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Founded 1982

## THE HILLS GRAMMAR SCHOOL

## YEAR 12

## Trial HSC EXAMINATION 2015

## MATHEMATICS GENERAL

Time Allowed:
Teacher Responsible:

Two and a half hours (plus five minutes reading time)
Mr O'Neill

## Instructions

- Attempt all questions.
- Answer all questions in Section I on the Multiple Choice Answer sheet provided at the back of the examination paper.
- Answer all questions in Section II in the spaces provided.
- Write using blue or black pen.
- Diagrams are not to scale.
- Marks may be deducted for careless, untidy or badly arranged work.

| Section I | Section II <br> Question 26 | Section II <br> Question 27 | Section II <br> Question 28 | Section II <br> Question 29 | Section II <br> Question 30 | TOTAL |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |

## Section I

## 25 marks

Attempt Questions 1-25
Allow about 35 minutes for this section
Use the multiple-choice answer sheet for Questions 1-25

1 Simplify $6 x^{2}+2 x+4 x^{2}$
(A) $12 x^{2}$
(B) $12 x^{4}$
(C) $10 x^{2}+2 x$
(D) $10 x^{4}+2 x$

2 Which of the following represents 0.5 MB to nearest 1000 ?
(A) 430000 B
(B) 524000 B
(C) 530000 B
(D) 525000 B

3 Which of these functions represents exponential growth?
(A) $y=2\left(5^{x}\right)$
(B) $y=3(0.2)^{x}$
(C) $y=\frac{3}{x}$
(D) $y=4 x^{3}$

4 The normal distribution shows the results of a mathematics assessment task. It has a mean of 60 and a standard deviation of 10


What percentage of results lies in the shaded region?
(A) $16 \%$
(C) $34 \%$
(B) $32 \%$
(D) $68 \%$

5 Use the table below to calculate the present value of an annuity where $\$ 12,000$ is contributed each year for six years into an account earning 3\% per annum compound interest.

| Present value of \$1 |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| End of year | $3 \%$ | $4 \%$ | $5 \%$ | $6 \%$ |  |
| 5 | 4.5797 | 4.4518 | 4.3295 | 4.2124 |  |
| 6 | 5.4172 | 5.2421 | 5.0757 | 4.9173 |  |
| 7 | 6.2303 | 6.0021 | 5.7864 | 5.5824 |  |
| 8 | 7.0197 | 6.7327 | 6.4632 | 6.2098 |  |

(A) $\$ 15,183.83$
(B) $\$ 54,956.40$
(C) $\$ 65,006.40$
(D) $\$ 72,000.00$

6 Zac is driving at a speed $(\mathrm{V})$ of $80 \mathrm{~km} / \mathrm{h}$. It takes Zac two seconds to react to a dangerous situation before applying the brakes. The stopping distance ( $d$ ) is given by the formula:

> Stopping distance: $d=\frac{5 V t}{18}+\frac{V^{2}}{170}$
> Where $\boldsymbol{V}=$ speed of vehicle
> $\boldsymbol{t}=$ reaction time in seconds

How far will Zac travel in his car after applying the brakes using this formula?
(A) 60 m
(B) 82 m
(C) 164 m
(D) 246 m

7 The angle of depression from $A$ to $C$ is $40^{\circ}$. The distance from $B$ to $C$ is 15 metres.


Not to scale

How high above $B$ is $A$, to the nearest metre?
(A) 10 m
(B) 11 m
(C) 13 m
(D) 18 m

8 Jasmine purchased a used car for $\$ 7500$ and depreciated it by $\$ 700$ each year. What is its depreciated value after three years?
(A) $\$ 4700$
(B) $\$ 5400$
(C) $\$ 6100$
(D) $\$ 6800$

9 Adelaide is located at $\left(35^{\circ} \mathrm{S}, 139^{\circ} \mathrm{E}\right)$ and Yokohama is located at $\left(35^{\circ} \mathrm{N}, 139^{\circ} \mathrm{E}\right)$. What is the distance between Adelaide and Yokohama to the nearest kilometre? (Assume the radius of the earth is 6400 km ).
(A) 559
(B) 3910
(C) 7819
(D) 15526

10 What is the gradient of the least-squares regression line given $r=0.561, s_{x}=1.987$ and $s_{y}=4.579$ ?
(A) 0.24
(B) 1.29
(C) 7.13
(D) 16.21

11 In the diagram, $O$ represents the centre of the earth, and $Q$ lies on both the Equator and the Greenwich Meridian.


Not to scale

What is the latitude and longitude of point $P$ ?
(A) $35^{\circ} \mathrm{N} 105^{\circ} \mathrm{E}$
(B) $35^{\circ} \mathrm{N} 105^{\circ} \mathrm{W}$
(C) $55^{\circ} \mathrm{N} 105^{\circ} \mathrm{E}$
(D) $55^{\circ} \mathrm{N} 105^{\circ} \mathrm{W}$

12 A 120 watt ceiling fan is run for 24 hours each day. If electricity is charged at $24.8 \mathrm{c} / \mathrm{kWh}$, what is the cost of running the ceiling fan for 30 days, to the nearest cent?
(A) $\$ 15.68$
(B) $\$ 21.43$
(C) $\$ 86.40$
(D) $\$ 2142.73$

13 Lachlan earns $\$ 81500$ in a year. His allowable deductions total $\$ 4000$. Using the table below, which of the following expressions represents his total tax payable?

| Taxable income | Tax payable |
| :--- | :--- |
| $0-\$ 18200$ | Nil |
| $\$ 18201-\$ 37000$ | Nil +19 cents for each $\$ 1$ over $\$ 18200$ |
| $\$ 37001-\$ 80000$ | $\$ 3572+32.5$ cents for each $\$ 1$ over $\$ 37000$ |
| $\$ 80001-\$ 180000$ | $\$ 17550+37$ cents for each $\$ 1$ over $\$ 80000$ |
| $\$ 180001$ and over | $\$ 54550+45$ cents for each $\$ 1$ over $\$ 180000$ |

(A) $\$ 3572+\$ 40500 \times 0.325$
(B) $\$ 3572+\$ 44500 \times 0.325$
(C) $\$ 17550+\$ 1500 \times 0.37$
(D) $\$ 17550+\$ 5500 \times 0.37$

14 What is the best description between living standards and life expectancy?
(A) Constant correlation
(B) Negative correlation.
(C) Positive correlation.
(D) Zero correlation.

15 What is the size of the smallest angle $(\theta)$ in the triangle below?


Not to scale
(A) $\cos \theta=\frac{16^{2}+18^{2}-10^{2}}{2 \times 10 \times 16}$
(B) $\cos \theta=\frac{10^{2}+18^{2}-16^{2}}{2 \times 10 \times 18}$
(C) $\cos \theta=\frac{10^{2}+18^{2}-16^{2}}{2 \times 10 \times 16}$
(D) $\cos \theta=\frac{16^{2}+18^{2}-10^{2}}{2 \times 16 \times 18}$

16 Oscar borrows $\$ 800$ over 3 years at an interest rate of $9.5 \%$ p.a. Calculate the simple interest?
(A) $\$ 29$
(B) $\$ 86$
(C) $\$ 228$
(D) $\$ 343$

17 A factory produces bags of cashews. The weights of the bags are normally distributed, with a mean of 900 g and a standard deviation of 50 g . What is the best approximation for the percentage of bags that weigh more than 1000 g ?
(A) $0 \%$
(B) $2.5 \%$
(C) $5 \%$
(D) $16 \%$

18 The number of residents at Ashcroft is expected to increase using the formula $N=3000 t^{3}$, where $N$ is the number of residents and $t$ is the time in years. What is the expected number of residents of Ashcroft after three years?
(A) 9000
(B) 27000
(C) 78000
(D) 81000

19 Which of the following correctly expresses $c$ as the subject of $A=b c^{2}+d$ ?
(A) $c= \pm \sqrt{\frac{A-d}{b}}$
(B) $c= \pm \frac{\sqrt{A-d}}{b}$
(C) $c= \pm \sqrt{\frac{A}{b}}-d$
(D) $c= \pm \sqrt{\frac{A}{b}-d}$

20 The speed ( $v$ ), in $\mathrm{km} / \mathrm{h}$, of a ski lift is inversely proportional to the weight ( $w \mathrm{~kg}$ ) it carries. A ski lift carrying a weight of 320 kg can travel at $16 \mathrm{~km} / \mathrm{h}$. What is the speed of the ski lift if weight decreases to 250 kg ?
(A) $\frac{16 \times 250}{320}$
(B) $\frac{16 \times 320}{250}$
(C) $\frac{250 \times 320}{16}$
(D) $\frac{320}{16 \times 250}$

21 What is the correlation between the variables in this scatterplot?

(A) Low negative
(B) Low positive
(C) High negative
(D) High positive

22 A radial survey is shown below.


Find the area of the $\triangle R O S$ correct to the nearest square metre.
(A) $5 \mathrm{~m}^{2}$
(B) $9 \mathrm{~m}^{2}$
(C) $11 \mathrm{~m}^{2}$
(D) $12 \mathrm{~m}^{2}$

23 A field is bordered by three straight fences and a garden as shown below.

Not to Scale


What is the area of the field (in square metres)?
(A) $\frac{11}{3}(12+7+10)$
(B) $\frac{11}{3}(12+28+10)$
(C) $\frac{22}{3}(12+7+10)$
(D) $\frac{22}{3}(12+28+10)$

24 Sascha measured a piece of material for her HSC major work as 200 mm correct to the nearest millimetre. What is the percentage error in her measurement?
(A) $\pm 0.0025 \%$
(B) $\pm 0.005 \%$
(C) $\pm 0.25 \%$
(D) $\pm 0.5 \%$

25 A car depreciates in value from $\$ 39000$ to $\$ 12250$ in four years using the declining balance method. What is the annual rate of depreciation to the nearest whole number?
(A) $17 \%$
(B) $18 \%$
(C) $25 \%$
(D) $26 \%$

## Section II

75 marks
Attempt Questions 26 - 30
Allow about 1 hour and 55 minutes for this section
Answer the questions in the spaces provided.
Your responses should include relevant mathematical reasoning and/or calculations.

Question 26 (15 marks)
Marks
(a) Sophie borrowed $\$ 192000$ at an interest rate of $5.25 \%$ per annum compounded monthly. The repayments have been set at $\$ 900$ per month.

| Months (n) | Principal (P) | Interest (I) | Repayment | Balance |
| :---: | :---: | :---: | :---: | :---: |
| 1 | $\$ 192000$ | $\$ 840$ | $\$ 900$ |  |
| 2 |  |  | $\$ 900$ |  |

(i) Show by calculation that the interest charged for the first month is
$\$ 840$.
$\qquad$
$\qquad$
(ii) How much is owed at the end of the first month?
$\qquad$
$\qquad$
(iii) What is the interest charged for the second month? Answer to the nearest cent.
$\qquad$
$\qquad$
(b) Simplify $\left(6 x y^{2}\right)^{3}$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(c) Draw the graph of $y=-2 x+3$ and find the gradient and $y$-intercept.

(d) An energy company charges for gas over a 3-month period are shown below.

| Usage charge | First 2000 MJ | $\$ 0.02580$ per MJ |
| :--- | :--- | :--- |
|  | Additional MJ over 2000 | $\$ 0.01620$ per MJ |

(i) Savannah used 5000 MJ of gas in this period. What is the charge?
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(ii) Gabriel has decided to reduce his energy bills. He has a target of \$80 for gas. What is the maximum number of MJ he is allowed in this period? Answer correct to the nearest megajoule.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(e) A credit card has a daily simple interest rate of $0.059 \%$ per day (no interest free period). Find the interest charged on $\$ 2210$ for 14 days. Answer correct to the nearest cent.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

Question 27 (15 marks)
(a) Alex is planning a trip to Toronto $\left(44^{\circ} \mathrm{N} 79^{\circ} \mathrm{W}\right)$ from Sydney $\left(34^{\circ} \mathrm{S} 151^{\circ} \mathrm{E}\right)$.
(i)
What is the difference in time between Toronto and Sydney to the nearest minute?
(ii) Alex plans to leave Sydney at 6 am on a Sunday. What is the time in Toronto when he leaves Sydney?
$\qquad$
$\qquad$
$\qquad$
(b) The diagram shows a semicircle, from which a trapezium has been cut.

All measurements are in centimetres.


What is the shaded area, to the nearest square centimetre?
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(c) Clark's rule $\left(\right.$ Dosage $\left.=\frac{\text { Weight }(\mathrm{kg}) \times \text { Adult dose }}{70}\right)$ is used to calculate dosages of medicine for children. What is the medication dose for Tyler, if he weighs 28 kg and the adult dose is 15 mL ?
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(d) The diagram shows three towns. Town $A$ is due west of town $B$ and the bearing of town $C$ from town $B$ is $025^{\circ}$.


Not to scale
(i) What is the size of $\angle A B C$ ?
$\qquad$
$\qquad$
(ii) Find the distance (to nearest kilometre) from town $A$ to town $C$.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(iii) What is the bearing of town $C$ from town $A$ ?
$\qquad$
$\qquad$
$\qquad$
$\qquad$

Question 28 (15 marks)
Marks
(a) Solve the following equations:
(i) $9 x-2=-11$

2
(ii) $\frac{4 y}{3}+3 y-5=0$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(b) A rain gauge registered 60 mm of rain during a storm. The rain fell on a shed with a rectangular roof that measures 25 metres by 10 metres.
(i) How many litres of water fell on the shed? Answer to the nearest litre
$\qquad$
$\qquad$
$\qquad$
(ii) The water that fell on the shed was collected in an empty cylindrical tank with a diameter of 6 metres. What depth of water will be in the tank? Answer correct to two decimal places.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(c) The table below shows forearm length and hand length.

| Forearm (in cm) | 25.0 | 25.5 | 26.0 | 26.5 | 27.0 | 27.5 | 28.0 | 28.5 | 29.0 | 29.5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Hand (in cm) | 17.3 | 17.6 | 18.2 | 18.4 | 19 | 19.4 | 19.8 | 20.1 | 20.4 | 20.6 |

(i) Draw a scatterplot using the above table.

1

(ii) Draw a line of best fit on the scatterplot.
(iii) Calculate the correlation coefficient between the forearm length and 1 hand length.
$\qquad$
$\qquad$
(d) Emily borrows $\$ 2700$ to buy a dining table. Her repayments are $\$ 135$ a month for two years.
i) What interest does Emily pay?
ii) What flat rate of interest per annum has Emily been 2 charged?

Question 29 (15 marks)
(a) Young's rule is used to prescribe medicine for children. The formula is:

Young's rule: $D=\frac{y A}{y+12}$
Where $\mathrm{D}=$ child dosage
$y=$ age in years
$A=$ adult dosage
Owen buys a prescription for 1200 mg of medicine. The adult dose is 50 mg and the recommended dose for Owen's child is 10 mg .
(i) How old is Owen's child using Young's rule?
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(ii) How many doses for Owen's child are contained in the prescription?
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(iii) It is recommended the medicine be taken at most 4 times a day. How many days will the prescription last at this rate for Owen's child?
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(b) The capture-recapture technique was used to estimate the population of penguins in 2013.

- 50 penguins were caught, tagged and released.
- Later, 110 penguins were caught at random.
- 20 of these 110 penguins had been tagged.

The estimated population of the penguins in 2013 was $13 \%$ less than the estimated population for 2012.
What was the estimated population for 2012?
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(c) Jack has a mobile phone contract that charges a monthly access fee of $\$ 79, \$ 250$ worth of calls are free, flag fall $\$ 0.35$ and call rate of $\$ 0.45$ per 30 second. What is the monthly charge if Jack made 400 calls whose duration was less than 30 seconds?
$\qquad$
$\qquad$
$\qquad$
(d) Charlotte is 57 kg and has consumed 5 standard drinks in the past four hours. She was stopped by police for a random breath test.
What would be Charlotte's blood alcohol content? Answer correct to 2 decimal places.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
e) Solve the following pair of simultaneous equations.

$$
\begin{gathered}
2 x-3 y=-1 \\
x=10-2 y
\end{gathered}
$$

$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(f) Sally's recent results in hospitality and timber are recorded in the table

| Course | Class Mean | Class Standard Deviation | Sally's Result |
| :--- | :---: | :---: | :---: |
| Hospitality | 55 | 10 | 85 |
| Timber | 55 | 15 | 85 |

(i) What is Sally's $z$-score for timber?
$\qquad$
(ii) What hospitality mark would be equivalent to a $z$-score of -1 ?
$\qquad$
$\qquad$
(iii) What percentage of students in Sally's class scored a mark between 45 and 75 for Hospitality?
$\qquad$
$\qquad$
$\qquad$

Question 30 (15 marks)
(a) Michael invests $\$ 3125$ at $12 \%$ per annum compounding quarterly.

How much will he have after 4 years? Answer to the nearest dollar.
2
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(b) Using the compound interest formula, calculate the present value of an annuity whose future value is $\$ 480,000$ over 8 years with an interest rate of $8.2 \%$ per annum compounded monthly. Answer correct to the nearest cent.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(c) The table below shows the future value on $\$ 1$ compounding at the interest rate per period.

| Future value of \$1 |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| End of <br> period | $4 \%$ | $6 \%$ | $8 \%$ | $10 \%$ |  |
| 1 | 1.00 | 1.00 | 1.00 | 1.00 |  |
| 2 | 2.04 | 2.06 | 2.08 | 2.10 |  |
| 3 | 3.12 | 3.18 | 3.25 | 3.31 |  |
| 4 | 4.25 | 4.37 | 4.51 | 4.64 |  |

Calculate the future value of a $\$ 32000$ annuity for 2 years at $8 \%$ p.a. compounded half yearly.
(d) An asteroid reached earth and exploded in the Pacific Ocean. The distance (in km) it travelled through the earth's atmosphere varied directly as the square of the time ( $t \mathrm{sec}$ ) it had been travelling. The asteroid travelled 384 kilometres in the first 16 seconds.
i) How far did the asteroid travel in the first 10 seconds?

2
$\qquad$
$\qquad$
$\qquad$
$\qquad$
ii) How long will it take for the asteroid to travel 294 kilometres?
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(e) Calculate the capacity of the following in Megalitres using Simpson’s Rule for volume. 2

$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
f) The angle of elevation from a boat out to sea to the top of a 220 -metre cliff is $37^{\circ}$. After travelling directly towards the cliff the angle of elevation from the boat to the top of the cliff is $56^{\circ}$.
i) Draw a diagram representing the information above 1
ii) How far did the boat travel towards the cliff? Answer correct to the nearest metre. 2
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

End of paper

## FORMULAE AND DATA SHEET

## Financial Mathematics

## Simple interest

$$
I=P r n
$$

$P$ is initial amount
$r$ is interest rate per period, expressed as a decimal
$n$ is number of periods

## Compound interest

$$
A=P(1+r)^{n}
$$

$A$ is final amount
$P$ is initial amount
$r$ is interest rate per period, expressed as a decimal
$n$ is number of compounding periods

## Present value and future value

$$
P V=\frac{F V}{(1+r)^{n}}, F V=P V(1+r)^{n}
$$

$r$ is interest rate per period, as expressed as a decimal
$n \quad$ is number of compounding periods

## Straight-line method of depreciation

$$
S=V_{0}-D n
$$

$S \quad$ is salvage value of asset after $n$ periods
$V_{0}$ is initial value of asset
$D$ is amount of depreciation per period
$n$ is number of periods

## Declining-balance method of depreciation

$$
S=V_{0}(1-r)^{n}
$$

$S \quad$ is salvage value of asset after $n$ periods
$V_{0}$ is initial value of asset
$r$ is depreciation rate per period, expressed as a decimal
$n \quad$ is number of periods

## Data Analysis

## Mean of a sample

$$
\bar{x}=\frac{\text { sum of scores }}{\text { number of scores }}
$$

## z-score

For any score $x$,

$$
z=\frac{x-\bar{x}}{s}
$$

$\bar{x}$ is mean
$S$ is standard deviation

## Outlier(s)

score(s) less than $Q_{L}-1.5 \times I Q R$ or
score(s) more than $Q_{U}+1.5 \times I Q R$
$Q_{L}$ is lower quartile
$Q_{U}$ is upper quartile
$I Q R$ is interquartile range

## Least-squares line of best fit

$$
y=\text { gradient } \times x+y \text {-intercept }
$$

gradient $=r \times \frac{\text { standard deviation of } y \text { scores }}{\text { standard deviation of } x \text { scores }}$

$$
y \text {-intercept }=\bar{y}-(\text { gradient } \times \bar{x})
$$

$r$ is correlation coefficient
$\bar{x}$ is mean of $x$ score
$\bar{y}$ is mean of $y$ scores

## Normal distribution

- approximately $68 \%$ of scores have z-scores between -1 and 1
- approximately 95\% of scores have
z-scores between -2 and 2
- approximately $99.7 \%$ of scores have $z$-scores between -3 and 3


## Spherical Geometry

## Circumference of a circle

$$
C=2 \pi r \text { or } C=\pi d
$$

$r$ is radius
$d$ is diameter
Arc length of a circle

$$
l=\frac{\theta}{360} 2 \pi r
$$

$r$ is radius
$\theta$ is number of degrees in central angle

## Radius of Earth

(taken as) 6400 km

## Time differences

For calculation of time differences using longitude: $15^{\circ}=1$ hour time difference

## Area

## Circle

$$
A=\pi r^{2}
$$

$r$ is radius

## Sector

$$
A=\frac{\theta}{360} \pi r^{2}
$$

$r$ is radius
$\theta$ is number of degrees in central angle

## Annulus

$$
A=\pi\left(R^{2}-r^{2}\right)
$$

$R$ is radius of outer circle
$r$ is radius of inner circle

## Trapezium

$$
A=\frac{h}{2}(a+b)
$$

$h$ is perpendicular height
$a$ and $b$ are the lengths of the parallel sides

## Area of land and catchment areas

unit conversion: 1 ha $=10000 \mathrm{~m}^{2}$

## Surface Area

Sphere

$$
A=4 \pi r^{2}
$$

$r$ is radius

## Closed cylinder

$$
A=2 \pi r^{2}+2 \pi r h
$$

$r$ is radius
$h$ is perpendicular height

## Volume

Prism or cylinder

$$
V=A h
$$

$r$ is radius
$h$ is perpendicular height

## Pyramid or cone

$$
V=\frac{1}{3} A h
$$

$A$ is area of the base
$h$ is perpendicular height

## Volume and capacity

unit conversion: $1 \mathrm{~m}^{3}=1000 \mathrm{~L}$

## Approximation Using Simpson's Rule

Area

$$
A \approx \frac{h}{3}\left(d_{f}+4 d_{m}+d_{l}\right)
$$

$h \quad$ distance between successive measurements
$d_{f}$ is first measurement
$d_{m}$ is middle measurement
$d_{l}$ is last measurement

## Volume

$$
V \approx \frac{h}{3}\left(A_{L}+4 A_{m}+A_{R}\right)
$$

$h \quad$ distance between successive measurements
$A_{L}$ is area of left end
$A_{M}$ is area of middle
$A_{R}$ is area of right end

## Trigonometric Ratios



$$
\begin{aligned}
& \sin \theta=\frac{\text { opposite side }}{\text { hypotenuse }} \\
& \cos \theta=\frac{\text { adjacent side }}{\text { hypotenuse }} \\
& \tan \theta=\frac{\text { opposite side }}{\text { adjacent side }}
\end{aligned}
$$

## Sine rule

In $\triangle A B C$

$$
\frac{a}{\sin \mathrm{~A}}=\frac{b}{\sin \mathrm{~B}}=\frac{c}{\sin \mathrm{C}}
$$

## Area of Triangle

$$
A=\frac{1}{2} a b \operatorname{Sin} C
$$

## Cosine rule

In $\triangle A B C$

$$
\begin{gathered}
c^{2}=a^{2}+b^{2}-2 a b \cos C \\
\text { or } \\
\cos C=\frac{a^{2}+b^{2}-c^{2}}{2 a b}
\end{gathered}
$$

## Units of Memory and File Size

1 byte $=8$ bits
1 kilobyte $=2^{10}$ bytes $=1024$ bytes
1 megabyte $=2^{20}$ bytes $=1024$ kilobytes
1 gigabyte $=2^{30}$ bytes $=1024$ megabytes
1 terabyte $=2^{40}$ bytes $=1024$ gigabytes

## Blood Alcohol Content Estimates

$$
\begin{aligned}
& B A C_{\text {Male }}=\frac{(10 N-7.5 H)}{6.8 M} \text { or } \\
& B A C_{\text {Female }}=\frac{(10 \mathrm{~N}-7.5 \mathrm{H})}{5.5 \mathrm{M}}
\end{aligned}
$$

$N$ is number of standard drinks consumed
$H$ is number of hours of drinking
$M$ is person's mass in kilograms

## Distance, Speed and Time

$$
D=S T, S=\frac{D}{T}, T=\frac{D}{S}
$$

average speed $=\frac{\text { total distance travelled }}{\text { total time taken }}$
stopping distance $=\left\{\begin{array}{c}\text { reaction-time } \\ \text { distance }\end{array}\right\}+\left\{\begin{array}{c}\text { braking } \\ \text { distance }\end{array}\right\}$

## Probability of an Event

The probability of an event where outcomes are equally likely is given by:

$$
P(\text { event })=\frac{\text { number of favourable outcomes }}{\text { total number of outcomes }}
$$

## Straight Lines

## Gradient

$$
m=\frac{\text { vertical change in position }}{\text { horizontal change in position }}
$$

## Gradient-intercept form

$$
y=m x+b
$$

$m$ is gradient
$b$ is $y$-intercept

## MATHEMATICS: MULTIPLE CHOICE ANSWER SHEET

Student: $\qquad$ Teacher: $\qquad$
Select the alternative A, B, C or D that best answers the question. Fill in the response circle completely.
Sample:
$2+4=$
A. 2
B. 6
C. 8
D. 9


If you think you have made a mistake, put a cross through the
 (C) incorrect answer and fill in the new answer.
If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word correct and drawing an arrow towards the correct answer.

| 1 | A | B | C | (D |
| :--- | :---: | :---: | :---: | :---: |
| 2 | A | B | C | (D |
| 3 | A | B | C | (D |
| 4 | A | B | C | (D |
| 5 | A | B | C | D |
| 6 | A | B | C | (D |
| 7 | A | B | C | (D |
| 8 | A | B | C | D |
| 9 | A | B | C | D |
| 10 | A | B | C | D |
| 11 | A | B | C | D |
| 12 | A | B | C | D |
| 13 | A | B | C | D |
| 14 | A | B | C | D |
| 15 | A | B | C | D |


| 16 | A | B | C | (D |
| :--- | :--- | :--- | :--- | :--- |
| 17 | A | B | C | (D |
| 18 | A | B | C | (D |
| 19 | A | B | C | D |
| 20 | A | B | C | D |
| 21 | A | B | C | D |
| 22 | A | B | C | (D |
| 23 | A | B | C | D |
| 24 | A | B | C | D |
| 25 | A | B | C | D |

## MATHEMATICS: MULTIPLE CHOICE ANSWER SHEET

Student: $\qquad$ Teacher: $\qquad$
Select the alternative A, B, C or D that best answers the question. Fill in the response circle completely.
Sample:

$$
2+4=
$$

A. 2
B. 6
C. 8
D. 9
(A)
(C)
(D)

If you think you have made a mistake, put a cross through the
 incorrect answer and fill in the new answer.
If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word correct and drawing an arrow towards the correct answer.



$$
\begin{aligned}
& \text { Most poorly attempted were } \\
& 3,11,12,14,20,22,24,25 .
\end{aligned}
$$

## Section II

## 75 marks

Attempt Questions 26-30
Allow about 1 hour and 55 minutes for this section
Answer the questions in the spaces provided.
Your responses should include relevant mathematical reasoning and/or calculations.

## Question 26 ( 15 marks)

(a) Sophie borrowed $\$ 192000$ at an interest rate of $5.25 \%$ per annum compounded monthly. The repayments have been set at $\$ 900$ per month.

| Months ( $\boldsymbol{n}$ ) | Principal ( $\boldsymbol{P}$ ) | Interest $(\boldsymbol{I})$ | Repayment | Balance |
| :---: | :---: | :---: | :---: | :---: |
| 1 | $\$ 192000$ | $\$ 840$ | $\$ 900$ | 191940 |
| 2 | 191940 | $\$ 839.74$ | $\$ 900$ |  |

(i) Show by calculation that the interest charged for the first month is

(ii) How much is owed at the end of the first month?

$$
\begin{aligned}
192000+840-900 & =191940
\end{aligned} \quad \text { Most wiorreet } 1
$$

(iii) What is the interest charged for the second month? Answer to the

2 nearest cent.

$$
\$ 839.74
$$

(b) Simplify $\left(6 x y^{2}\right)^{3}$

(c) Draw the graph of $y=-2 x+3$ and find the gradient and $y$-intercept.


| $m$ | $=-2$ |
| ---: | :--- |
| $b$ | $=3$ |

$\longrightarrow$

e) An energy company charges for gas over a 3-month period are shown below.

| Usage charge | First 2000 MJ | $\$ 0.02580$ per MJ |
| :--- | :--- | :--- |
|  | Additional MJ over 2000 | $\$ 0.01620$ per MJ |

(i) Savannah used 5000 MJ of gas in this period. What is the charge?
$\qquad$
*We ll Done.
(ii) Gabriel has decided to reduce his energy bills. He has a target of $\$ 80$
for gas. What is the maximum number of MJ he is allowed in this period? Answer correct to the nearest megajoule.

$$
\begin{aligned}
& \begin{aligned}
\text { * Fest incirres } \quad \begin{aligned}
\text { Frost } 2000 & =2000 \times 0.02580 \\
& =\$ .51-60
\end{aligned}
\end{aligned} \\
& \text { was } 1753 \mathrm{~kJ} \quad \therefore \quad 80-51.60=28-40 \\
& \begin{aligned}
& \text { forgot to } \\
& \text { and } 2000 \quad \therefore 2000+1753 \\
&=3753 \mathrm{KJ} .
\end{aligned}
\end{aligned}
$$

(f) A credit card has a daily simple interest rate of $0.059 \%$ per day (no interest free period). Find the interest charged on $\$ 2210$ for 14 days. Answer correct to the nearest cent.
$\qquad$

$$
I=2210 \times 0.059 \% \times 14
$$

$$
I=\$ 18.25 c^{\prime}
$$

$$
r
$$

$\qquad$ Done
(a) Alex is planning a trip to Toronto $\left(44^{\circ} \mathrm{N} 79^{\circ} \mathrm{W}\right)$ from Sydney $\left(34^{\circ} \mathrm{S} 151^{\circ} \mathrm{E}\right)$.
(i)

What is the difference in time between Toronto and Sydney to the nearest minute?

Most inccrrectwas
$\qquad$
(ii) Alex plans to leave Sydney at 6 am on a Sunday. What is the time in Toronto when he leaves Sydney?


* MC
(b) The diagram shows a semicircle, from which a trapezium has been cut. All measurements are in centimetres.


What is the shaded area, to the nearest square centimetre?

(c) Clark's rule $\left(\right.$ Dosage $\left.=\frac{\text { Weight }(\mathrm{kg}) \times \text { Adult dose }}{70}\right)$ is used to calculate dosages of medicine for children. What is the medication dose for Tyler, if he weighs 28 kg and the adult dose is 15 mL ?


$$
w_{e} \| D_{\text {ow }}
$$

(d) The diagram shows three towns. Town $A$ is due west of town $B$ and the bearing of town $C$ from town $B$ is $025^{\circ}$.

(i) What is the size of $\angle A B C$ ?
$\qquad$
(ii) Find the distance (to nearest kilometre) from town $A$ to town $C$. \& work well que.

(iii) What is the bearing of town $C$ from town $A$ ?

(a) Solve the following equations:
(i) $\quad 9 x-2=-11$

$$
\begin{aligned}
& 9 x+2=-9 \text { several students subtract } \\
& x=-1 \\
& \text { two }
\end{aligned}
$$

$\qquad$
(ii) $\frac{4 y}{3}+3 y-5=0 x^{3} x^{3}$

(b) A rain gauge registered 60 mm of rain during a storm. The rain fell on a shed with a rectangular roof that measures 25 metres by 10 metres.
(i) How many litres of water fell on the shed? Answer to the nearest litre

recognise $\operatorname{lm}^{3}=1 C C O L$
(ii) The water that fell on the shed was collected in an empty cylindrical tank with a diameter of 6 metres. What depth of water will be in the tank? Answer correct to two decimal places.

$$
\begin{aligned}
& \begin{array}{r}
15=\pi \times 3^{2} \times h \\
h=0.53 m .
\end{array} \\
& \text { * (using the wacrerect } \\
& \text { bemula } \\
& V=\pi r^{2} n \quad e c f(i) \\
& V=15 \mathrm{~m}^{3}
\end{aligned}
$$

(c) The table below shows forearm length and hand length.

| Forearm (in cm) | 25.0 | 25.5 | 26.0 | 26.5 | 27.0 | 27.5 | 28.0 | 28.5 | 29.0 | 29.5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Hand (in cm) | 17.3 | 17.6 | 18.2 | 18.4 | 19 | 19.4 | 19.8 | 20.1 | 20.4 | 20.6 |

(i) Draw a scatterplot using the above table.

(ii) Draw a line of best fit on the scatterplot.
Must be on line
(iii) Calculate the correlation coefficient between the forearm length and hand length.

in spat made to find $r$
(d) Emily borrows $\$ 2700$ to buy a dining table. Her repayments are $\$ 135$ a month for two years.
i) What interest does Emily pay?
$\qquad$
$\qquad$
ii) What flat rate of interest per annum has Emily been charged?


Question 29 ( 15 marks)
(a) Young's rule is used to prescribe medicine for children. The formula is:

Young's rule: $D=\frac{y A}{y+12}$
Where $\mathbf{D}=$ child dosage
$y=$ age in years
$A=$ adult dosage
Owen buys a prescription for 1200 mg of medicine. The adult dose is 50 mg and the recommended dose for Owen's child is 10 mg .
(i) How old is Owen's child using Young's rule?


cmmonesror $10 \times 12=120($ (ii $)$

How many doses for Owen's child are contained in the prescription?
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(iii) It is recommended the medicine be taken at most 4 times a day. How many days will the prescription last at this rate for Owen's child?

$$
120 \div 4=30 \text { day } 5
$$

$\qquad$
$\qquad$
(b) The capture-recapture technique was used to estimate the population of penguins in 2013.

- 50 penguins were caught, tagged and released.
- Later, 110 penguins were caught at random.
- 20 of these 110 penguins had been tagged.

The estimated population of the penguins in 2013 was $13 \%$ less than the estimated population for 2012.
What was the estimated population for 2012 ?
Calculation 2013 pop was well done.

(c) Jack has a mobile phone contract that charges a monthly access fee of $\$ 79, \$ 250$ worth of calls are free, flagfall $\$ 0.35$ and call rate of $\$ 0.45$ per 30 second. What is the monthly charge if Jack made 400 calls whose duration was less than 30 seconds?

(d) Charlotte is 57 kg and has consumed 5 standard drinks in the past four hours.

She was stopped by police for a random breath test.
What would be Charlotte's blood alcohol content? Answer correct to 2 decimal places.

$$
B A C=\frac{10 \times 5-7.5 \times 4}{5.5 \times 57}
$$


e) Solve the following pair of simultaneous equations.

$$
\begin{gathered}
2 x-3 y=-1 \\
x=10-2 y
\end{gathered}
$$


$\qquad$
(f) Sally's recent results in hospitality and timber are recorded in the table

| Course | Class Mean | Class Standard Deviation | Sally's Result | use formula |
| :--- | :---: | :---: | :---: | :---: |
| Hospitality | 55 | 10 | 85 | $\geq=\frac{x-x}{5}$ |
| Timber | 55 | 15 | 85 |  |

(i) What is Sally's $z$-score for timber?

$$
z=\frac{85-55}{15}=+2
$$

(ii) . What hospitality mark would be equivalent to a $z$-score of -1 ?
$\qquad$
(iii) What percentage of students in Sally's class scored a mark between 45 and 75 for Hospitality?


556575

Question 30 ( 15 marks)

## Marks

a) Michael invests $\$ 3125$ at $12 \%$ per annum compounding quarterly.

How much will he have after 4 years? Answer to the nearest dollar.


Convert to quarters

$r=12 \% \therefore 4=3 \%$
$n=4 \times 4=16$
2
convert to

$$
\begin{aligned}
& \text { montros } \\
& \text { Fr }=\operatorname{Pr}\left(1+\frac{r}{100}\right)^{n} \\
& 480000=P V(1+0.683 \%)^{96} \begin{array}{r}
r=52 \div 12 \\
r=0.653 \% \\
n=8 \times 12=96
\end{array} \\
& P V=\underline{480000}=\$ 249718^{2} .71 \\
& (\overline{1+6.685 \%})^{56}
\end{aligned}
$$

(c) The table below shows the future value on $\$ 1$ compounding at the interest rate per period.

| Future value of $\$ 1$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| End of <br> period | $4 \%$ | $6 \%$ | $8 \%$ | $10 \%$ |  |
| 1 | 1.00 | 1.00 | 1.00 | 1.00 |  |
| 2 | 2.04 | 2.06 | 2.08 | 2.10 |  |
| 3 | 3.12 | 3.18 | 3.25 | 3.31 |  |
| 4 | 4.25 | 4.37 | 4.51 | 4.64 |  |

convert to
half yearly
$r=5 \% \div 2$
$=4 \%$
$n=2 \times 2=4$
used 4.25
Calculate the future value of a $\$ 32000$ annuity for 2 years at $8 \%$ p.a. compounded half yearly.

$$
32000 \times 4.25=9136000
$$

d) An asteroid reached earth and exploded in the Pacific Ocean. The distance (in km ) it travelled through the earth's atmosphere varied directly as the square of the time ( $t \mathrm{sec}$ ) it had been travelling. The asteroid travelled 384 kilometres in the first 16 seconds.
Recognise Variation
i) How far did the asteroid travel in the first 10 seconds?

some students used speed = Distance
ii) How long will it take for the asteroid to travel 294 kilometres?

e) Calculate the capacity of the following in Megalitres using Simpson's Rule for volume. 2


$$
r=\frac{30}{3}[220+4 \times 350+290]
$$



$$
=19.1 \mathrm{ML}
$$

Most achieved 19100 KL
however found
a challenge?
f) The angle of elevation from a boat out to sea to the top of a 220 -metre cliff is $37^{\circ}$. After travelling directly towards the cliff the angle of elevation from the boat to the top of the cliff is $56^{\circ}$.
i) Draw a diagram representing the information above

ii) How far did the boat travel towards the cliff? Answer correct to the nearest metre. 2


End of paper

