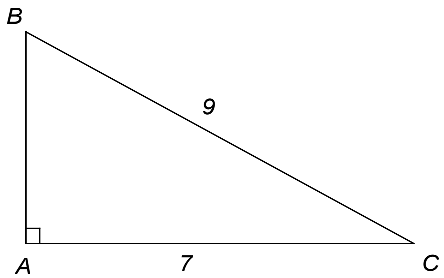
2004 YEAR 9 YEARLY

SECTION A (16 Marks)

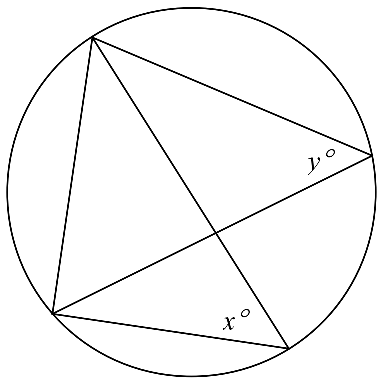
1. Evaluate x if $\sqrt{x} = 9$. **1**

2. Solve $x^2 = 4x$. **2**

3. Find $\angle ACB$ to the nearest degree. **2**



4. Using the diagram shown, write an equation connecting x and y giving a reason. **2**



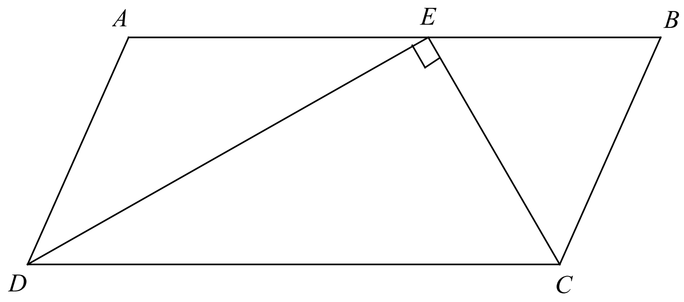
5. Solve for x: $8x = \dfrac{1}{\sqrt{x}}$. **2**

6. A bag contains five red and four blue marbles. Two are drawn together from the bag. Find the probability that

   (a) they are both red. **1**

   (b) they are both red if it is known that one marble is red. **2**

7. In the parallelogram $ABCD$ shown, $DE = 15$, $CE = 8$ and DE is perpendicular to CE. Find the area of the parallelogram. **4**



SECTION B (16 Marks)

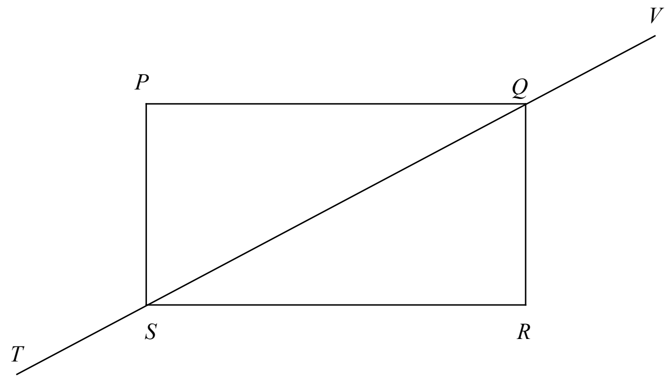
1. Given $P(x) = x^3 - x^2 + 9x - 9$,

   (a) find $P(-3)$. **1**

   (b) show that $(x - 1)$ is a factor of $P(x)$. **2**

   (c) briefly explain why there are no other real factors of $P(x)$. **2**

2. In the diagram shown, $PQRS$ is a rectangle and diagonal QS is extended to TV such that $TS = QV$. ***Copy the diagram onto your answer sheet*** and prove that $PVRT$ is a parallelogram. **3**



3. Fully factor $x^3 + 8y^3$. **2**

4. Find k if the line $2x - 4y + 1 = 0$ is parallel to the line $3x + ky - 7 = 0$. **2**

5. The diagonals AC and BD of a rhombus $ABCD$ are 12cm and 20cm.

   (a) Find its exact perimeter. **2**

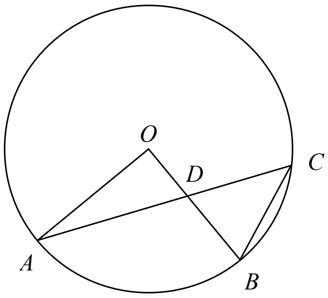
   (b) Find $\angle ABC$ to the nearest degree. **2**

SECTION C (16 Marks)

1. (a) On the same graph, neatly sketch the curves $y = 2^x$ and $y = 2^{-x}$. **3**

   (b) Find all solutions to $2^x = 2^{-x}$. **1**

2. In the diagram shown, $\angle AOB = 126°$ and $\angle OAD = 24°$.



3. Find $\angle CBD$ giving all reasons.

4. Solve for x: $x^2 - 2x - 4 = 0$. **2**

4. (a) Find the equation of the line through the points $A(0,4)$ and $B(3,2)$. **2**

   (b) Find l, the equation of the line through B and perpendicular to AB. **2**

   (c) The line l intersects the $y-$axis at C. Find the area of ΔABC. **2**

SECTION D (16 Marks)

1. Find all solutions to: **4**

$$5^{2x+y} = 1$$
$$4^{x-y} = 8$$

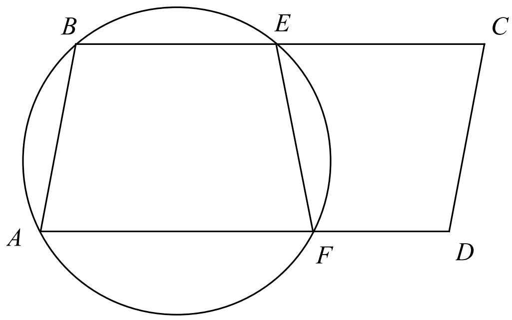
2. (a) Find the value of x for which $P(x) = x^3 - 3x^2 + 4$ has a double root. **2**

   (b) Neatly sketch $P(x) = x^3 - 3x^2 + 4$. **3**

   (c) Hence,or otherwise, solve $x^3 + 4 \le 3x^2$. **2**

3. Bill starts at A and walks $8\ km$. on a bearing of $039°T$ to B. How far east is he from A.(Answer to nearest km.) **3**

5. In the diagram shown, $ABCD$ is a parallelogram and EF a chord. ***Copy the diagram onto your answer sheet*** and prove that $AB = EF$. **2**



SECTION E (16 Marks)

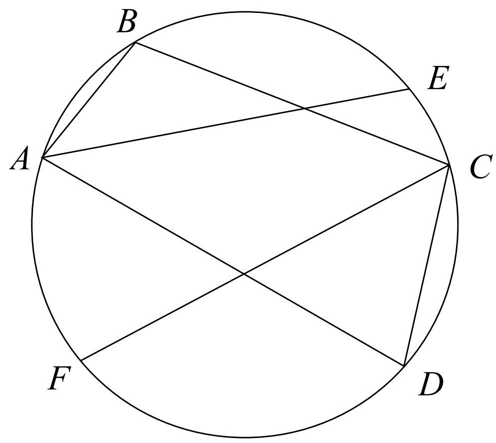
1. Find all solutions to $x^4 - 7x^2 + 12 = 0$. **3**

2. (a) Using a scale of 1cm. = 1 unit, graph the region defined by the inequalities: **3**

$$x + 3y \le 6$$
$$4x + 3y \le 12$$
$$x \ge 0$$
$$y \ge 0$$

(b) Find the co – ordinates of a point in this region which will maximise the value of z if $z = 5x - 2y$. **2**

3. In the diagram given, $ABCD$ is a cyclic quadrilateral where AE bisects $\angle BAD$ and CF bisects $\angle BCD$. ***Copy the diagram onto your answer sheet*** and prove that EF is a diameter. **3**



4. A square $ABCD$ has an area of 1 square unit. Point E is on side AB and point F is on side AD such that $AE = AF = x$.

(a) Draw a diagram showing the above information. **1**

(b) Show that A, the area of quadrilateral $CDFE$ is **2**

$$A = \frac{1}{2}\left(1 + x - x^2\right).$$

(c) Find the maximum area of quadrilateral $CDFE$. **2**

END of PAPER

YEAR 9 YEARLY
2004 SOLUTIONS

SECTION A

1. $\sqrt{x} = 9$

$\therefore x = 81$

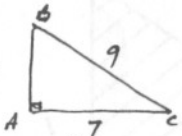
2. $x^2 = 4x$

$\therefore x^2 - 4x = 0$

$\therefore x(x-4) = 0$

$\therefore x = 0$ or $x = 4$

3.



$\cos A\hat{C}B = \frac{7}{9}$

$\therefore A\hat{C}B = 39^\circ$ (to nearest degree)

4. $x = y$

Angle at circumference in same segment are equal.

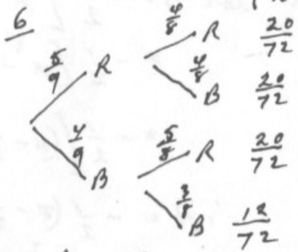
5. $8x = \frac{1}{\sqrt{x}}$

$\therefore x^{\frac{3}{2}} = \frac{1}{8}$

$\therefore x = (2^{-3})^{\frac{2}{3}}$

$\therefore x = \frac{1}{4}$

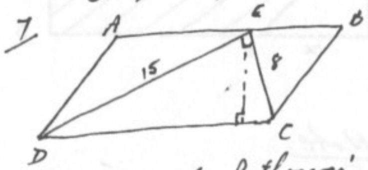
6.



(a) $P = \frac{20}{72} = \frac{5}{18}$

(b) $P = \frac{20}{72-12} = \frac{1}{3}$

7.



$CD = 17$ by Pythagoras' in ΔDEC

Area $= \frac{1}{2} \times 15 \times 8 = \frac{1}{2} \times 17 \times h$

$\therefore h = \frac{120}{17}$

$\therefore$ Area of parm. $= \frac{120}{17} \times 17$

$= 120\ u^2$

SECTION B

1. $P(x) = x^3 - x^2 + 9x - 9$

(a) $P(-3) = (-3)^3 - (-3)^2 + 9(-3) - 9$

$= -27 - 9 - 27 - 9$

$= -72$

(b) Let $P(x) = (x-1)\,Q(x)$

$P(+1) = +1 - 1 + 9 - 9$

$= 0$

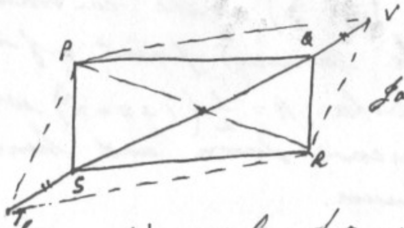
$\therefore (x-1)$ is a factor of $P(x)$

(c) $P(x) = x^2(x-1) + 9(x-1)$

$= (x-1)(x^2+9)$

Since x^2+9 has no real factors $\therefore P(x)$ has only one real factor.

2.



Join PR.

Since diagonals of a rectangle bisect each other, $\therefore$ PR bisects QS.

Now TS = QV given

$\therefore$ PR and TV bisect each other.

$\therefore$ PVRT is a parm.

3. $x^3 + 8y^3$

$= x^3 + (2y)^3$

$= (x+2y)(x^2 - 2xy + 4y^2)$

4. For $2x - 4y + 1 = 0$

let $m_1 = \frac{1}{2}$

For $3x + ky - 7 = 0$

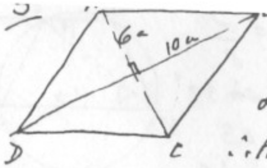
let $m_2 = -\frac{3}{k}$

Since lines parallel

$m_1 = m_2 \quad \therefore \frac{1}{2} = -\frac{3}{k}$

$\therefore k = -6$

5.



$\therefore$ Since diagonals of a rhombus bisect each other at right angles

$\therefore AB = \sqrt{6^2 + 10^2}$ by Pythagoras'

$\therefore AB = 2\sqrt{34}$ cm

$\therefore$ Perimeter is $8\sqrt{34}$ cm.

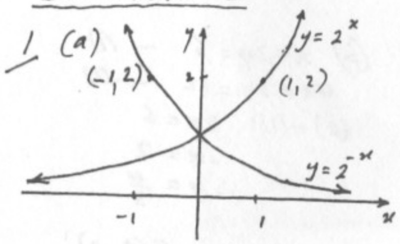
(b) Let $\tan A\hat{B}D = \frac{6}{10}$

$\therefore A\hat{B}D = 30^\circ 58'$

$\therefore A\hat{B}C = 62^\circ$ (to nearest degree)

SECTION C

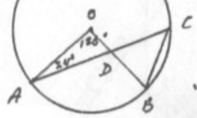
1. (a)



(b) $2^x = 2^{-x}$

$\therefore x = 0$

2.



$A\hat{D}O = 30^\circ$ (Angle sum of $\Delta AOD = 180$)

$\therefore C\hat{D}B = 30^\circ$ (vertically opposite angles eq…

$\therefore D\hat{C}B = 63^\circ$ (Angle at centre tw… angle at circumfe… subtended by same ar…

$\therefore C\hat{B}D = 87^\circ$ (Angle sum of $\Delta CBD = 180$)

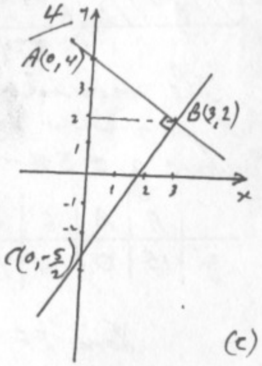
3. $x^2 - 2x - 4 = 0$

$\therefore x = \frac{2 \pm \sqrt{4+16}}{2}$

$= \frac{2 \pm 2\sqrt{5}}{2}$

$\therefore x = 1 \pm \sqrt{5}$

4.



(a) $y - 4 = -\frac{2}{3}(x - 0)$

$\therefore 3y - 12 = -2x$

$\therefore 2x + 3y - 12 = 0$

(b) $y - 2 = \frac{3}{2}(x - 3)$

$2y - 4 = 3x - 9$

$\therefore 3x - 2y - 5 = 0$ (line l…

(c) Line l through $C(0, -\frac{5}{2})$

Area of $\Delta ABC = \frac{1}{2}(4 + \frac{5}{2})3$

$= 9\frac{3}{4}\ u^2$

SECTION D

1. $5^{2x+y} = 1 = 5^0$

$4^{x-y} = 8$

$\therefore 2^{2x-2y} = 2^3$

$\therefore 2x + y = 0$ — (1)

$2x - 2y = 3$ — (2)

(1) − (2): $3y = -3$

$\therefore y = -1$

$x = \frac{1}{2}$

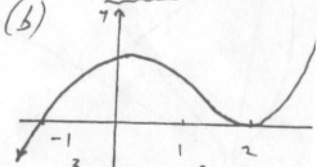
2. (a) $P(x) = x^3 - 3x^2 + 4$

$P(-1) = 0 \quad \therefore (x+1)$ is a factor.

Now $P(x) = (x+1)(x-2)^2$

$\therefore P(x)$ has a double root at $x = 2$

(b)

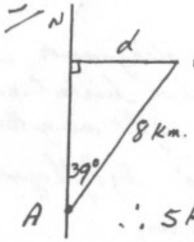


(c) $x^3 + 4 \le 3x^2$

$\therefore x^3 - 3x^2 + 4 \le 0$

$\therefore x \le -1$ or $x = 2$

1



$\sin 39° = \frac{d}{8}$

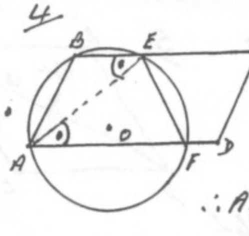
$\therefore d = 8 \sin 39°$

$= 5.03$

$\therefore$ 5 km east of A

(to nearest km)

4



Join AE

Since $A\hat{E}B = E\hat{A}F$ (alternate angles on $BE \parallel AF$)

$\therefore AB = EF$ (equal chords subtend equal angles at circumference)

Note: Other proofs may be found using the centre of the circle, or by proving that FECD is an isosceles trapezium.

SECTION E

1. $x^4 - 7x^2 + 12 = 0$

Let $u = x^2$

$\therefore u^2 - 7u + 12 = 0$

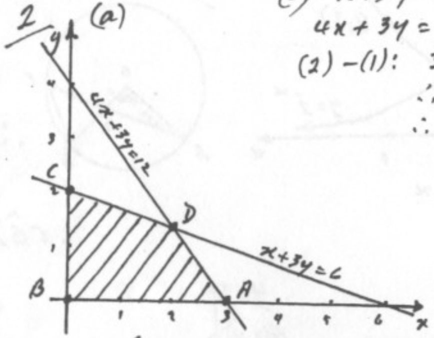
$\therefore (u-4)(u-3) = 12$

$\therefore u = 4$ or $u = 3$

$\therefore x^2 = 4$ or $x^2 = 3$

$\therefore x = \pm 2$ or $\pm\sqrt{3}$

2 (a)



(b) $x + 3y = 6$ — (1)

$4x + 3y = 12$ — (2)

(2) − (1): $3x = 6$

$\therefore x = 2$

$\therefore y = \frac{4}{3}$

A (3, 0)

B (0, 0)

C (0, 2)

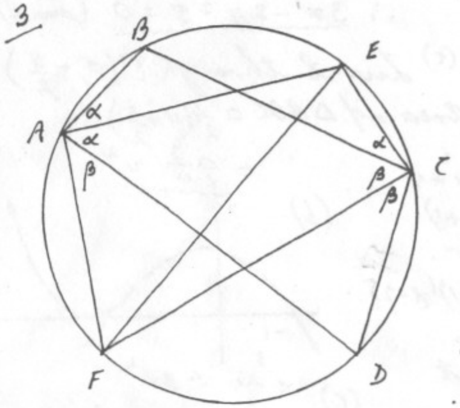
D $(2, \frac{4}{3})$

The maximum value of z is at a vertex of polygon ABCD, where $z = 5x - 2y$.

	A	B	C	D
z	15	0	−4	$7\frac{1}{3}$

Hence z is maximum at A(3, 0)

3



Join EF, AF and EC.

Now $E\hat{C}B = E\hat{A}B = \alpha$ (Angles at circumference in same segment)

Similarly $F\hat{A}D = F\hat{C}D = \beta$

In cyclic quad AECF

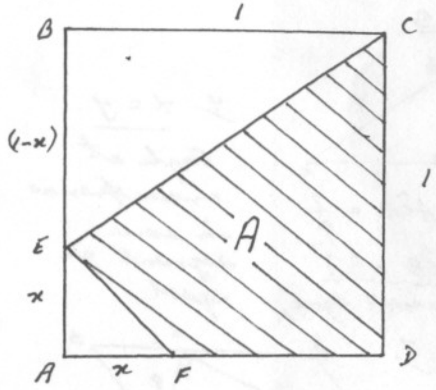
$2(\alpha + \beta) = 180$ (Opposite angles of cyclic quad supplementary)

$\therefore \alpha + \beta = 90$

$\therefore F\hat{A}E = 90°$

$\therefore$ EF is a diameter (angle in a semi-circle is a right angle)

4. (a)



(b) $A = 1 - \frac{1}{2}x^2 - \frac{1}{2}(1-x)$

$= 1 - \frac{x^2}{2} - \frac{1}{2} + \frac{x}{2}$

$= \frac{1}{2} + \frac{x}{2} - \frac{x^2}{2}$

$\therefore A = \frac{1}{2}(1 + x - x^2)$

(c) Let $A = -\frac{1}{2}\left[x^2 - x - 1\right]$

$= -\frac{1}{2}\left[x^2 - x + \frac{1}{4} - \frac{5}{4}\right]$

$= -\frac{1}{2}\left[\left(x - \frac{1}{2}\right)^2 - \frac{5}{4}\right]$

$\therefore A = -\frac{1}{2}\left(x - \frac{1}{2}\right)^2 + \frac{5}{8}$

Since $\left(x - \frac{1}{2}\right)^2 \geq 0$, $\therefore -\frac{1}{2}\left(x - \frac{1}{2}\right)^2 \leq 0$

$\therefore$ Maximum area is $\frac{5}{8}\ u^2$

Note:

If we use $x = -\frac{b}{2a}$ where $a = -1$ and $b = 1$

$\therefore x = \frac{1}{2}$ and

$A = \frac{1}{2}\left(1 + \frac{1}{2} - \frac{1}{4}\right)$

$\therefore A = \frac{5}{8}\ u^2$

where $\left(\frac{1}{2}, \frac{5}{8}\right)$ is the co-ordinate of the turning point of the parabola $A = \frac{1}{2}(1 + x - x^2)$ which is concave down and hence maximum.