

## 2011 <br> TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

## General Mathematics

## General Instructions

- Reading Time- 5 minutes.
- Working Time - $2^{1 ⁄ 2}$ hours.
- Write using a blue or black pen.
- Board approved calculators may be used.
- A Formulae Sheet is provided which may be used throughout the paper.


## Section I

Total marks (22)

- Attempt Questions 1-22.
- Answer on the Multiple Choice answer sheet provided.
- Allow about 30 minutes for this section.

Section II
Total marks (78)

- Attempt questions 23-28
- Answer in the booklets provided, unless otherwise instructed. Start a new booklet for each question.
- Allow about 2 hours for this section.


## Section 122 Marks

Attempt all questions
Allow about 30 minutes for this section
Use the multiple choice answer sheet for Questions 1-22

1. The following stem-and-leaf graph represents boys' and girls' heights

| Boys |  |  |  |  |  | Girls |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 99 |  |  |  |  | $15$$16$ | 69 |  |  |  |
|  |  |  |  |  | $\begin{array}{llll}0 & 2 & 8\end{array}$ |
|  |  | 8 | 7 | 5 |  | 17 | 114 |  |  |  |
| 77 | 5 | 3 | 3 | 1 | 18 | 0 |  |  |  |
|  |  | 6 | 4 |  | 19 |  |  |  |  |
|  |  |  |  | 0 | 20 |  |  |  |  |

What is the range of scores for the girls?
A. 11
B. 24
C. 31
D. 44
2. A dress is marked down by $20 \%$. If the original price of the dress was $\$ 70.00$, how much did it cost after the discount?
A. $\$ 14$
B. $\$ 50$
C. $\$ 56$
D. $\$ 90$
3. $2 a \times 2 a^{2}=$
A. $4 a^{2}$
B. $4 a^{3}$
C. $\quad 16 a^{2}$
D. $16 a^{6}$
4. Which data set satisfies the following three conditions?

$$
\text { Mode }=8, \text { Median }=4, \text { Range }=12
$$

A. $4,4,8,12$
B. $1,4,4,13$
C. $-4,8,8,16$
D. $-4,0,8,8$
5.


In how many ways can these 4 shapes be placed in a row?
A. 4
B. 10
C. 16
D. 24
6. Given the formula $Q=P^{2}$, when $P$ increases from -2 to 3 , how does $Q$ change?
A. A decreases of 5
B. An increase of 5
C. A decrease of 13
D. An increase of 25
7. A shop has three flavours of ice-cream to choose from: chocolate, vanilla and strawberry.


What is the probability that Jenny, Sarah and Robert would each choose a vanilla icecream from this shop?
A. $\frac{1}{9}$
B. $\frac{2}{9}$
C. $\frac{1}{3}$
D. $\frac{1}{27}$
8. $12-8(x-2)=$
A. $28-8 x$
B. $4 x+8$
C. $10-8 x$
D. $4 x+2$
9. The weights of a sample of four pieces of empty luggage were recorded as:

$$
3.3 \mathrm{~kg}, 3.8 \mathrm{~kg}, 4.2 \mathrm{~kg} \text { and } 4.5 \mathrm{~kg}
$$

What is the standard deviation (in kg ) of this sample, correct to 2 decimal places?
A. $\quad 0.37$
B. 0.39
C. 0.45
D. 0.52
10. Sarah works on weekdays after school for 2 hours, Monday to Friday and on Saturdays for 6 hours at time-and-a-half.

| Day | Mon | Tue | Wed | Thur | Fri | Sat |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Hours | 2 | 2 | 2 | 2 | 2 | 6 |

If her normal pay rate is $\$ 12.00$ per hour, how much does she earn in a normal week?
A. $\$ 126$
B. $\$ 168$
C. $\$ 192$
D. $\$ 228$
11. The probability that a new variety of seed will produce flowers in the first year of growth after being planted is 0.4 .


200 seeds of this variety are planted.
How many of the seeds are expected to produce flowers in the first year?
A. 8
B. 40
C. 80
D. 120
12. 8000 mail articles were collected from a post office mail box over a weekly period. The box-and-whisker plot shows the weights (in grams) of the article collected.


How many articles collected, weighed between 300 g and 350 g .
A. 1000
B. 2000
C. 2500
D. 4000
13. A tank is being emptied at a rate of 5 litres per minute. At this rate, how long will it take for a kilolitre of water to empty from the tank?
A. 3 minutes 20 seconds
B. 20 minutes
C. 33 minutes 30 seconds

D. 3 hours 20 seconds
14. Which of the diagrams shows the graph of $y=1-2 x$
A.

B.

C.

D.

15. Twelve players try out for a basketball team. There are seven people selected for the team.
How many possible teams can be selected?
A. 7
B. 12
C. 84
D. 792
16. The stopping distance $(d)$ of a car varies directly with the square of the speed $(v)$ of the car. The stopping distance of a car travelling at $90 \mathrm{~km} / \mathrm{h}$ is 45 metres.

Which of the following represents the correct relationship between $d$ (metres) and $v(\mathrm{~km} / \mathrm{h})$ ?
A. $2 d=45 v^{2}$
B. $\quad 45 d=2 v^{2}$
C. $\quad d=180 v^{2}$
D. $180 d=v^{2}$
17. The income tax rates below were applied in the 2008-2009 financial year.

| Taxable income | Tax on this income |
| :--- | :--- |
| $\$ 1-\$ 6000$ | nil |
| $\$ 6001-\$ 34000$ | 15 c for each $\$ 1$ over $\$ 6000$ |
| $\$ 34001-\$ 80000$ | $\$ 4200$ plus 30 c for each $\$ 1$ over $\$ 34000$ |
| $\$ 80001-\$ 180000$ | $\$ 18000$ plus 40 c for each $\$ 1$ over $\$ 80000$ |
| $\$ 180001$ and over | $\$ 58000$ plus 45 c for each $\$ 1$ over $\$ 180000$ |

Helen earned a gross income of $\$ 82000$ and had allowable deductions to the value of $\$ 2500$. Calculate the income tax payable for Helen's taxable income.
A. $\quad \$ 17850$
B. $\$ 18600$
C. $\$ 18800$
D. $\$ 32800$
18. Calculate interquartile range for the data presented in the cumulative histogram below.

A. 24
B. 28
C. 32
D. 36
19. A sporting event is being played in London and broadcast in Sydney.

| Location | Longitude |
| :--- | :--- |
| London | $0^{0}$ |
| Sydney | $150^{\circ} \mathrm{E}$ |

ABC Sydney broadcasts the start of play at 1:30 am Sunday local time live from London.
What time is it in London at the start of play?
A. $\quad 10: 00 \mathrm{pm}$ on Saturday
B. $\quad 3: 30 \mathrm{pm}$ on Saturday
C. $11: 30$ am Sunday
D. $11: 00 \mathrm{pm}$ on Sunday
20. In a floor design three identical square tiles of side length 15 cm are placed on the floor as shown below.


Each tile is placed exactly half way along the sided of the tile next to it.
What is the distance in centimetres between the corners A and B of the tiles in the pattern shown?
A. 21.2
B. 42.4
C. 45
D. 54.1
21. Steve borrows $\$ 5600$ to buy a car. The simple interest rate is $10.75 \% \mathrm{pa}$ and he takes the loan over 3 years. His monthly payment is
A. $\quad \$ 50.17$
B. $\$ 172.28$
C. $\$ 205.72$
D. $\$ 2468.67$
22. A farmer can determine the area (A) in square metres of a rectangular farm by using the formula:

$$
A=x(60-x)
$$

where $x$ represents the length of the farm (in metres).
The formula can be graphed as below:


What is the maximum area (in square metres) of the farm?
A. 60
B. 900
C. 1800
D. 3600

## Section 2

Questions 23-28 Total Marks (78)
Allow about 2 hours for this section

## Answer each question in a new booklet

Question 23 ( 13 Marks)
a. Electricity charges for households are based on the average daily usage of kilowatts per hour ( kWh ) over a period of 90 days.

A different rate for 'domestic' usage and 'off-peak' usage is applied:
Domestic Usage: $\quad \$ 0.119397$ per kWh for the first 1880 kWh
$\$ 0.124267$ per kWh for usage over 1880 kWh
Off Peak Usage: $\quad \$ 0.044988$ per kWh
A 10\% Goods and Services Tax (GST) is added to the total.
The graphs show a household's average daily usage of electricity for the last 90 days.
kWh per day

i. Calculate the charge for the 'domestic' usage of electricity for this household for the 90 days.
(b) A hexagonal prism made completely from thin glass is open at the top.

The prism has a height of 3 cm and a regular hexagonal base with edges 5 cm (shown below).

i. Explain why angle AOB is $60^{\circ}$
ii. Use the formula Area $=\frac{1}{2} a b \sin c$ to determine the area of triangle ABO to the nearest square centimetre.
iii. Find the area of the hexagonal base of the prism.
iv. Determine the volume of the prism
v. Determine the area ( $\mathrm{in} \mathrm{cm}^{2}$ ) of the glass used in the construction of the prism. 2
a. A company manager earns an annual salary of $\$ 80100$ and is paid each fortnight.

From each fortnight's gross salary, the manager has deductions of $\$ 440$ for superannuation, and $\$ 815$ in taxation.
i. What is the manager's gross fortnightly salary?
ii. Calculate the manager's normal net fortnightly salary.
iii. In the pay for the last fortnight of the year, the manager is paid an additional $17.5 \%$ of his gross salary for 2 weeks as his annual holiday loading.

Determine the gross amount of the holiday loading.
iv. If the holiday loading is taxed at a rate of $27.65 \%$, determine the manager's net salary for the last fortnight of the year.
b. The probability that a person will develop influenza if they have had a cold continuously for a number of days, is shown on the graph below:

i. What probability corresponds to one division on the vertical scale?
ii. After how many days with a cold will a person be certain to develop influenza?
iii. What is the probability that a person will develop influenza after 10 continuous days with a cold?
iv. Amy has had a cold for 2 days and her brother Tim has had a cold for 8 days.

What is the probability that both Amy and Tim will develop influenza?
v. When a doctor administers an annual influenza injection to patients, the probability of developing influenza is halved.

What is the probability that a patient, who has an injection, will develop influenza after 12 days with a cold?
vi. Comment on the likelihood of a patient who has had an injection after 1 day with a cold, developing influenza.
a. A hemispherical dome of diameter 58 metres forms the roof of a building.

i. Calculate the volume of the roof to the nearest cubic metre.
ii. The density of the hemisphere is given by the formula.

$$
\text { density }=\frac{\text { mass }}{\text { volume }} \quad \text { measured in tonnes } / \mathrm{m}^{3}
$$

Calculate the mass of the roof if its density is $75.5 \mathrm{~kg} / \mathrm{m}^{3}$
iii. The interior surface of the hemispherical roof is covered with mosaic tiles measuring 6 cm by 6 cm .

By calculating the surface area of the roof, determine approximately the number of tiles used.
b. Three flags on the roof of a building are shown in the diagram below.

i. Explain why $\angle P Q R$ is the largest angle in the triangle formed by the flags.
ii. Calculate the size of this angle to the nearest degree.
iii. Determine the size of $\angle P R Q$ to the nearest degree
iv. If the flag pole at $R$ is due east of the flag pole at $Q$, determine the bearing of the flag pole at $P$ from $R$.
a. The maximum temperature recorded each month for two towns is shown in the table below:

| Month | $\mathbf{J}$ | $\mathbf{F}$ | $\mathbf{M}$ | $\mathbf{A}$ | $\mathbf{M}$ | $\mathbf{J}$ | $\mathbf{J}$ | $\mathbf{A}$ | $\mathbf{S}$ | $\mathbf{O}$ | $\mathbf{N}$ | $\mathbf{D}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Town A | 31 | 30 | 28 | 26 | 23 | 18 | 17 | 19 | 24 | 27 | 28 | 30 |
| Town B | 36 | 34 | 30 | 26 | 22 | 17 | 13 | 20 | 25 | 27 | 32 | 35 |

i. Find for each town the
mean,
mode and
median temperature
ii. Find for each town the
range
interquartile range
standard deviation of the temperature
iii. Using the same scale, draw a box and whisker plot for each town
iv. Write a short description comparing the temperatures in each town
b. The length in metres of a sound wave varies inversely with the number if vibrations of the wave per second.

A wave of length 20.75 m produces 16 vibrations per second.
Calculate the number of vibrations per second produced by a wave of length 13.28 m
a. The graph below shows a comparison of petrol usage for two vehicles with the same fuel tank capacity over distances travelled.

i. What is the capacity of the fuel tanks of the two vehicles?
ii. Which vehicle uses less petrol per kilometre?
iii. What distance does vehicle B travel on a full tank of petrol?
iv. What is the fuel consumption in $\mathrm{L} / 100 \mathrm{~km}$ for vehicle A ?
v. What is the difference in petrol usage between the two vehicles after 200 km ? 1
vi. After 20 litres of petrol have been used in both vehicles, what extra distance has vehicle B travelled?
vii. The petrol ( P ) remaining in the fuel tank of vehicle A after travelling $d \mathrm{~km}$ can be represented by the equation $P=50-n d$.

What is the value of n in this equation?
b. The 140 metre walkway between the pylons of the bridge opens up to allow Marks ships to pass underneath.

The two sections of the walkway are elevated at an angle of $40^{\circ}$ and open to a height, $h$ metres above their original horizontal position, so that they are exactly in line with the top of the pylons.

i. Calculate the height ( $h$ ) that the sections of the walkway reach above their original horizontal positions. (Give your answer to the nearest metre).
ii. The walkway is 55 metres above the water line

Determine the height of the pylon from the water line.
iii. Determine the angle of elevation of the walkway from the water line at point $x$.
(Give your answer to the nearest degree).
a. $\quad \mathrm{A}$ formula relating $H$ and $k$ is given by:

$$
H=6+2 k^{3}
$$

i. Find the value of $k$ if $H=60$
ii. If $k$ doubles in value, determine the percentage change in $H$
b. The diagram shows a vertical cross-section of a river.

i. Use two applications of Simpson's rule to find the approximate area of the river's cross-section.
ii. Estimate the volume of water, in cubic metres, in a 50 -metre length of the river, assuming the cross-section is the same as above and uniform along the 50 -metre length. Give your answer to the nearest cubic metre.
iii. Estimate the volume of water, in litres, to pass through this cross-section in one hour if the river flows at a rate of $0.35 \mathrm{~m} / \mathrm{s}$.
Give your answer to the nearest thousand litres.
c. The city of Osaka, Japan (J) has co-ordinates $\left(37^{\circ} \mathrm{N}, 135^{\circ} \mathrm{E}\right)$ and Alice Springs in Northern Territory, Australia (A) has co-ordinates $\left(23^{\circ} \mathrm{S}, 135^{\circ} \mathrm{E}\right)$.
i. Determine the size of the angle JOA where O is the centre of the Earth,
ii. Calculate the distance between Osaka and Alice Springs to the nearest kilometre. (Assume the radius of the Earth is 6400 km )
iii. A plane travels between the two cities at an average speed of $685 \mathrm{~km} / \mathrm{h}$. Calculate the flight time in hours and minutes

1. B
2. C
3. B
4. D
5. D
6. B
7. D
8. A
9. D
10. D
11. C
12. B
13. D
14. B
15. D
16. D
17. A
18. A
19. B
20. D
21. C
22. B
23. a. (i) Usage $=65.3 \times 90$

$$
=5877 \text { kWh }
$$

Charge for domestic usage $=1880 \times \$ 0.119397+(5877-1880) \times \$ 0.124267$ $=\$ 721.17$
(ii) Off-peak usage $=15.4 \times 90 \times 0.044988=\$ 62.35$
(iii) Combined charge $=\$ 721.17+\$ 62.35$

$$
=\$ 783.52
$$

$$
\text { GST }=0.1 \times \$ 783.52
$$

$$
=\$ 78.35
$$

Total charge = \$783.52+\$78.35
= \$861.88
b. (i) Angle $A O B=60^{\circ}$ since $\triangle A O B$ is equilateral (all sides of length 5 cm )
(ii) Area $\triangle A O B=1 / 2 \times 5 \times 5 \times \sin 60^{\circ}$

$$
\begin{align*}
& =10.825 \ldots \\
& \approx 11 \mathrm{~cm}^{2} \tag{2}
\end{align*}
$$

(iii) Area of base $=6 \times 11$

$$
\begin{equation*}
=66 \mathrm{~cm}^{2} \tag{1}
\end{equation*}
$$

(iv) Volume $=66 \times 3$

$$
\begin{equation*}
=198 \mathrm{~cm}^{3} \tag{2}
\end{equation*}
$$

(v) Surface area $=66+6 \times 5 \times 3$

$$
\begin{equation*}
=156 \mathrm{~cm}^{2} \tag{2}
\end{equation*}
$$

24. a. (i) Fortnightly gross salary $=\$ 80100 \div 26-09$

$$
\begin{equation*}
=\$ 3070.14 \tag{1}
\end{equation*}
$$

(ii) Fortnightly net salary $=\$ 3070.14-(\$ 440+\$ 815)$

$$
\begin{equation*}
=\$ 1815.14 \tag{1}
\end{equation*}
$$

(iii) Holiday Loading $\quad=0.175 \times \$ 3070.14$
= \$537.27
(iv) tax on holiday loading $=0.2765 \times \$ 537.27$
= \$148.56

Net salary during holiday = \$1815.14 + \$537.27-\$148.56

$$
\begin{equation*}
=\$ 2203.85 \tag{2}
\end{equation*}
$$

b. (i) One vertical unit $=0.125$
(ii) certain flu after 16 days
(iii) P(flu after 10 days) $=0.625$
(iv) $\mathrm{P}($ Amy and Tim have flu) $=0.125 \times 0.5$

$$
\begin{equation*}
=0.0625 \tag{1}
\end{equation*}
$$

(v) $P$ (flu after 12 days and injection) $=0.5 \times 0.75$

$$
\begin{equation*}
=0.375 \tag{2}
\end{equation*}
$$

(v) Since the probability of getting flu after one day is half of one unit ie 0.0625, after having an injection this chance would be halved, ie 0.03125 . This would make the chance of getting the flu very unlikely.
25. a. (i) Volume of dome $=0.5 \times \frac{3}{4} \times \pi \times 29^{3}$

$$
\text { = } 51 \text { 080.2... }
$$

$$
\begin{equation*}
=51080 \mathrm{~m}^{3} \tag{2}
\end{equation*}
$$

(ii) $\quad 75.5 \mathrm{~kg}=0.075 \mathrm{~T}$

$$
0.0755=\frac{M}{51080}
$$

$$
M=0.0755 \times 51080
$$

$$
\begin{equation*}
=3856.54 \text { tonnes } \tag{2}
\end{equation*}
$$

$$
\begin{aligned}
\text { (iii) Surface area } & =0.5 \times 4 \times \pi \times 29^{2} \\
& =5284.1588 \ldots \\
& =5284 \mathrm{~m}^{2} \text { (to nearest metre) } \\
\text { Area tile } & =0.06 \times 0.06 \\
& =0.0036 \mathrm{~m}^{2} \\
\text { Number of tiles needed } & =5284.1588 \div 0.0036 \\
& =1467821.9 . \\
& =1468000 \text { to nearest thousand }
\end{aligned}
$$

b. (i) The largest angle is opposite the longest side. Since 18.5 cm is the longest side, $\angle \mathrm{PQR}$ is
the largest angle
(ii) $\cos Q=\frac{12.5^{2}+15^{2}-18.5^{2}}{2 \times 12.5 \times 15}$

$$
=0.104
$$

$$
\mathrm{Q}=84.03 \ldots
$$

$$
\begin{equation*}
\mathrm{Q}=84^{\circ} \text { to nearest degree } \tag{3}
\end{equation*}
$$

(iii) $\frac{\sin R}{12.5}=\frac{\sin 84^{\circ}}{18.5}$

$$
\sin R=0.67197 \ldots
$$

$$
R=42.2196 \ldots
$$

$$
\begin{equation*}
\mathrm{R}=42^{\circ} \text { to nearest degree } \tag{2}
\end{equation*}
$$

(iv) Bearing of P from $\mathrm{R}=270^{\circ}+42^{\circ}$

$$
\begin{equation*}
=312^{\circ} \tag{1}
\end{equation*}
$$

26. a. (i) Town $A \quad$ mean $=25.08$
mode $=28,30$
median $=26.5$
Town B mean $=26.42$
mode $=$ none
median $=26.5$
(ii)Town A range =31-17

$$
=14
$$

interquartile range $=29-21$

$$
=8
$$

standard deviation $=4.68$
Town B

$$
\begin{align*}
\text { range } & =36-13 \\
& =23
\end{aligned} \begin{aligned}
\text { interquartile range } & =33-21 \\
& =12 \\
\text { standard deviation } & =7.06
\end{align*}
$$

(iii)

(3)
(iv) While the median temperature for Town $A$ and Town $B$ is the same, there is a greater range and interquartile range for Town $B$ compared to Town $A$. This can also been seen in the differences in standard deviation.
b. $l=\frac{k}{v} \quad 20.75=\frac{k}{16}$

$$
\begin{aligned}
k & =20.75 \times 16 \\
& =332
\end{aligned}
$$

$l=\frac{332}{v}$

$$
13.28=\frac{332}{v}
$$

$$
\mathrm{V}=\frac{332}{13.28}
$$

$$
=25 \text { vibrations } / \mathrm{s}
$$

(3)
27. a. (i) 50 L
(ii) Vehicle B (iii) 620 km
(iv) $12 \mathrm{~L} / 100 \mathrm{~km}$
(v) 9 L
(vi) 90 km

$$
\text { (vii) } \begin{array}{rlr}
P & =50-n d & P=0, d=400 \\
0 & =50-n \times 400 & \\
n & =\frac{50}{400} & \\
& =0.125 & \text { (1) each part }
\end{array}
$$

b. (i) $\sin 40^{\circ}=\frac{h}{70}$

$$
\begin{align*}
\mathrm{h} & =70 \times \sin 40^{\circ} \\
& =45 \mathrm{~m} \tag{2}
\end{align*}
$$

(ii) Pylon height $=45+55$

$$
\begin{equation*}
=100 \mathrm{~m} \tag{1}
\end{equation*}
$$

(iii) $\tan \theta=\frac{55}{140}$ $\theta=21^{\circ}$
angle of elevation is $21^{\circ}$
(3)
28. a. (i) $H=6+2 k^{3}$

$$
\begin{align*}
60 & =6+2 k^{3} \\
56 & =2 k^{3} \\
27 & =k^{3} \\
k & =3 \tag{1}
\end{align*}
$$

(ii) if $k=6, \quad H=6+2 \times 6^{3}$

$$
=438
$$

Increase in $\mathrm{H}=438$ - 60

$$
=378
$$

$$
\text { changed }=\frac{378}{60} \times 100 \%
$$

$$
\begin{equation*}
=630 \% \tag{2}
\end{equation*}
$$

b. (i) $\mathrm{A}=\frac{2.1}{3}[0+4 \times 3+4.3]+\frac{2.1}{3}[4.3+4 \times 3.4+0]$

$$
\begin{equation*}
=23.94 \mathrm{~m}^{2} \tag{3}
\end{equation*}
$$

(ii) $V=50 \times 23.94$

$$
\begin{equation*}
=1197 \mathrm{~m}^{3} \tag{1}
\end{equation*}
$$

(iii) $0.35 \mathrm{~m} / \mathrm{s}$ for 1 hour

$$
0.35 \times 60 \times 60=1260 \mathrm{~m}
$$

$$
V=1260 \times 23.94
$$

$$
=30164 \mathrm{~m}^{3}
$$

$$
\begin{equation*}
=30164000 \mathrm{~L} \tag{2}
\end{equation*}
$$

c. (i) $\angle J O A=37+23$

$$
\begin{equation*}
=60^{\circ} \tag{1}
\end{equation*}
$$

(ii) distance $=\frac{60}{360} \times 2 \times \pi \times 6400$

$$
\begin{equation*}
=6702 \mathrm{~km} \tag{2}
\end{equation*}
$$

(iii) $\frac{6702}{685}=9.78 \mathrm{hrs}$

$$
\begin{equation*}
=9 \text { hours } 47 \text { minutes } \tag{1}
\end{equation*}
$$

