SCEGGS Darlinghurst

2007

**HIGHER SCHOOL CERTIFICATE
TRIAL EXAMINATION**

General Mathematics

This is a TRIAL PAPER only and does not necessarily reflect the content or format of the Higher School Certificate Examination for this subject.

General Instructions

- Reading time – 5 minutes
- Working time – 2½ hours
- Write using black or blue pen
- Diagrams should be drawn in pencil
- Calculators may be used
- A formulae sheet is provided at the back of this paper

Total marks – 100
Assessment Weighting – 40%

Section I | Pages 1–9
22 marks
- Attempt Questions 1–22
- Allow about 30 minutes for this section

Section II | Pages 10–21
78 marks
- Attempt Questions 23–28
- Allow about 2 hours for this section
- Start each question in a NEW booklet

Section I

22 marks
Attempt Questions 1-22
Allow about 30 minutes for this section

Use the multiple-choice answer sheet.

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A)   2   (B)   6   (C)   8   (D)   9

A ⬭   B ⬤   C ⬭   D ⬭

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ⬤   B ⬤✗   C ⬭   D ⬭

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word *correct* and drawing an arrow as follows.

correct

A ⬤✗   B ⬤✗   C ⬭   D ⬭

1 The probability of an event occurring is $\frac{89}{100}$.

Which statement best describes the probability of this event occurring?

(A) The event is likely to occur.

(B) The event is certain to occur.

(C) The event is unlikely to occur.

(D) The event will NOT occur.

2 The area, A of a trapezium is given by the formula:

$$A = \frac{h}{2}(a+b)$$

Calculate the area when $h = 12$, $a = 7$ and $b = 9$.

(A) 51

(B) 96

(C) 121

(D) 192

3 The table below shows the monthly repayments per $1000 on a bank home loan.

Term of Loan (years)	6.00%	6.25%	6.50%	6.75%	7.00%	7.25%	7.50%
5	$19.33	$19.45	$19.57	$19.68	$19.80	$19.92	$20.04
10	$11.10	$11.23	$11.35	$11.48	$11.61	$11.74	$11.87
15	$8.44	$8.57	$8.71	$8.85	$8.99	$9.13	$9.27
20	$7.16	$7.31	$7.46	$7.60	$7.75	$7.90	$8.06
25	$6.44	$6.60	$6.75	$6.91	$7.07	$7.23	$7.39

Determine the monthly repayment for a loan of $120 000 at 6.5% p.a. interest rate over 20 years.

(A) $7.46

(B) $89.52

(C) $895.20

(D) $7460

4   In a family of three children, the probability of having three boys is:

(A) $\dfrac{1}{8}$

(B) $\dfrac{1}{3}$

(C) $\dfrac{1}{2}$

(D) $\dfrac{3}{8}$

5   From a fridge containing 8 different cans of soft drink, Marco chooses two cans of soft drink.

How many different possible choices could he make?

(A)   2
(B)   16
(C)   28
(D)   56

6   The discount price of a DVD recorder is $638. The discount allowed was 12%.

What was the original marked price of the recorder?

(A) $76.56
(B) $561.44
(C) $725.00
(D) $714.56

7   The correct solution to the equation $\sqrt{3n-1} = 4$ is:

(A) $n = \dfrac{25}{3}$

(B) $n = 1$

(C) $n = \dfrac{\sqrt{5}}{3}$

(D) $n = \dfrac{17}{3}$

8 Two coins are tossed together 20 times.

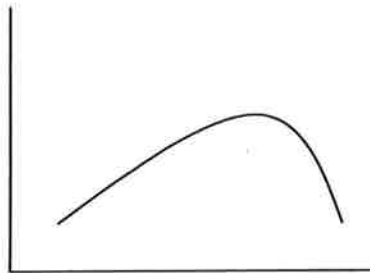Which calculation below illustrates the expected number of times you would get 2 heads?

(A) $\dfrac{1}{4} \times 20$

(B) $\dfrac{1}{2} \times 20$

(C) $\dfrac{1}{4} \times 40$

(D) $\dfrac{1}{2} \times 40$

9 Describe the type of distribution that best matches the diagram below.

(A) Positively skewed
(B) Normal
(C) Symmetrical
(D) Negatively skewed

10   The depreciation on my new laptop computer worth $4000 can be found using the declining balance method. The table below shows the value of the laptop at the end of each year.

Years	Salvage Value
0	$4000
1	$3200
2	$2560
3	$2048
4	$1638.40
5	$1310.72
6	$1048.58
7	$838.86
8	$671.09
9	$536.87
10	$429.50

In which year will the laptop first be worth half of its original value?

(A)   2nd

(B)   3rd

(C)   4th

(D)   5th

11   There are 12 scores in a set of data. One of the scores in an outlier. Which of the measures will be influenced by the outlier?

(A)   mean

(B)   interquartile range

(C)   mode

(D)   median

12   The sports club oval is in the shape of an ellipse. It is 200 m long and 160 m wide. At the end of the season the greenkeeper is going to spread fertilizer on the oval.

If one bag of fertilizer covers 150 m^2 how many bags of fertilizer will he need?

(A)   168

(B)   377

(C)   670

(D)   25 133

13   The heights (H cm) and weights (W kg) of a group of people were taken and plotted on a graph. A line of best fit has been drawn.
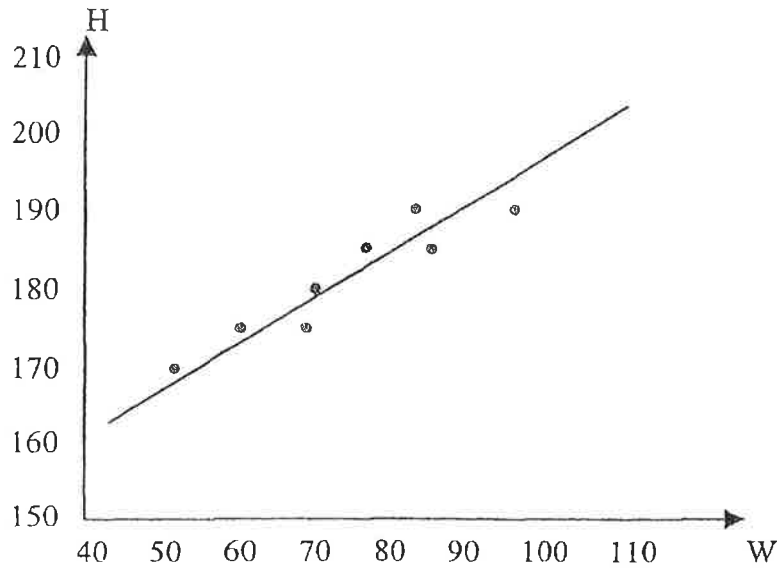
The equation of the line of best fit is:

(A) $H = 0.6W + 150$

(B) $W = 0.6H + 150$

(C) $W = 0.6H + 160$

(D) $H = 0.6W + 160$

14

| Taxable income	Tax on this income
$0 - $6000	Nil
$6001 - $21 600	17 cents for each $1 over $6000
$21 601 - $58 000	$2 652 + 30 cents for each $1 over $21 600
$58 001 - $70 000	$13 572 + 42 cents for each $1 over $58 000
Over 70 000	$18 612 + 47 cents for each $1 over $70 000

Yasmin has a gross income of $70 530 and allowable tax deductions that total $1 897.

What is the tax payable on Yasmin's taxable income?

(A) $18 861.10

(B) $16 964.10

(C) $17 892.20

(D) $18 037.86

15 A luxury health spa and resort in the Blue Mountains has used 1.084 million wall tiles. If each tile covers 225 cm^2, what is the total area that has been tiled?

(A) 243.9 m^2

(B) 24 390 m^2

(C) 243 900 m^2

(D) 2 439 000 m^2

16

2 cm

3 cm

4 cm

2 cm
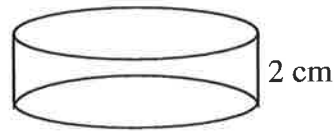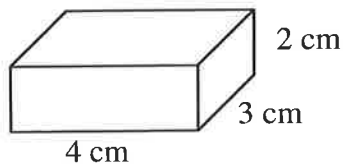
NOT TO SCALE

A block of chocolate in the shape of a rectangular prism 4 cm by 3 cm by 2 cm is melted down and poured into a cylindrical mould of height 2 cm. The volume of a cylinder $= \pi r^2 h$. The cylindrical mould is exactly filled with chocolate. Find the radius of the cylindrical mould to the nearest millimetre.

(A) 1.9 cm.

(B) 2.0 cm.

(C) 3.8 cm.

(D) 3.9 cm.

17 There are 70 boys and 80 girls in Year 12 at Wycombe High. In a stratified random sample based on gender, the number of girls who should be chosen for a sample of 60 students is:
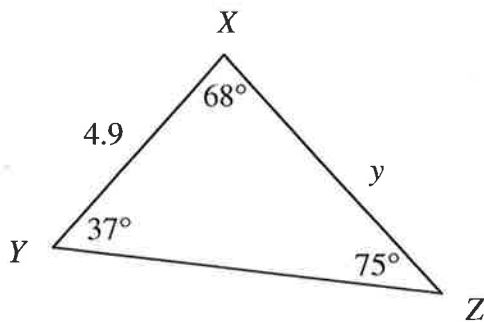
(A) 28

(B) 32

(C) 53

(D) 69

18 Determine the true bearing of A from C.

North

North ⬆ C

A 30°

40°

B
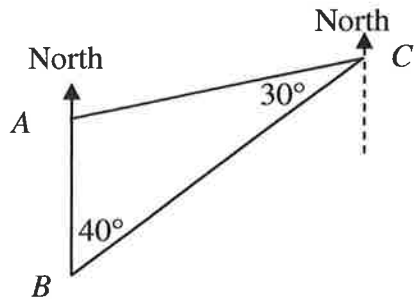
NOT TO SCALE

(A) 030°T

(B) 110°T

(C) 250°T

(D) 290°T

19 The value of y, correct to one decimal place is:

X

68°

4.9

y

37°

Y 75°

Z

NOT TO SCALE

(A) 2.4

(B) 3.1

(C) 3.2

(D) 4.7

20 Sydney has a latitude of 34° S and a longitude of 151° E. Wagga Wagga is 4° west of Sydney, 1° south of Sydney. Its co-ordinates will be:

(A) 33° S, 147° E

(B) 33° S, 155° E

(C) 35° S, 155° E

(D) 35° S, 147° E

21 The speed limit in the Harbour Tunnel is 80 km/h. This is equivalent to:

(A) 2.2 m/sec

(B) 22.2 m/sec

(C) 222.2 m/sec

(D) 2222.2 m/sec

22 A group of 6 students completed a spelling quiz. Their average score was recorded as 13, but one of the scores had been incorrectly recorded as 8 instead of 18.

Find the correct average.

(A) 5.2

(B) 14.7

(C) 15.5

(D) 16.0

End of Section I

Section II

78 marks
Attempt Questions 23–28
Allow about 2 hours for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

All necessary working should be shown in every question.

Marks

Question 23 (13 marks)

(a)   There are 12 teams in a netball competition. The probability that a particular   **1**
team will win the competition is $\dfrac{1}{12}$. Comment briefly on this statement.

(b)   What is the Greenwich or Prime Meridian?   **2**

(c)   Simplify $3x(y+x) - x(y-3x)$ **2**

Question 23 continues on page 11

Question 23 (continued)

(d)   Jessica and David have to do a comparative study on the weekend spending
        habits of male and female students in Year 12. They decide that a good
        starting point would be to record their own spending habits (in dollars) each
        weekend over a year and to show the results in the box-and-whisker plots below.

Amount spent ($) each weekend
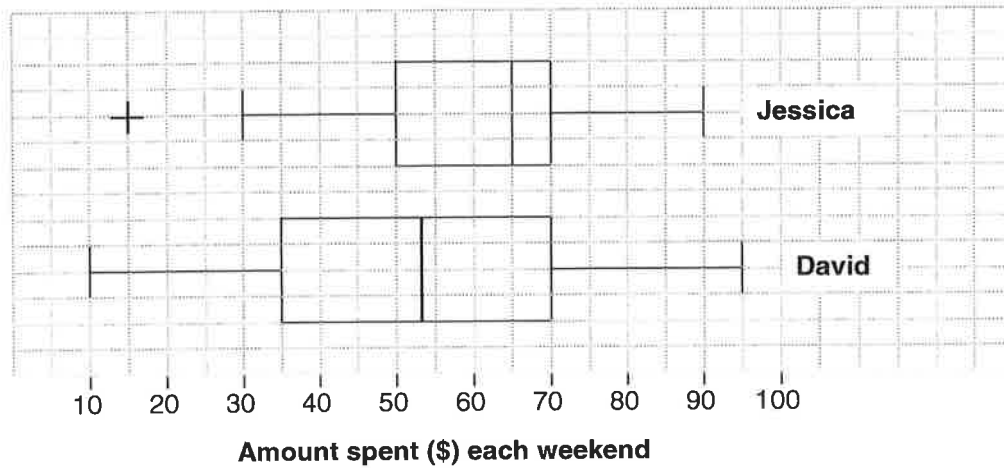
(i)   What is the interquartile range of the data for David?   **1**

(ii)   What is the value of the outlier in Jessica's scores?   **1**

(iii)   What are the medians for each set of data?   **1**

(iv)   Which of the two sets of data is likely to have the higher standard   **1**
        deviation? Give a reason for your answer.

(v)   Compare and contrast the two data sets, describing the main   **3**
        differences and similarities with reference to the shape of the
        distribution, and measures of location and spread.

(vi)   Based on her data, what is the probability that, on any one weekend,   **1**
        Jessica would spend between $70 and $90?

End of Question 23

Question 24 (13 marks) Use a SEPARATE writing booklet.

(a)   Make W the subject of $T = \dfrac{3W - a}{2a}$.   **2**

(b)   This table shows Surya's marks in two subjects in a recent Year 12 assessment block.

	Surya's Mark	Class Mean	Class Standard Deviation
Maths	65	60	5
Agriculture	64	50	7

   (i)   Calculate the z-scores for each subject.   **1**

   (ii)   In which subject has Surya performed best in comparison to the rest of the class? Give a clear reason for your answer.   **2**

   (iii)   We are told the results of each subject follow the normal distribution. What percentage of students have scored a mark higher than 55 in Maths?   **1**

(c)   A tennis player gets a second serve if the first serve does not go in.
Sally's first serve has a probability 0.6 of going in, and her second serve has a probability of 0.8 of going in.

   (i)   The tree diagram below shows the outcomes of the first serve. Copy the tree diagram and complete it for the second serve, showing the probability values on each branch.   **2**

First serve   **Second serve**

```
                    IN
         0.6
           /
          <
           \
            NOT IN
```
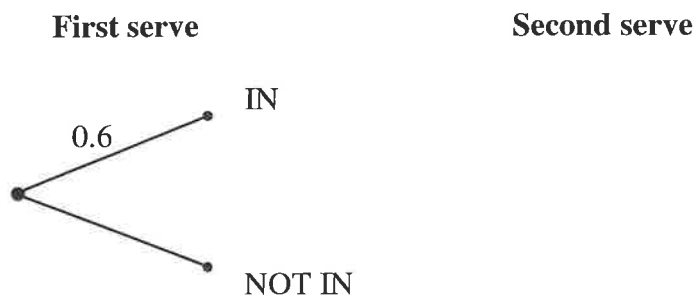
   (ii)   What is the probability that at least **one** of Sally's serves goes in?   **1**

Question 24 continues on page 13

Question 24 (continued)

(d)   In a game of dice, two dice are rolled together and the score is found by multiplying the numbers on each die. The table below shows the possible scores in any one game.

		1st Die					
		1	**2**	**3**	**4**	**5**	**6**
	1	1	2	3	4	5	6
	2	2	4	6	8	10	12
2nd	**3**	3	6	9	12	15	18
Die	**4**	4	8	12	16	20	24
	5	5	10	15	20	25	30
	6	6	12	18	24	30	36

(i)   What is the probability of scoring 36 in a game?   **1**

(ii) In this game, the prizes are as follows: **3**
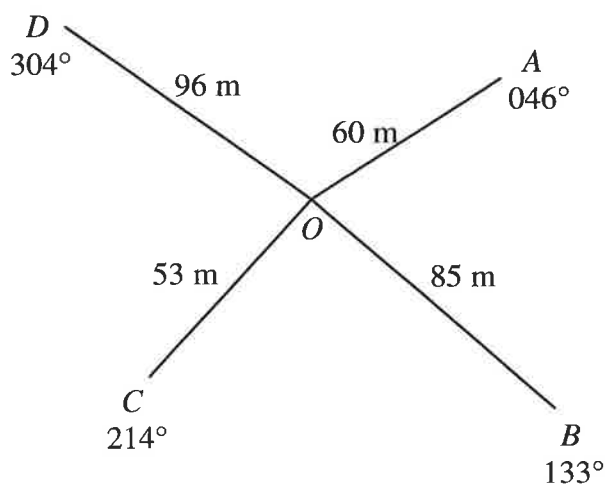
Score	Prize
36	Win $10
18 or more (18-35)	Win $5
Less than 18	Lose $1

The cost of playing a game is $1. Would you continue playing the game for an extended period? In your answer, comment on the financial expectation of the game.

End of Question 24

Question 25 (13 marks) Use a SEPARATE writing booklet.

(a)   The following notebook entry was made during a radial survey of a field.

**Not to
scale**

(i)   What is the size of angle AOB?   1

(ii)   Calculate the area of the triangle AOB. Round your answer to the   2
        nearest square metre.

(iii)   Find the distance from A to B. (Correct to 1 decimal place.)   2

Question 25 (continued)

(b) Athol and Yvonne have a circular pool, with a diameter of 3 m in their back yard. They have decided to concrete (shown in grey) around it. The concrete will be 1m wide.

Not to drawn scale

1.5 1

Pool
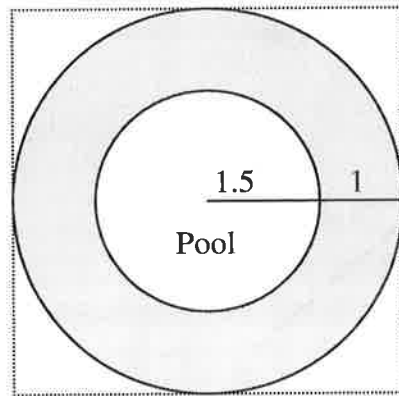
(i)   What will be the area of the annulus of concrete? (2 decimal places.)   **2**

(ii) Concrete is to be laid to a depth of 20 cm. What volume of concrete will   **2**
be needed? (Answer to nearest cubic metre.)

(iii) A square shadecloth cover (dotted line) will just cover the pool and path.   **2**
What will be the area of the shadecloth?

(c) O is the centre of the Earth. P has the co-ordinates $(15°S, 110°E)$, and Q has
the co-ordinates $(20°N, 110°E)$.

$110°E$

$20°N$

Q

O

$0°$

$15°S$

P
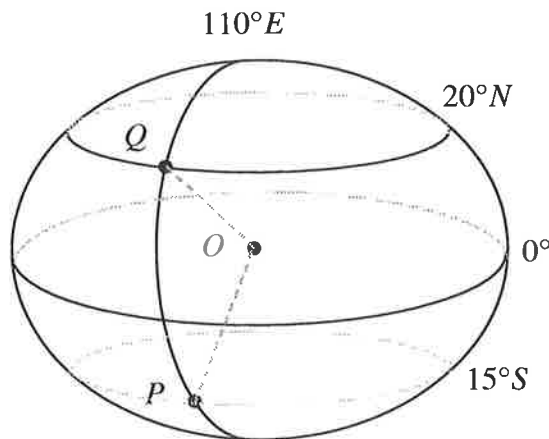
(i)   Find the size of the angle QOP.   **1**

(ii) Calculate the distance between P and Q correct to the nearest kilometre.   **1**
(The radius of the Earth is 6400 km.)

End of Question 25

Question 26 (13 marks) Use a SEPARATE writing booklet.

(a)   Write the number 1 030 000 000 in scientific notation.   **1**

(b)   In a normal working week Rachel works 35 hours. When she works an extra   **2**
6 hours overtime at time-and-a-half and 2 hours overtime at double time
she earns a total of $729.60.

How much is she normally paid per hour?

(c)   Simplify $24x^3 y^2 \div -\dfrac{3y^2}{2x^3}$ **1**

(d)   A blood test is used to diagnose diabetes. When the results of the test are
positive it indicates the patient has diabetes. When the results are negative
it indicates the patient does not have diabetes.

The results of 600 blood tests are shown in the table below.

	Blood tests positive	Blood tests negative	TOTAL
Patients who have diabetes	449	11	460
Patients who do not have diabetes	18	B	140
TOTAL	A	133	600

(i)   Find the missing values for A and B.   **1**

(ii)   How many patients were correctly diagnosed by the test to have diabetes?   **1**

(iii)   In what percentage of the tests were the results incorrect?   **2**
(Answer correct to one decimal place.)

Question 26 continues on page 17

Question 26 (continued)

(e)   Rosa contributed $300 at the end of each month to an investment fund which
        earned interest of 3% per annum compounded monthly.

    (i)   What was the balance in the fund after 12 years?   **2**

    (ii)   After 15 years the fund was worth $68 091.81   **1**
            How much interest had Rosa earned on her investment over 15 years?

    (iii)   Consider the case where Rosa invests a single amount into an   **2**
            investment account which earns interest of 3% per annum
            compounding monthly. What would this single amount be if it
            were to amount to $68 091.81 over 15 years?

End of Question 26

Question 27 (13 marks) Use a SEPARATE writing booklet.

(a)   Franko knows his bank PIN has four digits and he knows the digits
are 1, 3, 7 and 8, but he cannot remember the order.

   (i)   How many different four-digit PINs are possible?   1

   (ii)   What is the probability that Franko will be able to   1
correctly guess his PIN?

(b)   Joanne's gross weekly income is $700.

   (i)   Show that her monthly income, correct to the nearest dollar, is $3033.   1

   (ii)   The bank will not allow loan repayments to be more than 30% of a   1
customer's gross monthly income.

   What is the maximum amount Joanne can repay per month?

The table shown below can be used to calculate home loan repayments

Interest rate	Monthly repayments on a $1000 loan over			
	10 years	15 years	20 years	25 years
8.25% p.a.	$12.27	$9.70	$8.52	$7.88
8.50% p.a.	$12.40	$9.85	$8.68	$8.05
8.75% p.a.	$12.53	$9.99	$8.84	$8.22
9.00% p.a.	$12.67	$10.14	$9.00	$8.39
9.25% p.a.	$12.80	$10.29	$9.16	$8.56
9.50% p.a.	$12.94	$10.44	$9.32	$8.74

   (iii)   Joanne decides to purchase a unit worth $105 000. She settles on a loan   1
at 9.25% over 25 years.

   Calculate her monthly repayment.

   (iv)   How much in interest will Joanne pay over the term of the loan?   2
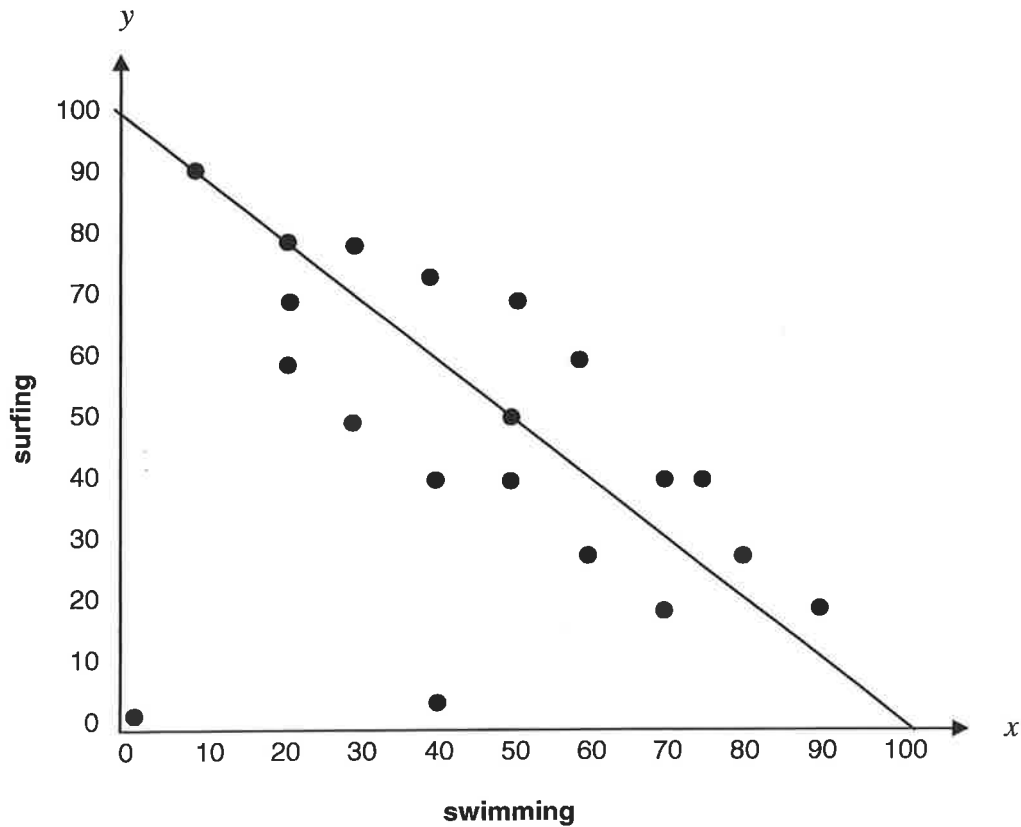
Question 27 (continued)

(c)   Felix and Felicity are planning a trip to Toronto in Canada when they leave
        school. They find that Toronto is $44°N, 79°W$ while Sydney is $34°S, 151°E$.

        (i)   What is the time difference between Toronto and Sydney?   **2**

        (ii)   Felix and Felicity plan to leave Sydney at 6am one Sunday.   **2**
               What time would it be in Toronto when they leave Sydney?

        (iii)   From Sydney, explain which direction (east or west) would be the   **2**
               shortest distance to fly to Toronto?

End of Question 27

Question 28 (13 marks) Use a SEPARATE writing booklet.

(a)   Simon is keen on both surfing and swimming. Over the Christmas holidays
        he asked his friends to record how many times they went surfing or
        swimming. He made a graph of the results.

(i)   Calculate the gradient of the line.   2

(ii)   What is the equation of the line of best fit drawn?   1

(iii) What is the correlation between the points?   1

(iv)   How many friends went swimming and surfing the   1
        same number of times?

Question 28 (continued)

(b)   A new optical laser machine for a local hospital was purchased for $45 000.

This machine will depreciate in value over time and the management of the hospital can use either the *"Straight Line"* method or the *"Declining Balance"* method to determine the machine's *"Salvage Value"* at the end of each year's use.

The following table gives the salvage value of the machine for the first 5 years of its use.

Depreciation of Optical Laser

	A	B
Year	Salvage Value $	Salvage Value $
New (0)	45 000	45 000
1	40 000	38 250
2	35 000	32 512.50
3	30 000	27 635.63
4	25 000	23 490.28
5	20 000	19 966.74

(i)   What information is conveyed in Column A that suggests the *"Straight Line"* method was used to depreciate the machine?   **1**

(ii)   Show that a Declining Balance of 15% was used in Column B to depreciate the machine.   **1**

(iii)   The management of the hospital has suggested that the optical laser should be replaced after 8 years. Find the amount by which the optical laser machine has depreciated at the end of 8 years if the declining balance method is applied.   **2**

(iv)   Hospital management decided to apply the "Straight Line" method of depreciation for the first 4 years. At the end of the fourth year they changed the method of depreciation to the declining balance method at the rate of 15% per annum.

1.   Find the value of the optical laser machine at the end of 8 years.   **2**

2.   Without further calculations, sketch a graph to show the value of the optical laser machine over the eight years.   **2**

Use the horizontal axis to represent time and the vertical axis to represent the value of the optical laser machine.

End of paper

FORMULAE SHEET

Area of an annulus

$$A = \pi\left(R^2 - r^2\right)$$

R = radius of outer circle

r = radius of inner circle

Area of an ellipse

$$A = \pi ab$$

a = length of semi - major axis

b = length of semi - minor axis

Area of a sector

$$A = \frac{\theta}{360}\pi r^2$$

θ = number of degrees in central angle

Arc length of a circle

$$l = \frac{\theta}{360}2\pi r$$

θ = number of degrees in central angle

Simpson's rule for area approximation

$$A \approx \frac{h}{3}\left(d_f + 4d_m + d_l\right)$$

h = distance between successive measurements

d_f = first measurement

d_m = middle measurement

d_l = last measurement

Simple Interest

$$I = Prn$$

Surface area

Sphere $\qquad\qquad A = 4\pi r^2$

Closed cylinder $\qquad A = 2\pi rh + 2\pi r^2$

r = radius

h = perpendicular height

Volume

Cone $\qquad V = \frac{1}{3}\pi r^2 h$

Cylinder $\quad V = \pi r^2 h$

Pyramid $\quad V = \frac{1}{3}Ah$

Sphere $\qquad V = \frac{4}{3}\pi r^3$

r = radius

h = perpendicular height

A = area of base

Sine rule

$$\frac{a}{Sin\,A} = \frac{b}{Sin\,B} = \frac{c}{Sin\,C}$$

Area of a triangle

$$A = \frac{1}{2}ab\,Sin\,C$$

Cosine Rule

$$c^2 = a^2 + b^2 - 2ab\,Cos\,C$$

or

$$Cos\,C = \frac{a^2 + b^2 - c^2}{2ab}$$

P = initial quantity

r = percentage interest rate per period, expressed as a decimal

n = number of periods

Compound Interest

$$A = P(1 + r)^n$$

A = final balance

P = initial quantity

n = number of compounding periods

r = percentage interest rate per compounding period, expressed as a decimal

Future value (A) of an annuity

$$A = M\left\{\frac{(1 + r)^n - 1}{r}\right\}$$

M = contribution per period, paid at the end of the period

Present value (N) of an annuity

$$N = M\left\{\frac{(1 + r)^n - 1}{r(1 + r)^n}\right\}$$

or

$$N = \frac{A}{(1 + r)^n}$$

Straight-line formula for depreciation

$$S = V_0 - Dn$$

S = salvage value of asset after n periods

V_0 = purchase price of the asset

D = amount of depreciation apportioned per period

n = number of periods

Declining balance formula for depreciation

$$S = V_0(1 - r)^n$$

S = salvage value of asset after n periods

r = percentage interest rate per period, expressed as a decimal

Mean of a sample

$$\bar{x} = \frac{\sum x}{n}$$

$$\bar{x} = \frac{\sum fx}{\sum f}$$

x = individual score

$\bar{x}$ = mean

n = number of scores

f = frequency

Formula for a z-score

$$z = \frac{x - \bar{x}}{s}$$

s = standard deviation

Gradient of a straight line

$$m = \frac{\text{vertical change in position}}{\text{horizontal change in position}}$$

Gradient-intercept form of a straight line

$$y = mx + b$$

m = gradient

b = y-intercept

Probability of an event

The probability of an event where outcomes are equally likely is given by:

$$P\,(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$