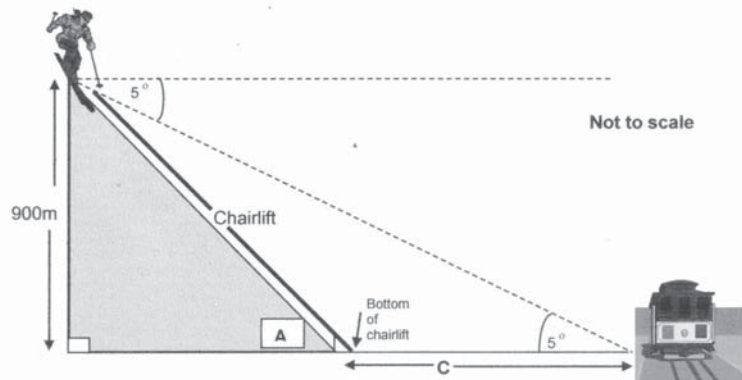
    (i)   What angle does the chairlift make with the horizontal (represented by A))?   **1**

    (ii)   Calculate the length of the chairlift correct to 3 significant figures.   **1**

    (iii)   How far is it from the bottom of the chairlift to the train (represented by C)? Answer to the nearest metre.   **2**

Question 27 Take a new booklet

(a) Polly was 25 days late in paying her credit card and paid $6.74 interest on an outstanding balance of $562.26.   **3**

What daily rate of interest was charged? Answer as a percentage correct to 4 significant figures.

(b) The declining balance method is used to calculate depreciation of refrigeration equipment every 6 months. The following formula is used to calculate the salvage value (S) in dollars and n is the number of 6 month time periods.

$$S = 7652(0.9)^n$$

    (i)   What was the initial value of the refrigeration equipment?   **1**

    (ii)   What will be the value of the equipment after 2.5 years? Answer to the nearest dollar.   **2**

    (iii)   What is the **annual** rate of depreciation used to calculate the salvage value?   **1**

    (iv)   If the salvage value is $1000, how many years has the equipment been in use? Answer to the nearest sufficient year.   **2**

(c) Polly borrows $200 000 to buy a house at 8% per annum interest over 20 years. She repays the loan in 20 annual instalments.

    (i)   Using a formula, find the annual instalment.   **2**

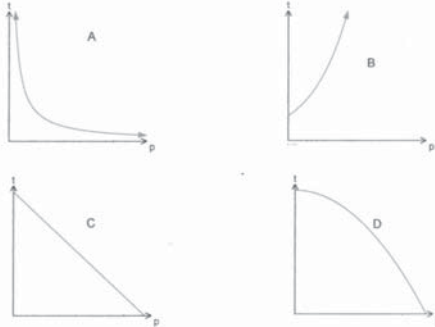   (ii)   How much of what Polly repays over the 20 years is interest?   **2**

Question 28   Take a new booklet

(a) The time (t) taken to complete a task varies inversely as the number of people (p) working.

Which of the graphs A, B, C or D could represent this?   **1**

(b) Car number plates consist of two alphabetical letters followed by two digits followed by two alphabetical letters e.g. PQ 40 RR.

   (i)   How many such number plates are possible?   **2**

   (ii)   What is the probability that a number plate of this format, chosen at random,   **2** would begin with TT and end with TT?

(c) An annual music festival attracts 8000 people when there is no charge. The organisers introduce a charge for tickets and it is found that for every dollar charged 500 fewer people attend the festival.

   (i)   For a ticket cost of $5, how many people will attend the festival?   **1**

   (ii)   What is the total income when the charge is $5?   **1**

   (iii) Find the lowest ticket cost for which no one will attend.   **1**

   (iv) If the ticket cost is $D, find an equation for the number of people P buying a ticket.   **2**

   (v)   Is there a maximum total income from the festival or is it the case that the higher   **3** the ticket price, the higher the income? Support your answer with calculations, a table of values or a graph.

End of Paper

Note that Q25(c) is on page 19

Student number ....

Detach this page and put it inside your answer booklet for Question 25

Question 25 (c)

The function for calculating the rectangular area that can be enclosed by a 56m wire fence is given by $A = 28x - x^2$ where x is the width of the enclosure in metres.

(i) Would the graph of this function be concave up or concave down? **1**
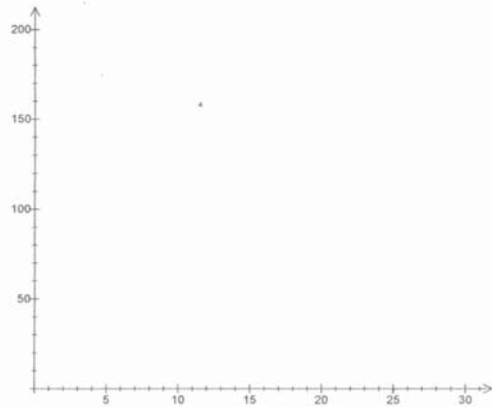
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(ii)

Complete this table of values and hence sketch the function on the axes below. **3**

x	0	7	14	21	28
A					

(iii) What is the maximum area of the enclosure? **1**

....

2011 General maths Trial Ascham.

1. A	5 B	9 A	13 C	17 D	21 C
2 A	6 C	10 D	14 B	18 A	22 C
3 B	7. B	11 D.	15 C	19 A	
4 C	8. D	12 D	16 B	20 D	

23a) Area $= \pi ab$ $a = 12, b = 9$
$= \pi \times 12 \times 9$
$= 339.29$
$= 340 \text{ cm}^2$ (2sf).

b)(i) Area $= \frac{h}{3}(d_F + 4d_m + d_L)$
$= \frac{5}{3}(7.2 + 4 \times 8.5 + 3.8)$
$= 75 \text{ m}^2$

(ii) $V = Ah$
$= 75 \times 1.5$
$= 112.5 \text{ m}^3$
N° of fish $= \frac{112.5}{1.75}$
$= 64.28$
∴ Max. of 64 fish.

c)(i) $C = 2\pi r$
$1.25 = 2\pi r$
$r = \frac{1.25}{2\pi}$
$= 0.1989 \text{ m}$
$= 19.89 \text{ cm}$
$\doteqdot 20 \text{ cm}$

(ii) $V = \pi r^2 h$
$= \pi \times (0.1989..)^2 \times 1$
$= 0.12433...$
$= 0.124 \text{ m}^3$ (to 3sf).

(iii) Capacity is 124 L (to nearest L).

d). (i) 15%
(ii) The percentage of glass goes down from 75% to 60%.

Q24 a) IQR $= 350 - 150$
$= 200$

(ii) N° between 300 and 350
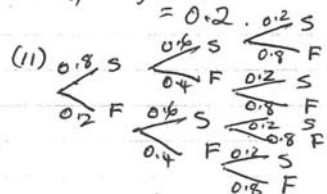is 25% of 2000
$= 500$.

(iii) Positively skewed.

(b) $\bar{x} = 68.5$
(i) $\sigma_n = 21.7$ (Pop^n st. dev.) (to 1dp).

(iii)
	$\bar{x}$	σ_n
Polly	68	16
Molly	68.5	21.7

Polly should receive the prize since her lower st. dev. indicates consistency and both means are similar.

(c) Total n° of students = 720
N° from y/12 in sample $= \frac{100}{720} \times 50$
$= 6.94$
∴ 7 students.

(d) (i) P(fails) = 1 - 0.8
$= 0.2$

(ii)

(iii) P(SSS) = 0.8 × 0.6 × 0.2
$= 0.096$
(iv) P(at least 1 solve) = 1 - 0.2 × 0.4 × 0.8
$= 0.936$.

Q25
(a)(i) $70000
(ii) 240 - 168 = 72 mths
$= \frac{72}{12}$ yrs
∴ Repays 6 years earlier

(iii) Amt saved = $640 × 240 - $740 × 168
$= $29 280.

(b)(i) concave down
(ii)
x	0	7	14	21	28
A	0	147	196	147	0
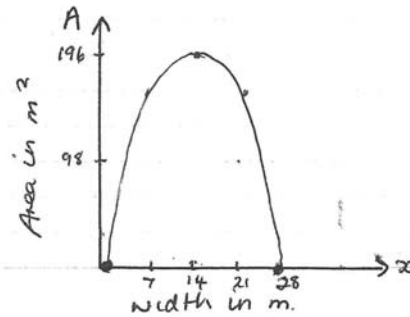
(iii) Max area is 196 m².

(b) $R = kI^2$
(i) $3.5 = k \times 1.5^2$
$k = \frac{3.5}{1.5^2}$
$= 1.555...$
$= 1.56$ (3 s.f.)

(ii) $2 = 1.56 \times I^2$
$I^2 = \frac{2}{1.56}$
$I = \sqrt{\frac{2}{1.56}}$
$= 1.13...$
$= 1.1$ (2sf)
∴ Current is 1.1 amps (2sf).

Q26
a)(i) 1° longitude → 4 mins
3h 24min = 204 mins
longitude diff = $\frac{204}{4}$
$= 51°$

(ii) Christmas Island is West of Sydney b/c sun rises later.
∴ Longitude $= (152° - 51°)$ E
$= 101°$ E

(b) (i) Angular dist = 31 + 39
$= 70°$
∴ Distance = 70 × 60 n.m.
$= 4200$ nm.

(ii) Time $= \frac{D}{S}$
$= \frac{4200}{240}$
$= 17.5$ hours

(iii) Perth & Beijing on same longitude so no time diff.
Time of arrival = 2pm Sat + 17.5h
$= 7.30$am Sun.

(iv) (α) Time in F. = 10^{25} am - 7h
$= 3^{25}$ am
(β) Time of arrival = 3^{25} am + 18h 45min
$= 22.10$
$= 10.10$ pm Mon.

(c). (i) $A = 30°$
(ii) sin 30° $= \frac{900}{d}$
$d = \frac{900}{\sin 30}$
$= 1800$ m.

(iii) $\frac{C}{\sin 25°} = \frac{1800}{\sin 5°}$
$C = \frac{1800}{\sin 5°} \times \sin 25°$
$= 8728.2...$
∴ Distance is 8728m (to n.m.).

Q27.

a) $I = Prn$

$6.74 = 562.26 \times r \times 25$

$r = \dfrac{6.74}{562.26 \times 25}$

$= 0.00047949$

∴ Daily rate is 0.04795% (4sf)

b)(I) Initial value = $7652

(II) $S = 7652(0.9)^5$

$= 4518.43..$

∴ Value after 2.5yrs is $4518

(III) Rate per 6mths $= 1-0.9$

$= 0.1$

$= 10\%$

Rate per annum $= 20\%$.

(IV) $1000 = 7652(0.9)^n$

$0.9^n = \dfrac{1000}{7652}$

$= 0.13068...$

$n \doteqdot 19.3$ (trial & error)

∴ It takes $\dfrac{19.3}{2}$ years $= 10$ years (nearest 5of.yr) using

c)(I) $N = M\left\{\dfrac{(1+r)^n - 1}{(1+r)^n r}\right\}$

$200000 = M\left\{\dfrac{(1.08)^{20} - 1}{(1.08)^{20} \, 0.08}\right\}$

$\approx M \times 9,818,...$

$M = \dfrac{200000}{9,818...}$

$= \$20370.44$

∴ Annual installment is $20370.44.

(II) Interest $= 20370.44 \times 20 - 200000$

$= \$207408.80$

Q28

a). Graph A.

b)(I) Numbers $= 26 \times 26 \times 10 \times 10 \times 26 \times 26$

$= 45697600$

(II) $Prob^y = \dfrac{1 \times 1 \times 10 \times 10 \times 1 \times 1}{26 \times 26 \times 10 \times 10 \times 26 \times 26}$

$= \dfrac{1}{26^4}$

$= \dfrac{1}{456976}$

c)(I) $h^0 = 8000 - 5 \times 500$

$= 5500$

(II) Income $= 5500 \times \$5$

$= \$27500$

(III) $8000 \div 500 = 16$

∴ If cost is $16 no one will attend.

(IV) $P = 8000 - 500D$

D	0	1	2	$\cdots\cdots$	16
P	8000	7500	7000		0

(V)

D	0	2	4	6	8	10	12	14	16
I	0	7000	etc		32000	etc			7000

∴ Max income occurs when cost is $8 per ticket.

Max income $= \$32000$.

Income $I = D(8000 - 500D)$ which is a quadratic function, concave down.

∴ A max value exists